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MATHEMATICAL MODELING OF THE THERMAL EXPANSION OF SPATIALLY REINFORCED COMPOSITES

A. F. Kregers

UDC 539.377.001:678.067

Composite materials (CM) are ever more widely used in structures subjected to mechanical as well as thermal loads. The implementation of spatial reinforcement schemes makes it possible to attain such combinations of thermophysical and mechanical properties which cannot be obtained with the aid of plane reinforcement schemes, e.g., macroisotropy of CM and combinations of properties similar to it. A peculiarity of CM is that the material and the structure are produced simultaneously; the development of structural models for calculating the thermophysical properties of spatially reinforced CM in connection with the problems under consideration takes up an important place, and so far it has not been sufficiently investigated.

In structural models the value of the coefficient of linear thermal expansion (CLTE) $\alpha^*_{\beta\gamma}$ of a composite, in distinction to the value of thermal conductivity, depends on the rigidity properties of the components of the CM, and this undoubtedly complicates the problem of determining the CLTE.

In the structural theories of deformation including the theory of thermoelasticity, an important place is taken up by the methods of determining the properties of unidirectionally reinforced CM; the thermophysical characteristics of these materials have been studied fairly extensively [e.g., 1—4] (a brief review of publications on this subject was presented in [2] and [4]).

The main object of the present work is the development of a structural approach to the evaluation of the CLTE of spatially reinforced fibrous composites with complex curvilinear arrangement of the reinforcement, the preparation of computer programs, and the solution of a number of specific problems. The investigation is based on the results of the publications [2, 5, 6]. In the first of them the mathematical apparatus is developed for evaluating the limits of measurement of the CLTE of multiphase macroanisotropic composites with anisotropic phases. In distinction to [2], by phase for a spatially reinforced composite we mean a unidirectionally reinforced transversally isotropic calculation element (CE) [5, 7]. The upper and lower boundaries (the fork) of the characteristics of elasticity of a hybrid composite reinforced by different monotropic fibers is determined with the aid of [5, 6]. Within the limits of a repeating element the fibers may have the most variegated paths (straight, curved in a plane or in space).

We introduce two systems of coordinates. The system 0, 1, 2, 3 is fixed with the CE, axis 1 is always directed along the fibers, and the plane 2, 3 is the plane of isotropy of the properties; in tensorial dependences these axes are denoted by Latin letters (except x, y, z). The chosen (arbitrary) axes of the CM are denoted x, y, z, and in tensorial dependences they are denoted by Greek letters (except α, ϕ, θ).

Since the CE is monotropic, it suffices to specify the mutual disposition of the axes 1, 2, 3 and x, y, z by the two angles ϕ and θ (Fig. 1), taking it that the axis 2 always lies in the plane y, z. In such a case the cosines of the angles $\ell_{i\beta}$ between the axes of both systems of coordinates are expressed through θ and ϕ (Table 1).

We represent the equations of thermoelasticity of CM in the form

Institute of the Mechanics of Polymers, Academy of Sciences of the Latvian SSR, Riga. Translated from Mekhanika Kompozitnykh Materialov, No. 3, pp. 433-441, May-June, 1988. Original article submitted October 23, 1987.

 0191-5665/88/2403-0316 $12.50 © 1988 Plenum Publishing Corporation

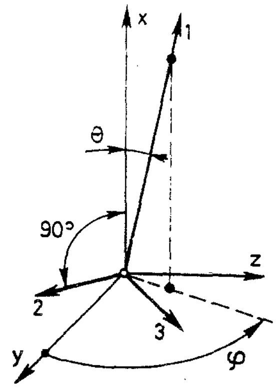

Fig. 1. Axes of coordinates of the composite x, y, z and CE 1, 2, 3.

TABLE 1. Cosines of the Angles between Axes of the Systems 1, 2, 3 and x, y, z (see Fig. 1)

