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ADOLESCENTS' ABILITY TO COMMUNICATE SPATIAL INFORMATION: ANALYZING AND EFFECTING STUDENTS' PERFORMANCE[1]

ABSTRACT. This study presents evidence which suggests that even though students in grades 6 through 8 are familiar with a variety of types of representation modes, they have great difficulty in successfully communicating spatial information. A Building Description Task, which consists of *a building* made up of ten small cubes taped together and a set of instructions, was presented to a sample of middle school children. They were asked to "help your friend to know what your building looks like." Students' attempts were classified by representation mode (verbal, mixed, graphic) and analyzed by grade and by sex. In addition, the effect of instruction in spatial visualization activities on students' preference for representation mode and rate of success on the task was assessed. The findings were examined relative to the practical teaching implications, to individual differences in spatial visualization ability and to design of spatial tests.

INTRODUCTION

Gaulin (1985) discusses the need and reasons for emphasizing various types of representations of spatial shapes and relations. He stresses the need of re-establishing the development of spatial intuition as one major goal for teaching geometry and the need for emphasizing a diversity of graphical representations of three-dimensional shapes and relations.

Graphical representation of various types are commonly used in a great number of practical situations and disciplines for conveying spatial information, for example maps, diagrams, flow-charts, and scientific or technical descriptive drawings. Consequently, the ability to represent and interpret three-dimensional geometric relations is a valuable skill for many school subjects and technical occupations. Providing all pupils with opportunity to explore a variety of types of representations of spatial and geometric information, as well as to communicate such representations should be a basic educational objective.

Goodnow's (1977) book on children's drawings, however, demonstrates that children have difficulties in representation of objects. In particular, Mitchelmore (1983) points out difficulties that adolescents have in representation of regular three-dimensional figures. These include children's difficulty in representing parallel and perpendicular lines in their drawings. Bishop (1979, 1983) and Parzysz (1988) remind us that the representation of a three-dimensional object by means of a two-dimensional diagram

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demands considerable conventionalizing which is not trivial. Yet, within Western cultures, we demand such conventionalizing of young children without any attempt to directly teach conventions such as dotted lines in drawings representing unseen edges and parallelograms representing square faces in drawings of cubes. The study reported in this paper will provide evidence to suggest that: (i) middle school students, boys and girls, have difficulties in successfully representing and communicating information on a three-dimensional building made up of cubes; and (ii) instruction in spatial visualization activities, including concrete experiences with cubes, is very helpful in improving students' ability to communicate and represent spatial information.

REVIEW OF RELATED STUDIES

The skill of representing three-dimensional objects is a part of spatial visualization, which is a particular sub-set of spatial skills. Whereas one group of researchers emphasizes the mental manipulation needed in spatial visualization tasks as its main critical aspect (McGee, 1979; Fennema, 1977; Guay, 1980), another group of researchers emphasizes the need for complicated, multi-step analytic processing of spatially presented information (Linn and Petersen, 1985). Correlational and logical-intuitive arguments abound for connection between mathematical thinking and mental manipulation of geometric images (Fennema and Sherman, 1977; Connor and Serbin, 1985; Smith, 1964; Ben-Chaim, Lappan and Houang, 1989).

Spatial visualization is not one of the usual components of the school curriculum. Therefore, spatial understanding is primarily informally acquired. Nevertheless, several studies of training programs to improve spatial visualization are reported in the literature. Among them, inconsistent results are found. For example, the studies of Blade and Watson (1955), Brinkmann (1966), Bishop (1973), Connors *et al.* (1978), McGee (1978) and Smith and Schroeder (1979) demonstrated effectiveness of training in improving performance on spatial tests. In contrast, Mendicino (1958), Myers (1958), Sedgwick (1961), Mitchelmore (1975) and Mundy (1987) found no improvement. Hence, the question of whether students can benefit from training in spatial visualization activities is still being addressed.

In addition, the typical, but not a universal, finding, that male performance on spatial visualization is superior to female performance (Harris, 1981; Liben, 1981; Maccoby and Jacklin, 1974; Bishop, 1983) has raised more questions. Such questions are regarding the relationship between sex differences in spatial ability and sex differences in mathematics achievement (Fennema, 1977; Harris, 1978; Sherman, 1967), the interaction of sex and instruction in affecting spatial ability (Eliot and Fralley, 1976; Maccoby and Jacklin, 1974; Harris, 1981), the interaction of sex and type of tasks and procedures used in assessment of spatial visualization (Linn and Petersen, 1985; Bishop, 1983) and regarding the sources of observed sex differences in spatial ability (Harris, 1978).

In conclusion, the mixed results of studies on training in spatial visualization and individual differences in this domain leave the field open for further research. In support, Mitchelmore (1975) says that "the greatest need is for the development of practical geometric and spatial teaching programs and for their experimental testing" (p. 172). Sherman (1979) argues that "methods for achieving this [improving spatial skill] . . . need to be devised, and their feasibility and advisability evaluated" (pp. 26–27). And finally, Bishop (1983) in his review of research on space and geometry provides further support for investigating training programs – "what are clearly needed now are more training studies using clinical testing procedures, and involving retention and transfer tasks" (p. 186). Furthermore, he strongly recommends that "figural and nonfigural stimuli need to be used" (p. 199) in assessing spatial ability. The nonfigural tasks should be visually provoking tasks.

PURPOSE OF THE STUDY

During a pilot study for developing spatial visualization activities for middle school students, it was found that students have difficulties in describing and representing three-dimensional solids. In order to evaluate the effectiveness of the instructional activities, especially the practical ones dealing with representation schemes and drawings, a "building description task" was created. There were three inter-related purposes in presenting this type of task. The first was to study types of representations used by middle school students in attempting to perform the task. The second was to investigate individual differences (by grade and by sex) in representation modes employed and to determine to what extent students were successful. The third purpose was to determine whether students' preference for representation mode and rate of success on the task would be affected by instruction in spatial visualization activities, and if so, whether the effect would vary by grade and by sex.

Several other aspects of the effectiveness of the instructional activities were measured by a spatial visualization multiple-choice test and have been reported elsewhere (Ben-Chaim, Lappan, and Houang, 1985, 1986, 1988).

THE BUILDING DESCRIPTION TASK

The Building Description Task consists of *a building* made up of ten small cubes taped together and placed on a stiff paper card and a set of instructions typed on the top half of an $8\frac{1}{2}'' \times 11''$ piece of paper with the remaining space for the student's response. Two corner isometric views of the building are shown in Figure 1. The instruction is shown in Figure 2.

