UNSTEADY MAGNETOHYDRODYNAMIC FLOWS IN A ROTATING ELASTO-VISCOUS FLUID

N. NANOUSIS
Technological Educational Institute, Koukouli, Patras, Greece

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Abstract. The unsteady flow of an incompressible electrically-conducting and elasto-viscous fluid (Walter's liquid B'), filling the semi-infinite space, in contact with an infinite non-conducting plate, in a rotating medium and in the presence of a transverse magnetic field is investigated. An arbitrary time-dependent forcing effect on the motion of the plate is considered and the plate and fluid rotate uniformly as a rigid body. The solution of the problem is obtained with the help of the Laplace transform technique and the analytical expressions for the velocity field as well as for the skin-friction are given.

1. Introduction

The study of the flow of non-Newtonian visco-elastic fluids has considerably gained importance due to its practical application in various disciplines. The elasto-viscous boundary-layer flow, near a Stagnation point based on a system of constitutive equations for elasto-viscous fluids known as Walter's liquid B', standed by Beard and Walter (1964). Soundalgekar (1974) studied the flow of an elasto-viscous fluid past an infinite impermeable plate moving impulsively in its own plane. Singh (1984) studied the flow of Walter's B' liquid past an infinite accelerated porous flat plate. The MHD flow of an elasto-viscous fluid past a porous flat plate, was presented by Sapria *et al.* (1990). In this paper we consider an incompressible, electrically-conducting, and elasto-viscous fluid filling the semi-infinite space in contact with an infinite non-conducting plate. The fluid and the plate rotate as a rigid body with a uniform angular velocity Ω perpendicular to the plate, in the presence of a uniform magnetic field of magnitude $\mathbf{B}_0$. The plate is assumed to be moving on its own plane with arbitrary velocity $U_0 f(t)$, where U_0 is a constant velocity and $f(t)$ a non-dimensional function of the time t. The effects on the velocity field and skin-friction of the various dimensionless parameters entering into the problem are discussed with the help of graphs.

2. Formulation of the Problem

Let us consider the three-dimensional flow of an electrically-conducting incompressible and elasto-viscous fluid (Walter's liquid B') in contact with a solid infinite plate, filling the semi-infinite space $z > 0$. On this plate an arbitrary point has been chosen as the origin O of a Cartesian coordinate system, with axes Ox and Oy fixed on the plate and Oz normal to it. The fluid and the plate rotate uniformly as a rigid body about the z-axis with an angular velocity $\Omega > 0$ in the presence of a uniform magnetic field of strength B_0 in the positive z-direction. Initially the plate and the fluid are at rest. Suddenly the

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plate is moved with a time-dependent velocity $U_0 f(t)$ in its own plate along the x-axis. We assume that the amplitude of the forcing effect $f(t)$ is small enough to permit us linearize the hydromagnetic equations relative to the rotating frame. Within the framework of these assumptions the equations which govern the flow are

$$\frac{\partial \mathbf{v}}{\partial t} + 2\Omega \times \mathbf{v} = -\frac{1}{\rho}\nabla\phi + \nu\frac{\partial^2 v}{\partial z^2} - k\frac{\partial^3 v}{\partial z^2\,\partial t} - \sigma\frac{B_0^2 \mathbf{v}}{\rho}, \tag{1}$$

$$\nabla \cdot \mathbf{v} = 0, \tag{2}$$

where $\mathbf{v} = (u, v, w)$denotes the velocity field ρ, the density ν the kinematic viscosity, ϕ the effective pressure field, σ the electrical conductivity of the fluid, and k the elastic parameter of the fluid. The z-component of (1) shows that the perturbation ϕ to the centrifugal pressure is independent of z, and can be set equal to zero (see Acheson, 1975; or Puri and Kulshrestha, 1976). Let us define

$$q = u + iv, \quad \text{with } i = \sqrt{-1} \tag{3}$$

and introduce the following non-dimensional quantities:

$$\begin{aligned} z' &= z\left(\frac{U_0}{\nu^2}\right)^{1/3}, \quad & \mathbf{v}' &= \frac{\mathbf{v}}{(\nu U_0)^{1/3}}, \quad & \Omega' &= \Omega\left(\frac{\nu}{U_0}\right)^{1/3}, \\ t' &= t\left(\frac{U_0^2}{\nu}\right)^{1/3}, \quad & k' &= k\left(\frac{U_0}{\nu}\right)^{2/3}, \quad & m &= \frac{\sigma \mathbf{B}_0}{\rho}\left(\frac{\nu}{U_0}\right)^{1/3}. \end{aligned} \tag{4}$$

