

Three-Gas Diffusion – Experimental and Theoretical Study*

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Summary. The purpose of this work was to compare experimental diffusion among three gases with the solution given by Stefan's equations to understand better how this phenomenon can work in the multi-component alveolar gas.

Experiments were performed in a cylinder full of beads open at one end and closed at the other in which a mixture of oxygen with helium or argon or sulphur hexafluoride could diffuse with ambient air through the open end. We solved Stefan's equations for the non-steady state by a finite-difference method and applied them to our experimental conditions with diffusion coefficients we had measured in binary experiments. We then made experiments and calculations to show the influence of the beads on gas transport.

Provided that diffusion is the only phenomenon, experimental and theoretical curves are very close together. Moreover beads nearly stop motions due to vortices or small differences of density.

We conclude that:

Stefan's equations should replace Fick's equations when more than two gases are involved.

One should bear in mind the possible influence of gravity and devise diffusion experiments accordingly.

In small spaces such as alveoli the influence of gravity must be negligible compared to diffusion.

Key words: Ternary diffusion – Binary diffusion – Gravity – Oxygen – Inert gases.

INTRODUCTION

The importance of gaseous diffusion in airways has been surmised as early as 1832 by Graham. In 1917 Krogh and Lindhard raised this idea again and

questioned whether mixing by diffusion was complete and whether ventilation of the lobes was equal. In 1946 Rauwerda tried to answer the first question with a simple model of the lung. Many other models were suggested by different authors which, although based on Weibel's data (1963), lead to different results. Complex mixtures including heavy or light gases were used to enhance the effect of diffusion. However, under conditions of more than two gases in a mixture, Fick's law (1855) does not apply any more. The appropriate equations for the diffusion of more than two gases have been available since Stefan (1871), and experiments by Wretschko (1870) showed special features of ternary diffusion.

Recently Chang et al. (1975) called attention to possible implications of ternary diffusion in respiratory physiology, and Modell and Farhi (1976) did in vitro studies in which the concentration of oxygen diffusing with helium and nitrogen was recorded.

The aim of the present work was to apply Stefan's law to conditions most often met during ventilation, i.e., to the non-steady state. Theoretical curves were compared with experimental findings obtained with mixtures of three different gases.

THEORY

According to Fick's law:

$$N_1 = -D_{12} S \frac{\partial C_1}{\partial x} \quad (1)$$

where:

N_1 : flux of gas 1 in moles per unit time

D_{12} : diffusion coefficient between gases 1 and 2

S : diffusion area

C_1 : concentration of gas 1 in moles per unit volume

x : distance.

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Differentiating with respect to x and taking into account the conservation of mass yields:

$$\frac{\partial C_1}{\partial t} = D_{12} \left\{ \frac{\partial^2 C_1}{\partial x^2} + \frac{1}{S} \frac{dS}{dx} \frac{\partial C_1}{\partial x} \right\} \quad (2)$$

which in the simple case of a constant diffusion area

($\frac{dS}{dx} = 0$) yields:

$$\frac{\partial C_1}{\partial t} = D_{12} \frac{\partial^2 C_1}{\partial x^2} \quad (3)$$

This equation can be integrated for cases with simple geometry. For the particular case of a cylinder of length l , closed at one end and initially filled with a gas of concentration C_A diffusing out at the other end into a medium of constant concentration C_B , concentration C_1 in the cylinder at distance x from the opening and at time t is given by Crank (1956):

$$C_1 = C_B + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(C_B - C_A)}{n} (\cos n\pi - 1) \sin \frac{n\pi x}{2l} \exp(-D_{12} n^2 \pi^2 t / 4l^2). \quad (4)$$

If the cylinder of length l is closed at both ends with an initial concentration C_A from 0 to $l/2$ and C_B from $l/2$ to l , concentration C_1 at distance x is given by Crank (1956):

$$C_1 = \frac{C_A + C_B}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(C_A - C_B)}{n} \cos \frac{n\pi x}{l} \sin \frac{n\pi}{2} \exp(-D_{12} n^2 \pi^2 t / l^2) \quad (5)$$

We now consider the case of three gases with:

C_1, C_2, C_3 : concentrations of the three gases in moles per unit volume

a_1, a_2, a_3 : ratios $C_1/(C_1 + C_2 + C_3), C_2/(C_1 + C_2 + C_3), C_3/(C_1 + C_2 + C_3)$ ranging from 0 to 1 with $a_1 + a_2 + a_3 = 1$.