The Building Description Task was designed to elicit the preferred mode of representation of a particular student and to gather data different in nature than what could be collected from a paper and pencil test, specifically, the MGMP Spatial Visualization multiple choice test (Ben-Chaim *et al.*, 1986). The language and wordings of the instructions were selected after careful examination of students' responses to similar tasks presented in pilot studies. For example, words such as "draw", "describe", "explain",

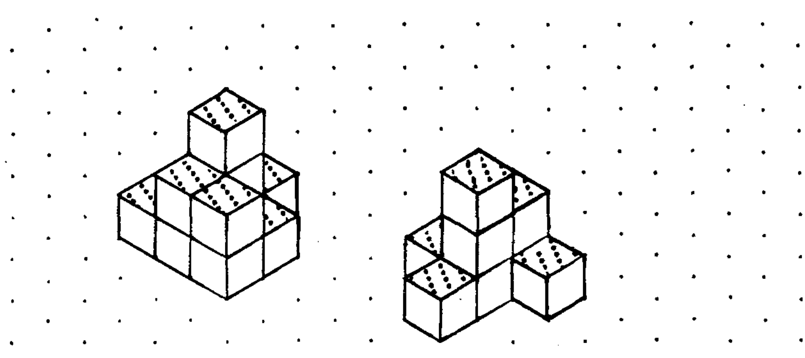


Fig. 1. Two corner isometric views of the model included in the Building Description Task.

You are seated on one side of a screen and your friend is seated on the other.
Your friend cannot hear what you say, but you may pass a piece of paper to him.
Your friend has a supply of cubes to work with.
Here is a building made of cubes. You are the only person that can see the building.
Your task is to help your friend to know what your building looks like.
Be as creative as you wish.

Fig. 2. The set of instructions for the Building Description Task.

"write", etc., were eliminated because they might bias the student to respond in favor of verbal or graphic descriptions. The building was selected from several options ranging from a flat building to a building with multiple towers in order to pose a task of moderate difficulty that most of the students can attempt.

According to Wattanawaha's DIPT (Dimension, Internalization, Presentation, Thought process) classification system for non-speeded spatial tasks (Wattanawaha and Clements, 1982), the Building Description Task falls into the highest values of each of the four independent characteristics. It is identified as D-3 which means "a question requiring 3-dimensional thought"; as I-2 which means that "there is not only a need for a visual image but in order to do the task the image must be operated upon (transform) in the mind"; as P-2 which means "the answer requires that the final visual image be represented by a drawing, or be described in words or by hand or other movements"; and as T-1 which means that "the task does not specify the mental operation but enough information is given for this to be determined" (Wattanawaha and Clements, 1982, p. 879).

The Wattanawaha's DIPT system classifies tasks rather than the responses to the tasks by individuals. If responses to tasks by individuals are to be classified, in particular the representation mode employed, then Richardson's notion of a verbalizer-visualizer continuum can be applied (Richardson, 1977). Moreover, Burden and Coulson (1981) make the suggestion to distinguish individuals according to the following three representation modes: verbal, graphic and mixed mode (which combines verbal and graphic representations).

THE STUDY

Sample and Data Collection

In order to include students in grades 6 through 8 from a broad range of socioeconomic status, three sites in and around a major mid-western city in the United States, were selected for the study. The three sites can be described as an urban inner city middle school that serves middle to low income families with approximately one-third blacks and Latinos; a rural middle school that serves a cross-section of middle-class families that are predominantly white; and a suburban middle school that serves a community of upper middle-class university and state government professionals and is predominantly white. The urban school had only 5th and 6th grade students, hence only the 6th grade students participated in the study; the rural school had all three grade levels (6th, 7th and 8th); and the suburban school had only 6th and 7th grade students.

Twenty-one classes, 7 of each grade level, participated in the study. Since several teachers had multiple classes at the same grade levels, 17 of the 21 participating classes were selected as representative of the whole group. Two weeks before the exposure to the spatial visualization activities or the multiple choice MGMP Spatial Visualization Test, three to four students were randomly selected from the class list of each of the 17 classes and were given the Building Description Task. About a month after the instructional intervention, the same selection procedure was followed on students from 15 out of 21 participating classes. Each student was individually administered the Building Description Task, with no time limit specified. The amount of time the students spent on the task ranged from 8 to 17 minutes with an average of 10 minutes, with no differences between the two administrations of the task. Table I shows the distribution of the entire sample by administration (pre- and post-intervention), grade level (6th, 7th and 8th) and sex.

TABLE I

Distribution of the sample by grade, sex, and administration (pre- and post-intervention)

	Pre-intervention		Post-intervention	
	# of students	# of classes	# of students	# of classes
Total	62	17	52	15
Boys	35		29	
Girls	27		23	
Grade 6	29	7	11	3
Boys	15		7	
Girls	14		4	
Grade 7	19	6	26	7
Boys	13		14	
Girls	6		12	
Grade 8	14	4	15	5
Boys	7		8	
Girls	7		7	

The Unit of Instruction

The Spatial Visualization unit of instruction was developed by the MGMP.[2] The unit includes ten carefully sequenced activities which require approximately three weeks of instructional time (12 to 15 class periods). The instructional unit involves representing three-dimensional objects in two-dimensional drawings and, vice versa, constructing three-dimensional objects with blocks, from their two-dimensional representations. The activities deal with flat views of buidings as well as with isometric drawings on dot paper (paper with dots arranged on diagonals rather than rows). In most of the activities, the students are asked to perform some fairly demanding orientation and visualization tasks. They are asked to mentally rotate a building and draw either flat views of the other sides or isometric drawings from other corners. Cubes are always available to help a student who needs to see the concrete object to be successful. It should be noted that none of the activities set for the students during the instruction, specifically involved tasks similar to the Building Description Task. See Lappan *et al.* (1984) for illustrations of some of the activities, and see Winter *et al.* (1986) for a complete teacher guide and the instructional model employed in the spatial visualization unit for instruction.

Classifying the Representation Modes

Following the Burden and Coulson (1981) suggestion, three representation modes were used to classify students' responses to the Building Description Task: verbal, graphic and mixed mode. A representation mode was classified as verbal if the students' message was carried by words. This could mean totally verbal or mainly verbal with a diagram which does not give any added information. For example, Figures 3 and 4 show three students' responses (from the administration of the Task prior to the intervention) that were classified as verbal mode, the first and second student (Figure 3) used only words, the third student (Figure 4) expressed her ideas mainly in words and the diagram did not add to the message.