Then, after suppressing the primes, the equations relevant to the problem reduce to

$$\frac{\partial q}{\partial t} + 2i\Omega q = \frac{\partial^2 q}{\partial z^2} - k\frac{\partial^3 q}{\partial z^2\,\partial t} - mq. \tag{5}$$

The initial and boundary conditions are:

$$\begin{aligned} q(z, t) &= 0, \quad && \text{for all } t < 0, \\ q(0, t) &= f(t), \quad && \text{for all } t \geq 0, \\ q(\infty, t) &\to 0, \quad && \text{for all } t > 0. \end{aligned} \tag{6}$$

The exact solution of Equation (5), subject to the boundary condition (6), is the key to the study of the present problem.

3. Solution

We shall find solutions of Equation (5) under its boundary conditions (6) for the following particular value of the function

$$f(t) = (t/t_0)H(t), \tag{7}$$

where t_0 is a constant and $H(t)$ is the Heaviside unit step function. Following Beard and Walter's (1964) method we assume that the solution of Equation (5) is of the form

$$q = q_0 + kq_1 , \tag{8}$$

which is valid for small values of k. If we substitute (8) in Equation (5) and equating the coefficient of the same powers of k, neglecting k^2, we get

$$\frac{\partial q_0}{\partial t} + 2i\Omega q_0 = \frac{\partial^2 q_0}{\partial z^2} - mq_0 , \tag{9}$$

$$\frac{\partial q_1}{\partial t} + 2i\Omega q_1 = \frac{\partial^2 q_1}{\partial z^2} - \frac{\partial^3 q_0}{\partial z^2\, \partial t} - mq_1 ; \tag{10}$$

with the boundary conditions

$$\begin{aligned} &q(0) = 0 , && q_1 = 0 \quad \text{for all } z \text{ and } r \leq 0 , \\ &q(0) = f(t) , && q_1 = 0 \quad \text{at } z = 0 , \\ &q(0) = 0 , && q_1 = 0 \quad \text{as } z \to 0 . \end{aligned} \tag{11}$$

The solution of Equations (9) and (10) using the boundary conditions (11) by the Laplace transform technique is given by

$$\begin{aligned} q(z, t) = \frac{H(t)}{t_0} \Bigg[\!\!\Bigg[& \exp(-z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{ht}\right) + \exp(z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} + \sqrt{ht}\right)\Bigg] - \\ & - \frac{z}{4\sqrt{h}} \left[\exp(-z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{ht}\right) - \exp(z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} + \sqrt{ht}\right)\right] + \\ & + k\left\{\frac{z}{2}\left[\exp(-z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{ht}\right) + \exp(z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} + \sqrt{ht}\right)\right] - \right. \\ & - \frac{z^2}{4}\left[\exp(-z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{ht}\right) - \exp(z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} + \sqrt{ht}\right)\right] + \\ & + \frac{1}{2}\left[\exp(-z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{ht}\right) + \exp(z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} + \sqrt{ht}\right)\right] - \\ & - \frac{1}{2}\left[\exp(-2z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{2\sqrt{t}} - \sqrt{ht}\right) + \exp(2z\sqrt{h}) \operatorname{erfc}\left(\frac{z}{\sqrt{t}} + \sqrt{ht}\right)\right] + \\ & \left. + \frac{z^2}{2\sqrt{\pi t}} \exp\left[-\left(ht + \frac{z^2}{4t}\right)\right]\right\}\Bigg]\!\!\Bigg] , \end{aligned} \tag{12}$$

$$\tau_w = \frac{H(t)}{t_0}\left[\left[t\left[\sqrt{h}\,(\operatorname{erfc}\sqrt{ht}) - 1) - \frac{1}{\sqrt{\pi t}}\exp(-\sqrt{ht})\right] - \right.\right.$$

$$\left.\left. - \frac{1}{2\sqrt{h}}\,[1 - (\operatorname{erfc}\sqrt{ht})] + k\left\{1 - [\sqrt{h}\,(\operatorname{erfc}\sqrt{ht}) - 1] - \frac{1}{\sqrt{\pi t}}\exp(-ht)\right\}\right]\right]; \tag{13}$$

where $h = m + 2i\Omega$.