N_1, N_2, N_3 : respective fluxes with $N_1 + N_2 + N_3 = 0$.

D_{12}, D_{23}, D_{31} : binary diffusion coefficients.

$$S \frac{\partial C_1}{\partial x} = \frac{1}{D_{12}} (a_1 N_2 - a_2 N_1) + \frac{1}{D_{31}} (a_1 N_3 - a_3 N_1)$$

$$S \frac{\partial C_2}{\partial x} = \frac{1}{D_{12}} (a_2 N_1 - a_1 N_2) + \frac{1}{D_{23}} (a_2 N_3 - a_3 N_2) \quad (6)$$

$$S \frac{\partial C_3}{\partial x} = \frac{1}{D_{31}} (a_3 N_1 - a_1 N_3) + \frac{1}{D_{23}} (a_3 N_2 - a_2 N_3)$$

These equations, worked out by Stefan (1871), reduce to equation (1) in the presence of two gases only. They can be used directly in the steady state as has been done by Toor (1957) and Chang et al. (1975). For the non-steady state which is the case in the lung, we have solved them by a finite difference method with an explicit scheme. Stability and convergence criteria are close to those involved in parabolic equations. For stability, one must choose such iteration steps Δx and Δt that the inequality $D \cdot \Delta t / (\Delta x)^2 < 1/2$ is satisfied. On the other hand it is desirable for the sake of convergence that this ratio be as close to $1/6$ as possible. Finally it must not be so low that it brings about round-off errors (Collins, 1973).

As will be seen later, only two different diffusion coefficients are involved in our experiments. Steps Δx and Δt will then be selected in such a way as to satisfy the preceding conditions for both coefficients.

Nevertheless, merely by inspection of equations (6), it can be seen that the flux of one component can exist without a concentration gradient for that component or even in the same direction as that concentration gradient. Those features, called by Toor osmotic diffusion and reverse diffusion, cannot be accounted for by Fick's law. We tried to demonstrate them.

EXPERIMENTAL METHOD

As described previously (Brès and Hatzfeld, 1975, 1976), we used a cylinder 1 m long and 96 mm in diameter which is open at one end. Oxygen concentration was recorded half-way along the length of the tube (50 cm from the open end) with an electrolytic cell¹ (Rost, 1970). The current through it is 0.25 μ A for every 10% of oxygen concentration at 101 kPa. By use of Faraday's law this corresponds to an oxygen consumption of 0.2 mm³ per h when the surrounding gas has a concentration of 20% oxygen. This oxygen intake is negligible when compared to the 1.8 l of gas in the cylinder as calculated from dimensions and equation (7) (given in discussion).

The linearity was evaluated by comparison with O₂ N₂ mixtures assayed by the Scholander method with an accuracy of 0.5% of the reading. Before each experiment the meter was calibrated with air (20.95% O₂). The drift cannot be estimated during an experiment but we rejected all recordings for which the error of the reading of air after the experiment was more than 0.3% O₂.

To reduce possible gas whirling motions which would accelerate equilibration the diffusion chamber was filled with glass beads 6 mm in diameter. This entails an increase in the path of the gas molecules which is difficult to estimate and thus precludes the use of

¹ Beckman model OM 10

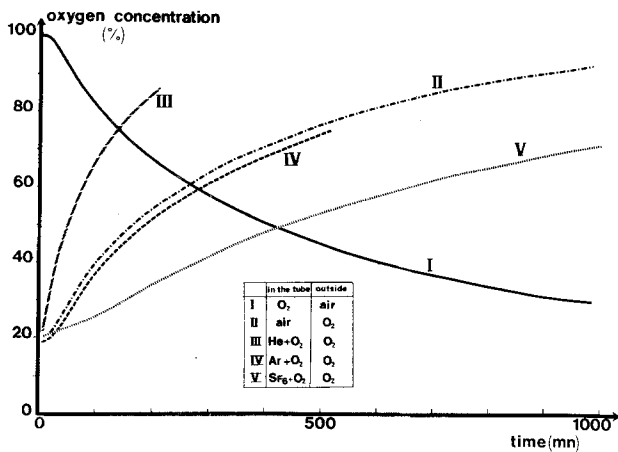


Fig. 1. Binary diffusion. Experimental curves of oxygen versus time half way along the cylinder (50 cm from both ends)

diffusion coefficients available in the literature. Therefore, we measured the apparent coefficients of binary experiments in our cylinder full of beads and used them in Stefan's equations.

