9. M. Madler, S. Ioshino, and F. Darms, "Boron-epoxy supporting struts for a fuel tank," in: Composite Materials in Aircraft Design [in Russian], Moscow (1975), pp. 120-132.
10. G. P. Zaitsev, V. M. Vasilevskii, I. A. Skoryi, and É. A. Moiseenko, "Investigation of the load-bearing capacity of the metal-plastic serrated joint of pipes made of composite materials by the method of planning experiments," Mekh. Kompozitn. Mater., No. 1, 162-168 (1981).
11. D. M. Karpinos, V. Kh. Kadyrov, and Yu. V. Krylov, "Comparative evaluation of the effectiveness of some types of end joints of tubular rods made of polymer composite materials," Mekh. Kompozitn. Mater., No. 5, 941-943 (1980).
12. I. Ya. Al'shits and B. N. Blagov, The Design of Plastic Components [in Russian], Moscow (1977).
13. V. T. Shcherbakov, V. K. Frolov, and V. N. Shcherbinin, "Strength and stability of thin-walled polymer rod-shaped struts," in: Design, Calculation, and Tests of Composite Materials [in Russian], Issue 10, TsAGI (1982), p. 120.
14. A. G. Popov, V. T. Shcherbakov, and V. N. Shcherbinin, "Stability and compressive strength of thin-walled shells and rods made of high-modulus polymer composite materials," in: Third Conference of Young Scientists and Specialists on the Mechanics of Composite Materials [in Russian], Riga (1981), pp. 122-123.
15. Yu. M. Tarnopol'skii and T. Ya. Kintsis, Static Test Methods for Reinforced Plastics [in Russian], Moscow (1975).
16. Yu. M. Tarnopol'skii, "Lamination of compressed rods made of composites," Mekh. Kompozitn. Mater., No. 2, 331-337 (1979).
17. Thermal Stability of Plastics for Structural Purposes [in Russian], Moscow (1980).
18. R. H. Knibbs and J. B. Morris, Composite, 9, No. 5, 209-218 (1974).
19. G. M. Gunyaev, I. G. Zhigun, Zh. G. Sorina, and V. A. Yakushin, "Shear resistance of composites based on whiskerized fibers," Mekh. Polim., No. 3, 492-501 (1973).
20. V. T. Shcherbakov, V. M. Muratov, R. G. Nafikov, and V. A. Lititskaya, "Experimental investigation of the load-bearing capacity of shells made of composite material and subjected to axial compression," Mekh. Kompozitn. Mater., No. 1, 93-97 (1981).
21. V. V. Khitrov and Yu. I. Katarzhnov, "The effect of the angle of the reinforcement on the load-bearing capacity of compressed wound rods," Mekh. Kompozitn. Mater., No. 4, 611-616 (1979).

MODELS FOR THE FORCE ANALYSIS OF COMPOSITE WINDING

A. I. Beil', A. R. Mansurov,
G. G. Portnov, and V. K. Trincher

UDC 678.2:678.067

1. A special force characteristic of the winding process consists in the fact that by developing a certain pressure on the underlying layers, each tension-wound loop results in their deformation and, consequently, variation in their tension. Studies of the winding process are devoted to determination of the relation between the stresses in the wound loops and the forces developed in the winding process with tensioning apparatus. The basic characteristics of the winding mechanics can be investigated on the simplest model – a ring – with subsequent generalization in the case of a cylinder. The layout of reinforcement in a radial section, as for all wound articles, assumes the form of a spiral. In designing wound articles, the spiral structure is replaced, as a rule, by a concentrically annular structure on the basis that the layer directions in these layouts differ only by a small rise angle of the spiral. In the annular model, the process of forced winding is schematized by sequential application with the initial application of annular layers on a mandrel, and then one on the other. In this case, it is assumed that a drop in stress in the wound layers occurs as a result of radial displacements of the semi-finished product.* Let us examine the case where

*The case of variation in tension in spirally arranged loops due to their slippage is discussed in [1].

Institute of Wood Chemistry, Academy of Sciences of the Latvian SSR, Riga. Institute of Mechanics, M. V. Lomonosov Moscow State University. Insitute of Polymer Mechanics, Academy of Sciences of the Latvian SSR, Riga.

Translated from Mekhanika Kompozitnykh Materialov, No. 2, pp. 303-313, March-April, 1983. Original article submitted September 15, 1982.

0191-5665/83/1902-0229$07.50

tensile winding is combined with pressure. Radial stresses can be developed on the surface of the article being wound by different methods — excessive pressure, continuous pressure applied by a steel strip, etc. (see, for example, [2, 3]). Both force parameters — the tension in the last loop and the pressure — can be varied in accordance with a preassigned program.

2. The stresses, deformations, and displacements in the i-th ring can be determined by the number of the ring and the overall number of wound layers j ($i \leqslant j$). Let us denote the radial stresses σ_r and the displacements u on the inner surface of the i-th ring for a total number of rings j by σ_r^{ij} and u_{ij}, and on the outer surface by $\bar{\sigma}_r^{ij}$ and $\bar{u}_{ij}$; the inside radius of the ring and the average circumferential stress in it by r_i and σ_θ^{ij}. Later on, the subscripts j will be dropped in certain cases. In an annular system, we have

$$\bar{\sigma}_r^i = \sigma_r^{i+1}; \quad \bar{u}_i = u_{i+1} \tag{1}$$

from the coupling condition of the rings. For the first ring laid over an elastic mandrel, we can assume

$$\lambda \sigma_r^{1j} - u_{1j} = 0, \tag{2}$$

where λ is a parameter characterizing the flexibility of the mandrel.

