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Evolution of the Balance Equations in Saturated Thermoelastic Porous Media Following Abrupt Simultaneous Changes in Pressure and Temperature

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Abstract. A mathematical model is developed for saturated flow of a Newtonian fluid in a thermoelastic, homogeneous, isotropic porous medium domain under nonisothermal conditions. The model contains mass, momentum and energy balance equations. Both the momentum and energy balance equations have been developed to include a Forchheimer term which represents the interaction at the solid-fluid interface at high Reynolds numbers. The evolution of these equations, following an abrupt change in both fluid pressure and temperature, is presented. Using a dimensional analysis, four evolution periods are distinguished. At the very first instant, pressure, effective stress, and matrix temperature are found to be disturbed with no attenuation. During this stage, the temporal rate of pressure change is linearly proportional to that of the fluid temperature. In the second time period, nonlinear waves are formed in terms of solid deformation, fluid density, and velocities of phases. The equation describing heat transfer becomes parabolic. During the third evolution stage, the inertial and the dissipative terms are of equal order of magnitude. However, during the fourth time period, the fluid's inertial terms subside, reducing the fluid's momentum balance equation to the form of Darcy's law. During this period, we note that the body and surface forces on the solid phase are balanced, while mechanical work and heat conduction of the phases are reduced.

Key words: Macroscopic mass, momentum and energy balance equations; Forchheimer term; saturated flow, thermoelastic porous media; abrupt change of temperature and pressure; nonlinear wave.

1. Introduction

The macroscopic model describing saturated flow of a Newtonian fluid in a thermoelastic porous medium, was developed, among others, by Bear and Bachmat (1990). This model contains mass, momentum and energy balance equations. However, they neglect the microscopic inertial terms at the fluid-solid interface. As a consequence, their averaged momentum balance equation does not contain the Forchheimer term (which is proportional to the velocity squared) which may be

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significant, especially for high velocities. Bear and Sorek (1990) start from this model, for an isothermal case, and describe the evolution of the averaged mass and momentum balance equations, to more simple but approximate forms, following an abrupt pressure impact exerted at the boundary of a porous medium domain. The same methodology is applied in the present study in order to investigate the evolution stages of the (averaged) mass, momentum, and energy balance equations, following an abrupt change in *both* the pressure and fluid temperature. This methodology involves mainly rewriting the (averaged) fluid and solid balance equations in nondimensional forms, and analyzing the relative order of magnitudes of the terms appearing in these equations during various time periods. This analysis yields four evolution of which the transport of mass, momentum, and energy are governed by different balance equations. In comparison to Nikolavskij (1990), the present theory is novel in two major aspects: (1) developing the evolving dominant balance equations of interacting fluid and solid phases following an abrupt change in fluid temperature and pressure and (2) accounting for Forchheimer terms for momentum and energy balances at the fluid-solid interface.

2. Macroscopic Balance Equations

2.1. MASS BALANCE EQUATIONS

The macroscopic mass balance equation for the fluid phase, neglecting the dispersive mass flux, is

$$\frac{\partial \phi \rho_f}{\partial t} = -\nabla \cdot (\phi \rho_f \boldsymbol{V}_f). \tag{1}$$

in which ϕ denotes porosity, $\boldsymbol{V}_f$ denotes the fluid's velocity vector, and $\rho_f [\equiv \rho_f(P, T_f)]$ denotes its density. We may rewrite (1) in the form

$$\begin{aligned} \rho_f \frac{\partial \phi}{\partial t} &+ \phi \rho_f \left(\beta_P \frac{\partial P}{\partial t} + \beta_T \frac{\partial T_f}{\partial t} \right) \\ &= -\rho_f (\boldsymbol{V}_f + \boldsymbol{V}_s) \cdot \nabla \phi - \phi \rho_f \nabla \cdot (\boldsymbol{V}_r + \boldsymbol{V}_s) \\ &\quad - \phi \rho_f (\boldsymbol{V}_r + \boldsymbol{V}_s) \cdot (\beta_P \nabla P + \beta_T \nabla T_f), \end{aligned} \tag{2}$$

in which $\boldsymbol{V}_r = \boldsymbol{V}_f - \boldsymbol{V}_s$ denotes the fluid's velocity relative to that of the solid, and

$$\beta_P = \frac{1}{\rho_f} \frac{\partial \rho_f}{\partial P}\bigg|_{T_f}; \qquad \beta_T = \frac{1}{\rho_f} \frac{\partial \rho_f}{\partial T_f}\bigg|_P, \tag{3}$$

with P and T_f denoting the fluid's pressure and temperature, respectively. We assume that $|\partial \phi / \partial t| \gg |\boldsymbol{V}_s \cdot \nabla \phi|$. This is equivalent to the statement that $\mathrm{St}^{\phi} =$

$L_c^\phi/t_c^\phi V_{sc} \gg 1$, in which St^ϕ is the Struhal number associated with the porosity, with L_c^ϕ and t_c^ϕ denoting the characteristic length and time intervals for changes in ϕ, respectively, and V_{sc} denotes the characteristic velocity of the solid.

Under the same assumption, the mass balance equation for the solid, assuming that the latter's density, ρ_s, is constant, is

$$\frac{\partial \phi}{\partial t} = (1-\phi)\nabla \cdot \boldsymbol{V}_s. \tag{4}$$

In view of (4), and the assumption that the porous medium is homogeneous, i.e. $\nabla\phi = 0$, we may rewrite (2) in the form

$$\phi\rho_f\left(\beta_P\frac{\partial P}{\partial t} + \beta_T\frac{\partial T_f}{\partial t}\right) + \phi\rho_f\nabla\cdot\boldsymbol{V}_r + \rho_f\nabla\cdot\boldsymbol{V}_s +$$
$$+\phi\rho_f(\boldsymbol{V}_r+\boldsymbol{V}_s)\cdot(\beta_P\nabla P + \beta_T\nabla T_f) = 0. \tag{5}$$

2.2. MOMENTUM BALANCE EQUATIONS

The macroscopic momentum balance equation for a Newtonian compressible fluid, neglecting dispersive momentum fluxes, was developed by Bear and Bachmat (1990). However, in their development, they failed to take into account the inertial term in the microscopic momentum balance equation at the solid-fluid interface. Hence, these effects do not appear in their macroscopic momentum balance equation. Without their assumption of isochoric flow, but with an assumption that the porous medium is homogeneous, i.e., $\nabla\phi = 0$, their averaged momentum balance equation would have taken the form:

$$\begin{aligned}\frac{\partial}{\partial t}(\phi\rho_f V_{f_i}) \\ = & -\frac{\partial}{\partial x_j}(\phi\rho_f V_{f_i}V_{f_i}) - \phi\left(\frac{\partial P}{\partial x_j} + \rho_f g\frac{\partial Z}{\partial x_j}\right)T^*_{ij} + \\ & +\mu_f\phi\frac{\partial^2 V_{f_i}}{\partial x_j\partial x_j} + (\mu_f + \lambda''_f)\phi\frac{\partial^2 V_{f_i}}{\partial x_i\partial x_j} - \mu_f\alpha_{ij}\frac{c_v}{\Delta_f^2}\phi V_{r_j}.\end{aligned} \tag{6}$$

In this equation, μ_f denotes the fluid's viscosity, $(\lambda''_f + (2/3)\mu_f)$ denotes the secondary viscosity, g denotes gravity acceleration, Z denotes elevation, and the tensorial coefficients α_{ij} and T^*_{ij} constitute constant macroscopic representations of the microscopic configuration of the solid-fluid interface, the latter is regarded as the tortuosity. Both are defined for an isotropic porous medium by

$$\alpha_{ij} \equiv \alpha_m\delta_{ij}, \qquad T^*_{ij} \equiv T^*_m\delta_{ij}, \tag{7}$$

where δ_{ij} denotes the ijth component of the Kronecker delta, and α_m and T^*_m are coefficients that represent solid matrix properties.

To include the inertial effect at the solid-fluid interface in their averaged momentum balance equation, Bear and bachmat (1990) should not have assumed that the drag and inertial forces at the microscopic solid-fluid interface are much smaller than the surface and body (gravity) forces. If, while developing an expression for the averaged pressure gradient at the solid-fluid interface, instead of their assumption (2.6.9), they would have made the assumption

$$\left|\left(\rho_f\frac{\partial V_{f_j}}{\partial t}-\frac{\partial \tau_{ij}}{\partial x_i}\right)v_j\right| \ll \left|\left(\frac{\partial P}{\partial x_j}+\rho_f g\frac{\partial Z}{\partial x_j}+\rho_f V_{f_i}\frac{\partial V_{f_i}}{\partial x_i}\right)v_j\right|, \tag{8}$$

their resulting macroscopic momentum balance equation would have included an additional term, known as the Forchheimer term. Note that in (8) we argue that the microscopic surface and body forces, together with inertial forces (associated with spatial velocity changes) are of higher magnitude than the viscous forces (associated with fluid shear stress tensor, τ_{ij}) together with the inertial forces due to the velocity rates. The inclusion of $\boldsymbol{V}\cdot\nabla\boldsymbol{V}$ in the RHS of (8) is significant for high and/or transient velocities. Many authors, e.g., Hsu and Cheng (1990), Nield (1991,1994), Olim (1994), often without a rigorous proof, include an expressions for the Forchheimer term in their macroscopic momentum balance equation. In the present work, we shall extend these expressions to the case in which the solid matrix is deformable. Thus, for the purpose of this paper, the Forchheimer term that we shall use, will have the form:

$$\sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}\phi\rho_f|\boldsymbol{V}_f-\boldsymbol{V}_s|(V_{f_i}-V_{s_i}) \tag{9}$$

where, Δ_f denotes the hydraulic radius of the pore space, and c_v denotes a shape factor. Note, however, that the additional Forchheimer term originates from a microscopic inertial term at the solid-fluid interface. This is in contradiction to the assumption made by other authors [Hsu and Cheng (1990), Nield (1991,1994), Olim (1994)] who refer to the Forchheimer term as a drag factor or in compensation for neglecting the inertial term associated with vorticity.

