A MODEL OF FAILURE OF A COMPOSITE PIPE IN COMPRESSION

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1. It is well known that failures of composite bodies subjected to uniaxial compressive loading along fibers often take place as a result of delamination. Various models of failure of solids in compression by means of delamination were constructed (see, for example [1-8]). However, because of the existence of a large number of failure mechanisms, on the one hand, and a large number of situations which can arise in practice, on the other hand, it is essential to examine the problem further.

Experimental results obtained for the failure of boron—aluminum pipes in compressive loading (it is expected that these results will be published slightly later) are not described qualitatively by the existing calculation method [2, 3]. The experiments were characterized by the strong dependence of the fracture load on the length of the pipe, by a large scatter of fracture loads, and by the sensitivity of the pipe to the defects formed in the manufacturing process.

Because of the high labor content of each separate test on a physically real pipe combined with the large number of design and structural parameters which greatly affect the results, the authors believe that it is essential to construct a model of the behavior of the compressed pipes suitable for analysis by computing experiments. Such a model is constructed in this work; the model is based on the existence and "evolution" of a system of technological (manufacturing) or service defects of the quasicrack type. The energy criterion is used for describing the splitting of unidirectionally reinforced pipes with buckling of strips formed in this phenomenon and by their growth leading to failure whose form resembles the "chinese lantern" (Fig. 1). The computing experiments were carried out to determine the mean values and variances of the main parameters which determine the defectiveness of the pipe and which ensure agreement within the calculations of physical experiments.

2. Initially, we examine a secondary, simpler problem of separation, buckling, and growth of a strip in a plate subjected to compressive loading. Similar problems (buckling and growth of the strip in compressive loading of a composite rod) were examined several times in the past [4, 5] using in most cases the beam approximation [9]. In addition to this, more exact approaches have also been used to determine the moment of the loss of stability

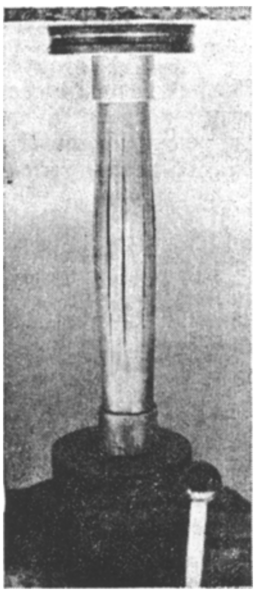

Fig. 1. Failure of a composite pipe in the form of "chinese lantern,"

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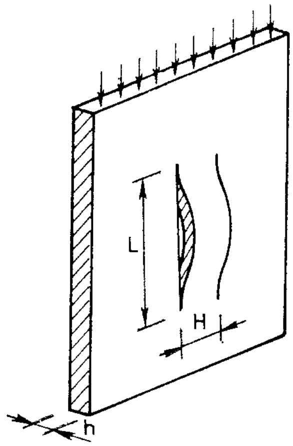

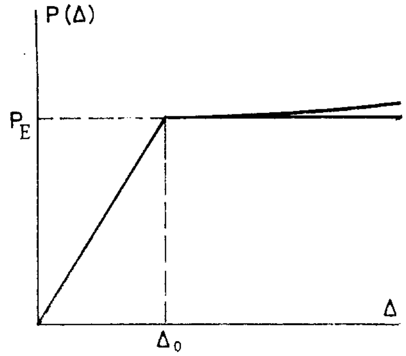

Fig. 2   Fig. 3

Fig. 2. Delamination process.