Axis	1	2	3
x	$\cos\theta$	0	$-\sin\theta$
y	$\sin\theta\cos\varphi$	$\sin\varphi$	$\cos\theta\cos\varphi$
z	$\sin\theta\sin\varphi$	$-\cos\varphi$	$\cos\theta\sin\varphi$

$$\varepsilon_{\beta\gamma}=a^*{}_{\beta\gamma\delta\eta}\sigma_{\delta\eta}+\alpha^*{}_{\beta\gamma}\theta;$$
$$\sigma_{\nu\rho}=A^*{}_{\nu\rho\psi\omega}\varepsilon_{\psi\omega}-\Gamma^*{}_{\nu\rho}\theta;\quad \Delta Q=c^*\theta, \tag{1}$$

where $\Gamma^*{}_{\nu\rho} = A^*{}_{\nu\rho\psi\omega}\alpha^*{}_{\psi\omega}$; $(A^*) = (a^*)^{-1}$; $\beta, \gamma, \delta, \eta, \nu, \rho, \psi, \omega = x, y, z$; ε, σ are strains and stresses averaged over the volume; θ is the temperature difference between two states of the CM; a^*, A^* are the tensors of compliance and rigidity, respectively, of the CM; α^* is the CLTE; Γ^* is the tensor of thermal stresses; ΔQ is the quantity of heat; c^* is the specific heat ($c^*{}_p$ at constant pressure, $c^*{}_V$ at constant volume); an asterisk denotes the thermophysical properties of spatially reinforced composites.

Unidirectionally Reinforced CM. To determine the CLTE of a monotropic CE we use the dependence [1, 2]

$$\overset{0}{\alpha}_{ij}=\langle\alpha_{ij}\rangle+P_{klmn}(\overset{0}{a}_{mnij}-\langle a_{mnij}\rangle)(\alpha_{kl}{}^{f}-\alpha_{kl}{}^{b}),$$

where the angle brackets denote averaging over the volume:

$$\langle a_{mnij}\rangle=\overset{f}{a}_{mnij}\mu_\Sigma+\overset{b}{a}_{mnij}(1-\mu_\Sigma);\quad \langle\alpha_{ij}\rangle=\alpha_{ij}{}^{f}\mu_\Sigma+\alpha_{ij}{}^{b}(1-\mu_\Sigma)$$

and

$$P_{klmn}(\overset{f}{a}_{mnrs}-\overset{b}{a}_{mnrs})=1/2(\delta_{kr}\delta_{ls}+\delta_{ks}\delta_{lr})\quad \text{or}\quad (P)=(a^{f}-a^{b})^{-1}. \tag{2}$$

The superscript 0 denotes here and henceforth the characteristics of CE. The symbol μ_Σ denotes the volume coefficient of reinforcement of a CE (it is the same for all CE and coincides with the total volume coefficient of reinforcement of a spatially reinforced hybrid CM); δ_{kr} is the Kronecker delta; the superscript f denotes fiber, b denotes binder (matrix); by dimensionality and nature P is the rigidity tensor.

According to (2) the components of the tensor P can be expressed through a^f-a^b, and since the CE is monotropic, we obtain

TABLE 2. Characteristics of Elasticity and CLTE of Fibers and Matrix Used for Numerical Computer Modeling

Fibers, matrix	E_{11}	E_{22}	G_{12}	ν_{21}	ν_{23}	α_{11}	α_{22}
Carbon high-modulus fibers	226	8,0	60,0	0,20	0,30	−1	18
Carbon low-modulus fibers	60	4,8	12,0	0,30	0,25	−1	18
Glass fibers	60	60,0	25,0	0,20	0,20	5	5
Carbon matrix	3	3,0	1,1	0,35	0,35	4	4
Epoxy matrix	3	3,0	1,1	0,35	0,35	45	45

Note. The values of E and G are given in GPa, α in 10^{-6} °K^{-1}.

$$P_{1111}=(\Delta a_{2222}+\Delta a_{2233})/f_1;\quad P_{1122}=\Delta a_{1122}/f_1;\quad P_{2222}=P_{3333}=(\Delta a_{1111}\Delta a_{2222}-$$
$$-\Delta a_{1122}^2)/f_2;\quad P_{1133}=P_{1122};\quad P_{2233}=(\Delta a_{1122}^2-\Delta a_{1111}\Delta a_{2233})/f_2;\quad P_{iijj}=P_{jjii};$$

where

$$f_1=\Delta a_{1111}(\Delta a_{2222}+\Delta a_{2233})-2\Delta a_{1122}^2;\quad f_2=f_1(\Delta a_{2222}-\Delta a_{2233});$$
$$\Delta a_{iijj}=a_{iijj}^{f}-a_{iijj}^{b}.$$