The process of applying the (j + 1)-th ring is associated with the advance of a given pressure (radial stress $\sigma_{r,j+1}^0$) on the surface of the j + 1 ring and with variation of the radial stress on the surface of the j-th ring. An increase in radial stress $\Delta\sigma_r^{j,j+1}$ can be represented in the form of the sum of the effects of certain components — removal of the stress $\sigma_{r,j+1}$ on the radius r_{j+1}, the application of a ring of thickness h with a negative clearance and a stress $\sigma_{\theta,j+1}^0$ in it, application of the stress $\sigma_{r,j+2}^0$ to the radius r_{j+2}, and variation of the tension $\Delta\sigma_\theta^{j+1,j+1}$ in the j + 1 loop as a function of application of the stress $\sigma_{r,j+2}^0$. Thus, we obtain

$$\Delta\bar{\sigma}_r^{j,j+1} = \Delta\sigma_r^{j+1,j+1} = -\overset{0}{\sigma}_{r,j+1} - \frac{h}{r_{j+1}} \overset{0}{\sigma}_{\theta,j+1} + \frac{r_{j+2}}{r_{j+1}} \overset{0}{\sigma}_{r,j+2} - \Delta\sigma_\theta^{j+1,j+1} \frac{h}{r_{j+1}} \tag{3}$$

for an increase in radial stress on the r_{j+1} radius. Setting $\sigma_{r,j+2}^0 = \sigma_{r,j+1}^0 + \Delta\sigma_{r,j+1}^0$ and $r_{j+2} = r_{j+1} + h$, we obtain

$$\Delta\sigma_r^{j+1,j+1} + \Delta\sigma_\theta^{j+1,j+1} \frac{h}{r_{j+1}} = \Delta\overset{0}{\sigma}_{r,j+1} + \frac{h}{r_{j+1}} (\overset{0}{\sigma}_{r,j+1} - \overset{0}{\sigma}_{\theta,j+1}) \tag{4}$$

from (3).

On applying stress increases to the boundary, stress and displacement variations, which, in addition to (4), are determined by a system consisting of an equilibrium equation, an equation of state, and Cauchy's relationship for increases in layer deformations, occur as a result of growth of the prestressed body over the entire system of j + 1 rings. Adopting the equations of an anisotropic nonlinear elastic body $e_{r,\theta} = F_{r,\theta}(\sigma_r, \sigma_\theta)$ as equations of state, we obtain

$$(\Delta\sigma_r^{i+1,j+1} - \Delta\sigma_r^{i,j+1}) + \frac{h}{r_i} (\Delta\sigma_r^{i+1,j+1} - \Delta\sigma_\theta^{i,j+1}) = 0; \tag{5}$$

$$\Delta e_{r,\theta}^{i,j+1} = \frac{\partial F_{r,\theta}(\sigma_r^{i,j}, \sigma_\theta^{i,j})}{\partial \sigma_r} \Delta\sigma_r^{i,j+1} + \frac{\partial F_{r,\theta}(\sigma_r^{i,j}, \sigma_\theta^{i,j})}{\partial \sigma_\theta} \Delta\sigma_\theta^{i,j+1}; \tag{6}$$

$$\Delta e_r^{i,j+1} = \frac{\Delta u_{i+1,j+1} - \Delta u_{i,j+1}}{h}; \quad \Delta e_\theta^{i,j+1} = \frac{\Delta u_{i,j+1}}{r_i} \quad (i = 1 \text{ to } j+1), \tag{7}$$

where $\sigma_{r,\theta}^{i,j}$ are the stresses in the i-th layer prior to application of the j + 1 layer ($i \leqslant j$).

A boundary condition, which is similar to (2), but written in terms of the increases in magnitude, is used in addition to (4). Relationships (2) and (4)-(7) form a system of 5(j + 1) + 1 linear equations to determine 5(j + 1) + 1 unknowns: $\Delta\sigma_{r,\theta}^i$, $\Delta e_{r,\theta}$ (i = 1 to j + 1); Δu_i (i = 1 to j + 2). The coefficients in (6) are determined, since the stress-strain state of a system of j rings at the moment when the j + 1 ring is applied is considered known. The accumulated stresses and displacements are determined from the relationships

$$\sigma_{r,\theta}{}^{i,j+1}=\sigma_{r,\theta}{}^{i,j}+\Delta\sigma_{r,\theta}{}^{i,j+1};\quad u_{i,j+1}=u_{ij}+\Delta u_{i,j+1}\quad (i=1 \text{ to } j);$$
$$\sigma_r{}^{j+1,\,j+1}=\sigma_{rj}{}^0+\Delta\sigma_r{}^{j+1,\,j+1};\quad \sigma_\theta{}^{j+1,\,j+1}=\overset{0}{\sigma_{\theta,\,j+1}}+\Delta\sigma_\theta{}^{j+1,\,j+1}. \tag{8}$$

As follows from the form of system (2) and (4)-(7), its right side is a vector with one nonzero component $\Delta\sigma_{r,j+1}{}^{o} + h/r_{j+1}(\sigma_{r,j+1}{}^{o} - \sigma_{\theta,j+1}{}^{o})$, which is small in the case of smooth pressure variation during winding, as a result of which the increases in stress for each step are also small as compared with $\sigma_{rj}{}^{o}$ and $\sigma_{\theta j}{}^{o}$. For the selected winding scheme, for example, the circumferential stress in the winding drops by a small amount $\Delta\sigma_\theta{}^{j+1,j+1}$, when the (j + 1)-th ring is applied. If we consider that $\sigma_\theta{}^{j+1,j+1} = \sigma_{\theta,j+1}{}^{o}$ after application of the pressure $\sigma_{r,j+1}{}^{o}$, $\Delta\sigma_\theta{}^{j+1,j+1}$ should be set equal to zero by convention (in this case, the two equations used to determine $\Delta e_r{}^{j+1}$ and $\Delta e_\theta{}^{j+1}$ cease to be independent). The difference between these methods of determining the boundary conditions vanishes when $h \to 0$. After winding the entire article, the external pressure can be removed. Variations of the stress—strain state, associated with the removal of pressure, are easily computed using the same difference scheme, if the law governing the state for unloading is known.