Fig. 3. Dependence of the load P carried by the compressed bar on the relative displacement Δ of its ends.

of compressed elements with delamination-type defects [6, 7]. In this work, we use an approximate approach which results in the full acceptable accuracy in this case. We assume that the strip is formed by two parallel continuous cracks with the length L. The distance between the cracks is H (Fig. 2). The compressive stresses $\sigma_y = -\sigma$ are applied at infinity; the thickness of the plate is h. The stress $\sigma = \sigma_E$ at which the strip loses its stability according to Euler (it is assumed that the foundation of the edges of the plate itself prevents its loss of stability), i.e.,

$$\sigma_E = \frac{\pi^2 EI}{S(\beta L)^2}.$$

Here E is Young's modules; $I = (Hh^3)/12$ is the moment of inertia of the cross section; $S = hH$ is the cross-sectional area of the strip. The coefficient β is determined by the method of securing the ends. Taking into account the fact that the results presented below are used for the case of buckling of a similar strip in the pipe, we assume that $\beta = 1/2$, i.e., we assume that the ends are fixed almost rigidly. Thus,

$$\sigma_E = \frac{4\pi^2 EI}{SL^2}. \tag{1}$$

We examined the behavior of the strip at $\sigma > \sigma_E$. It is evident that in the presence of a small deviation from the rectilinear form, the strips carries the same load $P = \sigma_E S$ and the displacement of its ends do not change (at the loss of stability the external forces do not carry out any work above the system). It is assumed that the difference of the displacements of the ends of the strip Δ is determined only by the external forces

$$\Delta = (\sigma/E)L.$$

Consequently, following Euler's analysis [10] for determining the coordinates of the points of the bent axis of the strip described in the Appendix, we obtain, at $\Delta \ll L$, the relationship between the force P acting on the strip and the displacement of its ends Δ in the form

$$P(\Delta) = \begin{cases} E\dfrac{\Delta}{L}S; & \Delta \leqslant \Delta_0; \\ \dfrac{64\pi^2 EI}{(4L+\Delta_0-\Delta)^2}; & \Delta > \Delta_0, \end{cases} \tag{2}$$

where

$$\Delta_0 = \frac{4\pi^2 I}{SL}.$$

The graph of the $P(\Delta)$ dependence is shown schematically in Fig. 3. Since, in the load variation range in which we are interested the value $\Delta(\sigma) \ll L$, the function (2) can be approximated

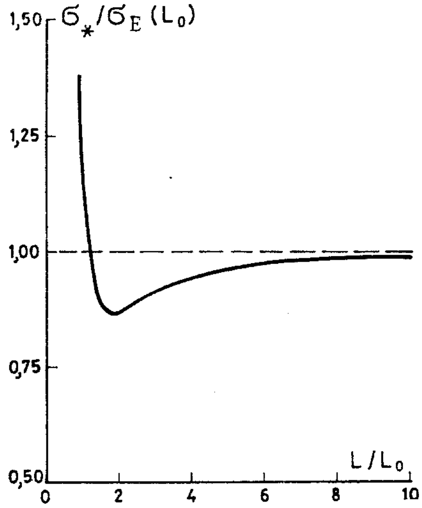

Fig. 4. Dependence of the critical separation stresses σ_* on the length L of the buckled strip.

by the dependence of the type of the deformation curve of an ideally plastic solid

$$P(\Delta) = \begin{cases} E\dfrac{\Delta}{L}S; & \Delta \leqslant \Delta_0; \\ P_E; & \Delta > \Delta_0 \end{cases}$$

or

$$P(\sigma) = \begin{cases} \sigma S; & \sigma \leqslant \sigma_E; \\ P_E; & \sigma > \sigma_E. \end{cases} \tag{3}$$

The dependence (3) will be used in subsequent considerations.

With increasing external loads the length L of the strip which has lost its stability (in Euler's sense) can increase as a result of the extension of the cracks which form this strip. In this case, we shall discuss the loss of stability according to Griffiths. Immediatedly after buckling the elastic energy U of the system does not change (as already mentioned) and the bending energy coincides with the elastic energy of the compressed strip: $U_0(L) = 1/2\, P_E\Delta_0$. With the increase of the external load as a result of the nonlinearity of the P–Δ diagram (Fig. 3), the variation of the elastic energy $\Delta U(L, \sigma)$ of the system is equal to

$$\Delta U = -U_0(L) - P_E(L)(\Delta - \Delta_0(L)) + 1/2\sigma S\Delta, \tag{4}$$