When we substitute this expression into (2), we obtain for the CE that

$$\alpha_{11}{}^0=\alpha_{11}{}^{f}\mu_\Sigma+\alpha_{11}{}^{b}(1-\mu_\Sigma)+(\alpha_{11}{}^{f}-\alpha_{11}{}^{b})g_1+(\alpha_{22}{}^{f}-\alpha_{22}{}^{b})g_2;$$
$$\alpha_{22}{}^0=\alpha_{33}{}^0=\alpha_{22}{}^{f}\mu_\Sigma+\alpha_{22}{}^{b}(1-\mu_\Sigma)+(\alpha_{11}{}^{f}-\alpha_{11}{}^{b})g_3+(\alpha_{22}{}^{f}-\alpha_{22}{}^{b})g_4; \tag{3}$$

where

$$g_1=P_{1111}\delta a_{1111}+2P_{1122}\delta a_{1122};\quad g_2=2[P_{1122}\delta a_{1111}+\delta a_{1122}(P_{2222}+P_{2233})],$$
$$g_3=P_{1111}\delta a_{1122}+P_{1122}(\delta a_{2222}+\delta a_{2233});\quad g_4=2P_{1122}\delta a_{1122}+$$
$$+(P_{2222}+P_{2233})(\delta a_{2222}+\delta a_{2233});$$
$$\delta a_{iijj}=a_{iijj}^{0}-a_{iijj}^{f}\mu_\Sigma-(1-\mu_\Sigma)a_{iijj}^{b}.$$

It is assumed in (3) that the binder is also monotropic but only on condition that the planes of isotropy of the binder and of the fibers of the CE coincide. Furthermore it was accepted in the work that the matrix is isotropic, i.e., $\alpha_{11}{}^b = \alpha_{22}{}^b = \alpha_c$.

Thus, to determine the CLTE of a calculation element we obtained the dependences in a more concrete form.

For the computer modeling of the thermophysical properties of the CE (3) we adopted the numerical values of the characteristics of elasticity and the CLTE for the fibers and the matrix [8-11] (Table 2). The calculated values of the thermoelastic properties according to (3) and [6] of three different CE for μ_Σ = 0.3 [carbon-carbon (C—C) CM with high-modulus (scheme 1H) and low-modulus (scheme 1L, Fig. 2) fibers and glass reinforced plastic] are presented in Table 3. The value μ_Σ = 0.3 was chosen with a view to the theoretical limit coefficients of reinforcement for the schemes of spatial reinforcement examined later on [12]. The effect of μ_Σ on the CLTE for CE, other conditions being equal, was determined according to (3) and is presented in Fig. 3. It attracts attention that there is relatively small deviation between the curves $\alpha-\mu_\Sigma$ for low-modulus and high-modulus C—C of unidirectional composites; this is due to the fact that for matrix and fibers the value of α is the same (see Table 2), regardless of the substantially different rigidity of the fibers.

We note that when we freely change the properties of the fibers and of the matrix in accordance with (3), we obtain continuously rising or descending curves $\alpha_{11}{}^0-\mu_\Sigma$ and $\alpha_{22}{}^0-\mu_\Sigma$; then these last-mentioned curves may not only have a point of maximum (curve 6 in Fig. 3) but also a point of minimum.

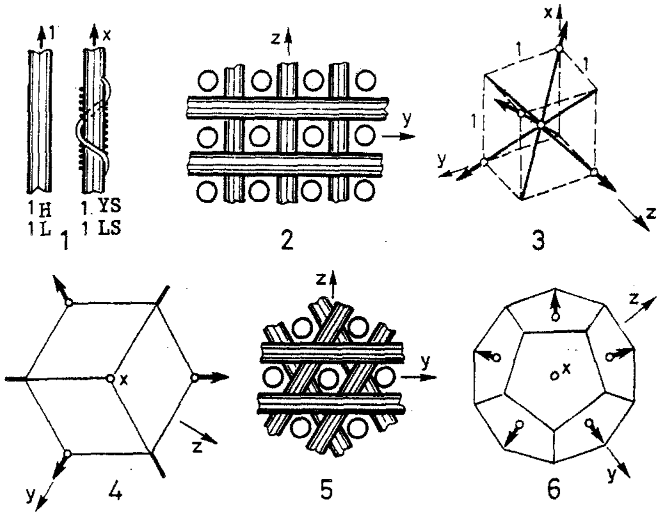

Fig. 2. Schema of reinforcement. Explanations in the text.

