Short Communication

Real-time image deblurring using four-wave mixing

Real-time mathematical operations using degenerate four-wave mixing (FWM), such as convolution and correlation [1–3], subtraction [4], division [5] and edge enhancement [2, 6–9] have been reported recently. The first three, i.e. convolution, correlation and subtraction are linear operations and the FWM used is operated in the linear region, so that the intensity of the output beam is essentially proportional to that of the incident object beam. The last two operations, i.e. division and edge enhancement are nonlinear operations and they utilize the nonlinear characteristic of FWM to obtain the desired results.

In this short communication, we propose a real-time image deblurring method using FWM, based on the well known deconvolution method, proposed and realized by Stroke and his co-workers [10–13]. The basis of their method involves spatial filtering using a holographic deconvolution filter where image division in the Fourier transform plane is performed. Since the deconvolution (division) filter is usually recorded on a negative photographic film by controlling γ, the slope in the linear region of the Hurter–Driffield curve of the film, the operation cannot be done in real-time.

In FWM, the recording of the hologram and the reconstruction of the image occur simultaneously and are almost in real-time (this time delay is usually very small, depending on the response time of the nonlinear medium used as the recording medium) because the time consuming processing of the film is not required.

In the following we first outline the general principle of image deblurring, and then describe how FWM operated in the nonlinear region can perform real-time image deblurring.

It is well known that for the incoherent-mode image case, the intensity of a blurred image (say, a photograph) can be expressed as [9, 10] (the coherent-mode image case is similar)

$$g(x',y') = \iint f(x,y)\, h(x'-x, y'-y)\, \mathrm{d}x\, \mathrm{d}y = f * h \qquad (1)$$

where $h(x, y)$ is the intensity of the impulse-response function of the imperfect imaging system which blurs the object (we assume that it is available in photographic form from experiments or by computation). $f(x, y)$ is the desired (deblurred) object intensity function and the symbol $*$ denotes spatial convolution. Applying the Fourier transform to Equation 1 we have

$$G = FH \qquad (2)$$

where G, F and H are the Fourier transform of g, f and h, respectively.

Multiplying Equation 2 with H^{-1}, it yields

$$F = G/H. \qquad (3)$$

After an inverse Fourier transform $\mathscr{F}^{-1}$, Equation 3 becomes

$$f = \mathscr{F}^{-1}(G/H). \qquad (4)$$

It is obvious that the desired (deblurred) object can be retrieved if image division of G/H in the Fourier transform plane can be realized.

Fig. 1 shows the proposed experimental set-up of FWM to perform real-time image deblurring, which is very similar to that used by White and Yariv [1, 2] to perform real-time convolution and correlation. On the other hand no external electric field is used here and our FWM is operated in the nonlinear region. A highly sensitive photorefractive crystal such as $BaTiO_3$ [7, 14] should be used as the recording medium, because, a thin crystal can then be used so that the four-wave mixing takes place only in the region where the Fourier transform is accurately realized [2]. The absence of an applied electric field avoids the amplitude distortion which otherwise occurs due to variation of local photoconductivity, and hence electric field strength, in different regions of the crystal [5]. The orientation of the $BaTiO_3$ crystal should be such that the transmission grating formed by beams g and h becomes dominant, utilizing the large electro-optic coefficient r_{42} [14]. Thus, the wavefront reflectivity of the FWM is adequate even without an external electric field.

In Fig. 1, three input beams, g, h and u (formed by passing through transparencies g, h and u, respectively) in the input planes are Fourier transformed by lens L_1 and L_2. The Fourier trans-

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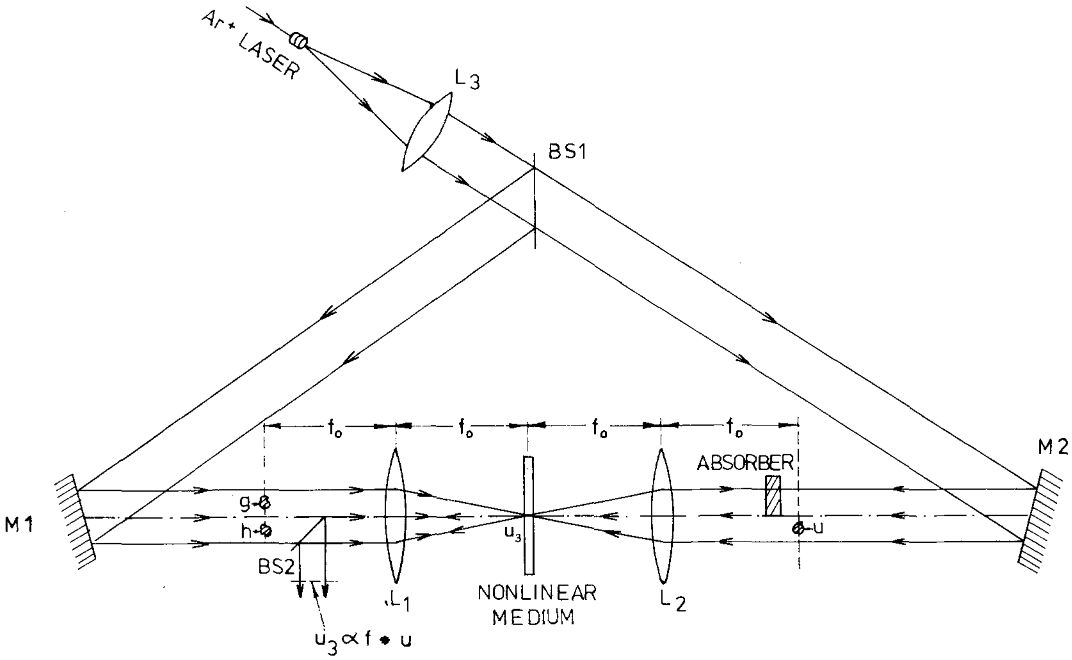

