J Popul Econ (1991) 4:233–255

Journal of
Population Economics

Life cycle savings and consumption constraints*

Theory, empirical evidence, and fiscal implications

Axel Börsch-Supan and Konrad Stahl

Department of Economics, University of Mannheim, Schloß, W-6800 Mannheim, Federal Republic of Germany

Received July 20, 1990 / Accepted March 11, 1991

Abstract. Recent tests of both the pure and the extended life cycle hypothesis have generated inconclusive results on the life cycle behavior of the elderly. We extend the life cycle model by introducing a constraint on the physical consumption opportunities of the elderly which, if binding, imposes a consumption trajectory declining in age. This explains much of the received evidence on the elderly's consumption and savings behavior, in particular declining consumption, and increasing savings and wealth with increasing age. Our analysis of German data gives additional support to our theory. We finally draw the implications of the theory on the incidence of consumption and income (wealth) taxes, and on the recent (inconclusive) tests of intergenerational altruism.

1. Introduction

The life cycle hypothesis, introduced by Ando, Brumberg and Modigliani in 1954, is a center piece of the current theory of consumption and savings. In its pure form, it is characterized by a simple but powerful prediction: people save until retirement, then dissave. Consequently, individual assets accumulate and then decumulate, producing the well-known triangular wealth trajectory (Modigliani 1966). All of this has deep and well discussed effects on the intergenerational incidence of income, consumption and wealth taxes. This paper investigates the validity of these effects for West Germany; it introduces an intuitive new explana-

* Research in this paper was supported by the National Institute of Aging, Grant No 3 PO1 AG 05842-01. The paper was read at the ISPE-Conference on Fiscal Implications of an Ageing Population, Vaalsbroek, The Netherlands, May/June 1990. We are grateful to the conference participants, and especially to Richard Bird, Maurice March and, Carol Popper, and Dave Wildasin for perceptive comments, and Ernst Seiler for able research assistance. Johannes Velling and two anonymous referees provided additional constructive critique.

tion for the failure of these predictions; and it discusses fiscal implications of our alternative explanation.

A large literature has emerged collecting evidence on the pure life cycle hypothesis. Hurd (1987) surveys the results of many cross-sectional studies that directly (by investigating the asset-age profile) or indirectly (by estimating variants of the Euler Equation implied by life-cycle optimization) reject the pure life cycle hypothesis, most prominently Mirer (1979), Danzinger et al. (1982) and Hall and Mishkin (1982). Hurd correctly points out that cross-sectional analysis is marred because age is confounded with differential cohort earnings and wealth is confounded with differential mortality risks. Hurd, Diamond and Hausmann (1984) and Bernheim (1987) employ panel data from the U.S. National Longitutinal Survey (NLS) and data from the U.S. Retirement History Survey (RHS). All three studies find dissaving after retirement in favor of the life cycle hypothesis. However, all three studies consider only the younger old, up to age 69 in the NLS and up to age 73 in the RHS. Feinstein and McFadden (1989) and Venti and Wise (1990) investigate housing wealth using panel data. Both do not find evidence in favor of a reduction of housing wealth among the elderly, even when elderly homeowners move. In summary, the evidence supporting the *pure* life cycle hypothesis in the United States is weak, if not negative.

Our paper serves three objectives. One is to provide a simple, yet intuitive explanation for the failure of the pure life cycle hypothesis. The second objective is to collect additional evidence on this phenomenon from West Germany. Finally, we discuss the fiscal implications that emerge if the elderly's behavior is governed by a modified rather than by the pure life cycle hypothesis.

Our study employs cross-sectional data from the 1983 West German Income and Expenditure Survey ("Einkommens- und Verbrauchsstichprobe 1983"). We observe a decline in wealth among the younger old, supporting the findings of Bernheim, Diamond and Hausman, and Hurd. However, we find the opposite among the older old who are not observed by these authors: savings rates increase again, and consequently, assets are accumulated rather than depleted. In fact, households with heads aged 80 and above have the largest wealth among all age groups. This finding is subject to Hurd's criticism of the use of cross-sectional data. However, we present some additional evidence which rule out that our results are produced solely by the confounding effects of differential mortality and differential cohort earnings. We also rule out by assumption an operative bequest motive.

Rather, our empirical findings point to a plausible, yet until now almost uninvestigated phenomenon which is the focus of this paper: a change in the attainable utility caused by age and deteriorating health that constrains consumption.

While the pure life cycle hypothesis predicts on average a more or less flat consumption profile, the elderly may find themselves unable to consume as much as younger people. It is well known that elderly have lower food consumption in real, as well as in monetary terms. In addition, mobility is impeded by deteriorating health, possibly reducing expenditures for travel and transportation. In other words, the elderly, particularly in very old age, may find themselves physically constrained in their consumption possibilities. They may not even be able to consume all of their annuity income. Hence, the oldest old may experience forced savings which they had otherwise consumed if they were not consumption-constrained, according to the trajectory predicted by the pure life cycle hypothesis.

Consumption constraints that emerge when people age and their health deteriorates are one way to formulate attainable utility that changes with age and health status. This notion is not absolutely new. In their review article, Clark et al. (1978, p. 548) briefly mention this phenomenon. In another context, Viscusi and Evans (1990) investigate health-dependent utility functions.

The notion of health and age-dependent consumption constraints represents another modification of the pure life cycle hypothesis which resurrects the general idea of life cycle behavior in spite of a more complicated shape of the wealth-age profile. It complements other modifications, such as the introduction of a bequest motive or of precautionary savings in order to lower the risks of unexpected poor health or unanticipated longevity not supported by annuity income.

The following Sect. 2 presents a model of pure life cycle behavior, augmented only by the introduction of health-dependent consumption constraints in old age. We delineate alternative consumption and wealth trajectories after retirement, depending on annuity income, initial wealth, and health status.

Section 3 presents empirical evidence for West Germany. We examine expenditure patterns, savings behavior, and asset accumulation.

Finally, Sect. 4 discusses fiscal implications. Consumption constraints alter the differential incidence of consumption versus income taxes between generations. We show how income and consumption taxes alter consumption and wealth trajectories, and we find a simple explanation for the apparent failure of recent tests of Ricardian equivalence. We finally discuss potential implications on the reverse annuity mortgage program in the United States, the social security program to be financed by a value-added tax in Japan, and the general equity-efficiency trade-off.

2. Theory

This section presents theoretical considerations on the impacts age and health related consumption constraints might have on the retired consumer's optimal consumption and savings plan over the rest of his life time. We first do this within a highly stylized discrete time optimal control model in which a consumer faced with a deterministic consumption constraint calculates his optimal consumption plan. This model is informally extended to include a stochastic consumption constraint that eventually is influenced by the consumer's investment into his health, a stock variable. A final extension involves investment in housing as an alternative to expenditure into consumption.

2.1 Basic model

The following simple discrete time control model describes the optimal consumption and wealth trajectories between a consumer/laborer's time of retirement, $t = 0$, and his certain time of death, $t = T$. The modelling context is not to be taken too literally. In particular, for ease of exposition, we abstract from a bequest motive and from the stochastic implications arising from a precautionary savings motive. However, before extending the discussion to a more general framework, let us first describe the model and derive the results.

Our representative consumer is supposed to maximize, at retirement time $t = 0$, the discounted present value of intertemporally separable utility obtained over the remaining T periods of his life, subject to the dynamic constraint that interest and retirement annuity incomes net of consumption add to his wealth; and the $T+1$ static conditions that initial wealth is exogenously given, and subsequent wealth can never become negative.

Formally, our consumer's optimization problem is defined as follows:

$$\max \sum_{t=0}^{T-1} \beta_t \, u(c_t, h_t, a_t) \tag{1}$$

$$\text{s.t.} \quad w_{t+1} - w_t = r w_t + y - c_t \qquad p_{t+1} \qquad t = 0, \ldots, T-1 \tag{2}$$

$$w_0 \quad - \bar{w}_0 = 0 \qquad \alpha_0 \tag{3}$$

$$-w_t \leq 0 \qquad \lambda_\tau \qquad \tau = 1, \ldots, T \ , \tag{4}$$

where $\beta_t \equiv s_t/(1+\varrho)^t$,
and s_t = probability of having survived into period t,
ϱ = subjective discount rate,
c_t = consumption during period t,
h_t = level of health at time t,
a_t = age at time t,
w_t = wealth at the beginning of t,
$\bar{w}_0$ = exogenous initial wealth, $\bar{w}_0 > 0$,
r = interest rate,
y = annuity income extending over the retiree's lifetime, and
p_t, α_0, λ_t = costate variable and Lagrange multipliers, respectively, associated with the constraints.