3. Various modifications of a discrete model have been used, for example, in solving the problem of the winding of a metallic cylinder with a composite in [4], and in analyzing the effect of binder seepage on the stress state during winding in [5]. A variant of the discrete model, where the wound filaments were considered a quasihomogeneous orthotropic linearly elastic body (in this case, computation of stress and displacement increases was considerably facilitated), and summation in the form of (8) was retained, however, was employed in [6, 7]. Further development of this approach in linearly elastic problems consists in the conversion from summation to integration in (8). This limit conversion in the linearly elastic problem has been used in a number of studies devoted to investigation of the winding of different materials [8-11]. As applies to composites, this solution was obtained by Tarnopol'skii and Portnov [12] and analyzed by Tarnopol'skii et al. [13] and Tarnopol'skii and Beil' [14]. It should be pointed out once again that the winding of a linearly elastic material was discussed in all of the studies that we have enumerated (with the exception of [5, 13-15]), and the conversion to continuous relationships was complete only within the framework of this material model.

4. The discrete approach that we have outlined can also be represented as a difference analogy of the differential statement of the winding problem. In this form, it becomes a special case of the problem of the stress—strain state of an expanding body with a given stressed state on the boundary [16, 17]. As applies to an axisymmetric ring, the solving equations include equilibrium and state equations (as before, let us examine the case of nonlinear elasticity) and Cauchy's relationships in terms of derivatives with respect to the expanding radius:

$$r\frac{d\sigma_r}{dr}+\sigma_r-\sigma_\theta=0; \tag{9}$$

$$e_{r,\theta}=F_{r,\theta}(\sigma_r,\sigma_\theta) \tag{10}$$

$$\text{or}\quad \sigma_{r,\theta}=H_{r,\theta}(e_r,e_\theta); \tag{11}$$

$$\frac{\partial e_r}{\partial R}=\frac{\partial^2 u}{\partial r\partial R};\quad \frac{\partial e_\theta}{\partial R}=\frac{1}{r}\cdot\frac{\partial u}{\partial R}, \tag{12}$$

where r is the current radius, and R is the expanding outside radius of the ring.

The Cauchy relationships are valid even in this special case of the problem only for rates of change of deformations and displacements in terms of the growth parameter R of (12). The physical side of the matter consists in the fact that the displacements of each elementary ring are initiated nonsimultaneously and are reckoned from the moment of its application to the underlying, already deformed rings. The situation is analogous to examination of the deformation process of a body with initial stresses and deformations, where deformations and displacements, which are added to the initial values, enter into the equations of the system determining the process. For the case in question, a continuous change in the initial (with respect to further growth of the ring) stress—strain state occurs together with steady growth of the ring due to the prestressed elements.

The statement that we have made follows directly from (12) on conversion to strain increments Δe_r and Δe_θ. Let us designate $\dot{u} = \partial u(r, R)/\partial R$; hereafter, we will so designate all partial derivatives with respect to the growth parameter. We then obtain

$$u=\int_r^R \dot{u}(r,\bar{R})d\bar{R};\quad \frac{1}{r}u=\frac{1}{r}\int_r^R \dot{u}(r,\bar{R})d\bar{R}=\int_r^R \dot{e}_\theta(r,\bar{R})d\bar{R}=e_\theta-e_\theta^0(r)=\Delta e_\theta; \tag{13}$$

$$\frac{du}{dr}=\frac{\partial}{\partial r}\int_r^R \dot{u}(r,\bar{R})d\bar{R}=\int_r^R \frac{d\dot{u}}{dr}(r,\bar{R})d\bar{R}-\dot{u}(r,r)=e_r-e_r^0(r)-\dot{u}(r,r)=\Delta e_r-\dot{u}(r,r) \tag{14}$$

for the strain increases. As is apparent from Eqs. (13) and (14), Cauchy's relationships are not met for increases in loop deformations, although they are valid for their strain rates. This situation also makes conversion to differentiation with respect to the outside radius R expedient in the remaining equations (9)-(11). After differentiation, we obtain

$$r\frac{\partial\dot{\sigma}_r}{\partial r}+\dot{\sigma}_r-\dot{\sigma}_\theta=0; \tag{15}$$

$$\dot{e}_{r,\theta}=\frac{\partial F_{r,\theta}(\sigma_r,\sigma_\theta)}{\partial\sigma_r}\dot{\sigma}_r+\frac{\partial F_{r,\theta}(\sigma_r,\sigma_\theta)}{\partial\sigma_\theta}\dot{\sigma}_\theta \tag{16}$$

or

$$\dot{\sigma}_{r,\theta}=\frac{\partial H_{r,\theta}(e_r,e_\theta)}{\partial e_r}\dot{e}_r+\frac{\partial H_{r,\theta}(e_r,e_\theta)}{\partial\sigma_\theta}\dot{\sigma}_\theta.$$