The macroscopic fluid momentum balance equation, (6), together with the Forchheimer term, (9), takes the form

$$\begin{aligned}\frac{\partial}{\partial t}(\phi\rho_f V_{f_i})\\ = & -\frac{\partial}{\partial x_j}(\phi\rho_f V_{f_i}V_{f_j})-\phi\left(\frac{\partial P}{\partial x_i}+\rho_f g\frac{\partial Z}{\partial x_i}\right)T^*_m+\mu_f\phi\frac{\partial^2 V_{f_i}}{\partial x_j\partial x_j}+\\ & +(\mu_f+\lambda''_f)\phi\frac{\partial^2 V_{f_j}}{\partial x_i\partial x_j}-\mu_f\alpha_m\frac{c_v}{\Delta_f^2}\phi V_{r_i}-\sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}\phi\rho_f|\boldsymbol{V}_r|V_{r_i}.\end{aligned} \tag{10}$$

In view of (5), this equation can be rewritten as

$$\phi\rho_f \frac{\partial}{\partial t}(V_{r_i} + V_{s_i}) + \phi\rho_f(V_{r_j} + V_{s_j})\frac{\partial}{\partial x_j}(V_{r_i} + V_{s_i})$$

$$+\phi\rho_f\beta_P(V_{r_i} + V_{s_i})\frac{\partial P}{\partial t}+$$

$$+\phi\rho_f\beta_T(V_{r_i} + V_{s_i})\frac{\partial T_f}{\partial t} + \rho_f(V_{r_i} + V_{s_i})\left(\phi\frac{\partial V_{r_j}}{\partial x_j} + \frac{\partial V_{s_j}}{\partial x_j}\right)$$

$$+\phi\rho_f\beta_T(V_{r_j} + V_{s_j})(V_{r_i} + V_{s_i})\frac{\partial T_f}{\partial x_j}+$$

$$+\phi\rho_f\beta_P(V_{r_j} + V_{s_j})(V_{r_i} + V_{s_i})\frac{\partial P}{\partial x_j} + \phi T^*_m\left(\frac{\partial P}{\partial x_i} + \rho_f g\frac{\partial Z}{\partial x_i}\right)$$

$$-\phi\mu_f\left[\frac{\partial^2 V_{r_i}}{\partial x_j\partial x_j} + \frac{\partial^2 V_{s_i}}{\partial x_j\partial x_j}\right] -$$

$$-\phi(\mu_f + \lambda''_f)\left[\frac{\partial^2 V_{r_j}}{\partial x_i\partial x_j} + \frac{\partial^2 V_{s_j}}{\partial x_i\partial x_j}\right] + \mu_f\alpha_m\frac{c_v}{\Delta_f^2}\phi V_{r_i}$$

$$+\sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}\phi\rho_f|V_r|V_{r_i} = 0. \tag{11}$$

By summing the macroscopic momentum balance equations for the fluid and the solid phases [see Bear *et al.* (1992), eqs. (12) and (13)], the rate of momentum exchange across the solid-fluid interface is eliminated, we obtain the momentum balance equation for the porous medium as a whole in the form

$$\begin{aligned}\frac{\partial}{\partial t}(\phi\rho_f \boldsymbol{V}_r + \rho_b \boldsymbol{V}_s) \\ = -\nabla\cdot[\phi\rho_f(\boldsymbol{V}_r\boldsymbol{V}_r + \boldsymbol{V}_r\boldsymbol{V}_s + \boldsymbol{V}_s\boldsymbol{V}_r) + \rho_b\boldsymbol{V}_s\boldsymbol{V}_s]+ \\ +\nabla\cdot(\boldsymbol{\sigma}'_s + \boldsymbol{\sigma}_f) - \rho_b g\nabla Z,\end{aligned} \tag{12}$$

in which $\rho_b = \phi\rho_f + (1-\phi)\rho_s$ is the bulk density of the porous medium, $\boldsymbol{\sigma}_\alpha$ denotes the intrinsic phase average stress in the α-phase ($\alpha \equiv f, s$), and

$$\boldsymbol{\sigma}'_s = (1-\phi)(\boldsymbol{\sigma}_s - \boldsymbol{\sigma}_f); \quad \text{with}\, \sigma'_{sij} = \sigma'_{sji}, \tag{13}$$

denotes the effective stress ($\equiv$ intragranular stress) in the porous medium [see Bear and Bachmat (1990) and Trezaghi (1925)].

The macroscopic (averaged) form of the constitutive relationship for fluid stress in a thermoelastic porous medium [Bear *et al.* (1992)] is

$$\sigma_f = \mu_f[\nabla q_r + (\nabla q_r)^T] + \lambda_f''\nabla\cdot q_r\delta + 2\phi\mu_f\frac{D_s\varepsilon_{\text{skel}}}{Dt} + \phi\lambda_f''\frac{D_s e_{\text{skel}}}{Dt}\delta - PI, \quad (14)$$

in which the $(\ )^T$ denotes the transpose matrix, and $q_r (\equiv \phi V_r)$ denotes the relative specific flux. Note that (14) includes terms that are associated with the solid skeleton. They express the influence of the solid strain on the fluid stress at their common interface.

The macroscopic strain tensor for the solid matrix, $\varepsilon_{\text{skel}}$, is defined for small deformations by the compatibility law

$$\varepsilon_{\text{skel}} = \tfrac{1}{2}(\nabla \mathbf{w}_s + (\nabla \mathbf{w}_s)^T), \quad (15)$$

in which $\mathbf{w}_s$ denotes the displacement vector of the solid matrix. The volumetric strain ($\equiv$ dilatation), e_{skel}, is given by

$$e_{\text{skel}} = \nabla \cdot \mathbf{w}_s \quad (16)$$

From (16) and (4), it follows that

$$\nabla \cdot V_s = \frac{1}{1-\phi}\frac{\partial \phi}{\partial t} = \frac{D_s e_{\text{skel}}}{Dt}, \qquad \frac{D_s(\ )}{Dt} \equiv \frac{\partial}{\partial t}(\ ) + V_s \cdot \nabla(\ ), \quad (17)$$

with

$$V_s \equiv \frac{D_s \mathbf{w}_s}{Dt} \quad (18)$$

The macroscopic constitutive relationship for a thermoelastic solid matrix takes the form, (Bear *et al.*, 1992)

$$\sigma_s' = \lambda_s'' e_{\text{skel}}\delta + 2\mu_s' \varepsilon_{\text{skel}} - \eta(T_s - T_{s_o})\delta, \quad (19)$$

where T_s and T_{s_o} denote the solid temperature and its value at a reference stage, respectively,λ_s'', μ_s' denote the Lame coefficients of the solid matrix that have to be determined experimentally, and η denotes the thermoelastic coefficient. In view of (5), (14), (15), (16) and (19), we rewrite the porous medium momentum balance equation, (12), in the form

$$\phi\rho_f\frac{\partial}{\partial t}(V_{r_i} + V_{s_i}) + (1-\phi)\rho_s\frac{\partial V_{s_i}}{\partial t} + (V_{r_i} + V_{s_i})\phi\rho_f\beta_P\frac{\partial P}{\partial t}$$

$$+(V_{r_i} + V_{s_i})\phi\rho_f\beta_T\frac{\partial T_f}{\partial t} +$$

$$+\phi\rho_f(V_{r_i}+V_{s_i})\frac{\partial V_{r_j}}{\partial x_j}+\rho_f(V_{r_i}+V_{s_i})\frac{\partial V_{s_j}}{\partial x_j}+$$

$$+\phi\rho_f\beta_P(V_{r_i}+V_{s_i})(V_{r_j}+V_{s_j})\frac{\partial P}{\partial x_j}+$$

$$+\phi\rho_f\beta_T(V_{r_i}+V_{s_i})(V_{r_j}+V_{s_j})\frac{\partial T_f}{\partial x_j}+\phi\rho_f(V_{r_j}+V_{s_j})\frac{\partial}{\partial x_j}(V_{r_i}+V_{s_i})+$$

$$+(1-\phi)\rho_s V_{s_j}\frac{\partial V_{s_i}}{\partial x_j}-$$

$$-\lambda_s''\frac{\partial^2 w_{s_j}}{\partial x_i\partial x_j}-\mu_s'\left(\frac{\partial^2 w_{s_i}}{\partial x_j\partial x_j}+\frac{\partial^2 w_{s_j}}{\partial x_i\partial x_j}\right)+$$

$$+\eta\frac{\partial T_s}{\partial x_i}-\mu_f\left(\frac{\partial^2 V_{r_i}}{\partial x_j\partial x_j}+\frac{\partial^2 V_{s_i}}{\partial x_j\partial x_j}\right)-$$

$$-(\mu_f+\lambda_f'')\left(\frac{\partial^2 V_{r_j}}{\partial x_i\partial x_j}+\frac{\partial^2 V_{s_j}}{\partial x_i\partial x_j}\right)+\frac{\partial P}{\partial x_i}+$$

$$+V_{s_i}\rho_s\frac{\partial(1-\phi)}{\partial t}+V_{s_i}\rho_s\frac{\partial}{\partial x_j}[(1-\phi)V_{s_i}]+\rho_b g\frac{\partial Z}{\partial x_i}=0. \tag{20}$$