The last constraint implies that borrowing against future annuities is not possible, as $-w_t \leq 0$, $t = 1, \ldots, T$ implies $(1+r)w_{t-1} + y - c_{t-1} \geq 0$, $t = 1, \ldots, T$. Hence current consumption must be financed out of current annuity income and current wealth. In addition, we assume the level of health h_t to be exogenously specified, and nonincreasing in t. The effect of both health $\{h_t, t = 0, \ldots, T\}$ and age $\{a_t, t = 0, \ldots, T\}$ on the consumer's utility is captured in the following assumptions:

$$\exists \, \bar{c}_t := \bar{c}(h_t, y_t), \; t = 0, \ldots, T \quad \text{with} \quad \frac{\partial \bar{c}}{\partial h_t} > 0, \; \frac{\partial \bar{c}}{\partial y_t} < 0 \ ,$$

$$\text{such that} \quad \frac{\partial u}{\partial c_t} > 0, \; \frac{\partial^2 u}{\partial c_t^2} < 0 \;\; \forall \; c_t \leq \bar{c}_t \tag{A1}$$

$$\text{and} \qquad \frac{\partial u}{\partial c_t} = 0 \qquad \forall \; c_t > \bar{c}_t \ .$$

Hence with decreasing health and increasing age, our consumer is constrained by an upper bound $\bar{c}_t$ on his consumption opportunities. We call $\bar{c}_t$ the constraint on consumption in period t. As health remains exogenous in the present model, we assume that $h_t \geq h_{t+1}$. All this implies that $\bar{c}_t \geq \bar{c}_{t+1}$, $t = 0, \ldots, T-1$. A more elegant formulation would involve an endogenous determination of the consumer's health level. However, as we will discuss in Sect. 2.2, this adds little to the analysis.

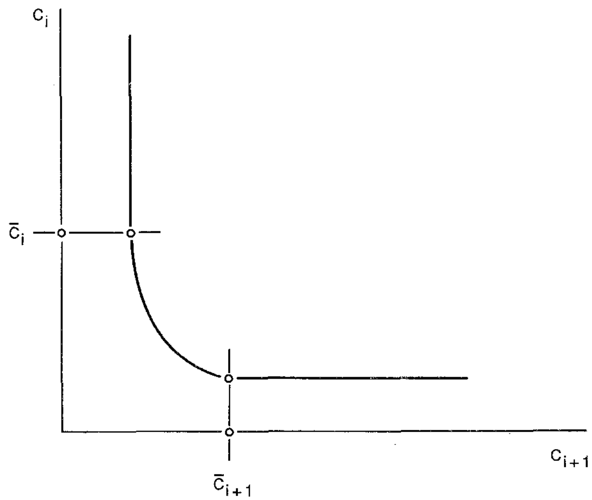

Fig. 1. Typical indifference curve

We also assume as usual that

$$\lim_{c_t \to 0} u'(c_t) = +\infty \ , \tag{A2}$$

and finally suppose that

$$s_t \geq s_{t+1} \quad \text{with} \quad \frac{s_t}{s_{t+1}} \leq \frac{s_{t+1}}{s_{t+2}} \ . \tag{A3}$$

Assumption A3 describes the increasing mortality risk associated with increasing age. Owing to A1 we can add to (1)–(4) the constraint and associated Lagrange multiplier

$$c_t - \bar{c}_t \leq 0 \ \ \gamma_t \quad t = 0, \ldots, T \ . \tag{5}$$

We also write $u_t = u(c_t)$. The typical indifference curve resulting from these assumptions is pictured in Fig. 1.

The Lagrangian function associated with problem (1)–(5) is then defined as

$$L = \sum_{t=0}^{T-1} \beta_t u(c_t) - \sum_{t=0}^{T-1} p_{t+1} \ [w_{t+1} - (1+r)w_t - y + c_t] - \alpha_0[w_0 - \bar{w}_0]$$
$$- \sum_{t=1}^{T} \lambda_t[-w_t] - \sum_{t=0}^{T-1} \gamma_t[c_t - \bar{c}_t] \ . \tag{6}$$

Any optimum must satisfy

$$\frac{\partial L}{\partial p_{t+1}} = 0 \Rightarrow w_{t+1} - w_t = rw_t + y - c_t \ , \qquad t = 0, \ldots, T-1 \ , \tag{7a}$$

$$\frac{\partial L}{\partial w_t} = 0 \Rightarrow p_t = (1+r)p_{t+1} + \lambda_t \qquad , \qquad t = 0, \ldots, T-1 \ , \tag{7b}$$

$$\frac{\partial L}{\partial c_t} = 0 \Rightarrow \beta_t u'(c_t) = p_{t+1} + \gamma_t \ , \qquad t = 0, \ldots, T-1 \ , \tag{7c}$$

$$p_0 = \alpha_0 > 0 \ , \tag{7d}$$

$$p_T = \lambda_T = 0 \ , \tag{7e}$$

$$\gamma_\tau \geq 0 \quad \text{and} \quad > 0 \Rightarrow c_t = \bar{c}_t \ , \quad t = 0, \ldots, T-1 \ , \tag{7f}$$

$$\lambda_t \geq 0 \quad \text{and} \quad > 0 \Rightarrow w_t = 0 \ , \quad t = 0, \ldots, T-1 \ , \tag{7g}$$

where all variables are evaluated at their optima. Since the Hamiltonian associated with (1)–(5) is strictly concave in the control c_t, and the Slater constraint qualifications hold, an optimal solution to the problem always exists. We therefore proceed directly with an analysis of the optimal consumption and wealth trajectories.

Optimal consumption trajectories are easily characterized in the case where consumption is unconstrained, i.e. $\lambda_t = \gamma_t = 0$. Conditions (7b) and (7c) together imply that

$$s_t u'(c_t) = \frac{1+r}{1+\varrho} s_{t+1} u'(c_{t+1}) \ . \tag{8}$$

Hence the expected marginal utility from consumption today must be equal to that obtained from consumption tomorrow, inflated by the rate of accumulation of wealth and discounted at the subjective discount rate. Hence,

$$c_t \gtreqless c_{t+1} \Longleftrightarrow (1+\varrho) s_t \gtreqless (1+r) s_{t+1} \ . \tag{9}$$

In particular, *optimal consumption decreases monotonically in the consumer's age, ie.,* $c_t \geq c_{t+1}$ *if* $\varrho \geq r$ *i.e., if the subjective discount rate is larger than the interest rate. Conversely, the consumption trajectory is strictly monotonically increasing in the consumer's age, i.e.* $c_t < c_{t+1}$ *if* $(1+\varrho)s_t < (1+r)s_{t+1}$. If the latter inequality holds for some t, a consumption trajectory that increases first and then decreases can emerge only if

$$\frac{s_{T-2}}{s_{T-1}} > \frac{1+r}{1+\varrho} \ . \tag{10}$$

In this case, there is a unique time period $\hat{t} > 0$ such that $(1+\varrho)s_{t-1} < (1+r)s_t$, $t = 0, \ldots, \hat{t}$, and $(1+\varrho)s_t \geq (1+r)s_{t+1}$ thereafter. This case is very likely to occur because the survival probability into T, s_{T-1}, is likely to be very small.

Thus much for the analysis of the unconstrained consumption trajectory, which is rather standard. The analysis of constrained trajectories involves more study. We start with the following technical

Remarks

(i) By (7b), $p_t = 0 \Rightarrow p_{t+1} = 0$; $\lambda_t = 0$.
(ii) By (7c) and (A1), either $p_{t+1} > 0$, or $\gamma_t > 0$, or both > 0 .
(iii) Suppose $\exists \tilde{t} < T$ with $\bar{c}_{\tilde{t}-1} > a$ and $\bar{c}_{\tilde{t}} < a$. Then $\lambda_t > 0$ can hold only for $t \leq \tilde{t} - 1$, with $\gamma_t = 0$. Conversely, $\gamma_t > 0$ implies $\lambda_t = 0$ for $t \leq \tilde{t} - 1$, so both consumption and wealth constraints cannot both be effective at the same time.

The following Lemmata help characterizing the constrained trajectories. Proofs are relegated to an appendix.

Lemma 1. *Suppose* $\exists \tilde{t} < T$ *with* $\bar{c}_{\tilde{t}-1} \geqslant$ y and $\bar{c}_{\tilde{t}} < y$. *Then* $p_t = 0$, $\lambda_t = 0$ $\forall t = \tilde{t}+1, \ldots, T$ *and* $\gamma_t > 0 \forall t = \tilde{t}, \ldots, T-1$.