A representation mode was classified as graphic if the student attempted to communicate by visual drawings accompanied with at most labels or numbers. Figures 5, 6, and 7 display several examples of students' responses (from the pre-intervention) that were classified as graphic mode. Figure 5 shows 3 different students' versions of what we refer to as a map plan. This idea essentially gives the base and information about how tall to make the stack on each part of the base. Once we observed students in the pilot study

The building is tall. It looks like there are stairs that go up around the sides to the top, which is tall. The other side has basically the same pattern. Each goes up one story. It is tall, yet there are a lot of parts which are low to the ground. The back is even all the way up but it is not the highest part of the building. In the front before the tower there is a platform like structure and there is another one on the side. Both platforms are about equal to one story high. The highest part of the building is the middle and about 3 stories high. There even could be a balcony on the other side.

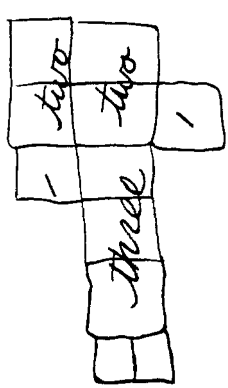
8TH GRADE GIRL

It is made up of 10 red blocks of about 1 inch by 1 inch. It is about 3 inches wide and 3 inches tall. The building has 3 levels of blocks. If you were to take 8 of its blocks and put them together you could make a cube. But, one block has been stacked upon another to make sort of smoke stack. The other 2 blocks stick out from 2 side.

8TH GRADE BOY

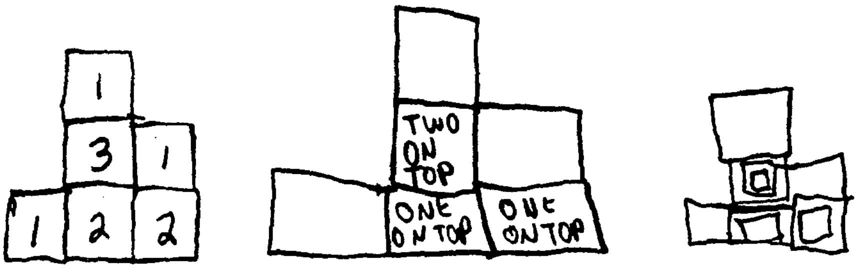
Fig. 3. Two students' attempts, prior to the instruction, classified as verbal mode.

There is a building that I would like to tell you about. First of all it has many different stories. 2 of them are 2 floors high, 3 of them are just 1 story, and one in the middle is 3. The building is red. The building is made of cubes. On the left hand side you don't put in a cube in the front then you put 1 in the middle and 2 behind that. It is easier if you do it in rows so that is the first row. In the second row it consists of 6 cubes there is one by itself in front. Then behind that there is 3 piled on top of each other then 2 cubes behind that. Then in the 3rd row there is only 1 cube and that is in the 3rd space back. The whole building consists of 10 cubes. Remember they are red. Here is sort of a picture but please remember this building is standing up.



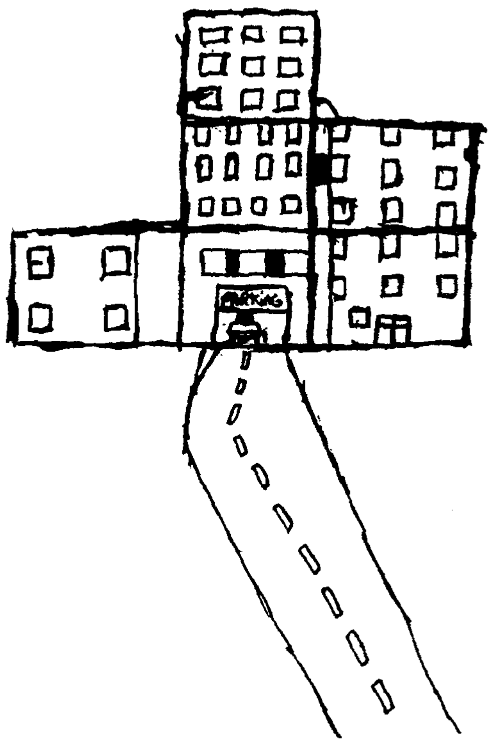
7TH GRADE GIRL

Fig. 4. A students' attempts, prior to the instruction, classified as verbal mode.



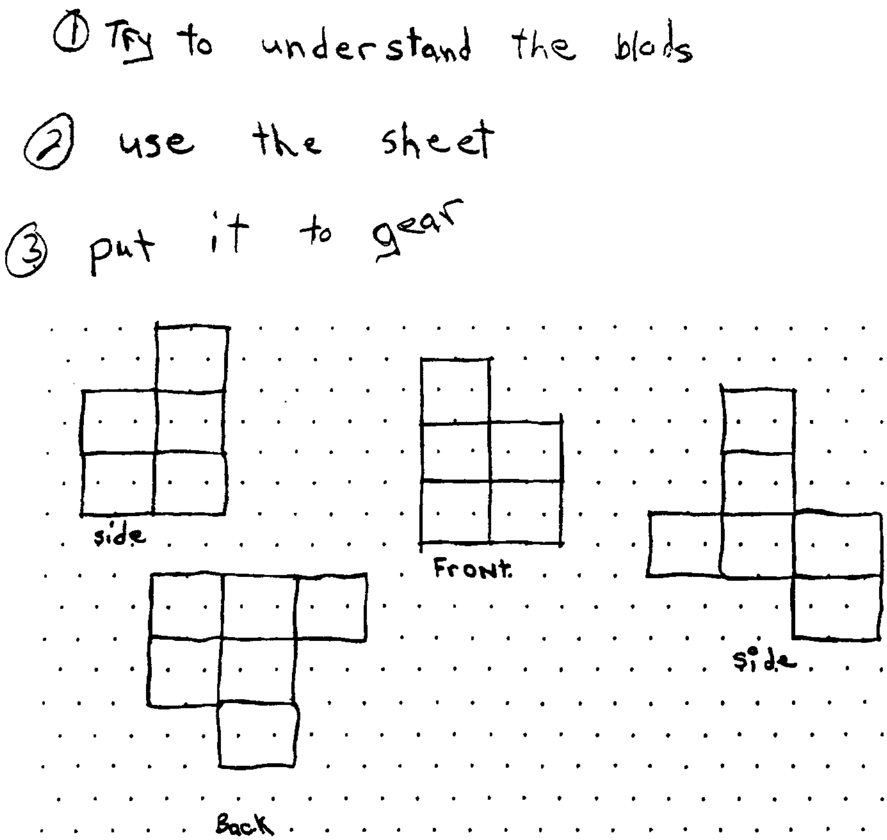
7TH GRADE BOY 6TH GRADE BOY 7TH GRADE BOY

Fig. 5. Three students' attempts, prior to the instruction, demonstrating the idea of a "map plan" of a building.