The resultant system of equations is solved for boundary condition (2), which is differentiated with respect to R, and the given full tensor of stresses on the expanding boundary

$$\sigma_r(R,R)=\sigma_r^0(R);\quad \sigma_\theta(R,R)=\sigma_\theta^0(R). \tag{17}$$

To use this boundary condition in the system of equations differentiated with respect to R, it must be transformed, substituting (17) in the equilibrium equation when r = R:

$$\sigma_r^0(R)+R\frac{\partial\sigma_r(r,R)}{\partial r}\Big|_{r=R}-\sigma_\theta^0(R)=0. \tag{18}$$

Let us hereafter use the derivative from the first of conditions (17) with respect to the direction r = R:

$$\frac{\partial\sigma_r(r,R)}{\partial r}\Big|_{r=R}+\dot{\sigma}_r(r,R)\big|_{r=R}=\frac{d\sigma_r^0(R)}{dR}. \tag{19}$$

Substituting (19) in (18), we obtain

$$\dot{\sigma}_r(r,R)\big|_{r=R}=\frac{d\sigma_r^0(R)}{dR}+\frac{\sigma_r^0(R)-\sigma_\theta^0(R)}{R}. \tag{20}$$

As is apparent, system (4)-(7) is a difference analogy (with a spacing h equal to the thickness of the strip being wound) of system of differential equations (12), (15), (16), and (20).

The system of equations described can be replaced by an equivalent equation of second order in rates of change in the stress σ_r or displacement u with respect to the expanding outside radius ($\dot{\sigma}_r$, $\dot{u}$):

$$\frac{\partial F_\theta}{\partial\sigma_\theta}\frac{\partial^2\dot{\sigma}_r}{\partial r^2}+\left[\frac{\partial^2 F_\theta}{\partial\sigma_\theta^2}\frac{\partial\sigma_\theta}{\partial r}+\frac{3}{r}\frac{\partial F_\theta}{\partial\sigma_\theta}+\frac{1}{r}\left(\frac{\partial F_\theta}{\partial\sigma_r}-\frac{\partial F_r}{\partial\sigma_\theta}\right)\right]\times$$
$$\times\frac{\partial\dot{\sigma}_r}{\partial r}+\left\{\frac{1}{r}\left(\frac{\partial^2 F_\theta}{\partial\sigma_r^2}+\frac{\partial^2 F_\theta}{\partial\sigma_r\partial\sigma_\theta}+\frac{\partial^2 F_\theta}{\partial\sigma_\theta^2}\right)\left(\frac{\partial\sigma_r}{\partial r}+\frac{\partial\sigma_\theta}{\partial r}\right)+\right. \tag{21}$$
$$\left.+\frac{1}{r^2}\left[\left(\frac{\partial F_\theta}{\partial\sigma_\theta}-\frac{\partial F_r}{\partial\sigma_r}\right)+\left(\frac{\partial F_\theta}{\partial\sigma_r}-\frac{\partial F_r}{\partial\sigma_\theta}\right)\right]\right\}\dot{\sigma}_r=0$$

for the boundary conditions

$$\dot{\sigma}_r=\frac{d\sigma_r^0}{dR}+\frac{\sigma_r^0-\sigma_\theta^0}{R}\quad(r=R);$$

$$\left[\lambda - r\left(\frac{\partial F_\theta}{\partial \sigma_r} + \frac{\partial F_\theta}{\partial \sigma_\theta}\right)\right]\dot{\sigma}_r - r^2 \frac{\partial F_\theta}{\partial \sigma_\theta} \cdot \frac{\partial \dot{\sigma}_r}{\partial r} = 0 \quad (r = r_0)$$

or

$$\frac{\partial H_r}{\partial e_r}\frac{\partial^2 \dot{u}}{\partial r^2} + \left[\frac{\partial^2 H_r}{\partial e_r^2}\frac{\partial e_r}{\partial r} + \frac{\partial^2 H_r}{\partial e_r \partial e_\theta} \cdot \frac{\partial e_\theta}{\partial r} + \frac{1}{r}\frac{\partial H_r}{\partial e_r} + \frac{1}{r}\left(\frac{\partial H_r}{\partial e_\theta} - \frac{\partial H_\theta}{\partial e_r}\right)\right]\frac{\partial \dot{u}}{\partial r} + \left[\frac{1}{r}\left(\frac{\partial^2 H_r}{\partial e_\theta^2}\frac{\partial e_\theta}{\partial r} + \frac{\partial^2 H_r}{\partial e_r \partial e_\theta} \cdot \frac{\partial e_r}{\partial r}\right) - \frac{1}{r^2}\frac{\partial H_\theta}{\partial e_\theta}\right]\dot{u} = 0 \quad (22)$$

for the boundary conditions

$$\lambda\left(\frac{\partial H_r}{\partial e_r}\frac{\partial \dot{u}}{\partial r} + \frac{\partial H_r}{\partial e_\theta}\frac{\dot{u}}{r}\right) - \dot{u} = 0 \quad (r = r_0);$$

$$\frac{\partial H_r}{\partial e_r}\frac{\partial \dot{u}}{\partial r} + \frac{\partial H_r}{\partial e_\theta}\frac{\dot{u}}{r} = \frac{d\sigma_r^0}{dR} + \frac{\sigma_r^0 - \sigma_\theta^0}{R} \quad (r = R).$$

The stresses, strains, and displacements in the wound ring are related to the rates of change of their values with respect to the expanding outside radius by the relationships

$$u(r, R) = \int_r^R \frac{\partial u(r, \bar{R})}{\partial \bar{R}} d\bar{R};$$

$$\sigma_{r,\theta}(r, R) = \int_r^R \frac{\partial \sigma_{r,\theta}(r, \bar{R})}{\partial \bar{R}} d\bar{R} + \sigma_{r,\theta}^0(r); \quad (23)$$

$$e_{r,\theta}(r, R) = \int_r^R \frac{\partial e_{r,\theta}(r, \bar{R})}{\partial \bar{R}} d\bar{R} + e_{r,\theta}^0(r).$$

Note that a special case of the approach described (for less general equations of the relation and boundary conditions) was used in [18] to investigate the opposite problem — the search for a winding regime that would ensure tensioning held above a minimal value in the wound loops.