Hence for $t \in \{\tilde{t}, \ldots, T\}$ *there are unique state and control trajectories with the properties* $c_t = \bar{c}_t \geq \bar{c}_{t+1} = c_{t+1}$, $t = \tilde{t}, \ldots, T$ *and* $0 < w_t < w_{t+1}$, $t = \tilde{t}+1, \ldots, T$.

Lemma 2. $p_t > p_{t+1} > 0$ $\forall t = 0, \ldots, \tilde{t}$ *(if* $\tilde{t}$ *exists), and* $= 0$ *thereafter.*

Lemma 3. *Let* $(1+\varrho)s_t < (1+r)s_{t+1}$, $t < \min = (\hat{t}, \tilde{t})$ *(if* $\hat{t}$ *and* $\tilde{t}$ *exist). Then* $\lambda_t > 0 \Rightarrow \lambda_{\tau+1} > 0$, $\tau = t, \ldots, \min(\hat{t}-1, \tilde{t}-1)$.

Hence if consumption is constrained at some t by $w_{\check{t}} = 0$, *it remains constrained this way if the discount rate is low relative to the interest rate. It follows that* $c_\tau = c_{\tau+1} = y$, $\tau = \check{t}, \ldots, \min(\hat{t}-1, \tilde{t}-1)$ *and* $c_\tau \geq c_{\tau+1}$ *thereafter.*

Remarks (i) to (iii) together with Lemmata 1–3 allows us to completely characterize the optimal consumption and wealth trajectories. Consider first the *consumption trajectory.* Observe first that *any optimal consumption trajectory either decreases from time zero on or it increases up to time* $t = \min(\tilde{t}, \hat{t}, \bar{t})$ *and decreases thereafter.* If constrained at $\check{t}$ by the wealth constraint (4), the consumption trajectory increases as long as $(1+\varrho)s_t < (1+r)s_{t+1}$ or otherwise decreases for $t = 0, \ldots, \check{t}-1$; it then jumps at $\check{t}$ from some level strictly above the level of annuity income y downwards to that level; it stays on the annuity income level y up to $\min(\hat{t}-1, \bar{t}-1)$, and decreases thereafter. Thus much on the consumption trajectory.

Let us now turn to the analysis of the optimal *wealth trajectory.* Suppose first that the consumption trajectory decreases monotonically. Then any wealth trajectory increases monotonically if $rw_0 + y \geq c_0$. Conversely, if $rw_0 + y < c_0$, then either w_t decreases monotonically if $rw_t + y < c_t \forall t$; or there is a $\underline{t}$ such that $rw_{\underline{t}} + y < c_{\underline{t}}$ and $rw_{\underline{t}+1} + y \geq c_{\underline{t}+1}$, in which case w_t decreases for $t < \underline{t}$ and increases thereafter.

Consider finally the case where the optimal consumption trajectory c_t increases first, and then decreases. Then w_t obviously increases monotonically as long as $rw_t + y \geq c_t$. If there exists a period $\{\underline{t}_1, \underline{t}_2\}$ with $rw_t + y \leq c_t$ for all $t \in \{\underline{t}_1, \ldots, \underline{t}_2\}$, then w_t decreases during this time period. In particular, $w_t = 0$, $t \in \{\check{t}, \ldots, \min(\hat{t}-1, \tilde{t}-1)\}$, if $\check{t}$ exists.

Figures 2–4 exemplify typical consumption and wealth trajectories. For simplicity, the trajectories are drawn in a continuous fashion. Rather than drawing the wealth trajectory directly, we have chosen to characterize the trajectory $\{rw_t + y, \; t = 0, \ldots, T\}$. While obviously moving in the same direction as $\{w_t, \; t = 0, \ldots, T\}$, it relates directly to the consumption trajectory, by virtue of the dynamic equation (2): The wealth trajectory increases (decreases) if $rw_t + y \; (\gtrless) \; c_t$, i.e. if consumption in period t is below (above) interest plus annuity income.

The general conclusions from our formal analysis are as follows:

1. *If initial wealth or annuity income is high enough,* then the elderly consumer is never willing (able) to fully exploit the returns to his wealth (or his annuity income). In this case, *wealth increases strictly for the rest of his life.*
2. *If towards the rest of his lifetime our elderly consumer happens to be constrained in his physical capabilities to consume, he will remain constrained for the rest of his lifetime.* Although constrained, he may be able to decumulate

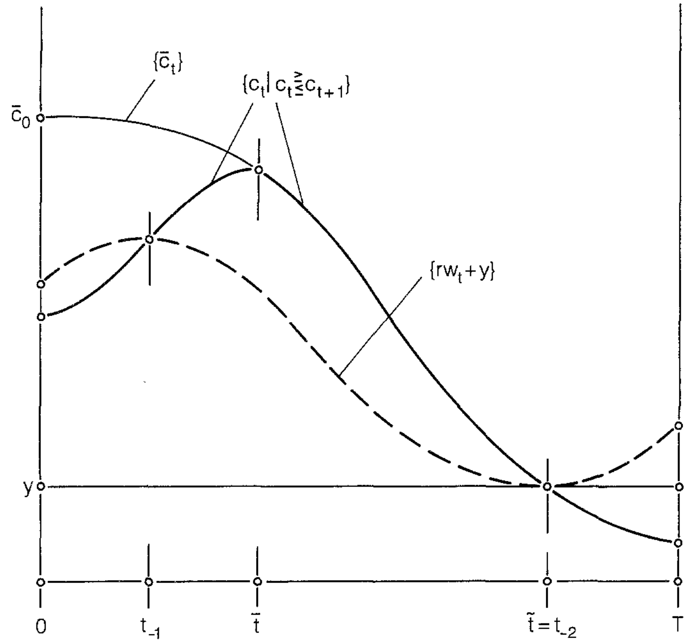

Fig. 2. Increasing/decreasing consumption trajectory and corresponding wealth trajectory

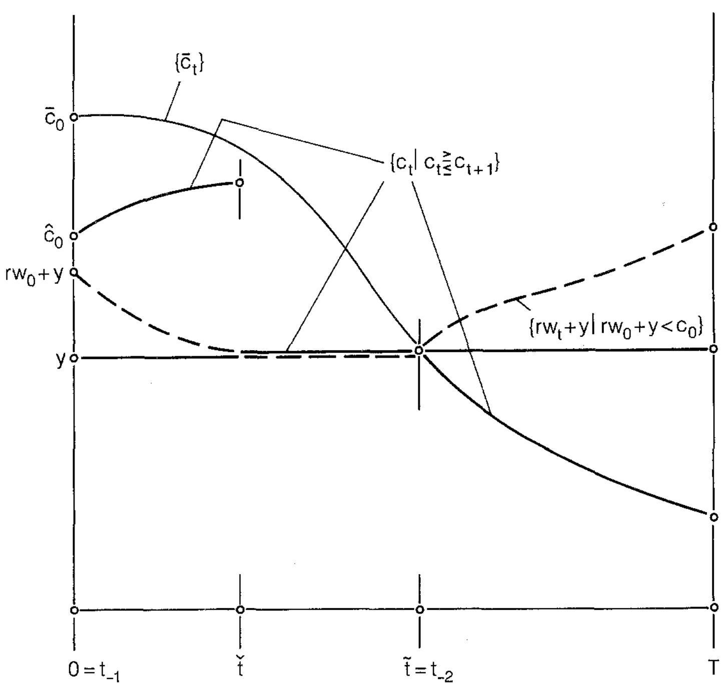

Fig. 3. Increasing/decreasing consumption trajectory and wealth trajectory with binding wealth constraint

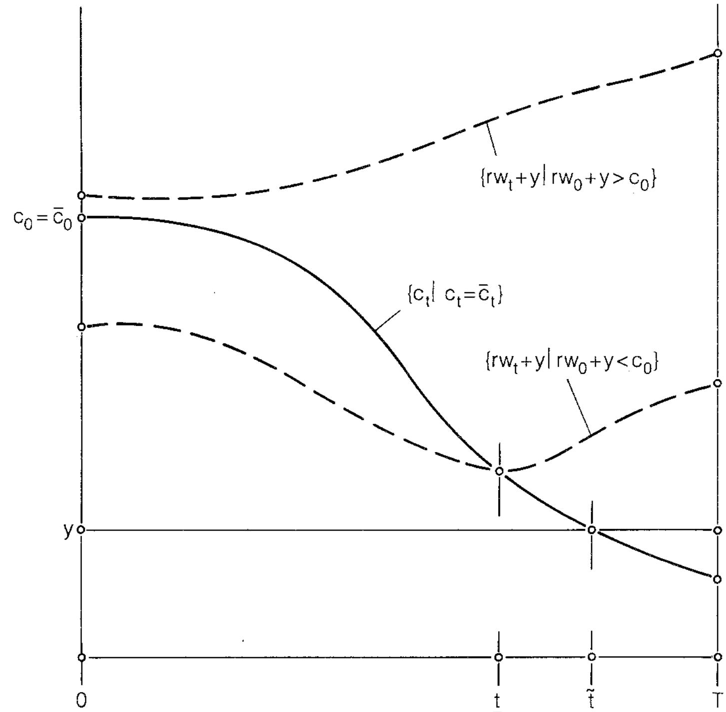

Fig. 4. Constrained consumption and corresponding wealth trajectories conditioned on high (low) initial wealth

his wealth as long as his consumption possibilities sufficiently exceed his annuity income. However, this *consumption pattern is* obviously *accompanied by an accumulation of wealth, if at any time his consumption is constrained at a level below his annuity income.*