where $\Delta = (\sigma/E)L$. The stability condition of the strip according to Griffith can be given in the form

$$\delta A \leqslant \delta\Delta U + 4h\gamma\delta L, \tag{5}$$

where δA is the work of the external forces when the length of the strip increases by δL; γ is the effective surface energy of the material. The thermal losses are ignored. The forces specify that the boundary (the value σ does not change in variation in respect of L), we have $\delta A = 2\delta\Delta U$, and the condition (5) is transformed to the form

$$\frac{\partial \Delta U}{\partial L} \leqslant 4h\gamma,$$

and, after substitution into (4), we have

$$\frac{\sigma^2 S}{2E} + \frac{4\pi^2 I\sigma}{L^2} - \frac{24\pi^4 EI^2}{SL^4} \leqslant 4h\gamma. \tag{6}$$

The $\sigma = \sigma_E$ the left-hand part of Eq. (6) is equal to zero and the stability condition is satisfied. Solving the equality (6) in relation to σ, we determine the critical stress σ_* at which the length L of the strip can increase

$$\sigma_*(L) = \sigma_E(L)\left(2\sqrt{1 + \frac{2\gamma E}{\sigma_E^2(L)}\frac{h}{S}} - 1\right). \tag{7}$$

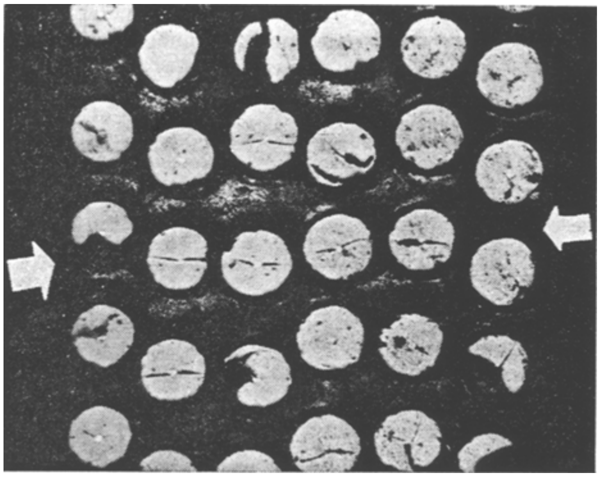

Fig. 5. Quasicracks (indicated by arrows) in the wall of Al—B composite pipe.

The $\sigma_*(L)$ dependence is shown in Fig. 4, where $L_0 = \pi I^{1/2}\left(\frac{2E}{\gamma hS}\right)^{1/4}$ is the characteristic length. If $L_S \le L_0$ (L_S is the initial length), the strip will propagate in an unstable manner up to the boundaries of the plate when the stresses reach the critical value of σ_*. At the initial length $L_0 < L_S < L_0\sqrt{3}$ the growth of the strip can be delayed only if its length increases to some value $L \ge L_0$. Finally, at $L_S \ge L_0\sqrt{3}$ the growth of the strip is stable with increasing external load up to the stress

$$\sigma_* = \sigma_E(L_0) = 2\sqrt{\frac{2\gamma Eh}{S}}.$$

We examine a case in which the strip is formed by blind cracks. We introduce the dimensionless connection parameter κ which is such that at $\kappa = 0$ we can examine the previously discussed case of continuous cracks, and at $\kappa = 1$ the cracks do not form. The defect characterized by $\kappa > 0$ will be referred to as a quasicrack.

If the energy losses in the process of formation of new surface in buckling of the strip are expressed by $4Lh\kappa\gamma$, the value σ^*_E obtained from the energy balance equation has the form

$$\sigma^*_E = \frac{4\pi^2 EI}{SL^2} + \sqrt{\frac{8h\kappa\gamma E}{S}}. \tag{8}$$

AT $\kappa = 0$ Eq. (8) is the same as Eq. (1). The stability condition (6) remains valid. The substitution of (8) into (6) shows that after buckling the strip remains stable only if its length satisfies the inequality

$$L^2 \geqslant \frac{\pi^2 I}{1-\kappa}\sqrt{\frac{32\kappa E}{\gamma hS}}.$$

Consequently, Eq. (7) remains valid and the quantity $\sigma_E(L)$ in this equation is determined by Eq. (1) as previously.