MEASUREMENT OF DIFFUSION COEFFICIENTS

We used the following mixtures for these binary experiments as well as for the ternary experiments which were done later: 78.8% He 21.2% O₂, 79% Ar 21% O₂, 78.24% SF₆ 21.76% O₂.

First we measured the diffusion coefficient between oxygen and nitrogen by filling the cylinder with oxygen which was allowed to diffuse with ambient air through the open end. This yielded a curve which we compared to the theoretical curves obtained from equation (4) with various diffusion coefficients. The experimental curve (I, Fig. 1) fits the computed curve for an oxygen-nitrogen diffusion coefficient of 0.15 cm²/s. We then carried out a similar experiment where the air in the cylinder diffused with oxygen passing through a small chamber at the open end (curve II). By again applying equation (4) we found an identical diffusion coefficient of 0.15 cm²/s.

Gas motions which might be produced by this external oxygen flow are therefore negligible and we were able to use this device to measure diffusion coefficients between helium and oxygen (curve III), argon and oxygen (curve IV), and sulphur hexafluoride and oxygen (curve V). We found values of 0.52 cm²/s, 0.14 cm²/s, 0.063 cm²/s respectively.

Our sensor allowed measurement of diffusion coefficients only between oxygen and another gas. Nevertheless since the masses and cross-sections of nitrogen and oxygen are very similar, one may assume

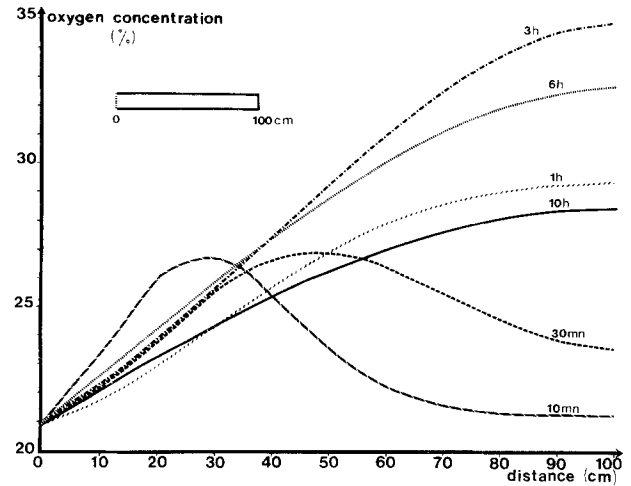


Fig. 2. Ternary diffusion between He, O₂, N₂: Theoretical curves of oxygen concentration versus distance at different times from the beginning of diffusion. 0 cm is at the open end

that their diffusion coefficients with a third gas are also very similar. From the theoretical formula given by Radford (1964) one gets the following ratios:

$$\frac{D_{N_2He}}{D_{O_2He}} = 0.96 \quad \frac{D_{N_2Ar}}{D_{O_2Ar}} = 1.$$

We then adopted the following values of the diffusion coefficients: O₂ N₂: 0.15 cm²/s, O₂ He and N₂ He: 0.52 cm²/s, O₂ Ar and N₂ Ar: 0.14 cm²/s, O₂ SF₆ and N₂ SF₆: 0.063 cm²/s.

THREE-GAS CURVES

We allowed each of our three mixtures filling the tube to diffuse with ambient air (79.05% N₂ 20.95% O₂). In each case we calculated the behavior of the three components with our computer program. As in those mixtures helium or argon or sulphur hexafluoride are assumed to have the same diffusion coefficient with oxygen and with nitrogen, they behave as though they were in a binary mixture (Stefan, 1871).

For the helium mixture Figure 2 shows the theoretical concentration of oxygen as a function of distance for different times. These curves display a maximum which increases and moves to the closed end up to a value of 34% at 3 h. Then the whole profile falls back to the equilibrium value. At the measuring point (50 cm from the open end) the oxygen reaches a maximum of 29.5% at 4 h.

