The last of these orientations is possible when  $|f_1/2f_2| \leq 1$ . An analogous condition for extremality was obtained in a two-dimensional problem in the theory of elasticity, namely orientation of the axes of orthotropy [5].

For obtaining approximate analytical solutions we will use the method of successive approximations. As the initial approximation will serve the solution to the problem of flexure for an anisotropic plate with rectilinear anisotropy. The subsequent approximations can be obtained through successive resolutions of the optimality condition with respect to angle  $\alpha$  and solution of the boundary-value problem of flexure for a plate with curvilinear anisotropy. Let us construct an analytical solution for the first approximation.

As an example we will consider the problem of optimizing the stiffness of an elliptical multilayer plate loaded by a uniform pressure. As the zeroth approximation for the deflection function we select the solution to the flexure problem for a plate with rectilinear anisotropy [2]:

$$w(x_1, x_2) = q_0 a^4 (1 - x_1^2/a^2 - x_2^2/b^2)^2/8H; \quad c = a/b; H = [3D_{2222} c^4 + 2D_{1122} c^2 + 4D_{1212} c^2 + 3D_{1111}];$$

where  $\alpha$  and b are the semiaxes of the ellipse. The directions of lay of reinforcing fibers in accordance with the three possible orientations of the axes of anisotropy are shown in Fig. 3.

### LITERATURE CITED

- 1. V. V. Bolotin and Yu. N. Novichkov, Mechanics of Multilayer Structures [in Russian], Moscow (1980).
- 2. S. G. Lekhnitskii, Anisotropic Plates [in Russian], Moscow (1957).
- 3. A. K. Malmeister, V. P. Tamuzh, and G. A. Teters, Resistance of Rigid Polymer Materials [in Russian], 2nd ed., Riga (1972).
- 4. J. Luveau, "Functional analysis and mechanics of continuous media," in: Theoretical and Applied Mechanics, Proc. Fourteenth Int. Congress IUTAM [Russian translation], Moscow (1979), pp. 323-345.
- 5. N. V. Banichuk, Shape Optimization of Elastic Bodies [in Russian], Moscow (1980).

### STABILITY OF LAMINATION IN COMPOSITES\*

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The fracture mechanisms of laminated and fibrous composites are very varied. Although a large number of authors dealt with this subject (reviews may be found in [1-3]), the processes of lamination, which are characteristic of the fracture of polymer composites, have not been sufficiently studied. In particular, there is no satisfactory theoretical description of stable and unstable brittle peeling near the fractures of the reinforcing elements of composites under arbitrary loads. Some special cases of lamination were investigated on continuous [4, 5] and simple discrete models [6-8] of the composite; an explanation of the stability through the effect of inelastic deformations was given in [3].

1. It follows from theoretical [9, 10] and experimental [2, 3] analyses that in a laminated composite with a crack transverse to the layers, with the composite being situated in an arbitrary uniform stress field, we must expect Z-shaped laminations (Fig. 1a).

The plane problem for a regular laminated medium with alternating hard and soft layers is described by the equations [1, 9]

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Fig. 1. Laminations at a transverse crack (a) and symmetrization of the problem (b. c).

$$Au''_{\alpha} + B \left[ u_{\alpha+1} - 2u_{\alpha} + u_{\alpha-1} + \frac{c}{2} (w'_{\alpha+1} - w'_{\alpha-1}) \right] = 0;$$

$$Dw_{\alpha}^{(4)} - C(w_{\alpha+1} - 2w_{\alpha} + w_{\alpha-1}) - B - \frac{c}{2} \left[ u'_{\alpha+1} - u'_{\alpha-1} + \frac{c}{2} (w''_{\alpha+1} + 2w''_{\alpha} + w''_{\alpha-1}) \right] = 0 \quad (-\infty < \alpha < \infty).$$
(1)

Here  $u_{\alpha}(x_1)$ ,  $w_{\alpha}(x_1)$  are the displacements of a point of the middle surface of the hard layer with the number  $\alpha$ ; A, D, tensile and flexural rigidities, respectively, of the hard layer; B, C, flexural rigidity and tensile rigidity transverse to the layers, respectively, of the soft layer; and c, distance between the middle surfaces of neighboring hard layers. A prime indicates here and henceforth differentiation with respect to  $x_1$ .