TABLE 3. Calculated Values of Thermoelastic Characteristics of Unidirectionally Reinforced CM (CE) with Straight Fibers for μ_Σ = 0.3

Characteristic	C–C of CM, carbon, schema		Glass reinforced plastic
	1H	1L	
E_{11}	69,9	20,10	20,50
E_{22}, E_{33}	4,43	3,80	5,92
G_{23}	1,53	1,33	2,18
G_{12}, G_{13}	2,76	2,22	2,49
ν_{21}	0,31	0,34	0,35
α_{11}	−0,84	−0,55	9,04
α_{22}, α_{33}	9,28	9,01	41,30
Γ_{11}	13,10	−31,30	−509
Γ_{22}, Γ_{33}	−74,60	−63,00	−462

Note. The values of E, G are given in GPa; α in 10^{-6} °K^{-1}; Γ in 10^{-6} GPa·°K^{-1}.

The numerical values of α_c for an isotropic polymer matrix can be established from experiments with an equal CM in the nonreinforced state. For fibers such a method is unsuitable, and here the following is recommended. On the basis of the investigated fibers we devise a unidirectionally reinforced model composite with previously known values of μ_Σ and E_c, ν_c, α_c. Then we determine experimentally the values of $\alpha_{11}{}^0$, $\alpha_{22}{}^0$, we solve the system of equations (3) for $\alpha_{11}{}^f$ and $\alpha_{22}{}^f$ and find

$$\alpha_{11}{}^f = D_1/D; \quad \alpha_{22}{}^f = \alpha_{33}{}^f = D_2/D;$$

where

$$D = d_{11}d_{22} - g_2 g_3; \quad D_1 = b_1 d_{22} + b_2 g_2;$$
$$D_2 = b_2 d_{11} + b_1 g_3; \quad d_{11} = 1 - \mu_\Sigma - g_1;$$

$$d_{22} = 1 - \mu_\Sigma - g_4; \quad b_1 = \alpha_{11}{}^0 - \alpha_c(\mu_\Sigma + g_1 + g_2);$$
$$b_2 = \alpha_{22}{}^0 - \alpha_c(\mu_\Sigma + g_3 + g_4);$$

g_1, g_2, g_3, g_4 are presented in (3).

Spatially Reinforced CM. Whereas the CLTE of unidirectional CM was determined unambiguously in accordance with (3), for any other CM with two or more directions of reinforcement (including spatially reinforced CM) in accorcance with the method [2, 5] under consideration,

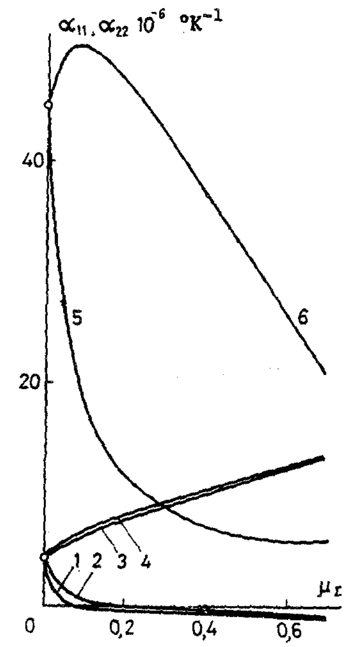

Fig. 3. Dependences of the CLTE of unidirectionally reinforced composites on μ_Σ: 1, 4) low-modulus and 2, 3) high-modulus carbon-carbon CM; 5, 6) glass reinforced plastic; 1, 2, 5) α_{11}; 3, 4, 6) α_{22}.

they are determined by two limit values, i.e., the fork of the solution of

$$\alpha^*_{\beta\gamma}=\frac{A_2B'_2+A'_2B_2}{A_2+A'_2}\pm\Delta\alpha^*_{\beta\gamma}, \tag{4}$$

where

$$\Delta\alpha^*_{\beta\gamma}=\frac{1}{A_2+A'_2}\{A_2A'_2[4(A_2+A'_2)(C'-C_1)-(B_2-B'_2)^2]\}^{1/2};$$

$$A_2=\frac{1}{2}(a^*_{\beta\gamma\beta\gamma}-a^A_{\beta\gamma\beta\gamma});\quad A'_2=\frac{1}{2}(a^a_{\beta\gamma\beta\gamma}-a^*_{\beta\gamma\beta\gamma});\quad B_2=a^A_{\beta\gamma\delta\eta}\langle\Gamma_{\delta\eta}{}^0\rangle;$$