3. To construct the model of failure in axial compression ($h \ll R$) of a longitudinal reinforced pipe we accept the following assumption based on experimental observations. We assume that the pipe contained a system of defects of the quasicrack type (see above) formed as a result of technological (manufacturing) process, as in service (heat changes, various types of corrosion, etc.). The nature of quasicracks can differ: fiber laying defects, separation at the fiber—matrix boundary, a chain of cracked fibers, longitudinal cracks in the matrix. Figure 5 shows the photograph of the cross section of part of the wall of the boron-reinforced aluminum pipe. Chains of longitudinal cracks (indicated by the arrows) in the fibers oriented in the radial direction are clearly visible. The length of such a quasicrack in the axial direction can reach large values and the transverse strength of the defective area of the wall can be very low.

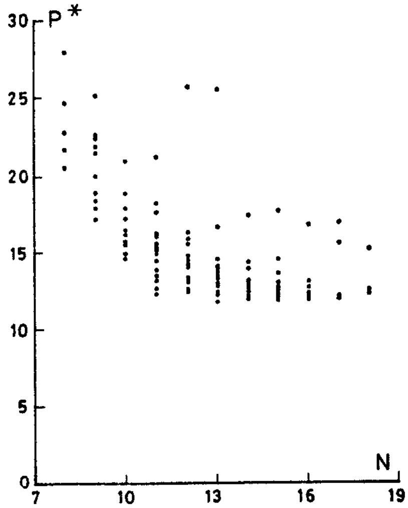

Fig. 6

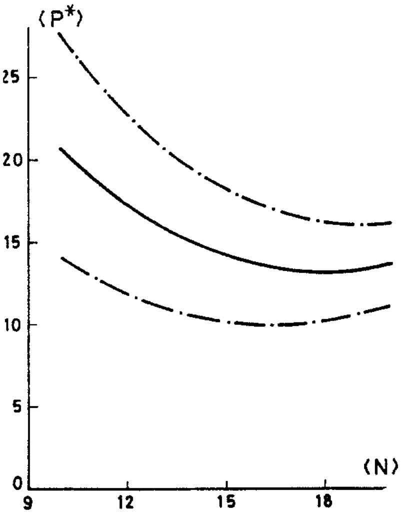

Fig. 7

Fig. 6. Dependences of strength P^* on the number of strips N in the computing experiments and $\langle N\rangle = 13$, $N_{max} = 25$, $N_{min} = 0$, $\langle \ell\rangle = 75$ mm, $\beta_\ell = 24$, $\langle z\rangle = 500$ mm, $\beta_z = 16$, $\beta_\varphi = 0.1$, $\gamma = 0.3$ kgf/mm, $\kappa = 0.5$, $L = 1000$ mm.

Fig. 7. Dependences of $\langle P\rangle^*$ on the mean number of the strips $\langle N\rangle$. The remaining parameters are the same as in Fig. 6.

The distribution of the quasicracks is random and the strength of each pipe is also a random quantity. Consequently, the quantitative model of failure must be statistical. In this connection, all the parameters are divided into two groups, determinate and statistical. The first group includes the pipe length L, wall thickness h, diameter 2R, Young' modules in the direction of the pipe axis E_z, the effective surface energy γ which characterizes the resistance to longitudinal cracks. The second group of the parameters characterizes the defects; these parameters are random quantities: the number of quasicracks N, the length of th j-th quasicrack ℓ_j, the coordinates of the center z_j and φ_j with the inequalities

$$l_j/2<z_j<L-l_j/2;\ \varphi_1<\varphi_2<\ldots<\varphi_{N-1}<\varphi_N, \tag{9}$$

satisfy, and the connection parameter κ_j introduced in point 2.