The regularity of the medium, and consequently also of Eqs. (1), is disturbed when  $\alpha = n$ , n + 1 and  $\alpha = -n$ , -(n + 1) in the zone of lamination. On  $x_1 \in [0, x_1^\circ]$  we introduce the function

$$\phi_{1}(x_{1}) = B \left[ u_{n+1} - u_{n} + \frac{c}{2} (w'_{n+1} + w'_{n}) \right];$$

$$\phi_{2}(x_{1}) = C (w_{n+1} - w_{n}).$$
(2)

These formulas determine the tangential and normal interlaminar stresses before lamination. After lamination we do not insert in (1)  $\varphi_1$ ,  $\varphi'_1$ ,  $\varphi_2$  for  $\alpha = \pm n$ ,  $\pm (n + 1)$  on the sections of opened-up ( $\varphi_2 > 0$ ) lamination cracks. For closed cracks ( $\varphi_2 < 0$ ) we put  $\int \varphi_2$  and  $f \varphi'_2$  instead of  $\varphi_1$  and  $\varphi'_1$  (f is the coefficient of Coulomb friction).

For  $\alpha = \pm n$ ,  $\pm (n + 1)$  we supplement on the left-hand and right-hand sides of Eqs. (1) the same terms, so that the matrix of the differential-difference operators of system (1) becomes regular. Assuming for the time being that  $\varphi_1$ ,  $\varphi_2$  are known, we obtain for the problem in Fig. 1a an inhomogeneous system of equations with homogeneous conditions on the lips of the transverse crack  $x_1 = 0$ ,  $|\alpha| \leq n$  (the longitudinal force, the moment, and the generalized transverse force in the hard layer):

$$Au'_{\alpha} = 0; \quad Dw''_{\alpha} = 0;$$
  
$$Dw'''_{\alpha} - B \frac{c}{2} \left[ u_{\alpha+1} - u_{\alpha-1} + \frac{c}{2} (w'_{\alpha+1} + 2w'_{\alpha} + w'_{\alpha-1}) \right] = 0$$
(3)

and the conditions at infinity ( $\chi$  is the coefficient of reinforcement):

$$\sigma_{11}^{\alpha} = \sigma_{11}^{\infty} / \chi; \quad \sigma_{13}^{\alpha} = \sigma_{13}^{\infty}; \quad \sigma_{33}^{\alpha} = \sigma_{33}^{\infty}.$$
(4)

The conditions (3) correspond to the case of an opened-up transverse crack.

We represent the solution of the problem by the sum of b and c (Fig. 1). For  $\alpha = \pm n, \pm (n+1), x_1 \in [0, x_1^0]$  we put in the right-hand sides of Eqs. (1) of the symmetric b and antisymmetric c problems the functions  $1/2 \varphi_1$  and  $1/2 \varphi_2$  (their signs are shown in Figs. 1b, c). The solutions of b and c have to satisfy the conditions  $\sigma_{11}^{\alpha} = \sigma_{11}^{\infty}/\chi, \sigma_{13}^{\alpha} = 0, \sigma_{33}^{\alpha} = \sigma_{33}^{\infty}$  and  $\sigma_{11}^{\alpha} = \sigma_{33}^{\alpha} = 0, \sigma_{13}^{\alpha} = \sigma_{13}^{\infty}$ , respectively, at infinity.

In accordance with [9], we introduce on the lips of the transverse crack  $|\alpha| \leq n$ ,  $x_i \in [0, x_i^0]$  the hitherto unknown distribution of the generalized distortions (discontinuities of generalized displacements): for the symmetric problem  $\zeta_{\alpha} = u_{\alpha}(0) \ \theta_{\alpha} = w'_{\alpha}(0)$ ; for the antisym-

metric problem  $n_{\alpha} = w_{\alpha}(0)$ . We represent the solutions of each of the problems b and c by superpositions of the solutions:  $\{u_{\alpha}, w_{\alpha}\} = \{u_{\alpha}^{\infty}, w_{\alpha}^{\infty}\} + \{u_{\alpha}^{0}, w_{\alpha}^{0}\} + \{u_{\alpha}^{*}, w_{\alpha}^{*}\}$ . Here  $u_{\alpha}^{\infty}, w_{\alpha}^{\infty}$  is the solution at infinity;  $u_{\alpha}^{\circ}, w_{\alpha}^{\circ}$  is the solution depending on the distribution of the distortions;  $u^{*}_{\alpha}, w^{*}_{\alpha}$  is the solution depending on  $\varphi_{1}, \varphi_{2}$  on the right-hand sides of Eqs. (1). The former are constructed elementarily:

$$u_{\alpha}^{\infty} = \sigma_{11}^{\infty} c x_1 A^{-1}; \quad w_{\alpha}^{\infty} = \sigma_{33}^{\infty} \alpha C^{-1} \quad (b);$$

$$u_{\alpha}^{\infty} = 0; \quad w_{\alpha}^{\infty} = \sigma_{13}^{\infty} x_1 B^{-1} \quad (c).$$
(5)