For the three mixtures, Figures 3, 4, 5 show the theoretical curve of oxygen concentration at 50 cm (A) and the experimental curves obtained in the following conditions: vertical tube with opening down (B), vertical tube with opening up (C), and horizontal tube (D). For argon the experimental curve is too close

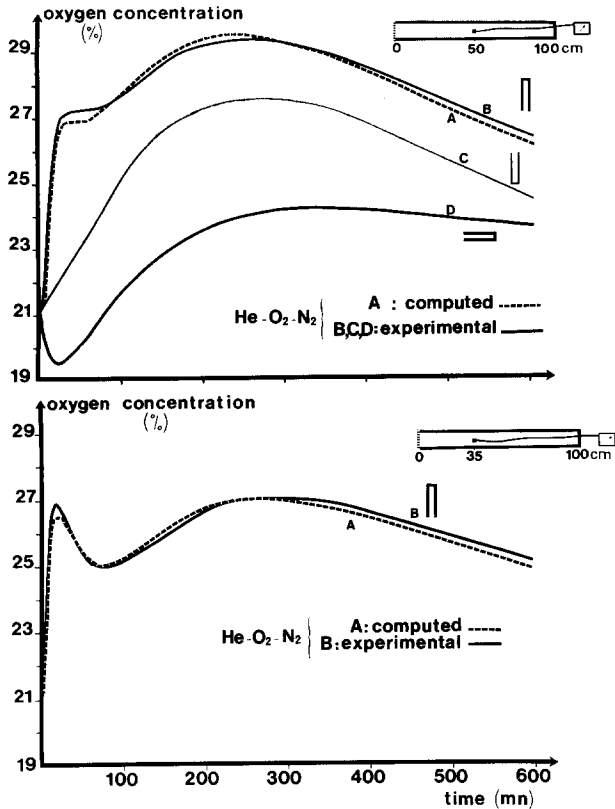


Fig. 3. Ternary diffusion between He, O₂, N₂: Theoretical and experimental curves of oxygen concentration at 35 cm (bottom) and 50 cm (top) from the open end. The He O₂ mixture filling the cylinder at time zero diffuses with ambient air through the open end. The cylinder position is symbolized at the right hand side

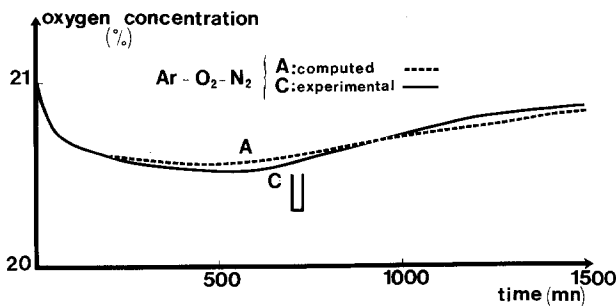


Fig. 4. Ternary diffusion between Ar, O₂, N₂: Theoretical and experimental curves of oxygen concentration at 50 cm from the open end. The Ar O₂ mixture filling the cylinder at time zero diffuses with ambient air through the open end. The cylinder is vertical with open end upwards

to the equilibrium value of 21% to lead to significantly different results concerning the tube position. For helium we also measured curve B at 35 cm (Fig. 3) because at this point, as can be seen from Figure 2, a double maximum is predicted. For each curve several experiments were performed with the same temperature and pressure conditions. Experiments are well reproducible. For example, in Figure 3, curve B re-

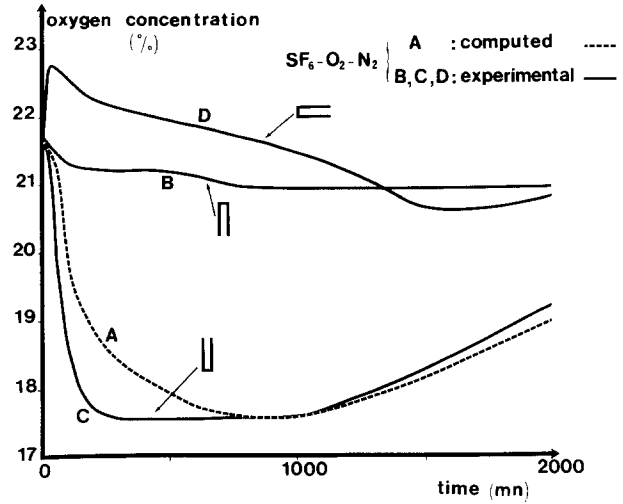


Fig. 5. Ternary diffusion between SF₆, O₂, N₂: Theoretical and experimental curves of oxygen concentration at 50 cm from the open end. The SF₆, O₂ mixture filling the cylinder at time zero diffuses with ambient air through the open end. Same symbols as in Figure 3

presents the average of four experiments with values deviating by $\pm 0.18\%$ from the mean.