The function of the distribution density $\chi(N)$ of the discrete quantity N, i.e., the number of quasicracks, is approximated by the binomial distribution

$$\chi(N)=\binom{N_{\max}-N_{\min}}{N-N_{\min}}\left(\frac{\langle N\rangle-N_{\min}}{N_{\max}-N_{\min}}\right)^{N-N_{\min}}\left(\frac{N_{\max}-\langle N\rangle}{N_{\max}-N_{\min}}\right)^{N_{\max}-N},$$

where N_{min}, N_{max}, $\langle N\rangle$ is respectively, the minimum, maximum, and mean number of strips in the pipe; the multiplier $\binom{N_{\max}-N_{\min}}{N-N_{\min}}=\frac{(N_{\max}-N_{\min})!}{(N-N_{\min})!\,(N_{\max}-N)!}$ is the binomial coefficient; the variance D_N is calculated from the equation

$$D_N=\langle N\rangle\frac{N_{\max}-\langle N\rangle}{N_{\max}-N_{\min}}.$$

The distribution of the angular coordinate of the j-th quasicrack is specified in the form

$$\varphi_j=\frac{2\pi}{N}(j+\chi\beta_\varphi),$$

where N is the number of quasicracks formed in the given numerical experiment; χ is a random quantity which is uniformly distributed in the section [−0.5, 0.5]. The constant β_φ characterizes the variance. To satisfy the second inequality in (9) we must have $0 \le \beta_\varphi \le 1$.

The length ℓ_j is governed by Weibull's distribution with the density $f(\ell|\beta_\ell, \langle\ell\rangle)$. The function f(x) has the form

$$f(x|\beta,\langle x\rangle)=\alpha\beta x^{\beta-1}\exp(-\alpha x^{\beta}), \tag{10}$$

where

$$\alpha=\left(\frac{\Gamma\left(1+\frac{1}{\beta}\right)}{\langle x\rangle}\right)^{\beta};\quad x>0;\quad \beta>0.$$

The quantity β characterizes the variance D_x:

$$D_x=\langle x\rangle\sqrt{\frac{\Gamma\left(1+\frac{2}{\beta}\right)}{\Gamma^2\left(1+\frac{1}{\beta}\right)}-1}.$$

The distribution of the centers of the quasicracks can be selected as having either the form of Eq. (10) with the parameters $\langle z\rangle$, β_z, or as uniform along the length of the pipe. If in the calculation experiments $\ell_j > L$ (or $z_j \bar{\in} (\ell_j/2, L - \ell_j/2)$, we redetermine ℓ_j (or z_j and ℓ_j).

The connection parameter κ_j for each j-th quasicrack is specified by the uniform distribution of the random quantity in the section $[\langle\kappa\rangle - \beta_\kappa, \langle\kappa\rangle + \beta_\kappa]$ with an allowance made for obvious restrictions: $0 \le \kappa_j < 1$. In the partial case $\beta_\kappa = 0$ and the uniform distribution is reduced to the determinate case $\kappa_j = \langle\kappa\rangle$ $(j = \overline{1, N})$.

We assume that two adjacent quasicracks form a strip. In stressing the pipe the strip can lose stability according to Euler and then increase its length when Griffith's condition is satisfied.

The algorithm of loading in the computing experiments has the following form. For each j-th strip we determined the external load P_j at which the loss of stability according to Euler can take place. We solved the minimum $P_k = \min_j P_j$ from this set of the loads. The pipe is loaded to the load P_k and the k-th strip is transferred to the buckled state. Subsequently, we verify with the Griffith condition for the k-th strip is satisfied and determine its new length L_k, if this condition is satisfied. The configuration of the entire system changes, the stresses are recalculated (and are possibly the lengths of the adjacent strips L_{k-1}, L_{k+1}), in the process of calculating the set of the external loads is repeated. If for any nonbuckled s-th strip the critical external stresses $\sigma_E(L_S)$ are higher than the compressive strength for the material σ_*, the value of σ_* is assumed to be critical for the s-th strip. The strips which lose stability according to Euler can subsequently grow only if specific conditions are fulfilled; in this case the load in each such strip can only be increased. The loading process of the pipe continues until all the strips lose the stability according to Euler. This condition of the pipe is regarded as critical.