The solutions  $u_{\alpha}^{\circ}$ ,  $w_{\alpha}^{\circ}$ , bounded at infinity, were constructed in [9]. For constructing the solutions  $u_{\alpha}^{*}$ ,  $w_{\alpha}^{*}$ , which are also bounded at infinity, we use the Fourier-Stiltjes transformation with the discrete argument  $\alpha$ ; this procedure was described in [9, 10]. In regard to the transforms u\*, w\* of the displacements  $u_{\alpha}^{*}$ ,  $w_{\alpha}^{*}$ , we obtain the boundary conditions for systems of inhomogeneous ordinary differential equations of eighth order with constant coefficients, homogeneous boundary conditions for  $x_1 = 0$ , and conditions of boundedness at infinity. The conditions for  $x_1 = 0$  correspond to the inhomogeneous conditions of the symmetric and the antisymmetric problems for determining  $u_{\alpha}^{\circ}$ ,  $w_{\alpha}^{\circ}$  [9].

The solutions u\*, w\* of the problems b and c are constructed by the method of variation of arbitrary constants. They are obtained as expressed through the definite integrals of  $\varphi_1, \varphi'_1, \varphi_2, \varphi'_2$ . A distinctive feature of these solutions is that they do not contain functions that increase with increasing x<sub>1</sub>. For instance, after integration by parts, to eliminate  $\varphi'_1$ , the solution for the problem b has the form

$$\{u^*, w^*\} = \sum_{j=1}^{3} \int_{-x_1^0}^{x_1^0} \mathbf{L}_j(x_1 - \bar{x}_1) \varphi(\bar{x}_1) \exp(t_j |x_1 - \bar{x}_1|) d\bar{x}_1.$$
(6)

Here  $\varphi = \{\varphi_1, \varphi_2\}$ ;  $t_j$  are the characteristic indices with negative real parts of the problem in the transforms [9];  $L_j(2, 2)$  are the matrices whose elements are discontinuous functions that depend on the parameter of the Fourier-Stiltjes transform, the properties of the medium and n. Discontinuities of  $L_j$  occur at zeros of  $\varphi_2$ , i.e., at the boundaries of the open sections of the lamination cracks.

Inversion of (6) and of the solution u\*, w\*, analogous to it, for the antisymmetric problem, the summation of all solutions of the problems b and c, yield the solution of the initial problem a. It depends on the unknowns: the distribution of the distortions  $\zeta_{\alpha}$ ,  $\theta_{\alpha}$ ,  $\eta_{\alpha}$  on  $x_1 = 0$ ,  $|\alpha| \leq n$  and of the functions  $\varphi_j$  on  $x_1 \in [0, x_1^0]$ . For their determination we use conditions (3) and formula (2) for  $\varphi_j$ . If we substitute  $u_{\alpha}$ ,  $w_{\alpha}$  into (2), (3), we obtain, in view of the symmetry of the solution, a system of integroalgebraic equations

$$G(x_{1})\xi + \int_{0}^{x_{1}^{0}} H(x_{1}, \bar{x}_{1})\varphi(\bar{x}_{1})d\bar{x}_{1} - \varphi(x_{1}) = p \quad (x_{1} \in [0, x_{1}^{0}]);$$

$$K\xi + \int_{0}^{x_{1}^{0}} M(\bar{x}_{1})\varphi(\bar{x}_{1})d\bar{x}_{1} = g.$$
(7)

Here  $\boldsymbol{\xi} = \{\zeta_0, \zeta_1, \dots, \zeta_n, \theta_0, \dots, \theta_n, \eta_0, \dots, \eta_n\}$ ; the matrices G (2, 3n + 3), H (2, 2) are determined through the influence functions  $u_{\alpha}^{\circ}, w_{\alpha}^{\circ}$  [9]; the matrices K (3n + 3, 3n + 3), M (3n + 3,2) are determined via the Fourier-Stiltjes integrals from the solutions of type (6). The vectors **p** (2) and **g** (3n + 3) depend on the external field  $\sigma_{11}^{\circ}, \sigma_{13}^{\circ}, \sigma_{33}^{\circ}$ . We do not present the formulas for the elements of these matrices here because they are too cumbersome.