2.3. ENERGY BALANCE EQUATIONS

In writing the macroscopic energy balance equations for the fluid and the solid, we assume linear thermodynamics. Furthermore, the specific heat at constant volume for the fluid, C_f, and the specific heat at constant strain for the solid, C_s, are assumed to be constant. In addition, we omit external energy sources and the energy associated with viscous dissipation is assumed to be negligible, i.e., $|\tau:\nabla \boldsymbol{V}_f| \ll |P\nabla\cdot \boldsymbol{V}_f|$. Hence, we write the macroscopic fluid energy balance equation

$$\frac{\partial}{\partial t}(\phi\rho_f(C_fT_f+\tfrac{1}{2}\boldsymbol{V}_f^2))$$

$$=-\nabla\cdot[\phi\rho_f(C_fT_f+\tfrac{1}{2}\boldsymbol{V}_f^2)\boldsymbol{V}_f-\phi\boldsymbol{D}^{*H}\nabla(\rho_fC_fT_f)-\phi\boldsymbol{\lambda}_f^*\nabla T_f]-$$

$$-\alpha^{*H}(T_f-T_s)-T_f\left.\frac{\beta_T}{\beta_P}\right|_{\rho_f}\nabla\cdot(\phi\boldsymbol{V}_s)-\phi T_m^*\boldsymbol{V}_f\cdot\nabla P-$$

$$-\sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}\phi\rho_f|\boldsymbol{V}_r|\boldsymbol{V}_r\cdot\boldsymbol{V}_s \tag{21}$$

in which, α^{*H} denotes the heat transfer coefficient associated with the rate of heat transfer between the solid and the fluid phases, $\boldsymbol{\lambda}^*_\alpha(\equiv \lambda_\alpha \boldsymbol{T}^*_\alpha = \lambda_\alpha T^*_m \boldsymbol{\delta} = \lambda_{\alpha m}\boldsymbol{\delta})$ denotes the thermal conductivity of an $\alpha(\equiv f, s)$ phase, and $\boldsymbol{D}^{*H}$ denotes the fluid's thermal dispersion tensor as suggested by Nikolaevskij (1959) defined by

$$D^*_{ij} = a^H_T |\boldsymbol{V}_f| \delta_{ij} + (a^H_L - a^H_T) \frac{V_{f_i} V_{f_i}}{|\boldsymbol{V}_f|} \tag{22}$$

in which a^H_L and a^H_T denote the longitudinal and transversal thermal dispersivity, respectively. Although α^H_L is about 10–20 times larger than a^H_T, for simplifing the present analysis we assumed, a^H_T to be equal to a^H_L, since we are interested only in the principle of thermal dispersion. Note that without Forchheimer term (The last term on the RHS), (21) can be obtained by following Bear and Bachmat (1990) upon including fluid mass and momentum balance equations. Forchheimer term maybe interpreted as a heat source at the solid-fluid interface, and was originated from the fluid momentum balance equation. In view of (22) and the fluid mass balance (2), by subracting the momentum balance equation multiplyed by the fluid's velocity term, the fluid's energy balance equation, (21), becomes

$$\begin{aligned}
&\phi\rho_f C_f T_f \beta_P \frac{\partial P}{\partial t} + \phi\rho_f C_f \frac{\partial T_f}{\partial t} + \phi\rho_f C_f \beta_T T_f \frac{\partial T}{\partial t} + \\
&+ \phi\rho_f C_f \beta_T T_f (V_{r_j} + V_{s_j}) \frac{\partial T_f}{\partial x_j} + \\
&+ \phi\rho_f C_f (V_{r_j} + V_{s_j}) \frac{\partial T_f}{\partial x_j} + \phi\rho_f C_f T_f \frac{\partial V_{r_j}}{\partial x_j} + \rho_f C_f T_f \frac{\partial V_{s_j}}{\partial x_j} + \\
&+ \phi\rho_f C_f \beta_P T_f (V_{r_j} + V_{s_j}) \frac{\partial P}{\partial x_j} - \\
&- \phi a^H_T \frac{(V_{r_l} + V_{s_l}) \frac{\partial}{\partial x_j}(V_{r_l} + V_{s_l})}{|\boldsymbol{V}_r + \boldsymbol{V}_s|} \times \\
&\times \left(\rho_f C_f T_f \beta_p \frac{\partial P}{\partial x_j} + \rho_f C_f (1 + \beta_T T_f) \frac{\partial T_f}{\partial x_j} \right) - \\
&- \phi a^H_T |\boldsymbol{V}_r + \boldsymbol{V}_s| \left[\rho_f C_f (2 + \beta_T T_f) \beta_T \left(\frac{\partial T_f}{\partial x_j} \right)^2 + \right. \\
&\qquad + 2\rho_f C_f \beta_P (1 + \beta_T T_f) \frac{\partial T_f}{\partial x_j} \frac{\partial P}{\partial x_j} +
\end{aligned}$$

$$+\rho_f C_f \beta_P^2 T_f \left(\frac{\partial P}{\partial x_j}\right)^2 + \rho_f C_f (1+\beta_T T_f)\frac{\partial^2 T_f}{\partial x_j \partial x_j} +$$

$$+\rho_f C_f \beta_P T_f \frac{\partial^2 P}{\partial x_j \partial x_j}\Bigg] +$$

$$+\phi \lambda^*_{fm} \frac{\partial^2 T_f}{\partial x_i^2} - \alpha^{*H}(T_s - T_f) + \phi T_f \frac{\beta_T}{\beta_P}\left(\frac{\partial V_{r_j}}{\partial x_j} + \frac{\partial V_{s_j}}{\partial x_j}\right) +$$

$$+\sqrt{\frac{c_v \alpha_m}{\Delta_f^2}}\phi \rho_f |\boldsymbol{V}_r| V_{r_j} V_{r_j} = 0. \tag{23}$$

For a thermoelastic isotropic solid, the macroscopic heat balance equation as obtained by Bear and Bachmat (1990) is

$$\rho_s C_s T_s \frac{\partial (1-\phi)}{\partial t} + (1-\phi)\rho_s C_s \frac{\partial T_s}{\partial t} + (1-\phi)\rho_s C_s V_{s_j}\frac{\partial T_s}{\partial x_j} -$$

$$-(1-\phi)\lambda^*_{sm}\frac{\partial^2 T_s}{\partial x_i^2} +$$

$$+\alpha^{*H}(T_s - T_f) + \eta T_s \frac{\partial V_{s_j}}{\partial x_j} + (1-\phi)\rho_s C_s T_s \frac{\partial V_{s_j}}{\partial x_j} = 0. \tag{24}$$

Altogether we have so far 8 variables $(P, T_f, T_s, \boldsymbol{V}_r, \boldsymbol{V}_s, \rho_f, \phi, \mathbf{w}_s)$. To solve for these variables, we have 6 balance equations: (4), (5), (11), (20), (23), (24); a constitutive relation for the pressure and definition (18).

Our objective in what follows, is to analyze the order of magnitude of the various terms of the balance equations, following an abrupt change in both the pressure and the temperature.

3. Nondimensional Forms of the Balance Equations

Our next objective is to analyze equations (4), (5), (11), (20), (23) and (24) in order to eliminate from them nondominating terms, i.e., ones that are much smaller then other ones in the same equation. To achieve this goal, we first rewrite the above equations in dimensionless forms. The various dependent variables that are appearing in the dimensionless equations as well as their derivatives, are of order one, if appropriate reference values are selected for them. Hence, the order of magnitude of a term is determined by the scalar factor that multiplies that term. By comparing these scalar factors for any two terms, we can determine the conditions under which one of these terms is much smaller or larger than the other. This condition takes the form of a requirement that a certain dimensional number be much smaller or much longer than a unit.

In the present case, since our aim is to compare all the terms with terms associated with the rate of pressure changes, we (1) select a scalar for every variable and parameter in the balance equations. The scalar factor gives the ratio between the dimensionless variable, $(\ )^*$, and the dimensional one [e.g.,$(\Delta x) = L_c(\Delta x^*)$]; (2) rewrite the balance equation by replacing variables and parameters by the product of the dimensionless variable and the corresponding scalar. In this way, each term in the balance equation takes the form of a product of a scalar factor involving various scales, and a dimensionless factor the latter is of order 1. We divided the equation by the scalar factor that appears in the term that includes the rate of pressure change.

To simplify our symbols, we use, e_c, as a characteristic value of the intensive quantity, e and L_c^e and t_c^e as characteristic increments in the spatial and time steps, respectively, both associated with e. Furthermore, we introduce the following dimensionless numbers which were obtained in the nondimensionalization process.

Struhal number, St^e, associated with the intensive quantity, e.

$$\mathrm{St}^e \equiv \frac{L_c^e}{t_c^e V_{\alpha c}}, \tag{25.1}$$

in which $V_{\alpha c}$ denotes the characteristic mass weighted velocity of the α-phase.

Euler number, Eu_α, associated with the α phase.

$$\mathrm{Eu} \equiv \frac{P_c}{\rho_c^\alpha (V_{\alpha c})^2}, \tag{25.2}$$

where ρ_c^α denotes the characteristic density of the α-phase.

Richardson number, Ri, is given by

$$\mathrm{Ri} = \mathrm{Fr}^{-2} \equiv \frac{g L_c^{V_f}}{(V_{f_c})^2}, \tag{25.3}$$

where Fr denotes the Froude number.