6TH GRADE BOY

Fig. 6. A students' graphic attempt, prior to the instruction, to represent the building with flat view.



6TH GRADE GIRL

Fig. 7. A students' graphic attempt, prior to the instruction, to represent the building with flat views.

using this efficient method of representing a building, we incorporated this idea into the instructional material. Figures 6 and 7 present 2 students' attempts to represent the building with flat views. Figure 8 displays a student's attempt to show the three-dimensionality of the building.

A representation mode was classified as mixed if the student used both words and drawings in attempting the Building Description Task. Figures 9 and 10 present 3 examples (also from the pre-intervention) which were classified as mixed mode.

Using the above criteria, the authors independently classified the students responses. In 97% of cases, complete agreement was found among the authors. Disagreements about the remaining 3% (4 students) were related

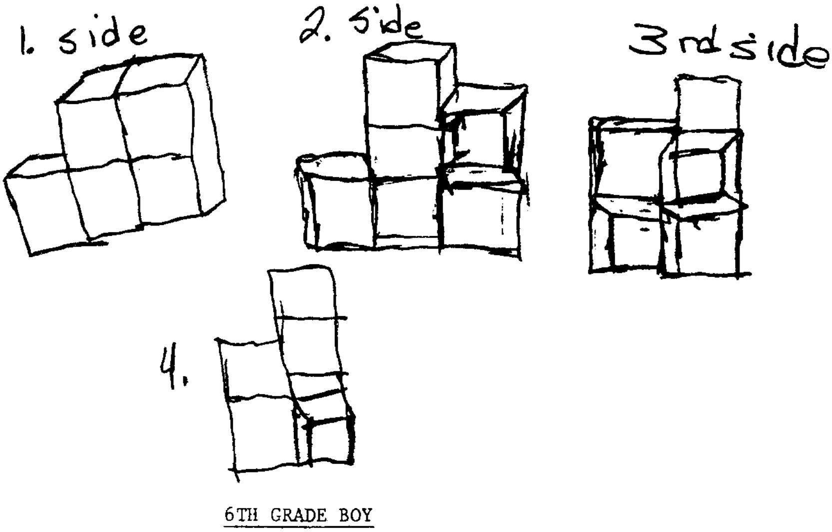


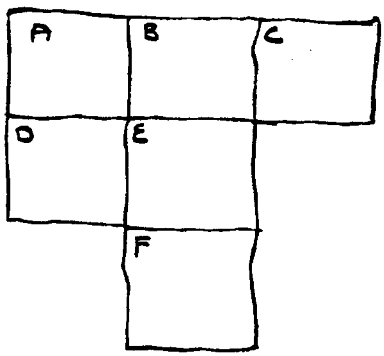
Fig. 8. A students' graphic attempt, prior to the instruction, to show the three-dimensionality of the building.

to classifying them as either mixed or graphic modes, and were resolved by reexamining the students papers.

As can be seen from the examples in Figures 3 to 8, the verbal and graphic modes may be considered as the ends of the visual/non-visual continuum of a type suggested by Richardson (1977). In contrast, the mixed mode encompasses a range of representation modes in-between the two ends of the continuum, as can be seen from the examples in Figures 9 and 10.

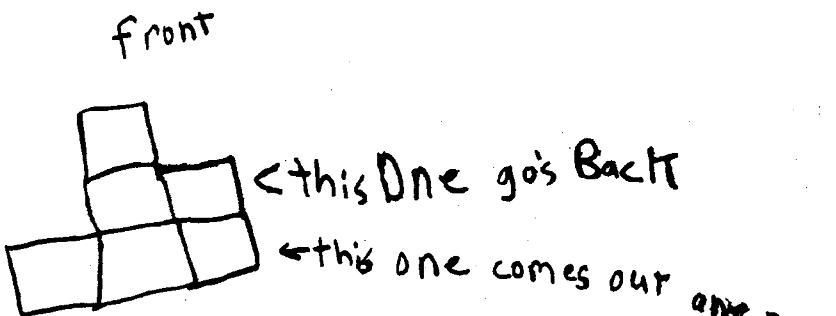
Determining Successful Performance

An attempt on the task was considered successful if the building could be re-created using the information communicated by the student. For example, Figures 11, 12, and 13 present 3 attempts (from the post-intervention administration) that were classified as successful, each with a different mode of representation. Additional examples of students' successful attempts are those presented in Figures 4, 5, and 9 (top). Using this criterion, the authors rated all the responses independently. Agreement was found on all the cases except for two which were verbal. In both cases, they were rated as successful after further examination. It should also be noted that four other cases were rated as successful by all the authors even though they contained some incorrect information among the isometric corner views



There are 6 blocks on the bottom layer placed in the form shown above. On the second layer there are 3 blocks placed on top of blocks A, B, and E. On the top layer there is 1 block placed on top of the block on top of block E.

8TH GRADE GIRL



7TH GRADE BOY

Fig. 9. Two students' attempts, prior to the instruction, classified as mixed mode.

given. Nonetheless, they included other correct descriptions by which one could re-create the building. All these four cases were from the post-intervention administration of the Building Description Task.

Results

A representative sample of student performance on the Building Description Task prior to the intervention is presented in Figures 3 to 10. These descriptions show that students ranged from straight forward attempts to draw and describe what they observed to be very fanciful interpretations of