2. In the system (7) the unknowns are, in addition to  $\xi$ ,  $\varphi(x_1)$ , also the zeros of the function  $\varphi_2$ . It was solved numerically with an EC-1033 computer by the iteration method according to these zeros. For the specified  $x_1^{\circ}$  we designated the first approximation for the zeros of  $\varphi_2$ , coinciding with the final distribution in the preceding variant for a smaller  $x_1^{\circ}$ . After the problem of the first approximation had been solved, we calculated the values of  $\varphi_2$  in "zeros" of this approximation. In dependence on the signs of  $\varphi_2$  we selected new values for the zeros of  $\varphi_2$ . If in some approximation for the interval between adjacent zeros of  $\varphi_2$  the condition  $\sup \varphi_2 < 10^{-2} [\varphi_2(0) + \varphi_2(x_1^0)]$  was fulfilled, then this interval was excluded from examination in the subsequent approximation.



Fig. 2. Dependences of the stresses in the soft layer of the composite at the tip of the lamination crack on its length (f = 0). The numbers next to the curves indicate the values of n; solid lines are for glass-fiber reinforced plastic; dashed lines are for carbon-fiber reinforced plastic.

Fig. 3. Dependences of the relative load necessary for the propagation of lamination (f = 0). The numbers next to the curves indicate the values of  $\sigma_{33}^{\infty}/\sigma_{11}^{\infty}$ ; solid lines are for glass-fiber reinforced plastic; dashed lines are for carbon-fiber reinforced plastic.



Fig. 4. Effect of friction on the stability of laminations with transverse (a) and longitudinal (b) loading by shear. The numbers next to the curves indicate the values of  $\sigma_{33}^{\infty}/\sigma_{13}^{\infty}$  (a) and of  $\sigma_{13}^{\infty}/\sigma_{11}^{\infty}$  (b); -----) f = 0,1; - . -) f = 0.2.

We investigated numerically the solutions of the following problems: biaxial tension of the medium  $(\sigma_{11}^{\infty} \neq 0, \sigma_{33}^{\infty} \neq 0, \sigma_{13}^{\infty} = 0)$ ; tension with shear  $(\sigma_{11}^{\infty} \neq 0, \sigma_{33}^{\infty} = 0, \sigma_{13}^{\infty} \neq 0)$  and  $\sigma_{11}^{\infty} = 0$ ,  $\sigma_{33}^{\infty} \neq 0, \sigma_{13}^{\infty} \neq 0$ . To the first problem correspond H-shaped laminations (Fig. 1b). Here there is no solution of c, but the factors 1/2 of  $\varphi_j$  have to be omitted everywhere. In all cases it was found that on the lamination crack there is either one section with friction  $(\varphi_2 < 0)$  adjacent to the crack tip, or else there are no such sections whatsoever.

The calculations were carried out for the parameters of a laminated medium characteristic of glass-fiber reinforced plastics [11]:  $Cc^2A^{-1} \approx E_3/E_1 = 0.3$ ;  $Bc^2A^{-1} \approx G_{13}/E_1 = 0.1$ ; and carbon fiber reinforced plastics:  $E_3/E_1 = 0.104$ ;  $G_{13}/E_1 = 0.042$ ; ( $\chi = 0.7$ ;  $E_1$ ,  $E_3$ ,  $G_{13}$  are the macroscopic elastic constants of the composite. The length  $x_1^{\circ}$  of the lamination crack and the friction coefficient f varied.

Figure 2 shows how the stresses change in the soft layer at the tip of the lamination crack  $(x_1 = x_1^{\circ})$  with increasing length of this crack. The curves correspond to uniaxial tension in the direction of thereinforcement with different lengths of the transverse crack. The stresses are referred to  $\sigma_{11}^{\circ\circ}$ ; the dimensionless coordinate is  $\xi_0 = 2x_1^{\circ}(B/A)^{1/2}$ . It can be seen that these stresses are strongly dependent on  $\xi_0$ , and  $\sigma_{33}$  with n 7 0 even change their sign. Since the comprehensive strength of polymer composites under transverse compression is much larger than the tensile strength, the lamination process is stable with all values of the parameters. The determination of stable and unstable crack growth is used here in the sense accepted in fracture mechanics.

