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Application of Newton's Law to Body Cooling

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Summary. Newton's law of cooling was used to analyze the fall in rectal temperature post mortem in 55 cases during refrigeration in a mortuary. As with solids of low thermal conductivity, there was an initial curvilinearity to the semilog plot lasting 1 to 11 hours (related to pelvic circumference) while the proper internal temperature distribution was becoming established. Thereafter there was a linear trend representing a constant percent cooling rate r_1 which lasted up to 24 hours for adults. This was followed by a less steep trend of cooling rate r_2, which was 37% less than r_1. This diminution in cooling rate occurred when the rectal temperature fell below 10°C and was ascribed to a reduction in the thermal diffusivity of fat. r_1 had the highest correlation with pelvic circumference. For the nude body in still air $r_1 = 15.6\%$ to 18.4% $(T_r - T_a)$/hour for children, 3.8% to 9.4% for adults. Wind increased r_1; the decrease due to clothing was questionable because of the paucity of cases.

Key-Words: Body Temperature — Body Cooling — Hypothermia — Thermal Constants — Cooling Constant.

It is of both theoretical and practical interest to know and understand the time course of body cooling, i.e., of the fall in body temperature [5, 6]. The rate of cooling in degrees/time, however, is not a constant but a diminishing variable as cooling proceeds and it is affected by several internal factors (size, specific heat, heat conductivity, and heat input) and several external factors (air temperature, wall temperature, air movement, vapor pressure, and clothing). The simplest trend possible for cooling is a geometric progression for the diminution of the difference between surface and air temperatures. In this case the geometric ratio serves as the measure of the rate of cooling. Newton [4] was the first to assume that cooling in general proceeds in this manner and hence it is known as his law of cooling. The application of this law to human body cooling, however, has not been systematically pursued and exploited much

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beyond passing references to the law. This paper attempts to do so for the simplest situation, namely, that of no heat input which is attainable only in the corpse. Preliminary reports have appeared elsewhere [7,8] but the data analyses were completed only during the current year. The first person to relate post mortem cooling to Newton's law was DOWLER in 1856 who wrote [1]:

"In this experimental inquiry, particularly in reference to the laws of heat in the recently dead body . . . it is of fundamental importance to keep in view the physical law of cooling first announced by Sir ISAAC NEWTON . . ."

"In a vast many human bodies, for hours after death, the calorific laws of increment and decrement, oscillation and uniformity present fundamental antitheses to the received physical and physiological theories of the day, being altogether peculiar, and not conformable to the Newtonian law until after the lapse of a period more or less prolonged, when true physical refrigeration predominates."

The meaning of much of this is obscure but DOWLER obviously thought he had found that there is a lapse of time between death and the onset of Newtonian cooling.

Dowler's studies had two short comings: 1. He scattered his measurements over too many parts of the body (axilla, rectum, abdominal cavity, chest cavity, heart, liver, brain, deep thigh and calf muscle) instead of pursuing a systematic series over only one or two parts. One reason for this lack of system was that he often made his measurements as he progressed with the dissection at an autopsy. 2. He did not pursue his measurements for a long enough interval of time; 7 hours was the longest time. Sometimes he had to stop because of "urgent circumstances". Sometimes "darkness put an end to further experiments". As a consequence, although he had thought of Newton's law of cooling, he could not begin to apply it to his data and neither can we.

Following DOWLER many post mortem studies were performed by several investigators but these were for medico-legal reasons and did not contribute directly to the application of Newton's law. Hence they will not be reviewed here.

Materials and Methods

Medico-legal cases taken to the Southern District Mortuary, Boston, Massachusetts, 1954—56, were used in this study. Of the 55 cases, 10 were women and 45 were men; 50 were white and 5 were black; 52 were adults (35 to 86 years) and 3 were children (1—6 years). Adult body weights ranged from 38 to 102 kg. The cause of death was heart failure in 40 cases, head injury in 3 cases, broken neck in 3 cases, pneumonia in 4 cases, assorted other causes in 5 cases. The shortest time interval between death and the start of measurements was about 1 hour (2 cases) and the longest was about 6.3 hours (1 case). Measurements were started within

1 to 2 hours on 29 cases and within 2—3 hours on 14 cases. The duration of measurements ranged from 8.7 to 110 hours; most were less than 24 hours.

The body lay supine on a plastic screen bed. Because the refrigerator contained 48 intercommunicating compartments through which cooled air was forced with a blower which turned on and off, it was necessary to provide for the control of the air currents around the corpse. This was done by sliding the bed into an aluminum-walled box which was in the refrigerator compartment. A gap of 1 cm existed between the end plates and the box but this space was apparently insufficient as a chimney hole for the escape of natural convection currents in still air experiments. The plates were removed and a blower was connected to the box at the head end when wind experiments were performed. All cases were exposed nude except three which were covered with one layer of wool clothing—socks and 2-piece underwear which overlapped at the waist. Subcutaneous fat thickness above the symphysis pubis was measured with a ruler in the 14 cases that were autopsied. In the remaining cases it was determined by inserting a needle until the resistance of the fascia was felt; the inserted segment was measured with a ruler.