$$(a^A)=(A^A)^{-1};\quad B'_2=\langle\alpha_{\beta\gamma}{}^0\rangle=\sum_{n=1}^{N}\alpha_{ij}{}^{0(n)}l_{i\beta}{}^{(n)}l_{j\gamma}{}^{(n)}v_n;$$

$$\langle\Gamma_{\delta\eta}{}^0\rangle=\sum_{n=1}^{N}\Gamma_{ij}{}^{0(n)}l_{i\delta}{}^{(n)}l_{j\eta}{}^{(n)}v_n;\quad \Gamma_{ij}{}^{0(n)}=A^{0(n)}_{ijkl}\alpha_{kl}{}^{0(n)};$$

$$C'=1/2\langle A_{\delta\eta\nu\rho}\alpha_{\delta\eta}{}^0\alpha_{\nu\rho}{}^0\rangle=1/2\sum_{n=1}^{N}A^{0(n)}_{\delta\eta\nu\rho}\alpha_{\delta\eta}{}^{0(n)}\alpha_{\nu\rho}{}^{0(n)}v_n;$$

$$A^{0(n)}_{\delta\eta\nu\rho}=A^{0(n)}_{ijkl}l_{i\delta}{}^{(n)}l_{j\eta}{}^{(n)}l_{k\nu}{}^{(n)}l_{l\rho}{}^{(n)};\quad C_1=1/2(a^A_{\delta\eta\nu\rho}\langle\Gamma_{\delta\eta}{}^0\rangle\langle\Gamma_{\nu\rho}{}^0\rangle).$$

The superscript A denotes that this characteristic of the CM was determined by the method of averaging the components of the rigidity tensors of the CE, a also by averaging of the components of the pliability tensors of the CE; n is the ordinal number of the CE; N is the number of CE; v_n is the weighting factor of the n-th CE, equal to the ratio of the volume of the

TABLE 4. Schema of Reinforcement

Schema	Number of direction											
	1		2		3		4		5		6	
	θ_1	φ_1	θ_2	φ_2	θ_3	φ_3	θ_4	φ_4	θ_5	φ_5	θ_6	φ_6
1	0	90										
2	0	90	90	0	90	90						
3	β	45	β	135	β	225	β	315				
4	0	90	γ	0	γ	120	γ	240				
5	0	90	90	0	90	120	90	240				
6	0	90	δ	0	δ	72	δ	144	δ	216	δ	288

Note. The values of θ, ϕ are given in degrees $\beta = 54.7356°$; $\gamma = 70.5288°$; $\delta = 63.4349°$; θ and ϕ according to Fig. 1 are specified for the axis of the filament.

reinforcement of the n-th CE to the total volume of the reinforcement in the CM; $\ell_{i\delta}^{(n)}$ is the cosine of the angle between the i-axis of the n-th CE and the axis of the composite δ; $\langle\alpha_{\beta\gamma}{}^0\rangle$ is the averaging of all $\alpha_{ij}{}^{0(n)}$ over the volume after they have been reduced to the axes x, y, z of the composite.

Through C_1 and C' we determine the fork of the values of effective heat capacity of the CM with constant pressure c^*_p (1) [2]:

$$C_1 < \frac{\langle c_V\rangle - c^*_p}{2T_0} \leqslant C'.$$

Let us consider two limit cases of dependence (4). If we adopt $a^* = a^A$, then $A_2 = 0$, and instead of (4) we obtain

$$\alpha^*_{\beta\gamma} = \alpha^{*A}_{\beta\gamma} = B_2, \tag{5}$$

and in the case $a^* = a^a$, $A'_2 = 0$ we obtain

$$\alpha^*_{\beta\gamma} = \alpha^{*a}_{\beta\gamma} = B'_2; \quad \alpha^{*A}_{\beta\gamma} \leqslant \alpha_{\beta\gamma} \leqslant \alpha^{*a}_{\beta\gamma}. \tag{6}$$

It can be seen that for the cases (5) and (6) the value of $\Delta\alpha^*_{\beta\gamma}$ in (4) is equal to zero. We note that in accordance with the properties of the tensors of second rank, composites with reinforcement arrangements 2 and 3 (see Fig. 2) are isotropic in regard to the CLTE, but by their rigidity characteristics they correspond to cubic symmetry. An additional numerical analysis of the value of $\Delta\alpha^*_{\beta\gamma}$ in the general case $a^A < a^* < a^a$ showed that for such composites it depends on the orientation of the axes of the composite whereas $\alpha^{*A}_{\beta\gamma}$ and $\alpha^{*a}_{\beta\gamma}$ are invariant to rotation of the axes. For this reason and for the purpose of simplifying the analysis of the results, we will henceforth represent the calculated values of the CLTE of different reinforcement schemata by the fork between $\alpha^{*A}_{\beta\gamma}$ and $\alpha^{*a}_{\beta\gamma}$.

