NONLINEAR COUPLING BETWEEN ELECTROMAGNETIC FIELDS IN A STRONGLY MAGNETIZED ELECTRON-POSITRON PLASMA

(Letter to the Editor)

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Abstract. The nonlinear coupling between electromagnetic fields in a strongly magnetized electron-positron plasma is considered. We point out that compressional magnetic field perturbations are excited by the rotational part of the nonlinear current, and derive a new nonlinear system of equations that is basic for studies of modulational instabilities and coherent nonlinear structures in magnetized electron–positron plasmas.

Recently, there has been a great deal of interest in studying collective processes involving wave motions (Shukla *et al.*, 1986; Michel, 1991) in electron–positron plasmas. The latter may be found in the early universe (Misner *et al.*, 1980; Rees, 1983), in active galactic nuclei (Miller and Witta, 1987), as well as in pulsar magnetospheres (Goldreich and Julian, 1969). In a strongly magnetized electron–positron plasma, we have the possibility of vortex formation (Yu *et al.*, 1986), which can play a very important role in cross-field particle and energy transport in pulsar magnetospheres.

In a previous paper (Yu *et al.*, 1986), a pair of coupled nonlinear equations that govern the interaction between low-frequency (in comparison with the electron or positron gyrofrequency) Alfvén waves in strongly magnetized electron–positron plasmas was derived. However, in that paper we have a system in which the scalar and the parallel component of the vector potential are highly coupled. Thus, it was assumed, a priori, that magnetic field-aligned magnetic perturbations are absent.

In this Letter, we incorporate the combined effects of the sheared as well as the compressional magnetic field perturbations and derive a system of nonlinear mode coupling equations for low-frequency electromagnetic fluctuations in a strongly magnetized electron–positron plasma. It is found that the compressional magnetic field perturbations can be driven nonlinearly in a cold magnetoplasma.

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We consider the nonlinear coupling between finite amplitude electromagnetic waves in an electron-positron plasma embedded in a uniform magnetic field $\boldsymbol{B}_0$, which is directed along the z-axis. In the presence of low-frequency ($|\partial_t| \ll \omega_c = eB_0/mc$; where e is the magnitude of the electron charge, m is the electron or positron mass, and c the speed of light) electromagnetic fields, the perpendicular (to z) component of the charged particle fluid velocity is

$$\boldsymbol{v}_{j\perp} \approx \boldsymbol{v}_E + \boldsymbol{v}_{jP} + v_{jz}\frac{\boldsymbol{B}_{1\perp}}{B_0} - \frac{B_{1z}}{B_0}\boldsymbol{v}_E \, , \tag{1}$$

where $\boldsymbol{v}_E = (c/B_0)\boldsymbol{E}\times\hat{\boldsymbol{z}}$, $\boldsymbol{v}_{jP} = (mc^2/B_0^2 q_j)[\partial_t + \boldsymbol{v}_{j\perp}\cdot\nabla_\perp + v_{jz}\partial_z]\boldsymbol{E}_\perp$, $\hat{\boldsymbol{z}}$ is the unit vector along the external magnetic field, $\boldsymbol{E}_\perp$ is the perpendicular component of the wave electric field vector, and $\boldsymbol{B}_1 = B_{1\perp} + \hat{\boldsymbol{z}}B_{1z}$ is the perturbed magnetic field. Here q_j is the charge of the particle species j (j equals e for the electrons and p for the positrons), i.e. $q_p = -q_e = e$.

The parallel component of the fluid velocity v_{jz} is given by

$$(\partial_t + \boldsymbol{v}_j\cdot\nabla)v_{jz} = \frac{q_j}{m}\left[E_z + \frac{1}{c}(\boldsymbol{v}_{j\perp}\times B_{1\perp})_z\right], \tag{2}$$

where $\boldsymbol{v}_j = v_{j\perp} + v_{jz}\hat{\boldsymbol{z}}$ and E_z is the component of the electric field vector along $\hat{\boldsymbol{z}}$.

For our purposes, we also need

$$\partial_t(n_p - n_e) + \nabla\cdot(\boldsymbol{j}/e) = 0, \tag{3}$$

the Poisson's equation

$$\nabla\cdot\boldsymbol{E} = 4\pi e(n_p - n_e), \tag{4}$$

as well as Faraday's law

$$\partial_t\boldsymbol{B}_1 = -c\nabla\times\boldsymbol{E}, \tag{5}$$

and the Maxwell's equation

$$\nabla\times\boldsymbol{B}_1 = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\partial_t\boldsymbol{E}, \tag{6}$$

where n_j is the number density and the plasma current is denoted by $\boldsymbol{j} = e(n_p\boldsymbol{v}_p - n_e\boldsymbol{v}_e)$.