Air and rectal temperatures were measured with thermocouples and a recording potentiometer. Temperature data were analyzed as in the previous reports [5,6]: the successive hourly differences between rectal temperature, T_r and air temperature, T_a, were divided by the initial difference and the ratios were plotted on semilog paper. Curves were fitted to the linear trends by the method of least squares to obtain the cooling constant k_1.

$$\ln [(T_r - T_a)_t/(T_r - T_a)_0] = \ln a - k_1 t \qquad \text{Eq. 1.}$$

$$(T_r - T_a)_t/(T_r - T_a)_0 = a\, e^{-k_1 t} \qquad \text{Eq. 2.}$$

where t = time in hours. The per cent rate of fall in temperature difference was then calculated by the equation:

$$r_1 = [1 - \text{antiln}\,(-k_1)] \times 100 \qquad \text{Eq. 3.}$$

The values of r_1 were correlated with body dimensions including weight, height, pelvic circumference, pubic antero-posterior diameter (measured with a pair of calipers), surface area [9], lean body mass [3], and total body fat (weight — lean body mass).

Results

Examples of rectal cooling curves are shown in Fig. 1 — one for a child in still air and one for an adult in a wind of about 3.8 m/sec. The semilog plots of the corresponding temperature ratios, $(T_r - T_a)_t/(T_r - T_a)_0$, are given in Fig. 2. These show an initial convexity. This was always present and lasted about 1 hour for children, 3 to 11 hours for adults. This interval, y, could be related to the pelvic circumference, x, by the simple equation:

$$y = 0.28\, e^{0.033x}. \qquad \text{Eq. 4.}$$

S.E. = $\pm$ 1.1 hour., correlation coefficient = 0.863. The curves then followed a linear trend (Fig. 2). This inflected to a less steep trend after 6 to 8 hours of cooling for the children, after about 24 hours for the adults. The cooling constant and per cent cooling rate for the first trend

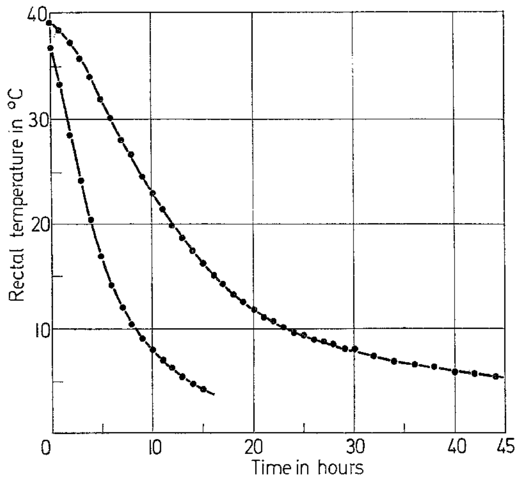

Fig. 1. Examples of the fall in rectal temperature during refrigeration. Upper curve: case 15 ♀, nude, in wind. Lower curve: case 26 ♂, a child, nude, in still air

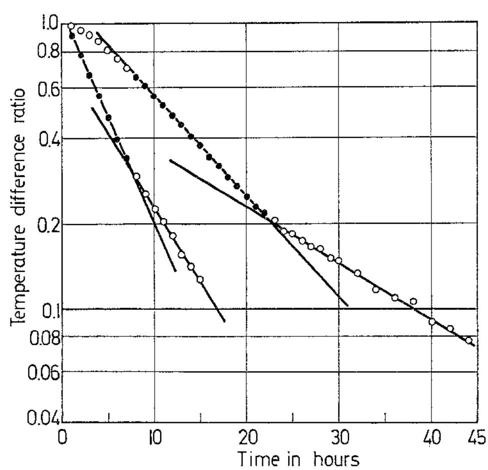

Fig. 2. Time course of the logarithm of $(T_r - T_a)_t/(T_r - T_a)_0$. Upper curve, case 15; lower curve, case 26; same data as in Fig. 1. Initial convexity shown by open circles. Points for k_1 shown by solid circles; for k_2 by open circles

were designated k_1 and r_1, respectively, and for the second trend, k_2 and r_2. Examples for the initial trend for adults in still air are shown in Fig. 3.