We will investigate by computer the dependence (4) for six different schemata of reinforcement (Table 4, Fig. 2); the characteristics of the components of the CM are presented in Table 2. By substantial supplementing and revision [6] a new program in the language PL/1 was prepared for these purposes and realized on an ES 1022 computer.

According to schema 1, the composites are reinforced by unidirectional straight complex filaments (1YS, 1LS) (straight filaments at the center, with filaments of the same or other material wound round them on a helical path). The mechanical properties of such CM were studied by the authors of [13–15]. The following designations were adopted: L) the schema contains solely low-modulus straight carbon fibers; H) high-modulus straight carbon fibers; YS) composite reinforced with hybrid complex filaments whose straight fibers have high modulus, and the winding is of low-modulus fibers; LS) composite reinforced with complex filaments whose straight and helical fibers are of low-modulus carbon.

When the axis 1 (see Fig. 1) has the direction of the straight fibers of the complex filament, then the equation of the helical filaments in parametric form looks as follows:

TABLE 5. Calculated Values of the Independent Characteristics of the Elasticity* of Spatially Reinforced C—C Composites, $\mu_{\Sigma} = 0.3$

Characteristics	No. and code of reinforcement schema									
	1YS	1LS	2H	2L	2YS	2LS	3H	3L	3YS	3LS
E_{xx}, GPa	42,7/11,3	14,3/9,18	27,2/6,45	9,65/5,21	18,4/5,58	8,12/4,99	6,58/5,55	5,13/4,65	7,30/5,28	6,34/4,78
E_{yy}, E_{zz}, GPa	5,14/4,45	4,67/4,06	27,2/6,45	9,65/5,21	18,4/5,58	8,12/4,99	6,58/5,55	5,13/4,65	7,30/5,28	6,34/4,78
G_{xy}, G_{xz}, GPa	3,12/2,43	2,82/2,17	2,35/2,18	1,93/1,82	2,69/2,08	2,45/1,88	9,13/2,44	3,36/1,99	6,30/2,17	3,01/1,94
G_{yz}, GPa	1,84/1,61	1,73/1,48	2,35/2,18	1,93/1,82	2,69/2,08	2,45/1,88	9,13/2,44	3,36/1,99	6,30/2,17	3,01/1,94
ν_{xy}	0,04/0,12	0,12/0,13	0,09/0,24	0,18/0,25	0,14/0,26	0,23/0,26	0,40/0,27	0,33/0,28	0,36/0,27	0,29/0,27

Characteristics	No. and code of reinforcement schema									
	4H	4L	4YS	4LS	5H	5L	5YS	5LS	6H	6L, 6LS
E_{xx}, GPa	21,4/6,12	8,26/5,01	15,1/5,47	7,54/4,92	21,7/5,79	8,30/4,77	15,2/5,24	7,31/4,72	16,1/5,87	7,07/4,86
E_{yy}, E_{zz}, GPa	13,7/5,96	7,17/4,92	11,9/5,42	7,20/4,88	21,7/6,37	8,45/5,19	15,4/5,62	7,77/5,04	16,1/5,87	7,07/4,86
G_{xy}, G_{xz}, GPa	3,13/2,26	2,25/1,87	3,33/2,11	2,61/1,90	2,30/2,12	1,89/1,78	2,64/2,04	2,41/1,85	6,42/2,33	2,79/1,92
G_{yz}, GPa	4,66/2,35	2,69/1,93	4,36/2,14	2,80/1,92	8,17/2,55	3,21/2,06	5,84/2,24	3,02/2,00	6,42/2,33	2,79/1,92
ν_{xy}	0,11/0,25	0,20/0,26	0,16/0,26	0,24/0,26	0,08/0,26	0,18/0,27	0,13/0,28	0,24/0,12	0,25/0,26	0,27/0,27

Note. The characteristics in the numerator were determined by the method of averaging the components of the rigidity tensors of the CE, in the denominator the same for the compliance tensors of the CE [5, 6].