4. We shall describe the operation of this model in a series of computing experiments carried out to determine the dependence of the strength of the pipe P^* on several parameters. In these experiments we "tested" pipes with the following characteristics: L = 1000 mm, R = 30 mm, h = 1 mm, E_z = 24,000 kgf/m², σ_* = 150 kgf/m². ~10² computing experiments were carried out for each selected set of the statistical parameters.

Typical example is shown in Fig. 6 which shows the dependence of P^* (in tonnes) of the number of strips N without averaging. This dependence is in satisfactory qualitative agreement with the real physical experiment. Both the $P^*(N)$ dependence and the averaged dependence $\langle P^*\rangle(\langle N\rangle)$ shown in Fig. 7 contain a minima. This indicates that the specific number of the strips N_0 is advantageous from the energy viewpoint; this conclusion was obtained previously [3] within the limits of the "deterministic" model. The dot-and-dash curves in Fig. 7-11 characterize the rms deviation of the strength P^* from the mean value $\langle P^*\rangle$ (——).

Within the limits of the proposed model we can derive the dependence of the compressive strength of the pipe on its length L (Fig. 8). It is evident that the strong dependence of $\langle P^*\rangle$ on the pipe length can be qualitatively explained by the scale effect. The required dependence can be obtained by, for example, the following method: instead of specifying the mean value of the number of cracks $\langle N\rangle$ along the length of the pipe L, we specified the mean unit density of the cracks: $\rho = \ell_\Sigma/(2\pi RL)$, where ℓ_Σ is the total length of the quasicracks. Thus, $\langle N\rangle = (2\pi RL\rho)/\langle\ell\rangle$. In this case, the centers of the quasicracks are distributed uniformly along the z axis and not in accordance with the law of the type of (10).