The relation of r_1 to pelvic circumference for all cases, nude in still air, is shown in Fig. 4. Two correlations were calculated: a linear one for the

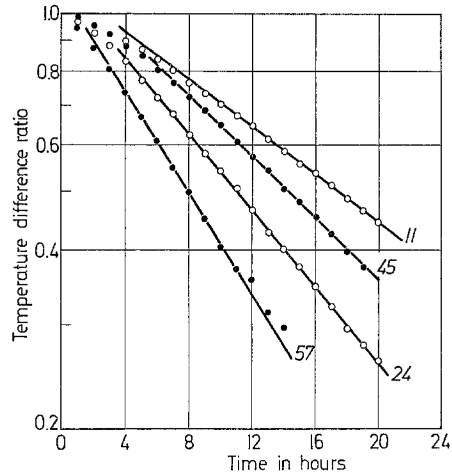

Fig. 3. Examples of semilog plots of rectal cooling of adults, nude in still air, showing extreme and intermediate rates. Top curve, case 11 ♀, $r_1 = 4.5\%\ (T_r - T_a)$/hr. Second curve, case 45 ♂, $r_1 = 5.8\%$. Third curve, case 24 ♂, $r_1 = 7.1\%$. Bottom curve, case 57 ♂, $r_1 = 9.4\%$

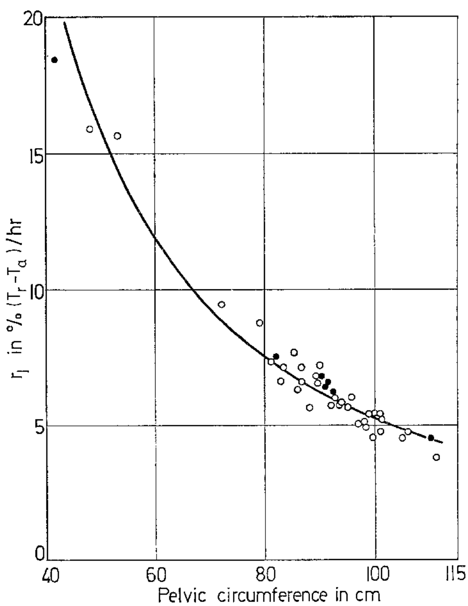

Fig. 4. Relation of r_1 to the circumference of the pelvis. ○ male. ● female. Nude, still air, adults and children. Correlation coefficient = 0.973

adult cases only and a curvilinear one for all cases by means of the equations:

$$r_1 = a + bC \qquad \text{Eq. 5.}$$

$$r_1 = aC^m \qquad \text{Eq. 6.}$$

where C = circumference and a, b, and m are constants. The coefficient for the curvilinear correlation was -0.973; for the linear correlation for adults, -0.925. Correlations for other body dimensions are summarized in the Table. The best correlation was with the circumference of the pelvis.

Table. *Relation of r_1 to body dimensions nude, still air*

	Linear (adults)				
	No. of cases	a	b	$\pm$ SE	r
Pelvic circumference	37	18.05	− 0.1290	0.46	− 0.925
Pubic AP-diameter	37	11.97	− 0.2903	0.89	− 0.671
Weight	37	11.23	− 0.0752	0.52	− 0.903
Total body fat	39	8.54	− 0.1277	0.52	− 0.899
Lean body mass	39	12.01	− 0.1211	0.79	− 0.789
Height	39	12.82	− 0.0405	1.12	− 0.329
Surface area	39	14.48	− 4.8053	0.64	− 0.843
	Curvilinear (all cases)				
	No. of cases	a	m	$\pm$ SE	r
Pelvic circumference	40	8000	− 1.5919	0.70	− 0.973
Pubic AP-diameter					
Weight	40	88.0	− 0.6410	0.76	− 0.969
Total body fat					
Lean body mass					
Height					
Surface area					

a, b, m = constants in Eq. 5 and 6; SE = standard error; r = correlation coefficient.

The effect of wind and of clothing on r_1 is shown in Fig. 5. The coefficient of correlation of r_1 to pelvic circumference was -0.977 in wind. There were not enough experiments with clothing to establish a correlation. The slope of the curve in wind is steeper than that in still air. The two curves intersect at a circumference of about 132 cm. According to the linear correlation of weight to pelvic girth (not shown here), this

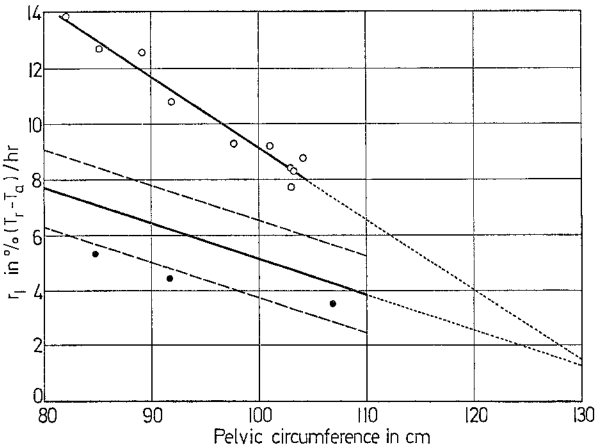

Fig. 5. Relation of r_1 for adults to the pelvic circumference. ∘ nude, wind. • clothed, still air. Bottom continuous curve = linear curve fitted to data for adults in Fig. 4. Dashed curves = $\pm$ 3 (S.E) of this curve