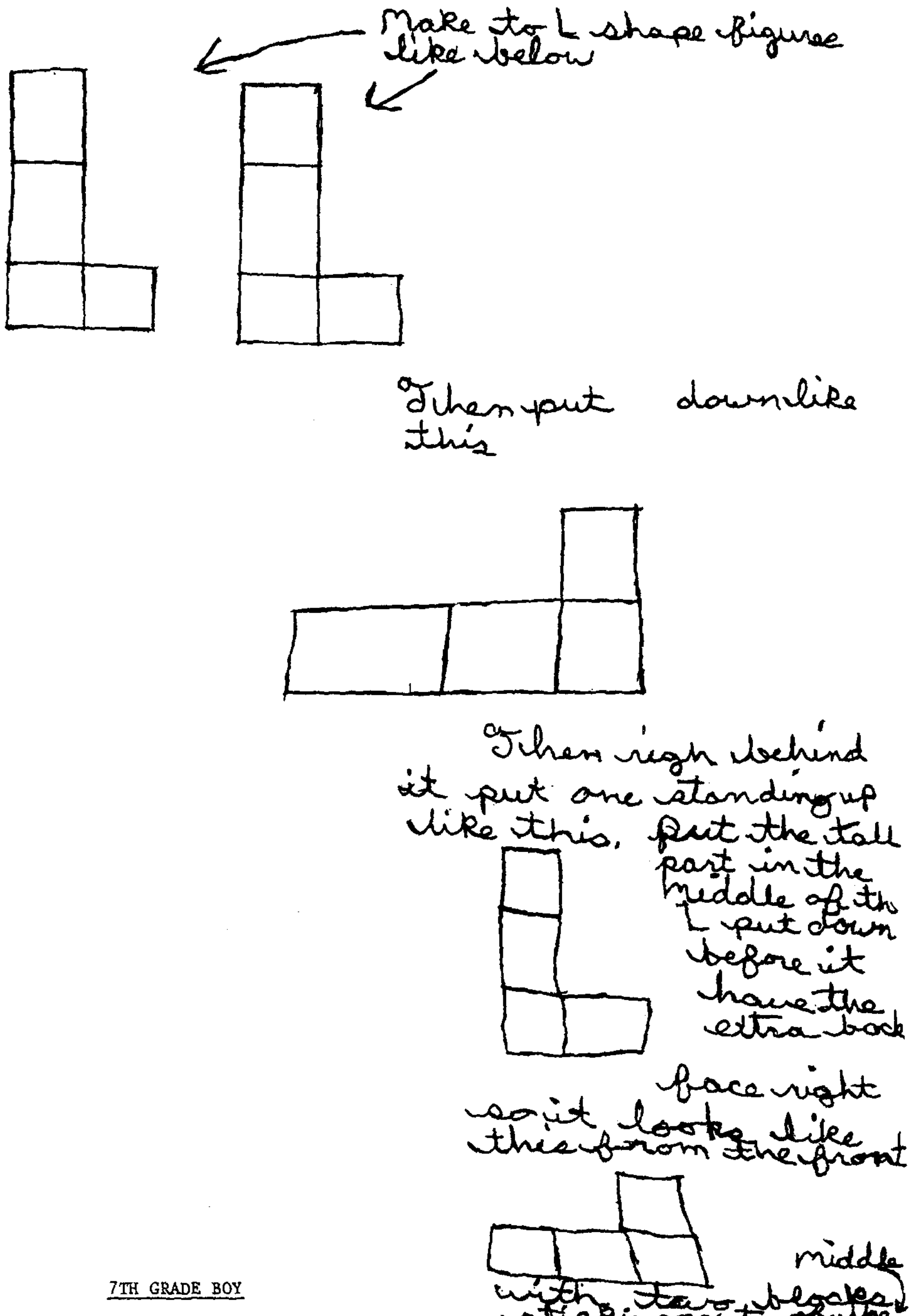


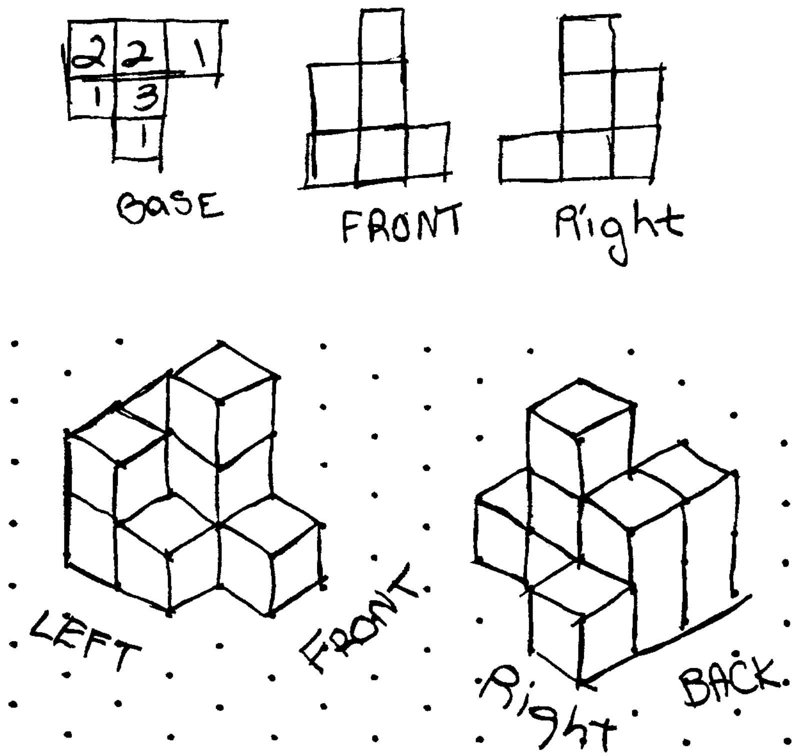
Fig. 10. A students' attempt, prior to the instruction, classified as mixed mode.

1. First I would write and tell my friend how many cubes there are in all.
2. Then I would write to that person to make one stack with three and two stacks with two and three with just one.
3. Then I would write to that person to take the stack of three, and one of two on any flat side of the stack of three.
4. Then I would write to that person to take one of the stack of one cube (one cube) and put it on the right side of the stack of two.
5. Now I would write to the person to take the other stack of two (the extra one) and put it on the left side of the stack of two connected to the stack of three.
6. Then, I would write to them to take one cube (stack of one), and place it on the left side, right next to the stack of three. (remember left side).
7. Then I would write to them to take the last cube and put it straight in back of the stack of three and then the building is built.
   During the whole writing they would be sending the paper back for me to write down more.

7TH GRADE GIRL

Fig. 11. A students' verbal attempt, after the instruction, rated as successful performance on the Building Description Task.

the building. One student, an 8th grade girl, described the building in words such as: "... stairs that go up around the sides to the top", "a tower", "a platform" and "a balcony" (see Figure 3). Another student, an 8th grade boy, fantasized: "It is one of (the) largest buildings in the world. It looks like something out of the future... the building itself has over two thousand offices in it." Other students drew buildings complete with windows, doors, paths, landscaping, and even television antennas. Students estimated heights: "My building is 30 feet tall. It has three stories. Each story is 10 feet tall, ...", or estimated area: "the base of the building covers fifteen acres of land", and other characteristics of the building. The students who stressed reality included such detail as "Its red cubes are taped together and have circles on the sides of them ... the cubes are hard and don't move." Furthermore, Figures 14, 15, and 16 show several student's graphic attempts to represent the building, each of which has some basic underlying misconception. In contrast, as regard to post-intervention, most of the students used map-plan, flat views and isometric views to describe the building. Figures 11, 12, and 13 include a sample of typical student responses from the post-intervention, in particular, the mixed and graphic mode.