2.2 Informal extensions

While the model discussed thus far gives us an intuitively appealing variant of the retirement portion of life cycle consumption and wealth, its ingredients remain somewhat simplistic. One might question the model set up primarily on three grounds. Firstly, both, the consumer's longevity, and his constraint on consumption possibilities can be considered a function of his health status, which in turn is influenced by his decision to invest in personal health care. Secondly, the constraint on consumption possibilities should not be perfectly predictable, and could in addition be made stochastically dependent on health investment. Thirdly and most importantly, the level of wealth at retirement time $t = 0$ should be made endogenous by backwards extending the model time over which the consumer controls consumption and accumulation of wealth.

To counter the first claim, we worked on an extension of the model along the lines of Grossman's (1972) seminal analysis, incorporating health as a second state variable whose evolution is influenced by the consumer's expenditures on personal health care. The productivity of these expenditures naturally decreases in the consumer's age. We incorporated this by supposing strictly monotonic decreases in age on the additional health generated from "large" (infinite) health expenditures. Assuming a monotonic (linear) relationship between health and consumption constraint, we arrived naturally at a trade off at any time between consumption and investment in health. While, as one might expect, the consumer heavily invests into health right after retirement in order to benefit from the thus highest productivity of health investment, and thus "postpones" the date at which the consumption constraint becomes effective, the alternative cases on the shape of consumption and wealth trajectories remain qualitatively the same. Thus the extension is of little use in bringing out the point of present concern.[1]

However, there is also an important empirical reason applying certainly to Germany for not taking all too seriously the wealth depleting effects of health investment. In contrast to countries such as the United States where the better part of health care expenditures has to be covered by the consumer himself, mandatory health insurance typically paid from annuity income withheld absorbs these expenditures.

The second of the three claims above on the stochastic nature of the consumption constraint has more serious implications on wealth and consumption trajectories. Suppose first that $\bar{c}_t$ decribes the expected, rather than the actual level at which the consumption constraint operates at t, and that $\tilde{c}_t$, $t = 0, \ldots, T$ are random variables reflecting the randomness of the constraint. Then we expect a risk averse consumer to deplete his wealth earlier inasmuch possible to him. He nevertheless might end up being constrained in his consumption opportunities at a later age, leading him to forced savings and wealth accumulation. Consider alternatively that the mean of the distribution governing $\tilde{c}_t$ is affected by the consumer's health and his investment into it. Then we expect the consumer to engage in *precautionary savings* (Kotlikoff 1986) to counteract the fate of a low realization of $\tilde{c}_t$. An analysis of this extension is technically rather involved and must be left for further work.

Let us now turn to the last of the above claims and contemplate about an extension of our model to a full life cycle one. Confronted with perfectly predictable consumption constraints, our consumer would unquestionably try to deplete his wealth as much as possible before retirement, as long as this strategy would contribute to life time utility. However, this would not completely invalidate our main point for two reasons. A first one is empirical: Ther are rather obvious cases involving the very rich who have accumulated wealth beyond any possibility of depletion. Their wealth follows the upper trajectory of Fig. 3.[2] More importantly, the consumption constraint may not allow to exploit the retirement annuity income, again a case likely to apply in West Germany with comparatively high retirement annuities. This would be of no concern in view of the pure life cycle hypothesis, if borrowing against that income would be possible. However, that option does not exist in the capital market, a clear case of market failure.[3]

---

[1] Ehrlich and Chuma (1990) published an interesting paper after this paper was written. They generalize Grossman's analysis by emphasizing the impacts of health investment on longevity. While this way T would become endogenous, an incorporation of this aspect into our model would not change the regimes and principal shapes of the trajectories derived from our model.

[2] In Hurd's (1989) theoretical model, such a trajectory follows from the very rich's bequest motive.

[3] We are grateful to David Wildasin for bringing forward this point. In fact, Boadway and Wildasin (1989) give an elegant public choice rationalization for the emergence of excessively high retirement annuity income levels.

Consider finally an alternative investment opportunity available to a retiree, namely housing investment. Opening that opportunity for a consumer faced with constrained consumption opportunities but positive wealth is likely to lead to an increase in housing investment *despite* the fact that he is only capable to reap parts of the return from it, because investing into utility generating housing stock is always a better alternative than investing into wealth that is of no use whatsoever.[4]

3. Evidence

The model presented in Sect. 2 produces several empirically testable predictions. If the elderly are indeed constrained in their consumption possibilities, we should observe consumption trajectories which decrease in old age. Corresponding to this, wealth trajectories should rise in old age, and indeed should rise monotonically if at retirement time the aged are endowed with sufficiently much wealth. We should also observe shifts in expenditure patterns away from commodities whose consumption is constrainted towards those whose consumption remains unconstrained. This section investigates the empirical content of these predictions.

3.1 Description of the data base

Our empirical analysis is based on the West German Income and Expenditure Survey ("Einkommens- und Verbrauchsstichprobe") in 1983. This survey is a representative cross-section of all West German households with annual gross income below DM 300000.–. It includes 43,050 households in 1983. Table 1 reports sample sizes by age group. The survey excludes the very wealthy, about 2% of the West German population. This is unfortunate but stacks the cards against our case as will become clear later. In addition to basic demographics, the data contain a detailed account of income by sources, wealth by asset categories, and household expenditures, computed from diaries. Unfortunately, the data do not contain information about the elderly's health status, nor about the elderly's children as long as they do not live with them.

Consumption expenditures are combined in ten categories: food, clothing, other consumer durables, housing expenditures (including rent, mortgage payments, maintenance, modernization, but excluding utilities), energy (including all utilities and heating), health (including insurance payments, out-of-pocket medical expenses, home care, prolonged nursing home stays not covered by health insurance), transportation (including commuting, car expenses, all other transportation costs for non-holiday reasons including family visits), travel (including air/rail/bus fares, accommodation and other expenses at destination for holiday trips, but excluding family visits), leisure (including education, phone, TV, movies, newspapers, pets, garden, etc.) and others (including mainly jewelry and smoking material).

Total net savings are split up into deposits and withdrawals to and from savings accounts, purchases and sales of stocks and bonds, and purchases and sales of partnerships. In addition, deposits and withdrawals from dedicated savings

---

[4] Such behavior is observed, among others, by Venti and Wise (1989).

Table 1. Sample size by age group

54 and younger	55–59	60–62	63–65	66–69	70–74	75–79	>79	55 and older	Total
28450	3528	2158	1853	2058	2713	1500	790	14600	43050

Source: West German Einkommens- und Verbrauchsstichprobe 1983
Note: Age refers to age of household head

accounts ("Bausparkassen") are reported. "Bausparkassen" are a West German-specific kind of savings- and loan institution that finance an important share of housing investments (including purchase and renovation).[5] The financial wealth categories correspond to these savings categories. Unfortunately, housing wealth is reported only poorly.[6]

Because the data are cross-sectional, measurement of age effects is confounded by differences amoung cohorts. Only panel data can separate pure age effects and pure cohort effects. Unfortunately, there are no panel data on expenditure shares and wealth trajectories available in West Germany. We are aware of the potential misinterpretations mentioned in Sect. 1. All we can do is to interpret age effects with this qualification in mind and to quote some aggregate wealth data in order to show the robustness of our findings, as we will do in Sect. 3.5.

3.2 Expenditures

Expenditure patterns clearly change with age. Table 2 presents total household expenditures in DM, classified by age-categories. Table 3 displays the corresponding expenditure shares as percentage of total expenditures[7], while Table 4 reports on expenditures as percentage of total household income. Finally, Table 5 presents per capita expenditures.