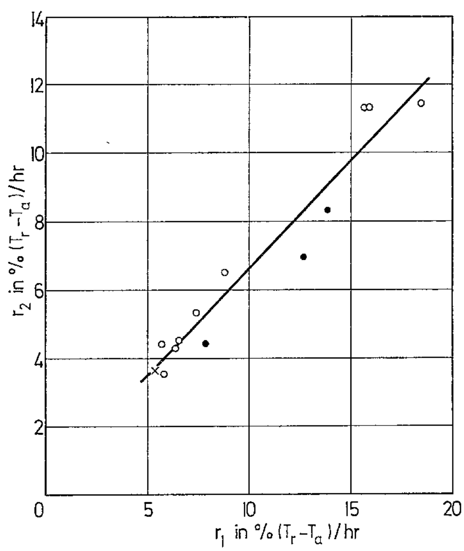

Fig. 6. Relation of r_2 to r_1, ∘ nude, still air. • nude, wind. × clothed, still air

corresponds to a weight of about 130 kg. A possible inference is that at greater circumferences or weights, wind would have an inappreciable effect on r_1.

Values for r_2 were obtained for 13 cases including 3 children. In one case the cooling trend depicted by r_2 continued until $(T_r - T_a) < 1°$ at about 80 hours of cooling. The relation of r_2 to r_1 is shown in Fig. 6. The trend is linear with slope of 0.63 regardless of body size or conditions — nude or clothed, wind or still air. Finally the cooling of the 5 black cases fell within the range of the white cases. Also no significant differences between male and female could be found as shown for example in Fig. 4.

Discussion

As explained in a previous report [6], the initial convexity of the curve on the semilog plot is due to the time required for the "cold front" to travel from surface to center, i. e., for a particular temperature distribution to develop from center to surface. After this has taken place, $(T_r - T_a)$ diminishes geometrically with a constant $-k$. It is reasonable that the time required for the "cold front" to move in should depend on the distance from surface to center as given by Eq. 4.

According to the formal mathematics for an infinite cylinder [6],

$$k = \beta_1^2 \, K / \varrho c a^2 \qquad \text{Eq. 7.}$$

where $\beta_1 = a$ constant dependent upon the heat transfer coefficient. (The subscript refers to the first term of an infinite series.)
K = thermal conductivity; ϱ = density; c = specific heat; a = radius.

Since the pelvic circumference is related to the radius, it is logical by Eq. 7 that the primary dimension with which k_1, and therefore r_1, should be inversely related should be the pelvic circumference. The correlations of r_1 with the other body dimensions in the Table are secondary and arise from the primary correlations of these dimensions with the pelvic circumference. For example it would not be expected that remote parts, such as arms and head, would effect the cooling of the rectum. Yet they do contribute to the weight of the body and r_1 was found to be correlated with weight ($r = -0.969$) because weight had a high correlation with circumference ($r = 0.956$).

As to the significance of the second per cent cooling rate, r_2, when first encountered it was considered to be an artifact due to some technical fault. None was found, however. Since thereafter r_2 was obtained whenever measurements were continued for more than about 24 hours, it become necessary to accept it as part of the course of cooling and to find an explanation for it. Since it occurred distinctly in three wind experiments (one shown in Fig. 2.), its appearance under still air conditions could not be ascribed to a possible diminution in the heat transfer coefficient as cooling progressed. The average T_r at the onset of the change from r_1 to r_2 was 9.4° C (5.0° to 13.6° C).

According to Eq. 7, k_1, and therefore r_1, is proportional to $K/\varrho c = \alpha$, the thermal diffusivity. A decrease in α, due either to a decrease in K

or an increase in ϱc or both, causes a decrease in k_1. Calculations based on the thermal constants in a handbook [2] disclose that α for water diminishes by 4.2% on cooling from 15° to 5° C. The relation of r_2 to r_1 in Fig. 6, however, has a slope of 0.63, i.e., r_2 was 37% less than r_1. Hence the decrease in α for water was only a minor factor causing the change from r_1 to r_2. This leaves the possibility that a decrease in α for fat is the primary factor. No data on the temperature dependence of the thermal constants for fat could be found in the literature, however, and so the possibility could not be readily tested.

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