Reynolds number, Re, is given by

$$\mathrm{Re} \equiv \frac{V_{fc}\sqrt{\kappa_c/\phi_c}}{v_c} = V_{f_c} L_c^{V_f} \mathrm{Da}^{1/2}, \tag{25.4}$$

in which the characteristic kinematic viscosity, v_c, is defined by

$$v_c \equiv \frac{\mu_c^f}{\rho_c^f}, \tag{25.5}$$

κ_c is a characteristic permeability given by

$$\kappa_c = \frac{\phi_c \Delta_f^2}{\alpha_m c_v}, \tag{25.6}$$

and Darcy's number, Da, is given by

$$\mathrm{Da} \equiv \frac{\kappa_c}{\phi_c (L_c^{V_f})^2}. \tag{25.7}$$

Fourier number, Fo_α, associated with the α-phase, is defined by

$$\mathrm{Fo}_\alpha \equiv \frac{\lambda_{\alpha m} t_c^{T_\alpha}}{\rho_c^\alpha C_\alpha (L_c^{T_\alpha})^2} = \frac{\alpha_\alpha t_c^{T_\alpha}}{\left(L_c^{T_\alpha}\right)^2} \tag{25.8}$$

Fourier number, $\mathrm{Fo}_{\mathrm{Dis}}$, associated with the fluid's thermal dispersion, is given by

$$\mathrm{Fo}_{\mathrm{Dis}} \equiv \frac{a_T^H V_{f_c} t_c^{T_\alpha}}{(L_c^{V_f})^2}. \tag{25.9}$$

Nusselt number, Nu, associated with the fluid phase, is given by

$$\mathrm{Nu} \equiv \frac{\alpha^{*H} \left(L_c^{T_f}\right)^2}{\lambda_f}. \tag{25.10}$$

Biot number, Bi, associated witht he solid phase, is given by

$$\mathrm{Bi} \equiv \frac{\alpha^{*H} \left(L_c^{T_s}\right)^2}{\lambda_s}. \tag{25.11}$$

We furthermore introduce some additional new scalar numbers. The characteristic strain, ε_c, is defined by

$$\varepsilon_c \equiv \frac{w_c^s}{L_c^{w_s}}. \tag{26.1}$$

The ratio, Q_c, between the solid's heat generated by deformation and its advective heat flux is given by

$$Q_c \equiv \frac{\eta L_c^{T_s}}{\lambda_{sm} t_c^{T_s}}. \tag{26.2}$$

The ratio, F_V, between the solid's surface force, per unit length of the solid, resulting from its deformation, to the fluid's inertial force, per unit volume of the fluid, is given by

$$F_V \equiv \frac{1}{\rho_c^f} \frac{\mu_s' / w_c^s}{V_{fc} / t_c^{V_f}} \tag{26.3}$$

The ratio, F_{VT}, between the solid's surface force (per unit length of the solid) resulting from its temperature change, to the fluid's inertial force (per unit volume of the fluid), is given by

$$F_{VT} \equiv \frac{\eta}{\rho_c^f} \frac{T_c^s / w_c^s}{V_{fc}/t_c V_f}. \tag{26.4}$$

The volume of the fluid added to the storage, per unit volume of the fluid, as the pressure changes, A_p, is defined by

$$A_p \equiv \phi_c \beta_p P_c. \tag{26.5}$$

The volume of the fluid added to the storage, per unit volume of the fluid, as the temperature changes, A_{T_f}, is defined by

$$A_{T_f} \equiv \phi_c \beta_T T_c^f. \tag{26.6}$$

In view of the scalar numbers (25) and (26), the balance equations (5) for fluid mass, (4) for solid mass, (11) for fluid momentum, (20) for the porous media momentum, (23) for the fluid energy, and (24) for the solid energy can be rewritten using products of nondimensional terms ([ ]*) and scalar factors.

The fluid's mass balance equation:

$$A_1 \left[\phi \rho_f \frac{\partial P}{\partial t}\right]^* + A_2 \left[\phi \rho_f \frac{\partial T_f}{\partial t}\right]^* + A_3 \left[\phi \rho_f \frac{\partial V_{r_j}}{\partial x_j}\right]^* + A_4 \left[\rho_f \frac{\partial V_{s_j}}{\partial x_j}\right]^* +$$
$$+ A_5 \left[\phi \rho_f (V_{r_j} + V_{s_j}) \frac{\partial P}{\partial x_j}\right]^* + A_6 \left[\phi \rho_f (V_{r_j} + V_{s_j}) \frac{\partial T_f}{\partial x_j}\right]^* = 0, \tag{27}$$

where

$$A_1 = 1, \qquad A_2 = \frac{A_{T_f}}{A_P} \frac{t_c^P}{t_c^{T_f}},$$

$$A_3 = \phi_c A_P^{-1} (\mathrm{St}^{V_f})^{-1} \frac{t_c^P}{t_c^{V_f}} \frac{L_c^{V_f}}{L_c^{V_r}} \left(1 - \frac{V_{sc}}{V_{fc}}\right), \qquad A_4 = A_P^{-1} (\mathrm{St}^{V_s})^{-1} \frac{t_c^P}{t_c^{V_s}},$$

$$A_5 = (\mathrm{St}^{V_f})^{-1} \frac{L_c^{V_f}}{L_c^P} \frac{t_c^P}{t^{V_f}}, \qquad A_6 = \frac{A_{T_f}}{A_p} (\mathrm{St}^{V_f})^{-1} \frac{L_c^{V_f}}{L_c^{T_f}} \frac{t_c^P}{t_c^{V_f}}. \tag{28}$$

The solid's mass balance equation:

$$B_1 \left[\frac{\partial \phi}{\partial t}\right]^* - B_2 \left[\frac{\partial V_{s_j}}{\partial x_j}\right]^* = 0, \tag{29}$$

where

$$B_1 = 1, \qquad B_2 = \frac{(1-\phi_c)}{\phi_c}\,\mathrm{St}^{V_s}\frac{t_c^{\phi}}{t_c^{V_s}}. \tag{30}$$

The fluid's momentum balance equation:

$$\begin{aligned}
&D_1\left[\phi\rho_f(V_{r_i}+V_{s_i})\frac{\partial P}{\partial t}\right]^* + D_2\left[\phi\rho_f(V_{r_i}+V_{s_i})\frac{\partial T_f}{\partial t}\right]^* \\
&\quad + D_3\left[\phi\rho_f\frac{\partial}{\partial t}(V_{r_i}+V_{s_i})\right]^* + \\
&\quad + D_4\left[\phi\rho_f(V_{r_j}+V_{s_j})\frac{\partial}{\partial x_j}(V_{r_i}+V_{s_i})\right]^* + D_5\left[\phi\rho_f(V_{r_i}+V_{s_i})\frac{\partial V_{r_j}}{\partial x_j}\right]^* + \\
&\quad + D_6\left[\phi T_m^*\frac{\partial P}{\partial x_i}\right]^* + \\
&\quad + D_7\left[\phi\rho_f(V_{r_j}+V_{s_j})(V_{r_i}+V_{s_i})\frac{\partial P}{\partial x_j}\right]^* + \\
&\quad + D_8\left[\phi\rho_f(V_{r_j}+V_{s_j})(V_{r_i}+V_{s_i})\frac{\partial T_f}{\partial x_j}\right]^* + \\
&\quad + D_9\left[\rho_f(V_{r_i}+V_{s_i})\frac{\partial V_{s_j}}{\partial x_j}\right]^* + D_{10}\left[\phi\rho_f T_m^*\frac{\partial Z}{\partial x_i}\right]^* + D_{11}[\phi V_{r_i}]^* - \\
&\quad - D_{12}\left[\phi\left(\frac{\partial^2 V_{r_i}}{\partial x_j\partial x_j} + \frac{\partial^2 V_{s_i}}{\partial x_j\partial x_j} + \frac{\partial^2 V_{r_j}}{\partial x_i\partial x_j} + \frac{\partial^2 V_{s_j}}{\partial x_i\partial x_j}\right) + \right. \\
&\quad \left. + \frac{\phi\lambda_f''}{\mu_{f_c}}\left(\frac{\partial^2 V_{r_j}}{\partial x_i\partial x_j} + \frac{\partial^2 V_{s_j}}{\partial x_i\partial x_j}\right)\right]^* + \\
&\quad + D_{13}[\phi\rho_f|V_r|V_{r_i}]^* = 0,
\end{aligned} \tag{31}$$

where

$$D_1 = 1, \qquad D_2 = \frac{A_{T_f}}{A_p}\frac{t_c^P}{t_c^{t_f}},$$

$$D_3 = \phi_c A_P^{-1}\frac{t_c^P}{t_c^{V_f}}, \qquad D_4 = \phi_c A_P^{-1}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}},$$

$$D_5 = \phi_c A_P^{-1}(\mathrm{St})^{-1}\frac{t_c^P}{t_c^{V_r}}\left(1-\frac{V_{sc}}{V_{fc}}\right), \quad D_6 = \phi_c A_P^{-1}(\mathrm{St}^{V_f})^{-1}\mathrm{Eu}_f\frac{L_c^{V_f}}{L_c^P}\frac{t_c^P}{t_c^{V_f}},$$

$$D_7 = (\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^P}\frac{t_c^P}{t_c^{V_f}}, \qquad D_8 = \frac{A_{T_f}}{A_P}(\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^P}\frac{t_c^P}{t_c^{V_f}},$$

$$D_9 = A_P^{-1}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_s}}, \qquad D_{10} = \phi_c A_P^{-1}\,\mathrm{Fr}^{-2}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}},$$

$$D_{11} = \phi_c A_P^{-1}\mathrm{Re}^{-1}\mathrm{Da}^{-1/2}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}}\left(1-\frac{V_{sc}}{V_{fc}}\right),$$

$$D_{12} = \phi_c A_P^{-1}\mathrm{Re}^{-1}\mathrm{Da}^{1/2}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}},$$

$$D_{13} = \phi_c A_P^{-1}\mathrm{Da}^{-1/2}(\mathrm{St}^{V_f})^{-1}\left(1-\frac{V_c^s}{V_c^f}\right)^2\frac{t_c^P}{t_c^{V_f}}. \tag{32}$$