Let us now derive the nonlinear mode coupling equations for finite amplitude waves. Substituting (1) into (3) and using (4), (5) and (6), we obtain to leading order

$$\begin{aligned}&\left(\nabla + \frac{\omega_P^2}{\omega_c^2}\nabla_\perp\right)\cdot\partial_t\boldsymbol{E} + \left(\frac{c}{B_0}\boldsymbol{E}\times\hat{\boldsymbol{z}}\cdot\nabla - \partial_t\frac{B_{1z}}{B_0}\right)\nabla\cdot E + \\ &+c\left\{\mathrm{d}_z\left[(\nabla\times\boldsymbol{B}_1)_z - \frac{1}{c}\partial_t E_z\right] - (\nabla\times\boldsymbol{B}_1)_z\partial_z\frac{B_{1z}}{B_0}\right\} + \\ &+\frac{\omega_P^2}{\omega_c^2}\frac{c}{B_0}\nabla\cdot[\boldsymbol{E}\times\hat{\boldsymbol{z}}\cdot\nabla\boldsymbol{E}_\perp] = 0,\end{aligned} \tag{7}$$

where we have introduced $\mathrm{d}_z = \partial_z + (\boldsymbol{B}_{1\perp}/B_0)\cdot\nabla$, and have assumed $\boldsymbol{v}_E\cdot\nabla \gg v_{jz}\partial_z$.

Furthermore, $\omega_P^2 = 8\pi n_0 e^2/m$ is the sum of the squared plasma frequencies, and n_0 is the equilibrium density. In (7), we have retained nonlinear terms up to second order in the wave fields and neglected the small term $\nabla\cdot[(n_j - n_0)\boldsymbol{v}_{jp}]$.

On the other hand, Equation (2) to leading order can be written as

$$\left(\partial_t + \frac{c}{B_0}\boldsymbol{E}\times\hat{\boldsymbol{z}}\cdot\nabla\right)\left[(\nabla\times\boldsymbol{B}_1)_z - \frac{1}{c}\partial_t E_z\right] = \frac{\omega_P^2}{c}\left(E_z + \boldsymbol{E}_\perp\cdot\frac{\boldsymbol{B}_1}{B_0}\right). \quad (8)$$

The compressional magnetic field perturbation B_{1z} is determined by taking the curl of (6) and using (1), and then considering the z component of the resulting equation. We obtain

$$\left[\nabla^2 - \left(1 + \frac{\omega_P^2}{\omega_c^2}\right)\frac{1}{c^2}\partial_t^2\right]B_{1z} = -\frac{4\pi}{c}(\nabla\times\boldsymbol{j}_{\perp nl})_z, \quad (9)$$

where

$$\boldsymbol{j}_{\perp nl} = \frac{c}{4\pi B_0}\left\{\boldsymbol{E}\times\hat{\boldsymbol{z}}\nabla\cdot\boldsymbol{E} + \frac{\omega_P^2}{\omega_c^2}\boldsymbol{E}\times\hat{\boldsymbol{z}}\cdot\nabla\boldsymbol{E}_\perp + \right.$$
$$\left. + B_{1\perp}\left[(\nabla\times B_1)_z - \frac{1}{c}\partial_t E_z\right]\right\}. \quad (10)$$

The magnetic field perturbation $\boldsymbol{B}_1$ appearing in (7) to (10) is naturally expressed in terms of $\boldsymbol{E}$ by means of (5). The latter also yields

$$\partial_t(\nabla\times\boldsymbol{B}_1)_z = c(\nabla_\perp^2 E_z - \partial_z\nabla\cdot\boldsymbol{E}_\perp). \quad (11)$$

We have thus derived four coupled nonlinear equations governing the evolution of the wave electric and magnetic fields in an electron–positron plasma embedded in a constant magnetic field. They generalize the previous equations (Yu *et al.*, 1986) by including the compressional magnetic field perturbation B_{1z}. It follows from (9) that the latter is driven on account of the rotational part of the nonlinear current arising from the mode coupling.

For illustrative purposes, we present the reduced nonlinear equations for the case in which one completely neglects the compressional magnetic field perturbation and the displacement current. Thus, when $\boldsymbol{E} = -\nabla\phi - (\hat{\boldsymbol{z}}/c)\partial_t A_z$, $\boldsymbol{B}_1 = \nabla A_z\times\hat{\boldsymbol{z}}$, $B_{1z} = 0$, and $\nabla_\perp \gg \partial_z$, where ϕ is the scalar potential and A_z is the parallel (to z) component of the vector potential, the SS-equations (7)–(11) reduce to (Yu *et al.*, 1986)

$$a\mathrm{d}_t\nabla_\perp^2\phi + c\mathrm{d}_z\nabla_\perp^2 A_z = 0, \quad (12)$$

and

$$\mathrm{d}_t\lambda_e^2\nabla_\perp^2 A_z - \partial_t A_z - c\mathrm{d}_z\phi = 0, \tag{13}$$

where $a = 1+c^2/v_A^2, v_A^2 = B_0^2/8\pi m n_0, \lambda_e = c/\omega_P, \mathrm{d}_t = \partial_t+(c/B_0)(\hat{z}\times\nabla\phi)\cdot\nabla$, and $\mathrm{d}_z = \partial_z + (1/B_0)(\nabla A_z \times \hat{z})\cdot\nabla$. Equations (12) and (13) govern the nonlinear mode coupling of low-frequency Alfvén fluctuations in a uniform electron–positron plasma when the compressional magnetic field perturbations are ignored.

In summary, we have derived a set of coupled nonlinear equations that governs the dynamics of finite amplitude low-frequency electromagnetic waves in a homogeneous electron–positron magnetoplasma. We have found that compressional magnetic field perturbations are present. Our equations are thus suitable for studies of spectrum cascading, modulational instabilities and the formation of coherent structures in a strongly magnetized electron-positron plasma.

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