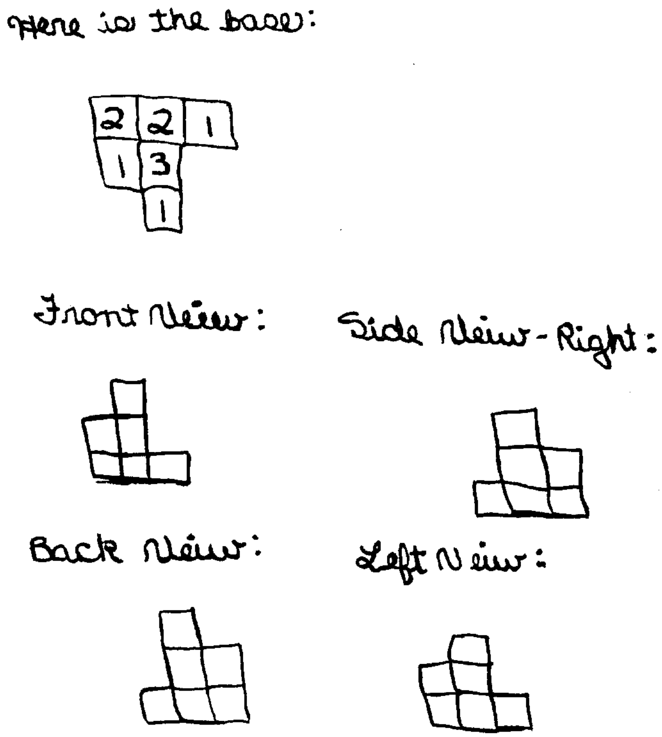
7TH GRADE GIRL

Fig. 12. A students' graphic attempt, after the instruction, rated as successful performance on the Building Description Task.

Overall, the attempts by students both pre- and post-intervention included a variety of types of representations. These can be considered as points lying somewhere along a continuum ranging from completely verbal to purely visual representation of the Building Description Task. Classification of the student responses by representation mode yielded the results shown in Table II. The percentages of students that were classified as using verbal, mixed or graphic representation mode are presented for pre- and post-intervention. The totals for the sample are reported and then are broken down by grade and by sex, separately. The number of boys and girls in each grade level did not warrant a meaningful break down by sex and grade.

Table II indicates that the total group on the pre-intervention data were fairly equally split among the three modes of representation. The intervention nearly eliminated the verbal mode and moved the group strongly

To construct the building you must use 10 cubes. First you take 3 cubes and stack them up. Then take 1 cube and put it right beside the other 3. Build a stack of 2 cubes. Put them next to the stack of 3 on the opposite side. Take another cube and put it on the left side of the stack of 3. Take 2 cubes, stack them up and put them on the left side of the other stack of two. Now take 1 cube and place it beside the stack of 2 that is behind the stack of 3.



7TH GRADE GIRL

Fig. 13. A students' mixed mode attempt, after the instruction, rated as successful performance on the Building Description Task.

toward the graphic end of the continuum. The grade by mode and sex by mode breakdowns show a similar pattern of movement toward the graphic representation. However, the grade by mode pre-intervention data shows significant differences in the distribution among modes of each grade level (chi-squared statistic is 11.91, with 2 df, $p < 0.018$). Sixth graders preferred

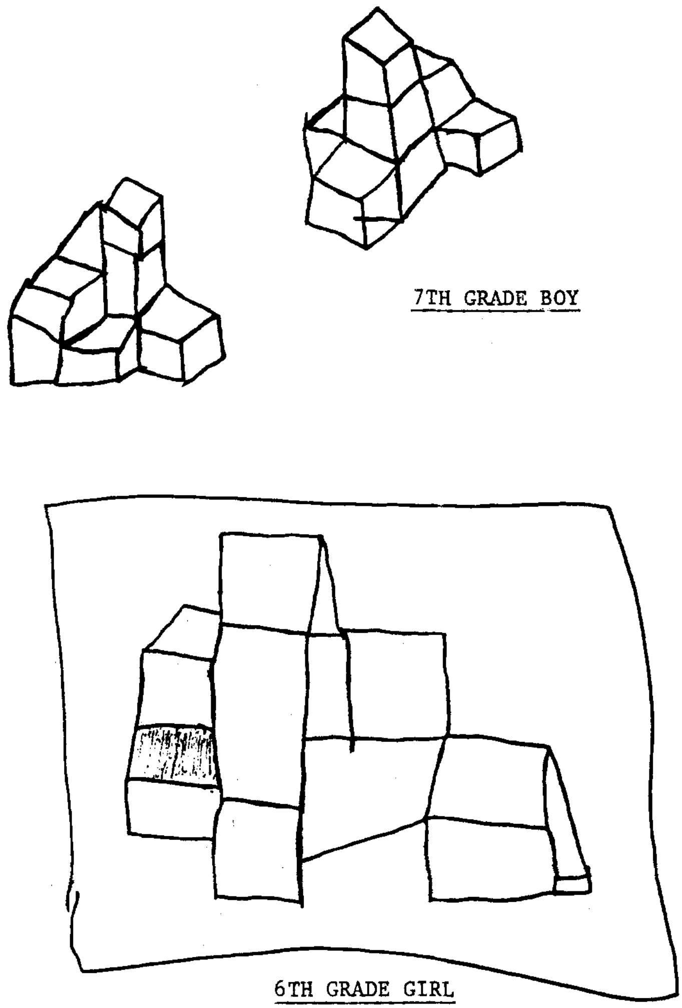


Fig. 14. Two students' graphic attempts, prior to the instruction, showing difficulties in representing parallels and perpendiculars in space figures.

mixed and graphic, while seventh graders preferred verbal and eighth graders preferred verbal and mixed. There are no significant differences by grade levels on the post-intervention data.