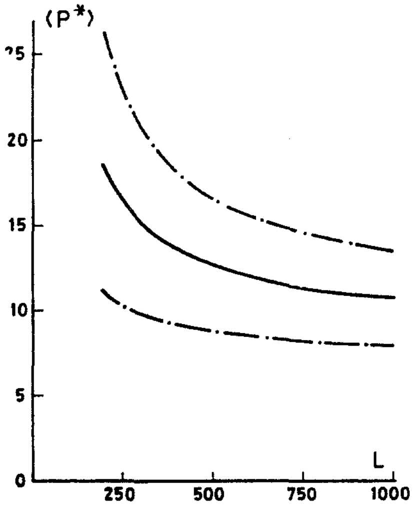

Fig. 8

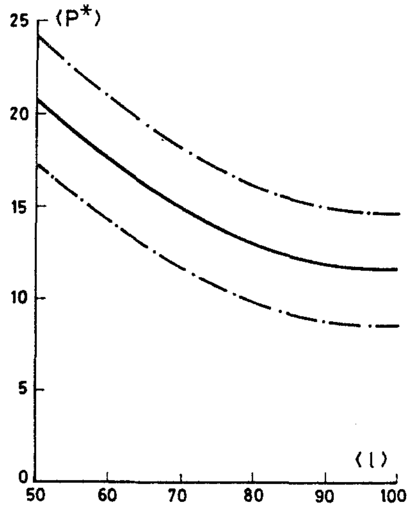

Fig. 9

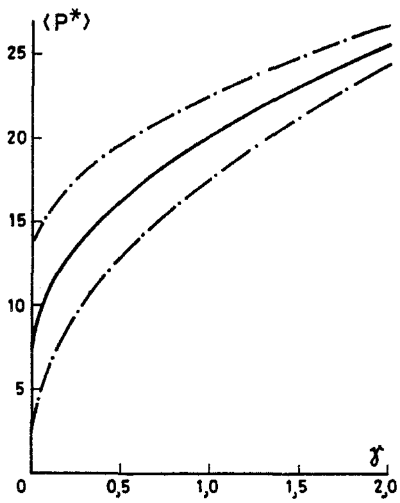

Fig. 10

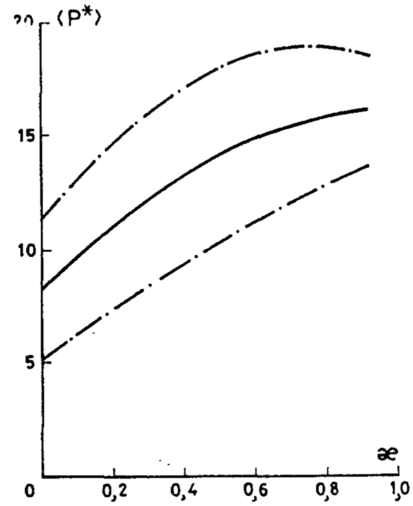

Fig. 11

Fig. 8. Dependences of $\langle P\rangle^*$ on the length of the pipe L at $N = 13$, $\langle \ell \rangle = 75$ mm, $\beta_\ell = 2$, $\langle z\rangle = 100$ mm, $\beta_z = 2$, $\lambda = 200$ mm, $\beta_\varphi = 0.1$, $\gamma = 0.3$ kgf/mm, $\kappa = 0.5$.

Fig. 9. Dependences of $\langle P\rangle^*$ on the mean length of the quasicrack $\langle \ell \rangle$ at $N = 13$, $\beta_\ell = 24$, $\langle z\rangle = 500$ mm, $\beta_z = 16$, $\beta_\varphi = 0.1$, $\gamma = 0.3$ kgf/mm, $\kappa = 0.5$, $L = 1000$ mm.

Fig. 10. Dependences of $\langle P\rangle^*$ on the effective surface energy γ at $N = 13$, $\langle \ell \rangle = 75$ mm, $\beta_\ell = 24$, $\langle z\rangle = 500$ mm, $\beta_z = 16$, $\beta_\varphi = 0.1$, $\kappa = 0.5$, $L = 1000$ mm.

Fig. 11. Dependences of $\langle P\rangle^*$ on the connection parameter κ at $N = 13$, $\langle \ell \rangle = 75$ mm, $\beta_\ell = 24$, $\langle z\rangle = 500$ mm, $\beta_z = 16$, $\beta_\varphi = 0.1$, $\gamma = 0.3$ kgf/mm, $L = 1000$ mm.

A simple variant used in this work and described later is based on introducing the characteristic dimension of the structural element of the pipe λ. We examined a pipe with the length L as consisting of k structural elements with the length λ: $L = k\lambda$. For each structural element we specified its own system of quasicracks with the same statistical parameters. We assume that the process of failure in each structural element takes place independently. The pipe is assumed to be failed if at least one of the structural elements is completely fractured. The characteristic dimensional structure element is selected on the basis of the experimental dependences $P^*(L)$. It is evident that the $\langle P^*\rangle$—L dependence becomes stronger with increasing variance of the distribution parameters.

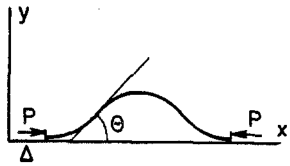

Fig. 12. Explanation is given in text.

It should be mentioned that the smallest length should be sufficiently large to eliminate the influence of the edge effect and failure by separation in the loss of stability of the system of fibers [8]. In addition to this, the value of L should be sufficiently small to ensure that the pipe as a whole does not lose the stability according to Euler.

Figures 9-11 show the dependences of the strength $\langle P^* \rangle$ on the mean length of the quasi-crack $\langle \ell \rangle$, the effective surface energy γ, and the connection parameter κ which are interesting from the viewpoint of the effect of technology and structure of the material on the strength of the structural member.