The momentum balance equation of the porous medium:

$$E_1\left[\phi\rho_f(V_{r_i}+V_{s_i})\frac{\partial P}{\partial t}\right]^* + E_2\left[\phi\rho_f(V_{r_i}+V_{s_i})\frac{\partial T_f}{\partial t}\right]^* +$$

$$+E_3\left[\phi\rho_f\frac{\partial}{\partial t}(V_{r_i}+V_{s_i})\right]^* +$$

$$+E_4\left[(1-\phi)\frac{\partial V_{s_i}}{\partial t}\right] + E_5\left[\phi\rho_f(V_{r_i}+V_{s_i})\frac{\partial V_{r_j}}{\partial x_j}\right]^* +$$

$$+E_6\left[\rho_f(V_{r_i}+V_{s_i})\frac{\partial V_{s_j}}{\partial x_j}\right]^* +$$

$$+E_7\left[\phi\rho_f(V_{r_i}+V_{s_i})(V_{r_j}+V_{s_j})\frac{\partial P}{\partial x_j}\right]^* +$$

$$+E_8\left[\phi\rho_f(V_{r_i}+V_{s_i})(V_{r_j}+V_{s_j})\frac{\partial T_f}{\partial x_j}\right]^* +$$

$$+E_9\left[\phi\rho_f(V_{r_j}+V_{s_j})\frac{\partial}{\partial x_j}(V_{r_i}+V_{s_i})\right]^* + E_{10}\left[(1-\phi)V_{s_j}\frac{\partial V_{s_i}}{\partial x_j}\right]^* -$$

$$-E_{11}\left[\frac{\partial^2 w_{s_j}}{\partial x_i \partial x_j} + \frac{\partial^2 w_{s_i}}{\partial x_j \partial x_j} + \frac{\lambda_s''}{\mu_s}\frac{\partial^2 w_{s_j}}{\partial x_i \partial x_j}\right]^* + E_{12}\left[\frac{\partial T_s}{\partial x_i}\right]^* -$$

$$-E_{13}\left[\frac{\partial^2 V_{r_i}}{\partial x_j \partial x_j} + \frac{\partial^2 V_{s_i}}{\partial x_j \partial x_j} + \left(1 + \frac{\lambda_f''}{\mu_{f_c}}\right)\left(\frac{\partial^2 V_{r_j}}{\partial x_i \partial x_j} + \frac{\partial^2 V_{s_j}}{\partial x_i \partial x_j}\right)\right]^* +$$

$$+E_{14}\left[\frac{\partial P}{\partial x_i}\right]^* + E_{15}\left[V_{s_i}\frac{\partial(1-\phi)}{\partial t}\right]^* +$$

$$+E_{16}\left[\phi\rho_f\frac{\partial Z}{\partial x_i}\right]^* + E_{17}\left[(1-\phi)\frac{\partial Z}{\partial x_i}\right]^* +$$

$$+E_{18}\left[(1-\phi)V_{s_i}\frac{\partial V_{s_j}}{\partial x_j}\right]^* = 0, \qquad (33)$$

where

$$E_1 = 1, \qquad E_2 = \frac{A_{T_f}}{A_P}\frac{t_c^P}{t_c^{T_f}},$$

$$E_3 = \phi_c A_P^{-1}\frac{t_c^P}{t_c^{V_f}}, \qquad E_4 = (1-\phi)A_P^{-1}\frac{\mathrm{Eu}_f \mathrm{St}^{V_s}}{\mathrm{Eu}_s \mathrm{St}^{V_f}}\frac{L_c^{V_f}}{L_c^{V_s}}\frac{t_c^P}{t_c^{V_f}},$$

$$E_5 = \phi_c A_P^{-1}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}}\frac{L_c^{V_f}}{L_c^{V_r}}\left(1 - \frac{V_{sc}}{V_{fc}}\right), \qquad E_6 = A_P^{-1}(\mathrm{St}^{V_s})\frac{t_c^P}{t_c^{V_s}},$$

$$E_7 = (\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^P}\frac{t_c^P}{T_c^{V_f}}, \qquad E_8 = \frac{A_{T_f}}{A_P}(\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^{T_f}}\frac{t_c^P}{t_c^{V_f}},$$

$$E_9 = \phi_c A_P^{-1}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}}, \qquad E_{10} = (1-\phi_c)A_P^{-1}\frac{\mathrm{Eu}_f}{\mathrm{Eu}_s}(\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^{V_s}}\frac{t_c^P}{t_c^{V_f}},$$

$$E_{11} = A_P^{-1}\varepsilon_c^2 F_V\frac{t_c^P}{t_c^{V_f}}, \qquad E_{12} = A_P^{-1}\varepsilon_c F_{VT}\frac{L_c^{w_s}}{L_c^{T_s}}\frac{t_c^P}{t_c^{V_f}},$$

$$E_{13} = A_P^{-1}\mathrm{Re}^{-1}\mathrm{Da}^{1/2}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}}, \qquad E_{14} = A_P^{-1}\mathrm{Eu}_f(\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^P}\frac{t_c^P}{t_c^{V_f}},$$

$$E_{15} = (1-\phi_c)A_P^{-1}\frac{\mathrm{Eu}_f \mathrm{St}^{V_s}}{\mathrm{Eu}_s \mathrm{St}^{V_f}}\frac{t_c^P t_c^{V_s}}{t_c^{\phi_c} t_c^{V_f}}, \qquad E_{16} = \phi_c A_P^{-1}\mathrm{Fr}^{-2}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}},$$

$$E_{17} = (1-\phi_c)A_P^{-1}\mathrm{Fr}^{-2}(\mathrm{St}^{V_f})^{-1}\frac{\rho_s}{\rho_c^f}\frac{t_c^P}{t_c^{V_f}}.$$

$$E_{18} = (1-\phi_c)A_P^{-1}\frac{\mathrm{Eu}_f}{\mathrm{Eu}_s}(\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^{V_s}}\frac{t_c^P}{t_c^{V_f}}. \tag{34}$$

The fluid's energy balance equation:

$$F_1\left[\phi\rho_f T_f\frac{\partial P}{\partial t}\right]^* + F_2\left[\phi\rho_f\frac{\partial T_f}{\partial t}\right]^* + F_3\left[\phi\rho_f T_f\frac{\partial T_f}{\partial t}\right]^* +$$

$$+F_4\left[\phi\rho_f T_f(V_{r_j}+V_{s_j})\frac{\partial T_f}{\partial x_j}\right]^* +$$

$$+F_5\left[\phi\rho_f(V_{r_j}+V_{s_j})\frac{\partial T_f}{\partial x_j}\right]^* + F_6\left[\phi\rho_f T_f\frac{\partial V_{r_j}}{\partial x_j}\right]^* + F_7\left[\rho_f T_f\frac{\partial V_{s_j}}{\partial x_j}\right]^* +$$

$$+F_8\left[\phi\rho_f T_f(V_{r_j}+V_{s_j})\frac{\partial P}{\partial x_j}\right]^* - F_{10}[T_s]^* +$$

$$+F_{11}\left[\phi T_f\left(\frac{\partial V_{r_j}}{\partial x_j}+\frac{\partial V_{s_j}}{\partial x_j}\right)\right]^* -$$

$$-F_{12}\left[\phi\frac{(V_{r_l}+V_{s_l})\frac{\partial}{\partial x_j}(V_{r_l}+V_{s_l})}{|\boldsymbol{V}_r+\boldsymbol{V}_s|}\rho_f T_f\frac{\partial P}{\partial x_j}\right]^* -$$

$$-F_{13}\left[\frac{(V_{r_l}+V_{s_l})\frac{\partial}{\partial x_j}(V_{r_l}+V_{s_l})}{|\boldsymbol{V}_r+\boldsymbol{V}_s|}\rho_f\frac{\partial T_f}{\partial x_j}\right]^* -$$

$$-F_{14}\left[\phi\frac{(V_{r_l}+V_{s_l})\frac{\partial}{\partial x_j}(V_{r_l}+V_{s_l})}{|\boldsymbol{V}_r+\boldsymbol{V}_s|}\rho_f T_f\frac{\partial T_f}{\partial x_j}\right]^* -$$

$$-F_{15}\left[\phi\rho_f|\boldsymbol{V}_n+\boldsymbol{V}_s|\left(\frac{\partial T_f}{\partial x_j}\right)^2\right]^* -$$

$$-F_{16}\left[\phi\rho_f T_f|\boldsymbol{V}_r+\boldsymbol{V}_s|\left(\frac{\partial T_f}{\partial x_j}\right)^2\right] - F_{17}\left[\phi\rho_f|\boldsymbol{V}_r+\boldsymbol{V}_s|\frac{\partial T_f}{\partial x_j}\frac{\partial P}{\partial x_j}\right]^* -$$

$$-F_{18}\left[\phi\rho_f T_f|\boldsymbol{V}_r+\boldsymbol{V}_s|\frac{\partial T_f}{\partial x_j}\frac{\partial P}{\partial x_j}\right]^* - F_{19}\left[\phi\rho_f T_f|\boldsymbol{V}_r+\boldsymbol{V}_s|\left(\frac{\partial P}{\partial x_j}\right)^2\right]^* -$$

$$-F_{20}\left[\phi\rho_f|\boldsymbol{V}_r+\boldsymbol{V}_s|\frac{\partial^2 T_f}{\partial x_j\partial x_j}\right]^* - F_{21}\left[\phi\rho_f t_f|\boldsymbol{V}_r+\boldsymbol{V}_s|\frac{\partial^2 T_f}{\partial x_j\partial x_j}\right]^* -$$

$$-F_{22}\left[\phi\rho_f T_f|\boldsymbol{V}_r+\boldsymbol{V}_s|\frac{\partial^2 P}{\partial x_j\partial x_j}\right]^* - F_{23}\left[\phi\frac{\partial^2 T_f}{\partial x_j^2}\right]^* +$$

$$+F_{24}[\phi\rho_f|\boldsymbol{V}_r|V_{r_j}V_{r_j}]^* = 0, \tag{35}$$

where

$$F_1 = 1, \qquad F_2 = \phi_c A_P^{-1}\frac{t_c^P}{t_c^{T_f}},$$

$$F_3 = \frac{A_{T_f}}{A_P}\frac{t_c^P}{t_c^{T_f}}, \qquad F_4 = \frac{A_{T_f}}{A_P}(\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^{t_f}}\frac{t_c^P}{t_c^{V_f}},$$