For the case of biaxial loading, Fig. 3 shows the stresses  $\sigma_{11}^{\infty}$  that are necessary for the propagation of the lamination crack; they are referred to these same stresses with  $x_1^0 = 0$ :  $\alpha_{11} = \sigma_{11}^{\infty} (\frac{\kappa}{50})/\sigma_{11}^{\infty}(0)$ . It is assumed that lamination increases when the criterion of fracture is fulfilled at the crack tip. As criterion of fracture we adopted the quadratic criterion constructed on the interlaminar strengths  $\Pi_{9^+}$ ,  $\Pi_{3^-}$ , and  $\Pi_{13}$ . In the calculations we adopted [11] for glass-fiber reinforced plastics:  $\Pi_{2^-}/\Pi_{3^+} = 7$ ,  $\Pi_{13}/\Pi_{3^+} = 2.5$ ; for carbon-fiber reinforced plastic:  $\Pi_{3^-}/\Pi_{3^+} = 3.17$ ,  $\Pi_{13}/\Pi_{3^+} = 0.91$ .

It follows from Fig. 3 that even with  $\sigma_{33}^{\infty} = 0$  the lamination process is stable. This conclusion, found in experiments [3], could not be explained on simpler models [7, 8]. Further application, even of small compressive  $\sigma_{33}^{\infty}$ , substantially increases the stability of the lamination process. With  $\sigma_{33}^{\infty} > 0$  the curves  $\alpha_{11}(\xi_0)$  have maxima: Up to some  $\xi_0$  the laminations are stable; with larger  $\xi_0$  they are unstable. This may be explained as follows: With increasing  $\xi_0$  at the tip of the lamination crack the stresses induced by  $\sigma_{11}^{\infty}$  decrease (Fig. 2), and those induced by  $\sigma_{33}^{\infty}$  increase approximately like  $\xi_0^{1/2}$ .

The obtained results (see Fig. 3) make it possible to explain the mechanism of the very stable nature of the lamination in uniaxial tension in the direction of the reinforcement that was observed in experiments. In fact,  $\epsilon_{33}^{\infty} \sim -\nu_{31}\sigma_{11}^{\infty}/E_1$  applies everywhere except in the region between lamination cracks, where  $\sigma_{11} \approx 0$ . This region is therefore compressed along the lamination surfaces by transverse normal stresses. This effect, in combination with the considerable friction on the lamination surfaces, is the principal cause of the stability of laminations for any  $\xi_0$ .

Analogous results for laminations in the field of shear stresses  $\sigma_{13}^{\infty}$  in combination with normal stresses  $\sigma_{33}^{\infty}$  are given in Fig. 4a, and in combination with  $\sigma_{11}^{\infty}$  in Fig. 4b. Here  $\alpha_{13} = \sigma_{13}^{\infty}(\xi_0)/\sigma_{13}^{\infty}(0)$ ,  $\alpha_{11} = \sigma_{11}^{\infty}(\xi_0)/\sigma_{11}^{\infty}(0)$  are the respective relative stresses necessary for the propagation of the lamination cracks. These results prove the strong stabilizing effect of the magnitude of the friction and of the compressive  $\sigma_{33}^{\infty}$ , and also the destabilizing effect of  $\sigma_{13}^{\infty}$  in tension in the direction of the reinforcement. However, with small  $\sigma_{13}^{\infty}$ (Fig. 4b) the regions of values  $\xi_0$  of stable growth of lamination cracks are the larger, the larger the friction coefficient is.

The results expressed in Figs. 4a, b enable us to draw conclusions about the change of the mechanisms of fracture of composites with transverse cracks when the ratios  $\sigma_{33}^{\infty}/\sigma_{13}^{\infty}$  or  $\sigma_{13}^{\infty}/\sigma_{11}^{\infty}$  change. The change of the nonmonotonic curves  $\alpha_{13}(\xi_0)$  into monotonic ones indicates a change of the mechanism of fracture of the composite from predominantly shear fracture to predominantly detaching failure (with  $\sigma_{33}^{\infty}/\sigma_{13}^{\infty} \ge 0.2$ , see Fig. 4a). An analogous change with  $\sigma_{13}^{\infty}/\sigma_{11}^{\infty} > 0.1$  (see Fig. 4b) indicates the change of fracture in consequence of accumulated fractures of the reinforcement with stable laminations to the mechanism of unstable shear lamination.

### CONCLUSIONS

1. We constructed analytical solutions of the problems of laminations near a transverse crack in a laminated composite in an arbitrary homogeneous field of external stresses.