Figure 1 Experimental arrangement of four-wave mixing for performing real-time image deblurring (deconvolution). M1 and M2 are the reflective mirrors, L_1, L_2 and L_3 are the lenses, and BS1 and BS2 are the beam splitters. f_0 is the focal length of lenses L_1 and L_2. The long dashed lines denote the input and output planes.

formed fields G, H and U will participate in the FWM process. If the intensity of beam h is much larger than that of g and u, then in the Fourier transform plane, the intensity of H will be much larger than that of G and U also.

From our previous result [5, 8], the generated beam U_3 at the surface of the crystal, travelling essentially in the opposite direction to beam h, will be of the form

$$U_3 = C\left(\frac{G}{H}\right)U \qquad (5)$$

where U is the Fourier transform of u and C is a constant. Applying an inverse Fourier transform to Equation 5 and using Equation 4, it yields

$$u_3 \propto f * u \qquad (6)$$

where u_3 is evaluated at the plane which locates a focal length's distance f_0 in front of lens L_1 (which performs the required inverse Fourier transform of Equation 5), as shown in Fig. 1.

If u is a δ function, Equation 6 becomes

$$u_3 \propto f * \delta \propto f. \qquad (7)$$

Then u_3 is the desired (deblurred) object function, because the convolution of a function with a δ function yields the function itself, though it is shifted along the transverse direction [15].

Note that in Fig. 1, since G and U are not exactly counterpropagating, the interaction between G, U, H and U_3 is not exactly phase matched. In our case, U is a plane wave (because u is a δ function), the phase matching condition will then put a constraint on the maximum angular spread of beam G [2]. This maximum angular spread is known to be [2, 16]

$$2\Delta\theta \simeq \Lambda/d \qquad (8)$$

where Λ is the fringe spacing and d the thickness of the crystal. Consequently, the maximum size of the incident beam g at the front focal plane of lens L_1 (see Fig. 1) will be $\Lambda f_0/d$ [2]. While this constraint is tolerable for transmission type FWM, it is very severe for reflection type FWM, due to the small fringe spacing and the consequent high angular selectivity [2, 17]. Unfortunately experimental verification of this method has not been obtained yet, because at present we do not have suitable photorefractive crystals such as $BaTiO_3$.

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References

1. J. O. WHITE and A. YARIV, *Appl. Phys. Lett.* **37** (1980) 5.
2. *Idem, Opt. Eng.* **21** (1982) 224.
3. L. PICHON and J. P. HUIGNARD, *Opt. Commun.* **36** (1981) 277.
4. Y. H. JA, *ibid.* **42** (1982) 377.
5. *Idem, ibid.* **44** (1982) in press.
6. J. P. HUIGNARD and J. P. HERRIAU, *Appl. Opt.* **17** (1978) 267.
7. J. FEINBERG, *Opt. Lett.* **5** (1980) 330.
8. Y. H. JA, in Proceedings of the Seminar on Image Processing, University of New South Wales, NSW, Australia, 23–25 June, 1982 (Radio Research Board of Australia).
9. *Idem, Opt. Quant. Elect.* **15** (1983) in press.
10. G. W. STROKE and R. G. ZECH, *Phys. Lett.* **25A** (1967) 89.
11. G. W. STROKE, F. FURRER and D. R. LAMBERTY, *Opt. Commun.* **1** (1969) 141.
12. G. W. STROKE and M. HALIOUA, *Phys. Lett.* **33A** (1970) 3.
13. G. W. STROKE, M. HALIOUA, F. THON and D. H. WILLASCH, *Proc. IEEE* **65** (1977) 39.
14. J. FEINBERG and R. W. HELLWARTH, *Opt. Lett.* **5** (1980) 519.
15. R. J. COLLER, C. B. BURCKHARDT and L. H. LIN, 'Optical Holography' (Academic Press, New York, 1971) p. 89.
16. H. KOGELNIK, *Bell Syst. Tech. J.* **48** (1969) 2909.
17. Y. H. JA, *Opt. Commun.* **41** (1982) 159.

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