It can be noticed that although oxygen concentration is initially the same inside and outside, oxygen concentration in the tube is predicted and observed not to be constant. In addition the curves obtained for one mixture differ from each other according to the tube position, one of them fitting the theoretical one.

INTERPRETATION

The differences between the experimental curves of one mixture for different positions of the tube are easily explained by the effect of gravity. This effect is microscopically negligible: the barometric formula obtained from Maxwell's distribution (for instance Landau, 1967, p. 137) shows that for a height of 1 m, the relative difference of concentration between a light gas like helium and a heavier one like oxygen due to gravity is about 10^{-4} ; with sulphur hexafluoride and oxygen this difference is about 5×10^{-4} ; both can then be neglected.

On the other hand, macroscopically the kinetic energy of gases is zero and, provided that they are not mixed, the heaviest gas sinks according to the buoyancy principle. This can be illustrated with chlorine, which is coloured and heavier than air (Brès and Hatzfeld, 1976): Three test-tubes about 25 cm long in three different positions are filled with chlorine. Caps are removed. The vertical tube with opening down empties at once; from the horizontal tube chlorine flows separated with air by a distinctly outlined surface. Five s later, the only tube which still holds chlorine is the vertical one with opening up. For the He O₂ mixture

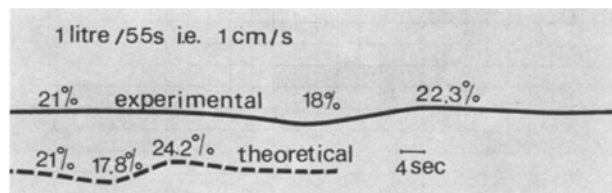


Fig. 6. Ternary diffusion: Recorded curve of oxygen concentration in the middle of the cylinder when the HeO₂ mixture, at a speed of 1 cm/s, displaces air which initially filled the cylinder. *Bottom left:* The theoretical curve drawn on the same scale

which is lighter than air with the open end of the tube pointing down (Fig. 3, curve B), no convective flow can be established by gravity and there is a flux of gas only as a result of diffusion. On curves C and D this gas is not confined and escapes from the tube. This convection tends to mask the effect of diffusion but not completely because it is hindered by the beads so that a maximum in oxygen concentration is still noticeable.

One additional feature of curves D and of curves C and F of Figure 2 in Modell and Farhi's paper (1976) is the existence of an extreme which is in opposite direction to the equilibrium value. Let us consider the helium-oxygen mixture: as it flows out because of its lightness it is replaced by air entering the tube which is deprived of part of its oxygen because of osmotic and reverse diffusion. When this air poor in oxygen comes near the sensor the curve shows a minimum. Then, as mixing proceeds, differences of density are reduced, diffusion predominates and entails the expected maximum of oxygen concentration.

To confirm this explanation we devised another experiment, in which flow in the tube is steady and known; instead of being due to gravity we created it by flowing the He-O₂ mixture into the air-filled tube. The linear velocity of the gas was calculated from the values of the flow and volume of gas in the tube given by equation (7). This velocity being known, gas concentration at any time and place in the tube can be predicted using our computer program slightly altered to take this translation into account. Figure 6 shows the result of such an experiment performed with a velocity of 1 cm/s. The recorded profile of oxygen concentration exhibits a minimum (18%) when the light mixture comes near the sensor, then passes to a maximum of 22.3%, and finally reaches equilibrium at 21.2%; the quantitative differences between the observed and theoretical curves are probably due to vortices. When the heavy mixture with SF₆ is used, the opposite phenomenon appears: the recorded oxygen concentration goes up to 24.4%, then passes through a minimum at 18%, and finally reaches equilibrium at 21.76%.