TABLE 6. Calculated Values of the Thermophysical Characteristics of Spatially Reinforced C–C Composites, $\mu_\Sigma = 0.3$

Characteristic	No. and code of reinforcement schema								
	1YS	1LS	2H, 3H, 4H, 6H	2L, 2LS, 3L, 3LS, 4L, 4LS, 6L, 6LS	2YS, 3YS, 4YS	5H	5L	5YS	5LS
α^*_{xx}	$\frac{-0,77}{14.39}$	$\frac{-0.36}{1,61}$	$\frac{1,39}{5,91}$	$\frac{3,45}{5,82}$	$\frac{1,92}{5,87}$	$\frac{2.13}{6,75}$	$\frac{4,51}{6,62}$	$\frac{2.71}{6.43}$	$\frac{4,28}{6,35}$
α^*_{yy}, α^*_{zz}	$\frac{7,59}{8,09}$	$\frac{7,22}{7,93}$	$\frac{1,39}{5,91}$	$\frac{3,45}{5,82}$	$\frac{1,92}{5,87}$	$\frac{1,10}{5,49}$	$\frac{2,99}{5,42}$	$\frac{1.60}{5.59}$	$\frac{3.07}{5,56}$
Γ^*_{xx}	$\frac{13,1}{56,5}$	$\frac{38,4}{52,7}$	$\frac{45,4}{50,2}$	$\frac{52,4}{61,2}$	$\frac{48,4}{67,2}$	$\frac{52,7}{73,3}$	$\frac{55,1}{61,9}$	$\frac{52,8}{67,8}$	$\frac{54,2}{61,6}$
Γ^*_{yy}, Γ^*_{zz}	$\frac{66.0}{69,0}$	$\frac{59,4}{62,7}$	$\frac{45.4}{50,2}$	$\frac{52,4}{61,2}$	$\frac{48.4}{67,2}$	$\frac{41,7}{60,8}$	$\frac{51,1}{67,0}$	$\frac{46.2}{60,9}$	$\frac{51,5}{69,0}$

Note. The values of α^* are given in 10^{-6} °K^{-1}; of Γ^* in 10^{-6} GPa·°K^{-1}; in the numerator and denominator the characteristics were determined in accordance with (5) and (6), respectively.

$k_1 = p_1 t$; $k_2 = p_2 \cos t$; $k_3 = p_3 \sin t$; where k_i are the coordinates of the axis of the helix on the axes 1, 2, 3, i = 1, 2, 3; p_1, p_2 are the parameters of the helix; $t = 0 \ldots 2\pi$. The number of filaments in the winding is 13, their diameter is 0.2 of the diameter of the core of the straight fibers; the angle of the axis of the helix with the axis of the complex filament is 44°; the volume of the fibers in the helixes is 43% of the total volume of the fibers. To the geometric characteristics explained above [12] there correspond the values $p_1 = 0.60$, $p_2 = 0.57$. In the subsequent schemata of reinforcement the axes of the complex filaments do not coincide with the axes of the composite x, y, z. Taking into account that the program for the calculation requires the derivatives of the equations of the paths of reinforcement to be specified in analytical form, we represent them for the helix (the winding) in the form

$$\partial x/\partial t = p_1 \cos\theta - p_2 \cos t \sin\theta;$$
$$\partial y/\partial t = p_1 \sin\theta \cos\varphi - p_2 \sin t \sin\varphi + p_2 \cos t \cos\theta \cos\varphi;$$
$$\partial z/\partial t = p_1 \sin\theta \sin\varphi + p_2 \sin t \cos\varphi + p_2 \cos t \cos\theta \sin\varphi.$$

The slope of the complex filament is specified by the angles θ and φ (see Fig. 1, Tables 1, 4). Schema 2 has three mutually perpendicular directions of reinforcement. In schema 3 the filaments are arranged along the body diagonals of a cube; schema 4 differs from schema 3 solely by the orientation of the axes x, y, z, here the x axis has the direction of one of the diagonals of the cube. In schema 5 there are altogether four directions of reinforcement, and by its thermomechanical properties such CM is transversely isotropic. Schema 6 has six directions of reinforcement: along the normals passing through the centers of the sides of a regular dodecahedron. In the range of linear elasticity such CM is macroisotropic. For all the six schemata we adopted $\mu_\Sigma = 0.3$ and the same specific amount of reinforcement in all the directions.