$$F_5 = \phi_c A_P^{-1}(\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^{T_f}}\frac{t_c^P}{t_c^{V_f}}, \qquad F_6 = \phi_c A_P^{-1}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}}\frac{L_c^{V_f}}{L_c^{V_r}}\left(1-\frac{V_{sc}}{V_{fc}}\right),$$

$$F_7 = A_P^{-1}(\mathrm{St}^{V_s})^{-1}\frac{t_c^P}{t_c^{V_s}}, \qquad F_8 = (\mathrm{St}^{V_f})^{-1}\frac{L_c^{V_f}}{L_c^P}\frac{t_c^P}{t_c^{V_f}},$$

$$F_9 = A_P^{-1}\mathrm{Nu}Fo_f\frac{T_c^s}{t_c^f}\frac{T_c^P}{t_c^{T_f}}, \qquad F_{10} = A_P^{-1}\mathrm{Nu}Fo_f\frac{t_c^P}{t_c^{T_f}},$$

$$F_{11} = \phi_c A_P^{-1}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}}\frac{\beta_T}{\rho_c^f C_f\beta_P}, \qquad F_{12} = \mathrm{Fo_{Dis}}\frac{L_c^{V_f}}{L_c^P}\frac{t_c^P}{t_c^{V_f}},$$

$$F_{13} = \phi_c A_P^{-1}\mathrm{Fo_{Dis}}\frac{L_c^{V_f}}{L_c^{T_f}}\frac{t_c^P}{t_c^{V_f}}, \qquad F_{14} = \frac{A_{T_f}}{A_P}\mathrm{Fo_{Dis}}\frac{L_c^{V_f}}{L_c^{T_f}}\frac{t_c^P}{t_c^{V_f}},$$

$$F_{15} = 2\frac{A_{T_f}}{A_P}\mathrm{Fo_{Dis}}\left(\frac{L_c^{V_f}}{L_c^{T_f}}\right)^2\frac{t_c^P}{t_c^{V_f}}, \qquad F_{16} = \frac{A_{T_f}^2}{\phi_c A_p}\mathrm{Fo_{Dis}}\left(\frac{L_c^{V_f}}{L_c^{T_f}}\right)^2\frac{t_c^P}{t_c^{V_f}},$$

$$F_{17} = 2\mathrm{Fo_{Dis}}\frac{\left(L_c^{V_f}\right)^2}{L_c^{T_f}L_c^P}, \qquad F_{18} = 2\frac{A_{t_f}}{\phi_c}\mathrm{Fo_{Dis}}\frac{\left(L_c^{V_f}\right)^2}{L_c^{T_f}L_c^P}\frac{t_c^P}{t_c^{V_f}},$$

$$F_{19} = \frac{A_P}{\phi_c}\mathrm{Fo_{Dis}}\left(\frac{L_c^{V_f}}{L_c^P}\right)^2\frac{t_c^P}{t_c^{V_f}}, \qquad F_{20} = \phi_c A_P^{-1}\mathrm{Fo_{Dis}}\left(\frac{L_c^{V_f}}{l_c^{T_f}}\right)^2\frac{t_c^P}{t_c^{V_f}},$$

$$F_{21} = \frac{A_{T_f}}{A_P}\mathrm{Fo_{Dis}}\left(\frac{L_c^{V_f}}{L_c^{T_f}}\right)^2\frac{t_c^P}{t_c^{V_f}}, \qquad F_{22} = \mathrm{Fo_{Dis}}\left(\frac{L_c^{V_f}}{L_c^P}\right)^2\frac{t_c^P}{t_c^{V_f}},$$

$$F_{23} = \phi_c A_P^{-1}\mathrm{Fo}_f\frac{t_c^P}{t_c^{T_f}},$$

$$F_{24} = \phi_c A_P^{-1}\mathrm{Eu}_f^{-1}\mathrm{Da}^{-1/2}(\mathrm{St}^{V_f})^{-1}\frac{t_c^P}{t_c^{V_f}}\frac{P_c}{\rho_c^f C_f T_c^f} \tag{36}$$

The solid's energy balance equation:

$$G_1\left[(1-\phi)\frac{\partial T_s}{\partial t}\right]^* + G_2\left[t_s\frac{\partial(1-\phi)}{\partial t}\right]^* + G_3\left[(1-\phi)V_{s_j}\frac{\partial T_s}{\partial x_j}\right]^* -$$

$$-G_4\left[(1-\phi)\frac{\partial^2 T_s}{\partial x_j^2}\right]^* + G_5[T_s]^* -$$

$$-G_6[T_f]^* + G_7\left[T_s\frac{\partial V_{s_j}}{\partial x_j}\right]^* + G_8\left[(1-\phi)T_s\frac{\partial V_{s_j}}{\partial x_j}\right]^* = 0, \tag{37}$$

where

$$G_1 = 1, \qquad G_2 = \frac{t_c^{T_s}}{t_c^{\phi_c}}, \qquad G_3 = (\mathrm{St}^{V_s})^{-1}\frac{L_c^{V_s}}{L_c^{T_s}}\frac{t_c^{t_s}}{t_c^{V_s}}, \qquad G_4 = \mathrm{Fo}_s,$$

$$G_5 = \frac{1}{(1-\phi_c)}\mathrm{Fo}_s\mathrm{Bi}, \qquad G_6 = \frac{1}{(1-\phi_c)}\frac{T_c^f}{t_c^s}\mathrm{Fo}_s\mathrm{Bi},$$

$$G_7 = \frac{1}{(1-\phi_c)}\mathrm{Fo}_s Q_c(\mathrm{St}^{V_s})^{-1}\frac{t_c^{T_s}}{t_c^{V_s}}, \qquad G_8 = (\mathrm{St}^{V_s})^{-1}\frac{t_c^{T_s}}{t_c^{V_s}}. \tag{38}$$

Next, we evaluate the relative magnitude of the terms appearing in the phase mass, momentum, and energy balance equations, following abrupt changes in both the pressure and the temperature of the fluid, at a point within the porous media. Actually, to allow the investigation of simultaneous effect of fluid's pressure and temperature, we assume that both are of the same order, (e.g., blast or shock waves).

4. Approximate Forms of Mass Momentum and Energy Balance Equations

To simplify the discussion, let us consider a fluid that is initially at rest. It should not affect the general methodology of determining the dominant forms of the balance equations at different evolution period.

Immediately following an abrupt change in both the pressure and the temperature, we determine the characteristic time of any intensive quantity. During a short period following a change, we may assume that the following time characteristic intervals are identical.

$$t_c^{V_r} = t_c^{V_s} = t_c^{P} = t_c^{T_f} = t_c^{T_s} = t_c^{\phi}. \tag{39}$$

Note, however, that after longer time periods, when inertia terms become smaller than those of dissipation, each of the e-quantities evolves at its characteristic pace. Hence, during a short time interval following the initiation of motion from rest, the average and characteristic velocities may be identical. They may be expressed in the form

$$V_{rc} = \frac{L_c^{V_r}}{t_c^{V_r}}, \qquad V_{sc} = \frac{L_c^{V_s}}{t_c^{V_s}}. \tag{40}$$

Note that in writing (40), we assume that $\mathrm{St}^e = 1(e \neq \phi)$. Actually, following (17), we note that $\varepsilon_{sk} = \varepsilon_{sk}(\phi)$. Yet, in view of (18), we define the relation between $\mathbf{w}_s$ and $\boldsymbol{V}_s$. Hence, we may conclude that L_c^{ϕ}, $L_c^{V_s}$ and w_c^s are related. However, to be consistent with the argumentation of choosing L_c^e related to each ∇_e, we choose to develop the dominant terms in the evolving balance equations on the basis of the above mentioned characteristic terms.

We now aim at constructing the various forms of the balance equations, considering the relative order of magnitude of $A_{mf}(mf = 1, 2, \ldots, 6)$, $B_{ms}(ms = 1, 2)$, $D_{Mf}(Mf = 1, 2, \ldots, 13)$, $E_{Mp}(Mp = 1, 2, \ldots, 18)$, $F_{Ef}(Ff = 1, 2, \ldots, 23)$ and $G_{Es}(Es = 1, 2, \ldots, 8)$ in (28), (30), (32), (34), (36) and (38), respectively. In assessing the magnitudes of these expressions, we consider the rates of change of fluid pressure and temperature to be of the same order of magnitude, e.g., blast or shock waves). This in view of (3), dictates the condition $\beta_p P_c = O(\beta_T T_c^f)$ and will manifest a unit order to the factor multiplying the fluid's temperature rate. Since we have assumed that the rate of changes of pressure and the fluid's temperature are of the same order of magnitude, it follows that A_p and A_{T_f} are also of the same order of magnitude

4.1. PERIOD OF UNIFORM DISTRIBUTIONS OF PRESSURE, STRESS AND TEMPERATURE

Immediately following an abrupt change in fluid's pressure and temperature, we may assume that

$$L_c^{V_r} \cong L_c^{P} \cong L_c^{T_f} \quad @ \quad t_c^{P} = t_c^{T_f} = 0^+. \tag{41}$$

In writing $t_c^P = t_c^{T_f} = 0^+$, we mean that the time distributions for the rise in fluid pressure and temperature are the same. During this period, we consider the nondimensional numbers A_P, A_{T_f}, Eu_α, F_{VT} and F_V to be of orders of magnitude greater than one. All other nondimensional numbers appearing in (25) and (26) ore of order one, except Fo_α which is of order zero. Accordingly, by virtue of (28) and (41) we obtain

$$A_1 \cong A_2 \cong A_5 \cong A_6 \cong \mathrm{O}(1), \tag{42}$$

while all the others A are of order zero. This, in turn, yields an approximate mass balance equation for the fluid in the form

$$\phi\rho\left(\beta_P\frac{\partial P}{\partial t} + \beta_T\frac{\partial T_f}{\partial t}\right) + \phi\rho_f(V_{r_j} + V_{s_j})\left(\beta_P\frac{\partial P}{\partial x_j} + \beta_T\frac{\partial T_f}{\partial x_j}\right) = 0, \tag{43}$$