5. Within the framework of the model described for the winding process, there are no major difficulties encountered where consideration is given to complex properties of the material being wound. The latter is accomplished most naturally in a continuous model of the winding process. Let, for example, the material possess viscoelastic properties, described by equations of the type (here, differentiation with respect to time is designated by a dot)

$$\dot{e}_{r,\theta} = \frac{\partial F_{r,\theta}(\sigma_r, \sigma_\theta)}{\partial \sigma_r}\dot{\sigma}_r + \frac{\partial F_{r,\theta}(\sigma_r, \sigma_\theta)}{\partial \sigma_\theta}\dot{\sigma}_\theta + D_{r,\theta}.$$

Here, $F_{r,\theta}$ takes into account the instantaneous nonlinearly elastic effects, while the components $D_{r,\theta}$ account for creep effects. Expressions for $D_{r,\theta}$ may be assigned in the form of functions of $e_{r,\theta}$ and $\sigma_{r,\theta}$, for example:

$$D_{r,\theta} = U_{r,\theta}\sigma_{r,\theta} + V_{r,\theta}e_{r,\theta} + \tilde{U}_{r,\theta}\sigma_{\theta,r} + \tilde{V}_{\theta,r}e_{\theta,r},$$

where $U_{r,\theta}$, $V_{r,\theta}$, $\tilde{U}_{\theta,r}$, and $\tilde{V}_{\theta,r}$ are constants. In this case, they reflect a two-dimensional variant of the law governing the deformations of a type of "typical body," but with nonlinear instantaneous components. In using hereditary type theories, $D_{r,\theta}$ may be functionals of the history of the stressed state.

Replacing equation of state (16) by Eqs. (24) and converting from derivatives with respect to R to derivatives with respect to t, we can convert from a system of type (2), (12), (15), and (16) equations to a solving equation of type (21) and (22) in terms of r and t with respect to $\dot{\sigma}_r = \partial\sigma_r(r, t)/\partial t$ or $\dot{u} = \partial u(r, t)/\partial t$, which is complicated by the presence of $D_{r,\theta}$ components, with the boundary conditions

$$\dot{\sigma}_r(r, t)\,|_{r=R(t)} = \frac{\partial \sigma_r^0(t)}{\partial t} + \frac{dR(t)/dt}{R(t)}\left[\sigma_r^0(t) - \sigma_\theta^0(t)\right]$$

when r = R(t).

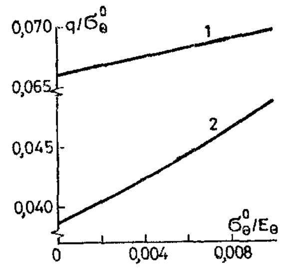

Fig. 1

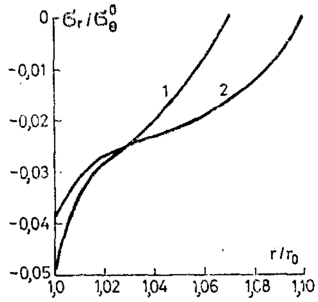

Fig. 2

Fig. 1. Effect of stress $\sigma_\theta{}^o$ in strip being wound on relative pressure $q/\sigma_\theta{}^o$ on absolutely rigid mandrel during winding of n = 100 loops. Curve 1 corresponds to $\beta = \sqrt{E_\theta/E_r} = 20$; 2) to $\beta = 40$. Ratio of initial loop thickness during winding to inside radius of article $h_o/r_o = 0.001$.

Fig. 2. Curves of relative radial stresses, computed with (1) and without (2) allowance for variation in geometric parameters during winding. $\beta = 40$; $n = 100$; $h_o/r_o = 0.001$; $\sigma_\theta{}^o/E_\theta = 0.01$.

For this case, the discrete model can be constructed as a finite-difference analogy of the continuous problem statement. Difference schemes employed to compute the relaxation process in cylindrically orthotropic rings have been used, for example, in [19, 20]. Bolotin et al. [21] utilized a time-and-radius-difference scheme to analyze the effect of seepage in the winding process; in this case, the application of a loop was considered instantaneous, while the time of its winding was divided conditionally into stages. Note that the hardening process of a semifinished product, which can be described by methods of nonlinear elasticity or the strain theory of plasticity [15], is the most significant process during winding as applies to composites. The viscoelastic effects accompanying the winding of these materials can be classed as less significant.

6. In addition to selection of a computational model, another problem requiring analysis for the problem statement of a composite winding is the selection of a standard for radial deformation and a coordinate system. The fact is that radial deformations associated with hardening of the material may reach tens of percents; in principle, this may require rejection of the hypothesis of the smallness of the deformations. It is necessary to note that during winding, large deformations are permissible only in the radial direction; they should not exceed the small deformations of pretensioning in the circumferential direction, while flexure of the layers will occur in the opposite case. The latter situation is associated with the smallness of the displacements of the loops as compared with the radius; this justifies solution of the winding problem in the geometrically linear statement. The errors that can be introduced with this approach must, however, be analyzed. It should be noted that deformations developing during winding of a single current loop are small. These small deformations apply to the current strain state; this results in the natural conversion to logarithmic radial deformation (Hencky's standard) in describing the finite strain state.