The two most important expenditure components are food and housing, covering more than half of all expenditures. The expenditure share for housing plus utilities and heating increases from about a quarter among the 50–54-year-old to almost 32% among households with heads aged 80 and above. The increase in the share of housing expenditures reflects primarily an increase in per capita space consumption. Since mobility costs are high, housing consumption is frequently not adjusted in spite of a decrease in household size. Also, housing consumption is not constrained by increasing impediments to mobility. The observed increase in the expenditure share may even underestimate actual consumption changes among elderly renters because the real price of rented housing tends to decline with age due to an increase in the so-called tenure discounts.[8]

---

5 Details on the Bausparkassen system can be found in Börsch-Supan and Stahl (1991).

6 Only assessed values are recorded which vary greatly by region and by year of last assessment. Quite frequently, they have not been updated since the purchase of the house. They do not represent potential sales prices and are therefore of little use in our empirical analysis.

7 Expenditures do not include transfers (such as taxes and gifts) and savings.

8 Tenure discounts are the different between actual rent and spot market rent for comparable dwelling units as a function of length of tenure. According to Behring et al. (1987), a 15-year tenure implies about a 15% rent discount in West Germany. Length of tenure is highly correlated with age. Almost 50% of the households aged 50 and above are renters.

Table 2. Total household consumption expenditures by age of household head 1983 (DM)

Age [years]	Food	Clothing	Durables	Housing	Energy	Health	Transp.	Travel	Leisure	Other	Total
50–54	10401	3722	4200	6733	2955	1809	7046	2161	3189	594	42810
55–59	8941	2972	3703	6063	2592	1680	5776	2084	3194	414	37419
60–62	8446	3054	3809	5491	2619	1723	5106	1928	2478	457	35111
63–65	7877	2710	3497	6013	2470	1783	5147	2123	2467	470	34557
66–69	6755	2658	2804	5041	2320	1858	3548	1988	2342	473	29787
70–74	6104	1919	2330	5027	2211	1744	2630	1638	1920	347	25870
75–79	6227	1887	2831	4699	2001	1754	2909	1599	1874	560	26341
80+	5272	1731	2371	4741	2072	1735	1427	1355	1843	545	23092

Source: West German Einkommens- und Verbrauchsstichprobe 1983

Table 3. Expenditure shares by age of household head 1983 (percentages)

Age [years]	Food	Clothing	Durables	Housing	Energy	Health	Transp.	Travel	Leisure	Other	Total
50–54	25.6	8.5	9.1	17.0	7.7	3.9	15.0	4.8	7.0	1.3	100.0
55–59	25.1	8.0	9.2	18.0	7.9	4.2	13.7	5.3	7.5	1.1	100.0
60–62	24.9	8.7	10.0	17.6	8.4	4.5	12.7	5.2	6.7	1.3	100.0
63–65	24.1	8.0	9.2	19.8	8.2	4.8	12.2	5.5	7.0	1.3	100.0
66–69	24.0	8.8	9.4	18.8	8.7	5.4	10.3	5.9	7.4	1.4	100.0
70–74	24.9	7.4	7.9	21.6	9.6	5.7	8.7	6.0	6.9	1.2	100.0
75–79	25.2	7.3	10.1	19.8	8.6	5.5	9.1	5.3	7.1	1.9	100.0
80+	23.7	6.8	10.0	23.0	9.8	6.3	5.8	5.0	7.5	2.0	100.0

Source: West German Einkommens- und Verbrauchsstichprobe 1983

Table 4. Consumption expenditures as percent of household net income 1983 (percentages)

Age [years]	Food	Clothing	Durables	Housing	Energy	Health	Transp.	Travel	Leisure	Other
50–54	16.5	5.4	6.1	10.8	4.9	2.7	10.1	3.0	4.7	0.9
55–59	16.5	5.2	6.1	11.5	5.1	2.9	9.6	3.4	5.3	0.7
60–62	18.7	6.4	7.6	12.6	5.9	3.6	9.7	3.9	5.1	1.0
63–65	19.5	6.5	7.9	15.3	6.4	4.0	10.8	4.6	5.7	1.1
66–69	19.6	7.3	7.9	15.2	7.1	4.6	9.4	4.9	6.2	1.1
70–74	20.1	6.1	6.5	16.8	7.6	4.8	7.3	4.8	5.9	1.0
75–79	19.9	5.8	8.3	14.8	6.4	4.5	7.9	4.5	5.6	1.8
80+	17.5	5.3	7.3	16.7	6.8	4.6	4.2	3.6	5.5	1.6

Source: West German Einkommens- und Verbrauchsstichprobe 1983

The only other expenditure category that exhibits a clear increase is health expenditures. The corresponding share almost doubles from 3.9 to 6.3%. In terms of net income, health expenditures peak for the 70–74-year-old, reaching 4.8%. It is notable that this is much lower than comparable United States figures: On average over all ages, United States households spend 5.7% of their net income on health (Börsch-Supan 1991). This large difference is mainly due to the difference in health insurance systems. Compared to the United States, the German health insurance system features a very broad coverage of most health expenses.

Table 5. Per capita consumption expenditures by age of household head 1983 (DM)

Age [years]	Food	Clothing	Durables	Housing	Energy	Health	Transp.	Travel	Leisure	Other	Total
50–54	3324	1191	1398	2194	967	609	2226	761	1027	203	13900
55–59	3732	1284	1667	2671	1136	736	2371	939	1346	192	16074
60–62	3960	1583	2016	2851	1315	939	2419	1025	1255	260	17623
63–65	3910	1479	1885	3280	1330	963	2570	1219	1345	244	18225
66–69	3693	1517	1583	2943	1328	1047	1830	1075	1271	248	16535
70–74	3663	1196	1336	3155	1361	1099	1468	1025	1178	223	15704
75–79	3772	1215	1693	3019	1232	1120	1810	1064	1182	336	16443
80+	3428	1071	1596	3315	1411	1148	863	817	1299	372	15320

Source: West German Einkommens- und Verbrauchsstichprobe 1983

And because the system is compulsory, it generates large implicit intergenerational transfer which flatten the age-health expenditure profile. Not covered in the German health insurance system are only self-medication, home care (with some exceptions), and prolonged nursing home stays.

All other expenditure items decline with age or stay approximately constant. Food consumption declines strongly by age. Absolute food expenditures, Table 2, are reduced to almost a half of the level at age 50, when the elderly reach age 80. This is partly due to decreasing household size. But also per capita food consumption declines strongly, as do the corresponding expenditure and income shares.

Clothing exhibits a declining expenditure share as well. The most dramatic change, however, are in the travel and transportation categories. The transportation share decreases sharply from almost a sixth of all expenditures among the 50–54-year-old households to less than 6% among the over 80-year-old households, i.e., to almost a third. Travelling features an interesting hump-shaped profile: the share first increases up to a peak shortly after the customary retirement ages, then declines.

We reach the conclusion that the patterns of expenditure shares clearly support our abstract notion of age and health related consumption constraints. Food consumption declines with age, basically for physiological reasons. Deteriorating health in particular impedes mobility, reduces the utility from travelling, or even renders travelling impossible.

3.3 Savings

Table 6 presents age-savings profiles by asset category. Net savings are computed as the difference between purchases and sales of bonds and stocks, plus the difference between deposits to and withdrawals from savings accounts, plus the corresponding differences of the smaller asset categories. Savings accounts, bonds and stocks are by far the most important asset categories. Total net savings display a surprisingly irregular cross-sectional age-profile: net savings are lowest between the ages of 35 and 45, then increase to a peak at about or just after retirement age. This peak is followed by a massive decline after the West German costumary retirement age (i.e., in the group of 66–69-year-old households). Thereafter, savings increase again to reach a life-time maximum at the oldest age category.

Table 6. Gross and net savings by age of household head (1983)

Age [years]	Savings accounts		Bausparkasse		Stocks and bonds		Partnerships		Net savings
	Depositis	Withdrawls	Depositis	Withdrawls	Purchase	Sales	Purchase	Sales	
30–34	4039	5264	2337	986	1048	636	77	10	605
35–39	4406	5706	2320	1067	1104	722	80	15	397
40–44	4790	6226	2071	850	1314	762	70	13	393
45–49	5053	6170	1837	601	1504	739	93	70	907
50–54	5300	6732	1630	599	2002	1097	72	6	544
55–59	5526	5981	1273	454	2184	1207	47	26	1360
60–62	5639	5753	887	354	2192	1174	137	14	1423
63–65	5162	5194	721	322	2274	975	48	62	1515
66–69	4451	5023	562	186	2116	1014	18	11	912
70–74	4317	3898	453	316	2169	1251	41	0	1496
75–79	4725	3915	345	98	2733	1418	18	0	2390
>79	5204	3844	281	41	3570	1934	102	0	3338

Source: West German Einkommens- und Verbrauchsstichprobe 1983

Most of the increase in net savings at old age is due to a rising balance of savings accounts. Withdrawals from savings accounts are almost constant after age 70. Purchase of less liquid stocks and bonds also increase, but so do sales. The data show a distinct change in portfolio composition from the less liquid assets (stocks, bonds, and dedicated savings) to simple passbook savings.[9]

The increase of savings activities until retirement and the decrease immediately thereafter could be interpreted as evidence in favor to the pure life cycle hypothesis. However, the savings pattern of the very old is in strong contrast to it. There is not only no dissaving after age 65; on the contrary, net savings strongly increase. This increase in savings is, of course, a reflection of the decline of total expenditures reported in the preceding subsection.