2. We investigated the stability and instability of processes of lamination and the effects of the magnitude of friction on the fracture surfaces and of the type of load on these processes.

# LITERATURE CITED

- 1. V. V. Bolotin and Yu. N. Novichkov, The Mechanics of Multilayered Structures [in Russian], Moscow (1980).
- A. S. Tetelman, "Fracture process in fiber composite materials," in: Compos. Mater. Test and Design, ASTM STP 460, Philadelphia, Pa. (1969), pp. 473-502.
- 3. Inelastic Properties of Composite Materials [in Russian], Moscow (1978).
- 4. V. D. Kuliev, Yu. N. Rabotnov, and G. P. Cherepanov, "Inhibition of cracking at the interface of elastic media," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 4, 120-128 (1978).
- 5. A. I. Zobnin, "Cleavage in unidirectional composites," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 2, 168-171 (1977).
- 6. P. van Dyke and J. M. Hedgepth, "Stress concentrations from single-filament failure in composite materials," Textile Res. J., <u>39</u>, No. 7, 618-626 (1969).

- 7. A. Akbarzaden, "Effect of broken fibers in the strength of unidirectional composite materials," Fibre Sci. Technol., 11, No. 3, 217-228 (1978).
- 8. A. M. Mikhailov, "Shear cracks in unidirectional glass-fiber reinforced plastic," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 1, 103-110 (1975).
- 9. V. V. Partsevskii, "Plane deformation of a laminated composite with transverse crack," Mekh. Polim., No. 4, 632-636 (1978).
- V. V. Partsevskii, "Stress distribution in a discrete model of a laminated medium near the section," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 3, 103-108 (1977).
   A. K. Miller, "Thermal expansion coefficients for laminates obtained from invariant
- 11. A. K. Miller, "Thermal expansion coefficients for laminates obtained from invariant lamina properties," Fibre Sci. Technol., <u>13</u>, No. 5, 397-409 (1980).

## REGULARITIES OF SHORT-TERM AND LONG-TERM STRENGTH OF COMPOSITES

### IN STATES OF COMPLEX STRESS\*

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The objects of the investigation were: 1) to verify the acceptability of most of the existing criteria of short-term strength (CSS) for describing the known and most representative experiments concerning nonuniaxial fracture of composites; 2) to work out a criterion of long-term strength (CLS) and its subsequent comparison with the existing conditions of time-dependent fracture using the known and fairly detailed long-term experiments; 3) to work out practical recommendations for the application of criteria of strength, and also of anisotropy formulas (AF) describing the change of the strength characteristics of composites in dependence on the coiling angle of the reinforcement, i.e., the angle  $\varphi$ .

1. Figure 1 presents the experimental data on the fracture of laminated tubes of a carbon-fiber composite used in rocket engineering [1]. The authors succeeded in predicting satisfactorily the fractive load in dependence on the angle  $\varphi$  only with the aid of a fairly cumbersome cubic tensor polynominal containing ten strength tensor components. It can be demonstrated that in this case the Gauss test

$$S = \sqrt{\sum_{i=1}^{n} (1 - R_i / R^*_i)^2 / n}$$

as the measure of the "efficiency" of the conditions of fracture is S = 17.2%. In formula (1)  $R_i = q \varphi_i / q_0$  and  $R_i^* = q \varphi_i / q_0^*$  are the theoretical and experimental values, of the fracture load for  $\Psi = \varphi_i$ ; the case  $\Psi = 0$  corresponds to a composite tube whose axis 1 coincides with the principal direction of the reinforcement u;  $q^*\varphi_i$  is the experimentally found internal fracture pressure; n = 6 is the number of varied angles.

For orthotropic sheets of composite, the quadratic tensor-polynominial CSS [2] is at present generally accepted; it usually contains six strength tensor components and leads to a considerably poorer result: S = 35.4%. Even poorer is the correlation of these experiments with the tensor-invariant criterion of Gol'denblat and Kopnov: S = 67.5%. Equally unsatisfactory for the experiments under examination are also other CSS: for them S > 50%. An exception is the dispersion approach [3-5], for which S = 18.7%. On the one hand, it may be viewed as a generalization of the ideas of Mises [6] concerning materials "with different strength" that are anisotropic and hydrostatically compressed, and on the other hand it may be viewed as a peculiar modification of the tensor-polynomial approach (TPA) by not operating with the nominal, but with the normalized stresses (NS):

$$|\eta_j| = |\sigma_j/\sigma^*_j| \leq 1, \tag{2}$$

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