Thus, the entire computational scheme described above is completely transported into a geometrically nonlinear problem with the exception of the fact that the grid acquires a variable spacing h_{ij} and loop radius r_{ij}:

$$h_{ij}=h_{i,j-1}+u_{i+1,j-1}-u_{i,j-1};$$

$$r_{ij}=r_{i,j-1}+\sum_{k=1}^{i-1}(h_{kj}-h_{k,j-1})=r_{i,j-1}+\sum_{k=1}^{i-1}(u_{k+1,j-1}-u_{k,j-1}).$$

As in the geometrically linear case, the radial stresses will be determined by summing, since the areas on which they act vary by not more than a small magnitude of the pretension in one loop (in eliminating distortions). It is expedient to replace summation of the increases in circumferential stresses by summation of the increases in the circumferential tensile forces in each of the loops, which develop during winding of the next loop. The circumferential stresses are computed from these forces and the current thicknesses of the loops.

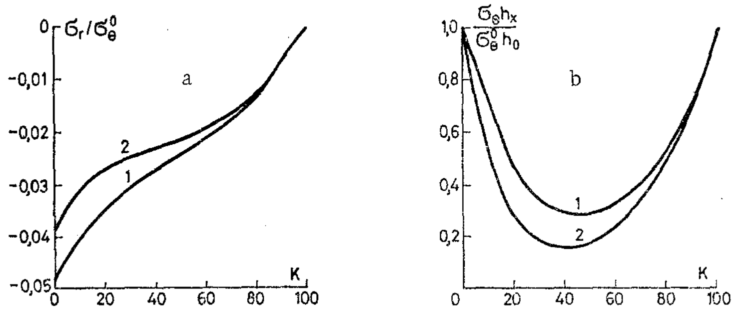

Fig. 3. Curves showing relative radial stresses (a) and relative circumferential forces (b) as function of coil number k with (1) and without (2) allowance for geometric dimensions of ring during winding. $\beta = 40$; $n = 100$; $h_0/r_0 = 0.001$; $\sigma_\theta^{\circ}/E = 0.01$.

In the case of linear (both physically and geometrically) winding with a constant tension applied to the strip, the relative pressure on the mandrel (q/σ_θ°) is independent of the tensioning force. This condition is not observed in solving the problem in the geometrically nonlinear, but physically linear statement [linear radial-compression diagram in stress σ_r vs logarithmic strain standard ($\ln h_{ij}/h_{ii}$) coordinates] (Fig. 1). The correction becomes significant, however, only for extremely high anisotropy of the strip being wound ($E_\theta/E_r > 1000$). The distribution of radial stresses is presented in Fig. 2 in comparison with the solution in the geometrically linear statement. As is apparent, the difference in the stress distributions may be significant for large E_θ/E_r. The size of the wound ring in the example under consideration, as is apparent from Fig. 2, is appreciably smaller than the thickness computed on the assumption of loop nondeformability. The difference in the stressed states corresponding to the two approaches diminishes significantly if we convert from the radii to the ordinal numbers of the loops. These relationships are presented in Fig. 3. As is apparent, the difference in stressed states diminishes appreciably in these coordinates. It should be noted that in the example cited, the anisotropy of the strip and the tensioning force are exceptionally large and go beyond the limits characteristic for the winding of the composites. For smaller values, the variation in geometric characteristics during winding can, as computations have shown, be neglected, especially in using the ordinal number of the layer as an independent variable, and the problem of composite winding can be solved in a simpler, geometrically linear statement.

7. The physical schematization that we adopted for the winding process is discrete in terms of an increasing outside radius. From this standpoint, a discrete model is more effective. In this model, however, the equilibrium equations and Cauchy's relationships for the wound loops are replaced by their finite-difference analogy. This circumstance serves as the source of a specific error in describing the winding process. The continuous model is free of the latter drawbacks; in this model, however, the process of a discrete increase in the outside radius is replaced by its continuous growth. For numerical realization of this method, it is therefore expedient that the interval in terms of the expanding outside radius be equal to the thickness of the strip being wound. Thus, both models approximately describe the physical scheme adopted for the winding process.

Determination of the stress--strain state for the discrete statement of the winding problem is associated with direct numerical realization of the discrete model. Its use results in a finite-difference scheme, i.e., repeating the solution of an increasing number of linear equations over and over (right up to the finite number of layers), the coefficients of which are varied in each step in conformity with the solution in the preceding interval. The thickness h of the layer being wound is actually adopted as the interval. As has already been indicated, consideration of time in the equation of state is not associated with primary complexities of the solution, making it only more cumbersome.

This method is characterized by an accuracy that is not very high, but that can be improved just by selecting a narrower interval and taking readings with increased accuracy (by retaining a large number of significant digits). Reduction of the interval to values significantly smaller than the thickness of the strip being wound, as we have already indicated, will give rise to distortion of the winding pattern, and is therefore undesirable. Another

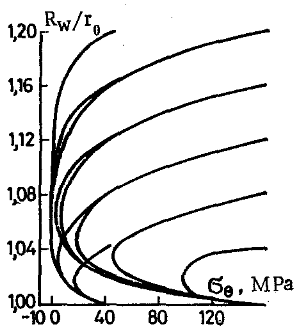

Fig. 4

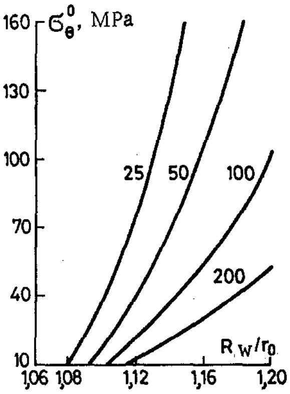

Fig. 5

Fig. 4. Variation of circumferential stresses over thickness with growth in relative size R_w/r_0 of article being wound (R_w is radius of ring after completion of winding) during winding with constant stress $\sigma_\theta{}^0$ = 40 and 160 MPa; B_r = 100 MPa.