3.4 Wealth

The increase in net savings among the very old is also reflected in the increase in non-housing wealth (Table 7). As predicted by the pure life cycle hypothesis, wealth is accumulated until retirement age, then decumulated up to age 69. However, after age 70, wealth increases again to reach a maximum in the oldest age category.

In addition to total wealth, wealth composition by asset categories displays also a strong age dependency. The elderly hold most of their wealth in savings accounts, followed by stocks and bonds. Bausparkassen wealth is highest at ages 30–34, prime age for first home buyers. Bausparkassen wealth is the only wealth category featuring a steady decline with age. The category "other net wealth" comprises partnerships and debt. Until age 50, debt (in particular mortgages), outweighs shares in partnerships. At very old age, average debt is almost zero and partnership shares are very small.

Table 7 is based on averages. It should be noted, however, that the wealth distribution is highly skewed. This is evident in Table 8. Average net wealth among

9 Partnership shares also increase with old age but are very small on average.

Table 7. Non-housing net wealth by age of household head (1983)

Age [years]	Savings accounts	Bausparkassen savings	Stocks and bonds	Other net wealth	Total non-housing wealth
30–34	7221	10090	3828	–2010	19069
35–39	8115	9963	4845	–1368	21555
40–44	9341	8552	6043	–306	23630
45–49	10400	7408	7113	–878	24043
50–54	12008	5929	8710	23	26670
55–59	12198	4717	10005	151	27071
60–62	13897	3272	11097	1082	29348
63–65	14249	2648	10629	1043	28569
66–69	13310	1888	10880	1423	27501
70–74	13269	1654	12504	1293	28720
75–79	13508	1408	13163	833	28912
>79	15913	1210	14275	470	31868

Source: West German Einkommens- und Verbrauchsstichprobe 1983
Note: "Other net wealth" are partnership shares minus net debt

Table 8. Distribution of net wealth among households with head aged 50 and above (1983)

Negative net wealth	2.1%
0 .... 1000 DM	6.1%
1000 .... 5000 DM	15.8%
5000 ...... 10000 DM	11.2%
10000 ..... 33000 DM	38.5%
33000 ..... 67000 DM	15.5%
67000 .... 100000 DM	4.1%
100000 DM and above	5.0%
Median wealth	DM 14000
Average wealth	DM 28142

Source: West German Einkommens- und Verbrauchsstichprobe 1983

households with head aged 50 and above is about DM 28000,–, while median wealth is only about DM 14000,–. 8.2% of households with head aged 50 and above report no or very little wealth (net wealth between DM 0,– and DM 1000,–). 2.1% of these households report negative net wealth. Note that the data do not include the upper two percent tail of the income distribution, hence probably an even large part of the wealth distribution.

It is interesting to observe that the wealth levels are substantially larger than those reported in the United States. These are on average about three times larger (see Wise 1990, Table II). However, even the higher German wealth levels do not support the notion that wealth is accumulated in order to cover living expenses at old age. More than a half of all elderly could not cover one year's living expenses by decumulating wealth, and only 1.7% of the elderly surveyed here would be able to cover their living expenditures for five years.

There is another piece of evidence against the pure life cycle hypothesis. Social security retirement income in West Germany is relatively large: the replacement rate with respect to net income is on average 71%, while in the United States, the corresponding figure is only 53%.[10] Therefore, the much more generous German social security system should produce ceteris paribus substantially lower wealth levels in West Germany compared to the United States, if wealth were held in order to equilibrate life cycle consumption. This is clearly not the case.[11]

3.5 Conclusions from the empirical analysis

In the preceding subsections, we have collected three pieces of evidence against the pure life cycle hypothesis: neither expenditure patterns, nor savings behavior, nor asset accumulation among the oldest elderly are in accordance with the pure life cycle hypothesis. Consumption does not stay flat but declines with age. In turn, there is no dissaving in old age, much rather, savings rates are at a peak among the elderly aged 80 years and older. Finally, while asset trajectories follow the path predicted by the pure life cycle hypothesis until age 70 (i.e., asset accumulation until retirement age, asset decumulation during retirement), a reversal is evident after age 70.

In contrast, all three pieces of evidence can be explained by age and health related changes in the consumption constraint as presented in Sect. 2. The observed up-down-up-again profile of wealth over age corresponds to that of Fig. 1. The observed wealth, savings and expenditure patterns fit well to this figure. The case of Fig. 1 is the case in which elderly are constrained in their consumption possibilities to levels below current income. In this case, the elderly are forced to save their excess income, and accumulate wealth.

Table 9 gives strong evidence that this case appears to be an empirically relevant one. 31.6% of all households with head aged 50 and above have consump-

Table 9. Consumption constraints (1983) (Percentage of households, consumption expenditures in relation to annuity income)

Age [years]	Percentage of elderly in age group with a ratio of annuity income to consumption expenditure			
	<1.0	1.0 to 1.2	1.2 to 1.5	>1.5
50–54	97.7%	1.5%	0.5%	0.3%
55–59	92.0%	3.9%	2.9%	1.2%
60–64	69.3%	13.4%	11.4%	5.9%
65–69	47.3%	23.5%	18.0%	11.2%
70–74	42.9%	22.7%	20.6%	13.8%
75–79	38.1%	19.2%	21.6%	21.1%
>79	30.5%	17.3%	23.2%	29.0%

Notes: Annuity income includes public and private pensions, as well as payments from life insurance and private transfers

Source: West German Einkommens- und Verbrauchsstichprobe 1983, based on 18259 elderly of age 50 and above

[10] See Börsch-Supan (1991), Table 6. All households in West Germany are covered by social security, except for about 9% who are self-employed.

[11] Of course, ceteris paribus does not hold when comparing West German with American behavior. Historical reasons alone appear to generate much higher savings rates in Germany among all age groups.

tion expenditures below their annuity incomes. More importantly, this percentage increases from very low (2.3% for ages 50–54) to very high (almost 70% for elderly above age 80). The elderly may underreport their expenditures. Still, 29% of the elderly aged 80 and above consume less than two third of their annuity income. Incidentally, Hurd (1990) reports similar findings on American elderly's consumption from longitudinal data.

While the notion of a consumption constraint offers a simple, yet intuitive explanation for the failure of the pure life cycle hypothesis, there are also other possible explanations. They include cohort effects, mortality risks, an operative bequest motive, and precautionary savings. We will discuss these in the following paragraphs, and then attempt a synthesis.

The first objection might be that the observed age profiles of expenditures, savings and wealth in our 1983 cross-section are generated by *cohort effects*. There are two reasons why this appears unlikely. First, if changes in cohort savings attitudes had generated the age-savings profiles in Table 6 and the asset trajectories in Table 7 implied by it, earlier cohorts should have saved more than later cohorts. However, aggregate data indicate the opposite. In 1978, households headed by elderly (aged 65 and above) saved 53.9% of the average savings accounts, while five years later, these households saved relatively more, not less, namely 60.1% of the average savings levels. Micropanel data with a longer time horizon will shed more light on cohort effects than these coarse numbers. However, the direction of the aggregate change is clearly against the cohort explanation.

A second reason against a pure cohort explanation is the sheer magnitude of the consumption constraints evident in Table 9. Even if the change between the age groups in that table were generated by changing cohort attitudes toward consumption, a majority of elderly households would anyway be consumption constrained in the sense that the proportion of elderly with consumption expenditures substantially lower than annuity income is very large.