5. Thus, we propose the statistical model of failure of the composite pipes in axial compression which is suitable for numerical realization and computing experiments. The calculations are in satisfactory qualitative agreement with the physical experiments. The detected dependences of the strength of the pipe (both of the mean values and of the scatter of the data) on the internal parameters making it possible to select a new approach to optimizing the technological conditions.

6. APPENDIX

We determine the connection between the load P on a rod with the length L which loses its stability according to Euler, and the relative displacement of its ends Δ. The length of the arc of the median line of the rod s, the curvature of this line $\kappa = d\theta/ds$, where θ is the angle between the tangent to the bent axis of the rod and the axis passing through its ends (Fig. 12). The ends of the rod are in the rigid restraint conditions

$$\theta=0; \quad s=0; \quad L/2; \quad L. \tag{11}$$

For hinged support the coordinates of the points of the bent axis of the rod determined on the basis of Euler's analysis were published in [10]. Following this method, we write the bending equation in a form $d\theta/ds = -k^2 y$, where $k^2 = P/(EI)$. Differentiating this equation in respect of the s and taking into account that $dy/ds = -\sin\theta$, we obtain after simple transformation

$$\frac{1}{2}\frac{d}{d\theta}\left(\frac{d\theta}{ds}\right)^2 = -k^2 \sin\theta.$$

Integrating this equation we can easily obtain that

$$\left(\frac{d\theta}{ds}\right)^2 = 4k^2\left(\sin^2\frac{\theta_0}{2} - \sin^2\frac{\theta}{2}\right), \tag{12}$$

where the integration constant is denoted as $4k^2 \sin^2 \theta_0/2$. It can be seen that θ_0 is the maximum possible value of the angle θ which is obtained at the inflection points $\left(\frac{d^2\theta}{ds^2}=0\right)$, and namely, at $s = L/4, 3L/4$. Since $|\theta| \le \theta_0$, we replace the variables

$$\sin\frac{\theta}{2} = \sin\frac{\theta_0}{2}\sin\varphi,$$

where $0 \le \varphi \le \pi$ for $0 \le s < L/2$. Consequently, Eq. (12) can be written in the form

$$ds = -\frac{1}{k}\frac{d\varphi}{\sqrt{1-\sin^2\frac{\theta_0}{2}\sin^2\varphi}}.$$

Integration of the last relation with an allowance made for the boundary conditions (11) gives

$$s=\frac{1}{k}\int_0^{\varphi}\frac{d\varphi}{\sqrt{1-m^2\sin^2\varphi}}=\frac{1}{k}F(\varphi|m),$$

where $F(\varphi|m)$ is the elliptical integral of the first kind with a parameter $m = \sin^2\theta_0/2$. This parameter is determined from the condition $s = L/4$ at $\varphi = \pi/2$:

$$F(\pi/2|m)=k(L/4). \tag{13}$$

After determining the quantity m from Eq. (13) we can easily determine the relative displacement of the ends of the rod using the equation

$$\Delta=\Delta_0+\int_0^L\sqrt{1-(dy/ds)^2}ds-L,$$

where Δ_0 is the relative displacement of the ends prior to the loss of stability ($dy/ds = 0$). Since we are interested in the relative displacements Δ which are small in comparison with L (this corresponds to $|dy/ds| \ll 1$), we can expand the subintegral radical into a series and, after replacing the variable $ds \approx -\frac{1}{k}d\varphi$, we obtain

$$\Delta=\Delta_0+\frac{2}{k}\int_0^{\pi/2}\sin^2\theta d\varphi=\Delta_0+\frac{2\pi m}{k}. \tag{14}$$

Expanding in Eq. (13) the left part into a series in respect of the exponents m, at $m \ll 1$ we obtain

$$\pi/2(1+m/4)=k(L/4).$$

Excluding the quantity m from Eq. (14) using the above equation, we obtain

$$k=\frac{\pi}{\frac{L}{2}+\frac{\Delta_0-\Delta}{8}},$$

which specifies the connection between P and Δ at small deflections:

$$P=\frac{4\pi^2EI}{\left(L+\frac{\Delta_0-\Delta}{4}\right)^2}.$$

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