The pre-intervention findings also indicate that there are no significant differences between boys and girls in their mode preferences. In fact the boys and girls split very evenly among the three modes. In contrast, on the post-intervention results, significant differences among boys and girls are observed (chi-squared statistic is 9.08, with 2 df, $p < 0.011$). While girls

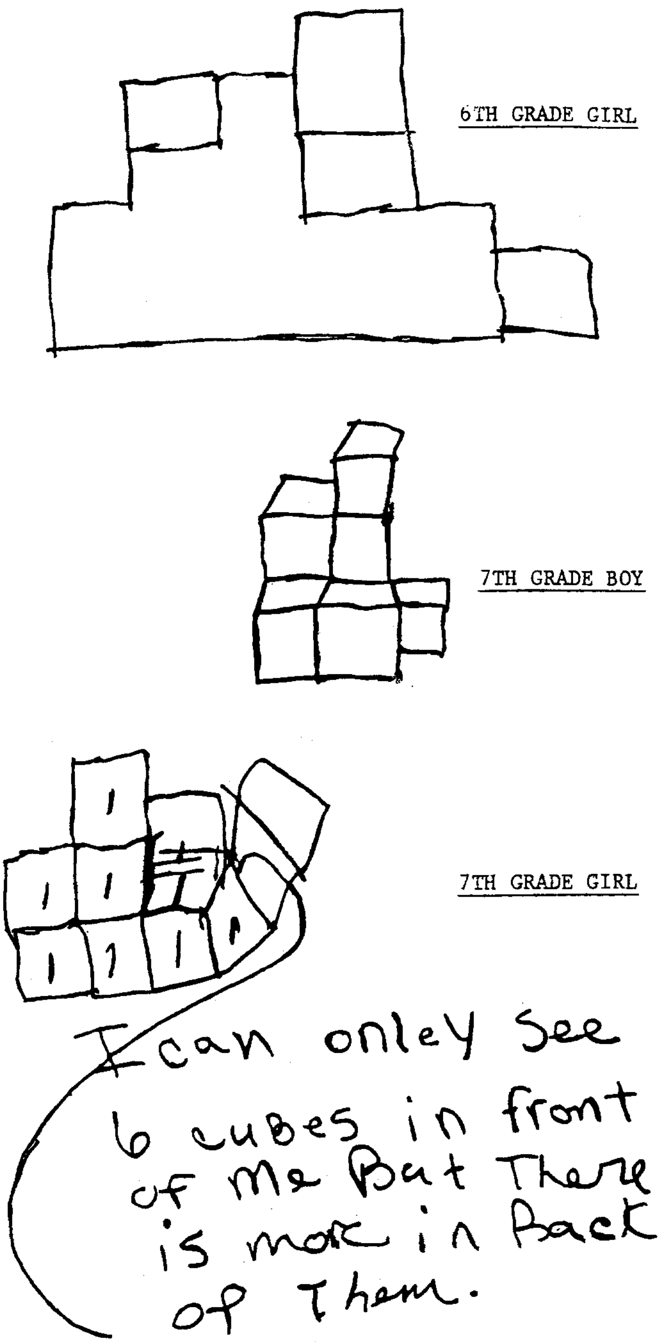


Fig. 15. Three students' graphic attempts, prior to the instruction, showing problems of awareness of hidden portions or the 3D of the figure.

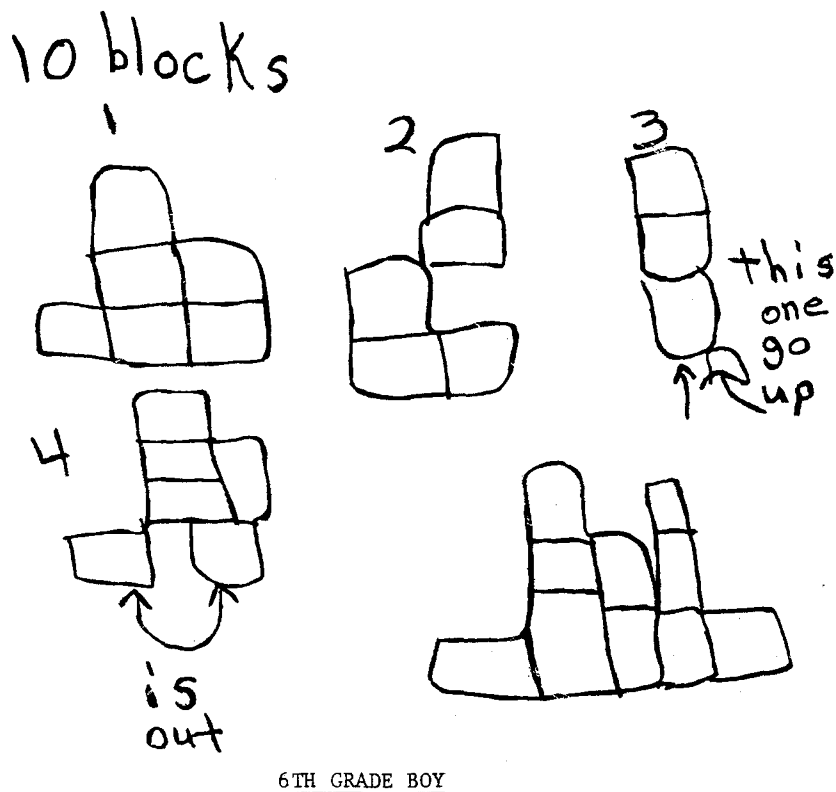


Fig. 16. A students' graphic attempt, prior to the instruction, showing problem with depth perception and flat views.

TABLE II

Mode of representation (% of students) by grade, by sex, and by administration (pre- and post-intervention)

	Pre-intervention			Post-intervention		
	Verbal	Mixed	Graphic	Verbal	Mixed	Graphic
Total	32.3	37.1	30.6	1.9	32.7	65.4
By grade						
6	17.3	37.9	44.8	0.0	63.6	36.4
7	52.6	21.1	26.3	3.8	23.1	73.1
8	35.7	57.1	7.2	0.0	26.7	73.3
By sex						
Boys	31.4	37.1	31.5	0.0	17.2	82.8
Girls	33.3	37.0	29.7	4.3	52.2	43.5

TABLE III

Success on the task (% of students) by grade, by sex, and by administration(pre- and post-intervention)

	Pre-intervention % correct	Post-intervention % correct
Total	25.8	82.7
By grade		
6	31.0	81.8
7	21.0	73.1
8	21.4	100.0
By sex		
Boys	25.7	89.7
Girls	25.9	73.9

employed fairly evenly mixed and graphic modes, boys strongly preferred the graphic mode.