Another competing explanation for a flatter asset decumulation path than predicted by the pure life cycle hypothesis is the realization of *mortality risks* by the optimizing household. Mortality rates vary by age. Increased mortality risk at old age shifts consumption to earlier ages, hence flattens the asset decumulation path. A reversal, however, as observed in the data, is impossible given the actual evolution of mortality rates. As Eq. (10) in Sect. 2 reveals, changing mortality risks can generate an increasing, then decreasing consumption pattern only if $s_t > (1+\varrho)/(1+r)s_{T-1}$. However, the survivor probability s_{T-1} must be at least as large as s_T, and $(1+\varrho)/(1+r)$ will be one on average with proper adjustments in the capital market. Hence, changing mortality risks cannot explain the majority of elderly with very low consumption that emerges from Table 9.[12]

In contrast, one may argue that a bequest motive could produce the asset trajectory observed in Table 7. Hurd's (1989) theoretical analysis, however, shows that wealth trajectories should decline even with such a bequest motive except possibly for the very wealthy. At any rate, we cannot test this motive with our data since it lacks information on children and family relations. Empirical evidence from other studies is mixed, see the Hurd (1987) versus Bernheim (1987) controversy. Both use the U.S. Retirement History Survey with much younger elderly than in our German data. Hurd's findings are particularly interesting in our con-

[12] Hamermesh (1984) reports a similar table with results contradicting ours, but excludes the older old.

text because he claims on one hand that the elderly dissave, but finds on the other hand convincing evidence against an operative bequest motive, since elderly with children have, on average, less wealth than those without children. In recent papers, Hurd (1989, 1990) reanalyzes these data, and again finds little evidence of a bequest motive. The same holds for the West German elderly sampled in the Socio-Economic Panel (Börsch-Supan 1991). Thus all direct tests of the bequest motive as it relates to living children reject the operation of such a motive. However, Hurd's (1987) finding that the elderly dissave may prove wrong also in the United States if the elderly investigated were older than in his 1987 study.

Yet another reason for not decumulating wealth as predicted in the pure life cycle hypothesis is *precautionary savings*, e.g., for unanticipated health expenditures or unexpected longevity. Again, our survey data lacks the health variables to test these hypotheses directly. We have two reasons not to believe that precautionary savings create the observed age-savings profiles. First, while precautionary savings may contribute to less dissavings during retirement, it appears implausible that precaution alone would produce such a steep increase beyond age 70, once households have dissaved between ages 60 and 70. Second, as already mentioned, West Germany features a compulsory health insurance that covers all acute health expenses. There is little need to save for unexpected health expenditures in West Germany except for long-term institutionalization that is not related to acute illness. Indeed, precaution should certainly not produce the higher financial wealth levels observed in West Germany vis-à-vis the United States.

In conclusion, we can pretty much rule out that cohort effects, mortality changes and an operative bequest motive have generated the departures in consumption, savings and wealth patterns from what the pure life cycle hypothesis would predict. The lack of reasonable health data prevents a test of the precautionary savings motive. Hence, although we do not want to claim that only age and health related consumption constraints explain completely the elderly's consumption and savings behavior, this simple and intuitive notion appears to be well reflected in the data, possibly in conjunction with a precautionary savings motive.

Finally, one might argue that the omission of the very rich biases our results. Unfortunately, almost all of the empirical studies found in the literature lack data about the behavior of this group.[13] Yet casual observation strongly suggests that they also behave in a way suggested by our theoretical model: Since their interest income from wealth exceeds all possibilities of consumption, their wealth increases, as the upper trajectory in Fig. 1 shows. Hence, inclusion of the very rich will only strengthen our results.

4. Fiscal implications

In this final section, we elaborate on the fiscal consequences of our life cycle model of the aged's constrained consumption. We do so by first going back to our theoretical model, and then generalize to include aspects not contained in our model.

---

[13] An exception is Hamermesh and Menchik (1987), who concentrate their analysis on bequests of the very rich rather than their age-wealth profile.

4.1 Fiscal implications: theory

Our model allows for the analysis of the impacts of three kind of taxes, namely a tax on consumption; a tax on or a change in the level of annuity income; and a tax on wealth. We compare the impacts of these taxes on elderly subject to consumption-constrained behavior to those generated from a standard model resting on the pure life cycle hypothesis, i.e., without the constraint set $\{\bar{c}_t, t = 0, \ldots, T\}$. At the outset we observe that *no (marginal change in any) tax has any effect in regimes where the consumer is constrained in his consumption opportunities.* Thus, consumer behavior in Fig. 4 is not affected at all by a tax. Hence the global effect of taxes on the aged is smaller in our model relative to the pure life cycle one.

More specifically, consider the impact of a *consumption tax* on a trajectory such as in Fig. 3. Our consumer will consume at a lower level below $\tilde{t}$, and $\check{t}$ will decrease. He continue to consume less under the regime where he can only draw from his annuity. Under the consumption tax, this period extends beyond $\tilde{t}$, and this leads to a slower accumulation of wealth thereafter. All these effects are exemplified in Fig. 5 where the primes on variable values and trajectories indicate the effects of the tax. Again, consumer behavior according to Fig. 4 under which $\bar{c}_t$ is binding for all t is not effected by (a marginal change in) the consumption tax. That tax only reduces "wasteful" savings and wealth. For a community involving both young and old members, all of this implies that any increase in a tax on consumption exercises relatively stronger negative allocative effects on the younger than on the older population. In fact, *taxing the aged in this case is a socially costless activity.*

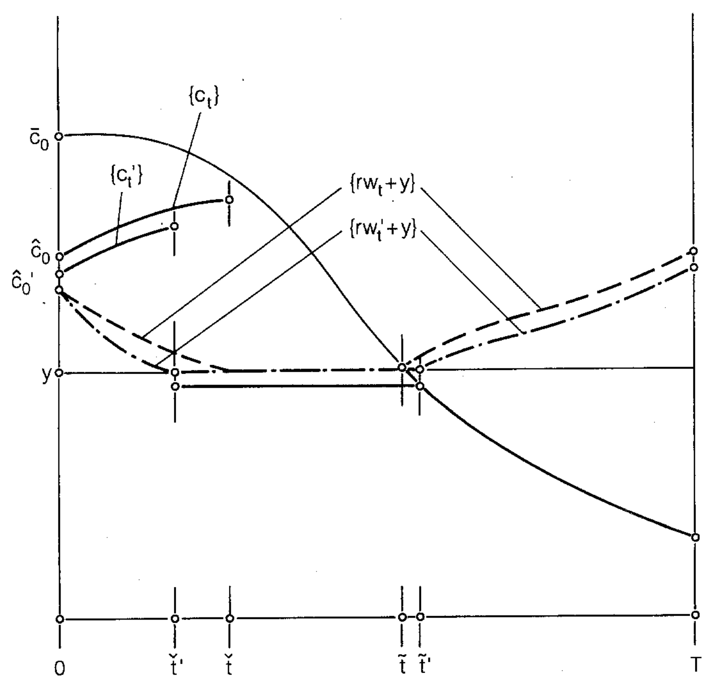

Fig. 5. Changes in consumption and wealth trajectories due to consumption tax

Similary, a *tax on annuity income* tends to generate less effects on the individual consumer constrained in his consumption opportunities. Referring again to a trajectory in Fig. 3, the period of unconstrained consumption will shorten, i.e. $\check{t}$ will decrease; and the period of consumption constrained by the reduced annuity income will increase beyond $\tilde{t}$. The impact of a *wealth tax* will be similar to that of the annuity tax. In conclusion, the effect of annuity or wealth taxes will again be moderated by the consumption constraint, implying in particular, that taxing (annuity) income or wealth hurts the aged less than the younger. This general result remains also valid in the more general case where the elderly can increase consumption opportunities by investing into their health.

In all, our theory implies that under effective consumption constraints the tax base for a regressive tax such as the consumption tax declines, and the tax base for a progressive tax such as an income or a wealth tax increases relative to the unconstrained, pure life cycle hypothesis case. Thus with the same tax structure, a given amount of tax proceeds can be collected in a more egalitarian way approaching a first-best solution, if consumption constraints are indeed effective.

Finally, there is a straightforward, yet strong implication of our theory on the recent debate about intergenerational atruism. Recent tests of Barro's (1974) theory by Abel and Kotlikoff (1988), or Altonji et al. (1989) are based on the theory's predictions that consumption growth of different generations in the extended family should be identical at any point in time; and that the intergenerational distribution of resources should not affect the intergenerational distribution of consumption. Both hypotheses cease to hold, if the older generation is constrained in its consumption opportunities. In light of our theory, it is therefore not surprising that the empirical evidence found by the above authors does not support the predictions from Barro's theory. However, the extension of pur theory into an overlapping generations model including an analysis of the intergenerational effects of taxes and debt must be left for further research.

4.2 Fiscal implications: policy

The concept of constrained consumption by the aged has several implications on policy. As far as *normative* aspects is concerned, the combination of two moves, namely a reduction in retirement annuity payments and a contigent contract providing small amounts of resources to aged in poor, and large ones to those in good health, would definitely conserve resources and thereby improve welfare. On the *positive* side, our theory provides a very straightforward explanation for the long observed failure of concepts generating cash income for the aged, such as the heavily subsidized reverse annuity mortgage program in the United States. In this program, retirees are offered to disinvest portions of their equity in their own housing units in lieu of a lifetime annuity. Despite heavy lauching efforts, the program was not accepted in so far as participation rates were very small. A possible reason for this, suggested by our theory, is that a large proportion of the aged simply do not need additional cash income because they are anyway not able to consume it.