This equation may be rewritten as

$$\phi\frac{D_f\rho_f}{Dt} = 0. \tag{44}$$

In view of (32) and (41) we find that

$$D_1 \cong D_2 \cong D_6 \cong D_7 \cong D_8 \cong \mathrm{O}(1), \tag{45}$$

other D factors are of order zero. Hence, if we subtract (45), multiplied by the fluid's velocity, from (31), and account for (45), we obtain the fluid's momentum balance equation in the form

$$\phi T_m^* \frac{\partial P}{\partial x_i} = 0. \tag{46}$$

By analyzing (34), in view of (41), we may conclude that

$$E_1 \cong E_2 \cong E_7 \cong E_8 \cong E_{11} \cong E_{12} \cong E_{14} \cong \mathrm{O}(1), \tag{47}$$

while the rest of the E factors are of zero order. If we now subtract (43), multiplied by the fluid's velocity, from (33), and also substitute (46) into (33), we obtain the porous media momentum balance equation, in view of (47), in the form

$$\nabla \cdot \sigma'_s = 0. \tag{48}$$

By virtue of (36) and (41), we write

$$F_1 \cong F_3 \cong F_4 \cong F_8 \cong F_{16} \cong F_{18} \cong F_{19} \cong \mathrm{O}(1), \tag{49}$$

while the rest of the F factors are of zero order. We now subtract (43), multiplied by $C_f T_f$, and substitute (46) in (35). We obtain the fluid's energy balance equation in the form

$$\phi \rho_f C_f T_f \alpha_T \beta_T^2 |\boldsymbol{V}_f + \boldsymbol{V}_s| \nabla T_f \cdot \nabla T_f = 0. \tag{50}$$

Since the factors in (46) and (50) are not zero, we use this result of constant spatial distributions of P and T_f, together with (43), and obtain

$$\beta_P \frac{\partial P}{\partial t} = -\beta_T \frac{\partial T_f}{\partial t} \tag{51}$$

or

$$\frac{\partial \rho_f}{\partial t} = 0. \tag{52}$$

Note that in view of (44) and (52), although the fluid is considered compressible, during this first time period it behaves as if it were incompressible, i.e.,

$$\rho_f = \text{const.} \quad @ \quad t = 0^+. \tag{53}$$

Examining (38) and in view of (41), we obtain

$$G_1 \cong G_2 \cong G_3 \cong G_8 \cong \mathrm{O}(1), \tag{54}$$

while all other G factors are of order zero. If we now subtract (4) (which all of its terms are of order one), multiplied by $C_s T_s$, we obtain the solid's energy balance equation in the form

$$(1 - \phi) \rho_s C_s \frac{D_s T_s}{Dt} = 0. \tag{55.1}$$

Hence, T_s will remain constant along characteristic curves defined by

$$\frac{D_s \boldsymbol{x}_s}{Dt} = \boldsymbol{V}_s, \tag{55.2}$$

where $\boldsymbol{x}_s$ is a position vector along the characteristic path associated with the solid matrix.

The pressure and the fluid temperature impulse propagate uniformly over a distance obtained from the factor D_6 in (32).

$$L_c^P \cong \mathrm{O}\left(\frac{t_c^P}{\rho_f \beta_P V_c^f} \right). \tag{56}$$

4.2. PERIOD OF NONLINEAR WAVE PROPAGATION

After the first time period, we let the order of magnitude of the Reynolds number, Re, be greater than one, while the remaining nondimensional numbers in (25) and (26) remain of order one. Consequently, following a procedure similar to that outlined in Section 4.1 above, by which the orders of magnitude of all the factors were determined, we conclude that the fluid's and the solid's mass balance equations, remain in the form (2) and (4), respectively.

The fluid's momentum balance equation, becomes

$$\phi\rho_f\frac{\partial}{\partial t}(V_{r_i}+V_{s_i})+\phi\rho_f(V_{r_j}+V_{s_j})\frac{\partial}{\partial x_j}(V_{r_i}+V_{s_i})+\phi T^*_m\frac{\partial P}{\partial x_i}+$$

$$+\phi\rho_f g\frac{\partial z}{\partial x_i}T^*_m+\sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}\phi\rho_f|\mathbf{V}_r|V_{r_i}=0. \tag{57}$$

To obtain the solid's momentum balance equation, we first consider the momentum balance of the porous medium as a whole, which was obtained under the 2nd period assumptions. The equation takes the form

$$\phi\rho_f\frac{\partial}{\partial t}(V_{r_i}+V_{s_i})+(1-\phi)\rho_s\frac{\partial V_{s_i}}{\partial t}+\phi\rho_f\beta_P(V_{r_i}+V_{s_i})\frac{\partial P}{\partial t}$$

$$+\phi\rho_f\beta_T(V_{r_i}+V_{s_i})\frac{\partial T_f}{\partial t}+$$

$$+\phi\rho_f\beta_T(V_{r_i}+V_{s_i})(V_{r_j}+V_{s_j})\frac{\partial T_f}{\partial x_j}+\phi\rho_f(V_{r_j}+V_{s_j})\frac{\partial}{\partial x_j}(V_{r_i}+V_{s_i})+$$

$$+(1-\phi)\rho_s V_{s_j}\frac{\partial V_{s_i}}{\partial x_j}-$$

$$-\lambda''_s\frac{\partial^2 w_{s_j}}{\partial x_i\partial x_j}-\mu'_s\left(\frac{\partial^2 w_{s_i}}{\partial x_j\partial x_j}+\frac{\partial^2 w_{s_j}}{\partial x_i\partial x_j}\right)+\eta\frac{\partial T_s}{\partial x_i}+$$

$$+\frac{\partial P}{\partial x_i}+V_{s_i}\rho_s\frac{\partial(1-\phi)}{\partial t}+$$

$$+(1-\phi)\rho_s V_{s_i}\frac{\partial V_{s_j}}{\partial x_j}+\phi\rho_f g\frac{\partial Z}{\partial x_i}+(1-\phi)\rho_s g\frac{\partial Z}{\partial x_i}=0. \tag{58}$$

We now subtract (57) from (58), while assuming that

$$|(1-\phi)\rho_s|\gg|\phi\rho_f(T^*_{sm}-T^*_{fm})|,$$

and obtain the momentum balance equation for the solid matrix, in the form

$$(1-\phi)\rho_s\frac{\partial V_{s_i}}{\partial t}+(1-\phi)\rho_s V_{s_j}\frac{\partial V_{s_i}}{\partial x_j}-\lambda_s''\frac{\partial^2 w_{s_j}}{\partial x_i\partial x_j}-$$

$$-\mu_s'\left(\frac{\partial^2 w_{s_i}}{\partial x_j\partial x_j}+\frac{\partial^2 x_{s_j}}{\partial x_i\partial x_j}\right)+\eta\frac{\partial T_s}{\partial x_i}+$$

$$+(1-\phi)T_{sm}^*\frac{\partial P}{\partial x_i}+(1-\phi)\rho_s g\frac{\partial Z}{\partial x_i}-\sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}\phi\rho_f|V_r|V_{r_i}=0. \qquad (59)$$

The fluid's energy balance equation, becomes

$$\phi\rho_f C_f T_f\beta_P\frac{\partial P}{\partial t}+\phi\rho_f C_f(1+\beta_T T_f)\frac{\partial T_f}{\partial t}+$$

$$+\phi\rho_f C_f(1+\beta_T T_f)(V_{r_j}+V_{s_j})\frac{\partial T_f}{\partial x_j}+$$

$$+\phi\rho_f C_f T_f\frac{\partial V_{r_j}}{\partial x_j}+\rho_f C_f T_f\frac{\partial V_{s_j}}{\partial x_j}+\phi\rho_f C_f\beta_P T_f(V_{r_j}+V_{s_j})\frac{\partial P}{\partial x_j}+$$

$$+\phi T_f\frac{\beta_T}{\beta_P}\left(\frac{\partial V_{r_j}}{\partial x_j}+\frac{\partial V_{s_j}}{\partial x_j}\right)+\sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}\phi\rho_f|V_r|V_{r_j}V_{r_j}=0. \qquad (60)$$

The solid's energy balance equation, becomes

$$(1-\phi)\rho_s C_s\frac{\partial T_s}{\partial t}+(1-\phi)\rho_s C_s V_{s_j}\frac{\partial T_s}{\partial x_j}+\eta T_s\frac{\partial V_{s_j}}{\partial x_j}=0. \qquad (61)$$

In view of the factors D_6 and D_{10} in (26), we may estimate the length of the time span during which Equations (57)–(61) are valid. We find

$$t_c^P\cong \mathrm{O}\left(\frac{\beta_P P_c V_c^f}{g}\right), \qquad (62)$$

and

$$\frac{L_c^P}{t_c^P}\cong \mathrm{O}\left(\frac{1}{\rho_c^f\beta_P V_c^f}\right). \qquad (63)$$