The calculated values of the independent elastic characteristics are presented in Table 5 in the form of a fork of the solution. Tests showed that for structural CM the experimental data lie within this fork, and closer to the results presented in the numerator. When we analyze these calculated values, we discover that for any of the first five schemata of reinforcement (see Fig. 2) in regard to the moduli of elasticity of the composite E or G we can find situations in which complex filaments (YS or LS) ensure greater rigidity of the CM than straight filaments (H or L, respectively). This result can find application in various problems of optimization.

It is interesting to note that for schema 4 the following nontraditional components of the compliance tensor of CM are nonzero: α^*_{yyxy}, α^*_{yyxz}, α^*_{zzxy}, α^*_{zzxz}, α^*_{xyyz}, α^*_{xzyz}, which had been pointed out in [16] already; on the other hand, $\alpha^*_{\beta\gamma}$ is isotropic.

The thermophysical characteristics α and Γ are presented in Table 6. It can be seen from the table that the values of the characteristics α and Γ for the schemata 2–4, 6 with the same reinforcing filament are identical because of the isotropy of the corresponding

TABLE 7. Experimental and Calculated Values of the Elastic Characteristics and CLTE of Spatially Reinforced Carbon-Organic Plastics

Value	E_{xx}	E_{yy}	ν_{yx}	ν_{zy}	α_{xx}	α_{yy}
Exptl.	148	11	0,14	0,21	0,30	18
Calc.	$\frac{154}{47}$	$\frac{12}{7}$	$\frac{0{,}18}{0{,}10}$	$\frac{0{,}32}{0{,}30}$	$\frac{0{,}20}{0{,}30}$	$\frac{14}{27}$

Note. For dimensionalities and notation see Tables 5 and 6.

properties. The seeming lack of influence of the rigidity properties of CM, which for the mentioned schemata of reinforcement are different, was investigated analytically. We succeeded in showing that when composites with different rigidity properties contain the same CE, the value of $\alpha^{*A}{}_{\beta\gamma}$ does not depend on $\langle\Gamma_{\eta\rho}\rangle$ only but also on the value of the expression $\alpha^{*A}_{xxxx} + 2\alpha^{*A}_{xxyy}$ (this last turned out to be the same for the CM dealt with above) (see Table 5). Such a peculiarity of the CM under consideration can also be made use of in problems of optimization.

We note that within the framework of the present publication we did not investigate the influence of factors such as the internal geometry of the helix itself, other variants of the components of the CM and the dependence of their properties on the temperature, the volume coefficient of the reinforcement, other schemata of reinforcement including flat ones, etc.; all this would have greatly broadened the limits of change of the thermoelastic properties of CM.

To compare the calculated results with the experimental ones we used the data of [4] corresponding to a hybrid, spatially reinforced polymer composite (Table 7). The carbon fibers are straight, their direction is that of the x axis; the organic fibers are curved in the plane y, z, and their path is described by the equation z = 0,23 sin t, y = t, x = 0, where t is the parameter in radians. The properties of the carbon fibers are the following: $E_{11}{}^f$ = 230 GPa; $E_{22}{}^f$ = 7.7 GPa; $G_{12}{}^f$ = 60.9 GPa; $\nu_{21}{}^f$ = 0.3; $\nu_{32}{}^f$ = 0.2; $\alpha_{11}{}^f = -0.55\cdot10^{-6}$ $°K^{-1}$, $\alpha_{22}{}^f = 13\cdot10^{-6}$ $°K^{-1}$, their share of the total volume of fibers is 94.3%; organic fibers: $E_{11}{}^f$ = 130 GPa; $E_{22}{}^f$ = 3.3 GPa; $G_{12}{}^f$ = 2.4 GPa; $\nu_{21}{}^f$ = 0.29; $\nu_{32}{}^f$ = 0.17; $\alpha_{11}{}^f = -6.7\cdot10^{-6}$ $°K^{-1}$; $\alpha_{22}{}^f = 57\cdot10^{-6}$ $°K^{-1}$; matrices: E_c = 4.5 GPa, ν_c = 0.35, $\alpha_c = 45\cdot10^{-6}$ $°K^{-1}$; μ_Σ = 0.7. It can be seen from Table 7 that the experimental values with the exception of ν_{zy} lie within the established values of the forks of the solutions, and the calculated results in accordance with (5) ($a^* = a^A$) are very close to the experimental results.

The submitted mathematical apparatus and its realization in the form of a computer program make it possible to evaluate the thermophysical properties (jointly with [6, 17, 18]) of spatially reinforced composites with very complex structure, and to use it for solving actual problems of optimization.

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