Fig. 5. Relationships between stresses $\sigma_\theta{}^0$ during winding and relative size R_w/r_0 of article being wound at which loops with $\sigma_\theta = 0$ appear. Numbers near curves indicate values of parameter B_r (MPa).

possibility for improvement of the discrete model consists in the use of more accurate discrete analogies of the continuous relationships. Moreover, the parameters of law (6) governing the state can be determined more accurately by computing, for example, the coefficients in (6) for each layer by the iterative method, i.e., first for the stresses and strains preceding superposition of the next loop, then after computing the increases in these values for moderate stresses and strains (prior to and after superposition of the loop), and then to define these increases more precisely, etc. The latter method has also been used in performing numerical computations.

Numerical realization of the continuous model is associated with the need to solve Eqs. (21) and (22) with variable coefficients given in tabular form for an expanding (with an interval h) boundary and corresponding boundary conditions.

Use of the finite-difference method for the solution results in a computational procedure analogous to that employed in the discrete model. Formulation of the problem within the framework of the continuous model, however, is better suited to use of the entire spectrum of numerical methods, including methods of the solution of edge problems for ordinary differential equations on a computer (see, for example, [22]), than its discrete analogy. Here, we can also resort to the above-described procedure for defining the coefficients of the equations more precisely by iteration. Conversion from differential type (21) and (22) equations to integro-differential substitution (23) is also possible when the continuous method is used.

8. Numerical computations illustrating the use of the models that we have described were performed for a physically nonlinear elastic semifinished product with a strain law in the form:

$$\sigma_r = H_r(e_r, e_\theta) = E_r e_r - B_r e_r^2 + A_{r\theta} e_\theta;$$
$$\sigma_\theta = H_\theta(e_r, e_\theta) = A_{\theta r} e_r + E_\theta e_\theta; \quad A_{r\theta} = A_{\theta r}.$$

We adopted values for the constants, which corresponded to those of fiberglasses in the reworked state: $E_\theta = 5 \cdot 10^4$ MPa and E_r = 4 MPa; the entire range of the strain curves for the semifinished fiberglass products compressed perpendicular to the layers, which are presented in [15, 23], could be encompassed by a variation in B_r in the 25-200-MPa interval. The value of the coefficient $A_{r,\theta}$ assumes little significance for solution of the problem (see Eq. (22)]; let us assume as a result of its smallness $A_{r\theta} = 0$.

Computations performed using the discrete and continuous models for the above-described finite-difference schemes indicated virtually coincident results (h/r_0 was assumed equal to 0.001). Characteristic dependences of the circumferential stresses on the thickness of the article being wound, with the wound strip under low (40 MPa) and high (160 MPa) tension, are

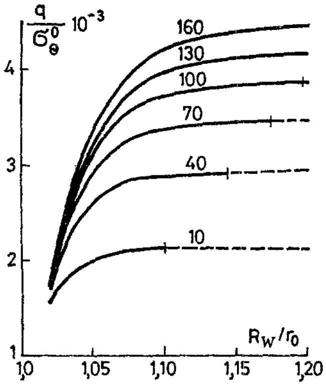

Fig. 6

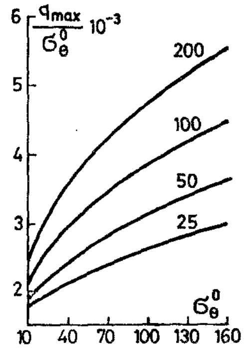

Fig. 7

Fig. 6. Relationships between relative pressure $q/\sigma_\theta{}^o$ on mandrel and relative size R_W/r_0 of ring being wound under different tensionings $\sigma_\theta{}^o$ (MPa) (numbers near curves). B_r = 100 MPa. Broken lines correspond to thicknesses at which distortions appear in rings.

Fig. 7. Relation between limiting relative pressure $q_{max}/\sigma_\theta{}^o$ on mandrel and stress $\sigma_\theta{}^o$ in strip being wound for different B_r (MPa) (numbers near curves).

presented in Fig. 4. As is apparent, complete loss of prestress by the layers, and even their transition into a region of compression, is possible with increasing thickness. In this case, the initial computational model remains, strictly speaking, valid only for a material that is rigid in the circumferential direction under compression. The circumferential properties of the layers of the semifinished product under compression are related to their curvature and become indeterminant. As is apparent, curvature of the layers is more likely under low tension during the winding of a material that is inflexible in the transverse direction under compression. This is illustrated in Fig. 5, where relationships between $\sigma_\theta{}^o$ and R_W/r_0 for which layers with $\sigma_\theta = 0$ first appear are shown.