DISCUSSION

Wretschko (1870) did experiments similar to ours with hydrogen, carbon dioxide and oxygen but could not monitor the oxygen concentration; he only showed an increase in its overall concentration in the part in which it was initially mixed with hydrogen. Thus, reverse diffusion was experimentally obtained a long time ago. To get correct experiments it is necessary to avoid gas motions due to gravity or whirls. The first experimenters who tackled problems of diffusion were aware of the influence of gravity. Accordingly, the heavier body was at the bottom of their vertical diffusion tube, whether it was brine and water (Fick, 1855) or gases (Graham, 1863; Loschmidt, 1870; Wretschko, 1870). To avoid whirls, Modell and Farhi (1976) used glass beads as we did, Graham (1863) cotton wool. As he stated: "In these experiments the glass cylinder was loosely packed with cotton wool, to impede the action of currents in the column of air; but this precaution was found unnecessary, as similar results were afterwards obtained in the absence of the cotton". Unfortunately he indicated only the height of his cylinder, 0.57 m, but not its diameter. As one may infer from Poiseuille's law, a narrow section is an obstacle to flow. Quoting Graham, Loschmidt did not use any beads or cotton in his tube which had a height of 0.975 m and a diameter of 26 mm only. As regards the influence of Graham's baffle, if the cotton had been effective, one would have expected that it would have influenced the results: being devised as an obstacle, it ought to have entailed an increase in the path of molecules. Probably the cotton was packed too loosely to exert sufficient drag on gas motions that may have been present. The ideal arrangement would seem to be a longitudinally honey-combed tube which would prevent whirls but not axial diffusion.

We devised the following binary experiments to show the influence of beads on vortices and gas motions due to gravity. Our cell was located near the bottom of our tube (94 cm from the open end), oxygen was introduced and then allowed to diffuse with ambient air. The experimental curves (all of which correspond to experiments done twice and giving the same results) are shown in Figure 7. The case of the tube with its open end upwards and full of beads corresponds to a diffusion coefficient of 0.15 cm²/s. If it is tilted horizontally the gas motions due to the difference of densities have little influence and the mixing towards equilibrium is only slightly faster (apparent diffusion coefficient: 0.17 cm²/s). On the other hand, when there are no beads these two experiments give different results. If the tube is vertical the curve does not follow Fick's law, because of vortices, but grossly corresponds to a diffusion coefficient of 0.39 cm²/s. If the tube is

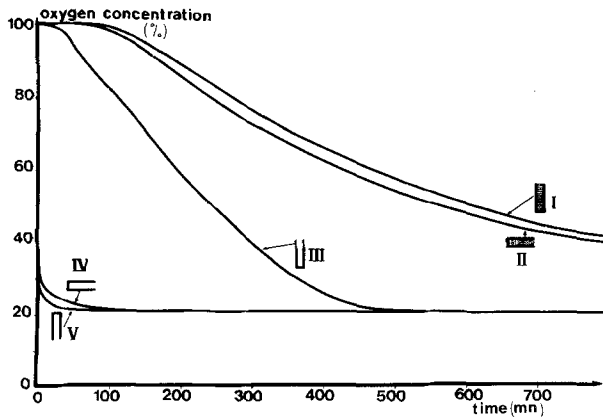


Fig. 7. Binary diffusion: Effect of beads on whirls and motions due to gravity: experimental oxygen concentration at 94 cm from the open end of the cylinder. Oxygen in the cylinder diffuses with ambient air. Same symbols as in Figure 3. In curves I and II the cylinder is filled with beads, in curves III, IV, V it is not

horizontal, gravity has a leading role and equilibrium can be assumed to be reached within a couple of minutes. If the tube is put bottom up, the equilibrium is reached even sooner.

We can deduce that, in the absence of beads, whirls and slight differences of density can bring about gas motions able to overshadow diffusion and that the presence of beads slows these motions.

The fact that our diffusion coefficients were measured with beads led us to estimate the longitudinal increase of the diffusion space. If one disregards defects and surface discontinuities and assumes that the cylinder is filled with layers of close-packed beads, the ratio between gas volume and total volume is:

$$\frac{\text{gas volume}}{\text{total volume}} = 1 - \frac{\pi}{3\sqrt{2}} \tag{7}$$

Because of defects all values of the angle between the tube axis and the perpendicular to these layers are equally probable. We found by a combination of elementary paths between the beads that the ratio R of the average real path of gas molecules L_r on the apparent path L_a is

$$R = \frac{L_r}{L_a} = \frac{1}{\pi/2} \left[\sqrt{3} - \frac{3}{2} + \frac{\alpha}{\sqrt{2}} \left\{ \frac{9}{4} - \sqrt{3} \right\} + \frac{\sqrt{3}-1}{\sqrt{2}} + \left\{ 2 + \frac{4}{\pi} \left\{ \frac{2}{3} - \alpha \right\} + \frac{\alpha\sqrt{3}}{2} \right\} \right] \tag{8}$$

where $\alpha = \text{Arc tg } \sqrt{8}$
 which yields $R = 1.2083$, $R^2 = 1.4601$.