During this period of evolution, we note the emergence of nonlinear wave equations. Actually, the procedure for obtaining these forms is based on time differentiation of the appropriate balance equations in which only the dominant terms remain. By time differentiation of (2), and substitution of (57) into the resulting equation, we obtain a nonlinear wave equation for the specific fluid mass:

$$\frac{\partial^2 \phi\rho_f}{\partial t^2} - \left(V_{fi}V_{fj} + \frac{T^*_{ij}}{\rho_f\beta_P}\right)\frac{\partial^2 \phi\rho_f}{\partial x_i \partial x_j}$$

$$= \left(2\frac{\partial(V_{fj}V_{fi})}{\partial x_j} + gT^*_{ij}\frac{\partial Z}{\partial x_j} + \frac{\phi T^*_{ij}}{\rho_f^2\beta_P}\frac{\partial \rho_f}{\partial x_j} + \sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}|\boldsymbol{V}_r|V_{r_i}\right)\frac{\partial \phi\rho_f}{\partial x_i} +$$

$$+\frac{\phi\beta_T T^*_{ij}}{\beta_P}\frac{\partial^2 T_f}{\partial x_i \partial x_j} +$$

$$+\phi\rho_f\left(\left(\frac{\partial V_{f_j}}{\partial x_j}\right)^2 + 2V_{f_j}\frac{\partial^2 V_{f_i}}{\partial x_j \partial x_i} + \frac{\partial V_{f_i}}{\partial x_j}\frac{\partial V_{f_j}}{\partial x_i} +\right.$$

$$\left. + \sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}\left(V_{r_i}\frac{\partial|\boldsymbol{V}_r|}{\partial x_i} + |\boldsymbol{V}_r|\frac{\partial V_{r_i}}{\partial x_i}\right)\right). \tag{64}$$

By time differentiation of (57), and substituting of (2) into the resulting equation, we obtain a nonlinear wave equation for the specific fluid velocity:

$$\frac{\partial^2 V_{f_i}}{\partial t^2} - V_{fk}V_{fj}\frac{\partial^2 V_i}{\partial x_k \partial x_j}$$

$$= 2V_{fk}\frac{\partial V_{fj}}{\partial x_k}\frac{\partial V_{fi}}{\partial x_j} - \frac{T^*_{ij}}{\rho_f}\frac{\partial^2 P}{\partial t \partial x_j} + \frac{T^*_{ij}}{\rho_f^2}\frac{\partial P}{\partial x_j}\frac{\partial \rho_f}{\partial t} +$$

$$+ \left(\frac{T^*_{kj}}{\rho_f}\frac{\partial P}{\partial x_j} + gT^*_{kj}\frac{\partial Z}{\partial x_j}\right)\frac{\partial V_{f_i}}{\partial x_k} +$$

$$+V_{fk}\left(\frac{T^*_{ij}}{\rho_f}\frac{\partial^2 P}{\partial x_k \partial x_j} - \frac{T^*_{ij}}{\rho_f^2}\frac{\partial P}{\partial x_j}\frac{\partial \rho_f}{\partial x_k}\right) -$$

$$-\sqrt{\frac{c_v\alpha_m}{\Delta_f^2}}\left(|\boldsymbol{V}_r|\frac{\partial V_{r_i}}{\partial t} + V_{r_i}\frac{\partial|\boldsymbol{V}_r|}{\partial t} - V_{fk}|\boldsymbol{V}_r|\frac{\partial V_{r_i}}{\partial x_k} - \right.$$

$$\left. -V_{fk}V_{r_i}\frac{\partial|\boldsymbol{V}_r|}{\partial x_k}\right). \tag{65}$$

By time differentiation of (59) we obtain a nonlinear wave equation for the solid velocity:

$$\frac{\partial^2 V_{si}}{\partial t^2} - \frac{\mu'_s + \lambda''_s}{(1-\phi)\rho_s}\frac{\partial^2 V_{sj}}{\partial x_i \partial x_j} - \frac{\mu'_s}{(1-\phi)\rho_s}\frac{\partial^2 V_{si}}{\partial x_j \partial x_j}$$

$$= -\frac{\partial V_{sj}}{\partial t}\frac{\partial V_{si}}{\partial x_j} - V_{sj}\frac{\partial^2 V_{si}}{\partial x_j \partial t} - \frac{\eta}{(1-\phi)\rho_s}\frac{\partial^2 T_s}{\partial t \partial x_i} -$$

$$-\frac{T^*_{ij}}{(1-\phi)\rho_s}\frac{\partial^2 P}{\partial t \partial x_j} + \frac{1}{(1-\phi)\rho_s}\frac{\partial V_{sk}}{\partial x_k}\times$$

$$\times\left((\mu'_s + \lambda''_s)\frac{\partial^2 w_{s_j}}{\partial x_i \partial x_j} + \mu'_s\frac{\partial^2 w_{s_i}}{\partial x_j \partial x_j} - \eta\frac{\partial T_s}{\partial x_i}\right) +$$

$$+\sqrt{\frac{c_v \alpha_m}{\Delta_f^2}}\frac{1}{(1-\phi)\rho_s}\left(\phi\rho_f V_{r_i}\frac{\partial |V_r|}{\partial x_i} + |V_r|\frac{\partial \phi\rho_f V_{r_i}}{\partial x_i}\right). \tag{66}$$

Substituting (18) into (59), results in a nonlinear wave equation for the solid's displacement:

$$\frac{\partial^2 w_{si}}{\partial t^2} - \frac{\mu'_s + \lambda''_s}{(1-\phi)\rho_s}\frac{\partial^2 w_{sj}}{\partial x_i \partial x_j} - \frac{\mu'_s}{(1-\phi)\rho_s}\frac{\partial^2 w_{si}}{\partial x_j \partial x_j}$$

$$= \frac{\partial w_{sj}}{\partial t}\frac{\partial^2 w_{si}}{\partial t \partial x_j} - \frac{\eta}{(1-\phi)\rho_s}\frac{\partial T_s}{\partial x_i} -$$

$$-\frac{T^*_{ij}}{(1-\phi)\rho_s}\frac{\partial P}{\partial x_j} - g\frac{\partial Z}{\partial x_i} +$$

$$+\sqrt{\frac{c_v \alpha_m}{\Delta_f^2}}\frac{\phi\rho_f}{(1-\phi)\rho_s}|V_r|V_{r_i} \tag{67}$$

4.3. PERIOD CHARACTERIZED BY THE INERTIAL AND DISSIPATIVE TERMS

During the third evolution period we refer to the case when all the scalar factor A_{mf}, B_{ms}, D_{Mf}, E_{Mp}, F_{Ef} and G_{Es} in (28), (30), (32), (34), (36), and (38), respectively, are of the same order of magnitude. Hence, the balance equations remain as (4), (5), (11), (20), (23), and (24).

4.4. PERIOD DOMINATED BY THE DISSIPATIVE TERMS

In this case, the drag associated with the viscosity dominates the fluid motion, after all the inertial effects have subsided. The nondimensional coefficients are of the following order of magnitude:

$$L, \mathrm{Fo}, \mathrm{Nu}, \mathrm{Bi}, \mathrm{St} \cong \mathrm{O}(1), \tag{68}$$

$$\varepsilon_c, \mathrm{Da}, \mathrm{Re}, A_p, A_{T_f} \ll \mathrm{O}(1), \tag{69}$$

and

$$F_V, F_{VT}, \mathrm{Eu} \gg \mathrm{O}(1). \tag{70}$$

Now that inertia terms are smaller than those of dissipation, we also postulate that the time duration associated with the velocity change is much greater than the one associated with the changes of any other property, e, namely

$$\frac{t_c^e}{t_c^V} \ll 1. \tag{71}$$

Under assumptions (68) to (71), we obtain the following new balance equations: Fluid mass balance.

$$\phi\rho_f \left(\beta_P \frac{\partial P}{\partial t} + \beta_T \frac{\partial T_f}{\partial t}\right) + \phi\rho_f \frac{\partial V_{r_j}}{\partial x_j} + \rho_f \frac{\partial V_{s_j}}{\partial x_j} = 0. \tag{72}$$

Fluid momentum balance equation (Darcy's law).

$$\mu_f \alpha_m \frac{c_v}{\Delta_f^2} \phi V_r = -\phi T_m^* (\nabla P + \phi\rho_f g \nabla Z). \tag{73}$$

Momentum balance for the porous medium.

$$\nabla P + (1-\phi)\rho_s g \nabla Z - \nabla \cdot \boldsymbol{\sigma}_s' = 0. \tag{74}$$

Fluid energy balance equation.

$$\phi\rho_f C_f \frac{\partial T_f}{\partial t} - \phi\lambda_{fm} \nabla^2 T_f - \alpha^{*H}(T_s - T_f) = 0. \tag{75}$$

Solid energy balance equation.

$$(1-\phi)\rho_s C_s \frac{\partial T_f}{\partial t} - (1-\phi)\lambda_{sm} \nabla^2 T_s + \alpha^{*H}(T_s - T_f) = 0. \tag{76}$$

5. Conclusion

A nondimensionalization procedure is applied to the macroscopic mass, momentum and energy balance equations describing saturated flow in a thermoelastic porous medium. Considering the possibility of high and rapid changes of the phases velocities, the analogous to Forchheimer terms were developed as additional momentum and energy components at the material boundary between the phases. The evolution of the dominating terms in these balances, in response to an abrupt change in both the fluid's pressure and temperature, was then analyzed.

Four distinct evolution periods were found.

- At the very first instant, the fluid's density remains constant. This in turn, constitutes a linear relation between the fluid's pressure and temperature rates of change. The fluid's pressure and temperature and the solid's matrix effective stress result in uniform spatial distributions. The spatial distribution of the solid's temperature and it's time rate of change are related by the solid's velocity. This is governed by the material derivative of the solid's temperature, being equal to zero.
- During the second time interval, the inertial terms dominate, and we note the occurrence of nonlinear waves forms for the fluid mass, the fluid velocity, the solid velocity and the solid's displacement.
- During the third evolution stage, the dissipative effects in the balance equations arise and become of the same order of magnitude as the inertial terms.
- As time proceeds, during the fourth stage, we note the domination of the dissipative terms while the inertial ones had subsided. The fluid's momentum is dominated by Darcy's law. The momentum balance equation of the porous medium represents forces equilibrium and energy balances are described by parabolic equations.

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