As is apparent, an increase in the tension and the winding of materials more rigid in the transverse direction significantly increases the thickness of the rings that can be wound in a single pass without fiber distortion under constant tension. Characteristic relationships between the relative pressure on the mandrel and the thickness of the ring being wound are presented in Fig. 6. They confirm two experimental facts observed during strain measurement of the mandrels — rapid convergence of the stress in the mandrel to a horizontal asymptote, which is associated with the significant anisotropy of the material being wound [24], and the relationship between the relative pressure on the mandrel and the tension [25]. The latter is caused by stiffening of the semifinished product in the transverse direction as the tension increases, i.e., when the radial stresses in the article being wound increase. This stiffening and the reduction in the anisotropy of the material being wound, which is associated with it, are illustrated in Fig. 7, where dependencies of the limiting pressure on the tensile force are presented for different deformation characteristics of the semifinished product in the transverse direction. Thus, allowance for nonlinearity makes it possible to explain experimental data on the winding process of composites.

CONCLUSIONS

1. It is shown that the discrete laminar model adopted in winding mechanics is a finite-difference analogy of the problem of a continuously expanding solid.

2. It is convenient to examine the winding of materials with any equation of state within the framework of the continuous model described in this study.

LITERATURE CITED

1. A. I. Beil' and G. G. Portnov, "Possible mechanism of the creep of laminar composite rings under internal pressure," Mekh. Polim., No. 5, 884-890 (1973).
2. Japanese Patent No. 53-31.
3. United States Patent No. 3226273.

4. V. V. Vasil'ev, "Elastoplastic deformations of metallic pressure vessels strengthened with unidirectional fiberglass," Mekh. Polim., No. 6, 1069-1074 (1969).
5. V. V. Bolotin and Yu. N. Novichkov, Mechanics of Multilayer Designs [in Russian], Moscow (1980).
6. V. P. Nikolaev and V. M. Indenbaum, "Computing the residual stresses in wound fiberglass articles," Mekh. Polim., No. 6, 1026-1030 (1970).
7. V. M. Indenbaum and V. G. Perevozchikov, "Computing the residual stresses in wound articles formed by the method of hardening by layers," Mekh. Polim., No. 2, 284-289 (1972).
8. R. Sauswell, Introduction to Elasticity Theory [Russian translation], Moscow (1948).
9. B. S. Koval'skii, "Theory of multilayer cable winding," Dokl. Akad. Nauk SSSR, 24, No. 3, 429-432 (1950).
10. S. V. Kozhin, "Drum loading during the multilayer winding of cable," in: Proceedings of the All-Union Scientific-Research Institute of Hoisting and Conveying Machinery [in Russian], No. 8(30) (1962), pp. 44-64.
11. B. Yablonskii, "Stressed state of a multilayer design during the winding of a strip on a cylinder," Prikl. Mekh., 7, No. 12, 130-133 (1971).
12. Yu. M. Tarnopol'skii and G. G. Portnov, "Variation in tensile force during the winding of fiberglass articles," Mekh. Polim., No. 2, 278-284 (1966).
13. Yu. M. Tarnopol'skii, G. G. Portnov, and A. I. Beil', "Technological problems in composite winding mechanics," in: Mechanics of Composites [in Russian], Moscow (1983), pp. 186-222.
14. Yu. M. Tarnopol'skii and A. I. Beil', "Problems of the mechanics of composite winding," in: Handbook of Composites [in Russian], Vol. 4, Fabrication of Composites, Amsterdam (1983), pp. 56-102.
15. G. G. Portnov and A. I. Beil', "Model used to account for nonlinearity of properties of semifinished product during force analysis of composite winding," Mekh. Polim., No. 2, 231-240 (1977).
16. V. A. Pal'mov, "On stresses developed during hardening of materials," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 4, 80-85 (1967).
17. V. K. Trincher, "Statement of problem of thermal stresses in expanding body with full stress and force-strain tensors assigned to expanding surface," in: Fifth All-Union Congress on Theoretical and Applied Mechanics, Annotation of Papers [in Russian], Alma-Ata (1971).
18. N. Yu. Ochan, "Programming the winding of articles made of composites nonlinearly elastic in the transverse direction," Mekh. Polim., No. 6, 987-993 (1977).
19. V. S. Ekel'chik and E. V. Nikiforova, "Determination of the stressed state of an orthotropic viscoelastic cylinder by the grid method," in: Problems of Shipbuilding [in Russian], No. 12, Leningrad (1976), pp. 143-147.
20. G. F. Lobanova, "Creep of an hereditary-elastic orthotropic tube loaded under internal pressures," in: Elasticity and Plasticity Studies [in Russian], No. 10, Leningrad (1974), pp. 120-129.
21. V. V. Bolotin, A. N. Vorontsov, and R. Kh. Murzakhanov, "Analysis of production stresses in wound composite articles over the duration of the entire fabrication process," Mekh. Kompozitn. Mater., No. 3, 500-508 (1980).
22. V. L. Biderman, "Some computational methods used to solve problems of structural mechanics which can be reduced to ordinary differential equations," in: Strength Computations [in Russian], No. 17, Moscow (1976), pp. 8-36.
23. Yu. M. Tarnopol'skii, G. G. Portnov, and A. I. Beil', "Mechanics of composite winding," Izv. Akad. Nauk LatvSSR, No. 12(401), 80-97 (1980).
24. R. É. Brivmanis and G. G. Portnov, "Investigation of the winding process of a binding formed from unidirectional glass-fiber-reinforced tape," Elektrotekhnika, No. 3, 53-57 (1968).
25. V. L. Blagonadezhin, P. V. Mishenkov, and V. G. Perevozchikov, "Investigation of the pressure on a mandrel during fabrication of wound articles by the method of measuring mandrel strain," in: Dynamics and Strength of Machinery [in Russian], Moscow (1970), pp. 133-138 (Proceedings of the Moscow Power Institute, No. 74).