According to this formula, we should expect real diffusion coefficients to be 46% higher than those measured in our bead-filled cylinder. It is difficult to check this value as diffusion coefficients vary in the literature. For instance, for 20°C and 101.3 kPa Radford (1964) quotes a coefficient between oxygen and nitrogen of 0.22 cm²/s, for 37°C Rauwerda (1946) uses 0.26 cm²/s, Cumming (1966) 0.25 cm²/s, Paiva (1972) 0.225 cm²/s.

We then measured it with a set-up taken from Boardman (1937) and with the help of formula (5). We found:

$D_{O_2N_2} = (0.206 \pm 0.009) \text{ cm}^2/\text{s}$ at 18°C and 101 kPa (8 experiments).

$D_{O_2N_2} = (0.229 \pm 0.01) \text{ cm}^2/\text{s}$ at 37°C and 101 kPa (3 experiments).

Our real coefficient of 0.206 cm²/s is 37% larger than the apparent one, rather than 46% as predicted by geometrical estimation. This tallies with the fact that every dislocation or surface discontinuity decreases the real path of gas molecules and thus lowers the difference between the diffusion coefficients.

CONCLUSION

Stefan's equations correctly describe the phenomena observed in the diffusion of three gases if one prevents undesirable motions. It is not possible to use either Fick's law (1855) or Graham's law of effusion (1832), the latter not being valid for the diffusion of gases between themselves. Models in respiratory physiology involving more than two gases should then be reconsidered inasmuch as they attempt to ascertain the importance of diffusion with equations which are not justified even for air breathing. Moreover in many experiments heavy or light test gases are used which enhance the features of ternary diffusion. Attempting to apply Fick's law to ternary situations can prove quite erroneous even qualitatively.

One may also wonder what is the influence of gravity versus diffusion in the lung. Let us consider two similar volumes (same shape but different sizes). Under the same conditions (same gases and same initial conditions) the time t_{diff} needed to obtain, by diffusion alone, the same profile of concentration is proportional to the square of the dimension l along which diffusion occurs:

$$t_{diff} \propto l^2 \tag{9}$$

On the other hand in the same two similar volumes the time t_{grav} required to obtain, by gravity alone, two similar profiles of concentration with the same initial conditions is proportional to l'/v where l' is the dimension along which the gas flows and v the average gas speed in this direction. If the flow due to gravity is

laminar, v is directly proportional to cross section S' perpendicular to l' , to the gradient of pressure causing the gas to move dp/dl'' , and inversely proportional to viscosity μ (see e.g. Brun, 1968, volume 2, page 6). l'' is the dimension along which gravity acts. dp/dl'' only depends on the differences of gas densities and is therefore a constant with our assumptions; μ is a constant too. One then can write:

$$\frac{t_{\text{diff}}}{t_{\text{grav}}} \propto \frac{l^2}{l'/S'} \quad (10)$$

Because of the similarity of shapes, $S \propto l^2$, $l' \propto l$, and one gets:

$$\frac{t_{\text{diff}}}{t_{\text{grav}}} \propto l^3 \quad (11)$$

The hypothesis of the flow due to gravity being laminar is valid in the lungs because, for volumes of the same order of magnitude as the larger airways, Reynold's number is much below the critical value and goes on decreasing with the cube of dimensions. Equation (10) shows that t_{diff} decreases and t_{grav} increases when dimensions decrease so that in small volumes diffusion acts chiefly. Let us consider the 23rd bronchial generation of the human lung as a cylinder of diameter 0.4 mm and length 0.5 mm (Weibel, 1963). One can compute a characteristic time for diffusion between nitrogen and oxygen to be 10 ms and in the same fashion a characteristic time for equilibration through gravity to be 4 s using $1.7 \cdot 10^{-5}$ Pa · s for the viscosity and $0.23 \text{ cm}^2/\text{s}$ for the diffusion coefficient. One can then conclude that in alveolar sacs and ducts gravity does not exert any appreciable influence before completion of diffusion.

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