4. Results and Discussion

For the purpose of discussing the results, numerical calculations are carried out for the velocity field, for different values of m, Ω, and k, respectively. Figure 1 shows that for a constant value of t, m, and k the velocity at any point increases as the value of non-Newtonian parameter k increases while it decreases with an increase in m, when t, Ω, and k are constants. It can also be seen that the velocity increases as Ω increases for fixed values of t, m, and k. Figure 2 shows the variation of the skin-friction τ_w for different values of t, Ω, m, and k. The shearing stress decreases with the increase in t

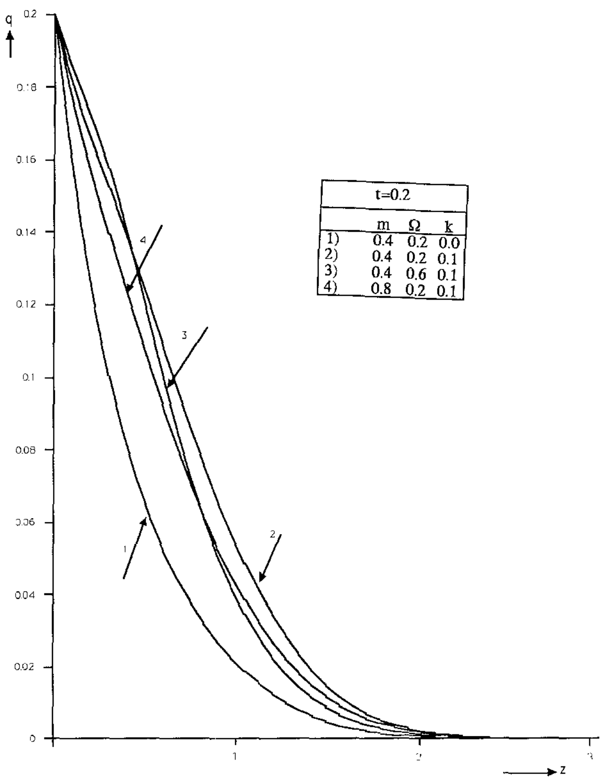

Fig. 1. Velocity profiles for different values of m, Ω, and k.

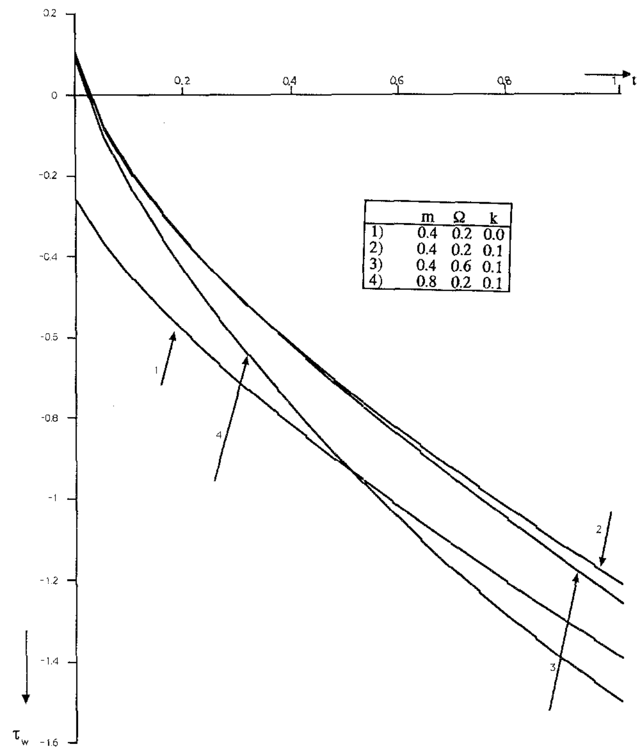

Fig. 2. Variation of the skin-friction for different values of m, Ω, and k.

and k for fixed values of m and Ω. It is also seen that shearing stress increases as the values of m or Ω increases.

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