Our theory has also quite strong implications when considering impacts of a policy proposal presently discussed in Japan. The Japanese government proposes to finance from a consumption tax the increase in social security payments required by the drastic change in the age distribution.[14] Because of the emerging demographic structure, the rationale for this proposal is that the better portion of the tax proceeds would be generated from the high consumption of the elderly themselves, with a relatively small negative redistributive impact on the younger generation. In view of our theory, however, the older elderly are in fact constrained in their consumption, consuming relatively little. Hence, we will see a negative redistributive impact on the younger households after all.

Finally, operative consumption constraints on the aged increase the divergence between equity and efficiency objectives in fiscal policy. While these consumption constraints shift more of the tax burden to the elderly relative to the pure life cycle hypothesis case, the elderly are also a captive population that can be taxed with little deadweight loss. It is now an interesting exercise in political economy to moderate between the two conflicting objectives.

Appendix

Lemma 1. Suppose $\exists \tilde{t}<T$ with $\bar{c}_{\tilde{t}-1} \geq y$ and $\bar{c}_{\tilde{t}}<y$. Then $p_t=0$ and $\lambda_t=0$ $\forall t=\tilde{t}+1, \ldots, T$ and $\gamma_t>0 \quad \forall t=\tilde{t}, \ldots, T-1$.

Proof. $p_T=0 \Rightarrow \gamma_{T-1}>0 \Rightarrow c_{T-1}=\bar{c}_{T-1}<y$ by (7c) and (A1).
Hence $w_T>0 \Rightarrow \lambda_T=0$. $p_T=0$, $\lambda_T=0 \Rightarrow p_{T-1}=0 \Rightarrow \gamma_{T-2}>0$.
By backwards induction, $p_{\tilde{t}+1}=0 \Rightarrow \gamma_{\tilde{t}}>0 \Rightarrow c_{\tilde{t}}=\bar{c}_{\tilde{t}}<a \Rightarrow w_{\tilde{t}+1}>0 \Rightarrow \lambda_{\tilde{t}+1}=0$.

Lemma 2. $p_t>p_{t+1}>0 \quad \forall t=0, \ldots, \tilde{t}$ (if $\tilde{t}$ exists), and $=0$ thereafter.

Proof. From Lemma 1, we know that $p_1=0$, $t=\tilde{t}+1, \ldots, T$.
Condition (7d)$\Rightarrow p_0>0$. Condition (7b)$\Rightarrow p_t>p_{t+1} \forall p_t>0$.
It is finally not possible that $p_t>0$, $p_{t+1}=0$, $t \in \{0, \ldots, \tilde{t}-1\}$. Suppose to the contrary that $p_t>0$, $p_{t+1}=0$. This implies $\lambda_t>0$ from (7b).
Hence $w_t=0 \Rightarrow c_t \leq a \Rightarrow \gamma_t=0$ by (A2). But (7c) together with (A1) imply that if $p_{t+1}=0$, then $\gamma_t>0$. Contradiction.

Lemma 3. *Let* $(1+\varrho) s_t<(1+r) s_{t+1}$, $t<\min(\hat{t}, \tilde{t})$ *(if* $\hat{t}$, $\tilde{t}$ *exist).*
Then $\lambda_\tau>0 \Rightarrow \lambda_{\tau+1}>0$, $\tau=t, \ldots, \min(\hat{t}-1, \tilde{t}-1)$.

Proof. (a) Suppose not. Then $\lambda_t>0$, $\lambda_{t+1}=0$. $\lambda_t>0 \Rightarrow w_t=0 \Rightarrow c_t=a$. By (7c) and Remark (iii), $\lambda_t>0 \Rightarrow \gamma_t=0$ and hence $\beta_{t+1} u'(y)=p_{t+1}$. Since $w_{t+1}=(1+r) w_t+y-c_t=0$, $\bar{c}_{t+1} \geq y \Rightarrow \gamma_{t+1}=0$, (c) implies $\beta_{t+1} u'(c_{t+1})=p_{t+2}$, where $c_t<y$ by virtue of assuming $\lambda_{t+1}=0$.
Using (7b), $\beta_t u'(y)=(1+r) \beta_{t+1} u'(c_{t+1})$

$$\Longleftrightarrow \frac{s_t}{s_{t+1}} \frac{1+\varrho}{1+r}=\frac{u'(c_{t+1})}{u'(y)} . \tag{*}$$

[14] The existence of that proposal was conveyed to us by Tatsuo Hatta, see also Hatta and Ogudi (1990).

But by (A1), RHS of (*) > 1 whereas $(1+\varrho)s_t<(1+r)s_{t+1} \Rightarrow$ LHS < 1. Contradiction.

(b) $\lambda_\tau>0$, $\tau=t,\ldots,\min(\hat{t},\tilde{t})$ follows from induction, as $(1+\varrho)s_\tau<(1+r)s_{t+1}$, $\tau=t,\ldots,\min(\hat{t},\tilde{t})$.

References

Abel A, Kotlikoff L (1988) Does the consumption of different age groups move together? A new nonparametric test of intergenerational atruism. NBER Working paper No 2490
Altonji J, Hayashi F, Kotlikoff L (1989) Is the exended family altruistically linked? Direct tests using micro data. Boston University, unpublished
Barro R (1974) Are government bonds net wealth? J Polit Econ 82:1095–1118
Behring K, Börsch-Supan A, Goldrian G (1987) Wohnungsnachfrageprognose 1995. Duncker & Humblodt, Berlin
Bernheim BD (1987) Dissaving after retirement: testing the pure life cycle hypothesis. In: Bodie Z et al (eds) Issues in pension economics. University of Chicago Press, Chicago
Börsch-Supan A (1991) Implications of an aging population: problems and policy options in West Germany and the United States. Econ Policy 12:104–139
Börsch-Supan A, Stahl K (1991) Do dedicated savings programs increase personal savings? An analysis of the West-German Bausparkassen System. J Publ Econ 44:265–297
Boadway R, Wildasin D (1989) A median voter model of Social Security. Int Econ Rev 30:307–328
Clark R, Kreps J, Spengler J (1978) Economics of aging: a survey. J Econ Liter 16:919–962
Danziger S, Gaag J van der, Smolensky E, Taussig M (1982) The life cycle hypothesis and the consumption behavior of the elderly. J Post Keynesian Econ 5:208–227
Diamond P, Hausman J (1984) Individual retirement and savings behavior. J Publ Econ 23:81–114
Ehrlich I, Chuma H (1990) A model of the demand for longevity and the value of life extension. J Polit Econ 98:761–782
Feinstein JS, McFadden D (1989) The dynamics of housing demand by the elderly. I. Wealth, cashflow and demographic effects. In: Wise DA (ed) The economics of aging. University of Chicago Press, Chicago
Grossman M (1972) The demand for health: a theoretical and empirical investigation. Columbia University, Press, New York
Hall RE (1978) Stochastic implications of the life cycle-permanent income hypothesis: theory and evidence. J Polit Econ 86:971–987
Hall RE, Mishkin F (1982) The sensitivity of consumption to transitory income: estimates from panel data on households. Econometrica 50:461–481
Hamermesh D (1984) Consumption during retirement: the missing link in the life cycle. Rev Econ Stat 66(1):1–7
Hamermesh D, Menchik P (1987) Planned and unplanned bequests. Econ Inquiry 25:55–66
Hatta T, Ogudi N (1990) Switching the Japanese Social Security system from pay-as-you-go to fully-funded. Manuscript
Hurd MD (1987) Savings of the elderly and desired bequests. Am Econ Rev 77:298–312
Hurd MD (1989) Mortality risk and bequests. Econometrica 57:779–813
Hurd MD (1990) Wealth depletion and life cycle consumption by the elderly. NBER Working Paper no 3472
Kotlikoff L (1986) Health expenditures and precautionary savings. NBER Working Paper no 2008
Mirer T (1979) The wealth-age relation among the aged. Am Econ Rev 69:435–443
Modigliani F (1966) The life cycle hypothesis of saving, the demand for wealth and the supply of capital. Soc Res 33:160–217
Venti SF, Wise DA (1989) Aging, moving and housing wealth. In: Wise DA (ed) The economics of aging. University of Chicago Press, Chicago
Venti SF, Wise DA (1990) But they don't reduce housing equity. In: Wise DA (ed) Issues in the Economics of Aging. University of Chicago Press, Chicago
Viscusi WK, Evans WN (1990) Utility functions that depend on health status: estimates and economic implications. Am Econ Rev 80:353–374
Wise DA (1990) Saving for retirement: The US case. J Jpn Int Econ 2:385–416