Table III shows the percentage of students who were successful on the Building Description Task by grade, by sex and by administration (pre- and post-intervention). The pre-intervention data indicate that the success rate was only 26% whereas after the intervention, the success rate rose remarkably to 83%. Similar gains are found in the by grade and by sex breakdowns with no significant differences among grades or between sexes. Further examination of mode by success reveals that for students using mixed mode, only 8 of 23 (35%) were successful before the intervention, whereas 13 of 17 (76%) were successful after the intervention. For students using the graphic mode, only 4 of 19 (21%) were successful before the intervention, whereas 29 of 34 (85%) were successful after the intervention. Only one student, a 7th grade girl, employed the verbal mode after the intervention and her attempt was successful (see Figure 11).

DISCUSSION AND CONCLUSIONS

Prior to Instructional Intervention

Evidence obtained from the students' attempts to perform the Building Description Task prior to the instruction indicates that middle school students, grades 6 through 8, can and do use spontaneously a variety of types of representations. These included verbal description, a graphic drawing of the side views or perspective drawings, descriptions by layers, coded orthogonal views and mixed strategies. Similar tasks were presented by Gaulin (1985) to 10–18 year old school children in Quebec and by Burton *et al.* (1986) to mathematics teachers-trainees in London, Sydney and Melbourne. They both report a variety of types of representations, consistent with those of the present study.

The pre-intervention results show that there are significant differences among students in different grades in their preference of representation modes, with 6th graders preferring the more "efficient" visual end of the visual/non-visual continuum. The slightly higher rate of success of the 6th graders versus 7th and 8th graders may be explained by the 6th graders' preferences of the mixed and graphic mode. However, there are no differences between boys and girls in their choice of representation modes, which are in agreement with the findings reported by Burden and Coulson (1981) and Burton *et al.* (1986). Interestingly, the pre-instructional data of the present study tend not to support Willis (1980) contention (based on a review of the spatial ability and hemispheric processing literature) that males and females tend to prefer different modes of analysis, with females preferring a verbal mode and males a non-verbal mode. Approximately the same proportion of males versus females preferred verbal representation at each of grades 6, 7 and 8. (The post-intervention data show a difference with girls splitting between mixed and graphic and boys strongly favoring graphic representation.)

Another important finding is that regardless of grade and sex, only 26% of the sample are able to successfully perform the Building Description Task prior to the instructional intervention. When the MGMP Spatial Visualization Test was administered to the same population of students, a similar overall poor performance was found (Ben-Chaim *et al.*, 1988). However, in contrast to the present study, there were significant grade and sex differences. These conflicting findings on grade and sex differences could be attributed to the differences in the type of tasks posed, in particular, the mode of task presentation. In support, in a review of sex differences in spatial ability, Linn and Petersen (1985) conclude that "the magnitude of the [sex] difference depends on the test used" (p. 1488). In addition, they indicate that "sex differences may result from differential rate of rotation, differential efficiency in strategy application, differential use of analytic processes, or differential caution" (p. 1489).

The results of students' attempts to draw the building, particularly prior to the instructional intervention, remind us, as indicated by Bishop (1983) and Parzysz (1988), that problems with visual symbolism are not trivial.

The three-dimensional drawings attempts exemplify the type of difficulties that students have in the plane representation of spaec figures. For example, Figure 14 provides evidence to support Mitchelmore's (1983) notion on the problem that children have in representing parallels and perpendiculars in their drawings in space. Furthermore, Figure 15 illustrates the problem identified by Ben-Chaim *et al.* (1985) as the students' lack of awareness of either the three-dimensionality or the hidden portions of the building. Figure 16 shows another difficulty related to depth perception and drawing flat views of the building. There is evidence that these types of difficulties persist even for adults (Burton *et al.*, 1986; Baldy, 1988).

After Instructional Intervention

One of the purposes of this study was to determine whether students' preference for representation mode and rate of success on the Building Description Task would be affected by training in spatial visualization activities. The results of this study demonstrate that middle school students, grade 6 through 8, regardless of grade level and sex, dramatically improved their performance after three weeks of instruction in spatial visualization activities. These activities did not specifically involve tasks similar to the Building Description Task. However, students were taught to represent buildings by three graphic schemes: the map plan, flat views and isometric views. Obviously, the post-intervention data indicates a "near" transfer (as defined by Mayer, 1974) in that most of the students moved toward the graphic mode and that the verbal mode was almost eradicated. In fact, four weeks after the instruction, many students provided successful flat views and satisfactory isometric views in addition to a correct map plan. Both "near" and "lateral" transfer of the training were evidenced from the post-intervention administration of the MGMP Spatial Visualization Test (see Ben-Chaim *et al.*, 1988). The language and activities used in the instructional unit seemed very natural to the students. This could partly account for the positive effect of the intervention.

Relative to the conflict in the professional literature in regard to the training effects, the effect found in this study is in agreement with those claiming that spatial visualization skills can be improved by training (see previous section on review of related studies). It should be noted that a significant positive training effect was also evident in students' performance on the MGMP Spatial Visualization Test. In addition, a persistence over time of the effects was demonstrated. Given an opportunity to make concrete representations with cubes, middle school students, regardless of grade level and sex, remarkably improved their ability to successfully communicate spatial information such as a description of a building made up of small cubes. In view of the poor performance prior to the instructional intervention, it is suggested that such concrete experience should be provided for children in the middle grades.

As to the issue of sex differences in spatial visualization, the post-intervention data indicates that even though both boys and girls made substantial gain, it appears that they responded differentially to the training, particularly with respect to mode preferences. Boys moved more sharply than girls toward the graphic mode, which is considered as the more efficient mode. The girls tended to split almost evenly between the mixed and graphic modes. Linn and Petersen (1985) attempt to explain observed sex differences on spatial tasks by relating to the selection and efficient applications of solution strategies. They claim that the "pattern of sex differences could result from a propensity of females to select and consistently use less efficient or less accurate strategies for these tasks" (p. 1492). The present sex differences in mode preferences may be explained similarly. However, further studies are needed to investigate the relationship among modes of representation, strategies used and efficient application of the solution strategies.

SUMMARY AND IMPLICATIONS

The evidence from the present study strongly suggest that even though students in grades 6 through 8 are familiar with a variety of types of representation modes, they have a great difficulty in successfully communicating spatial information related to a building made up of cubes. However, after instructional intervention in spatial visualization activities, students' rate of success rose dramatically regardless of grade level or sex. This suggests that spatial visualization training in particular concrete experiences with cubes – building, representing in two-dimensional drawings, and reading such drawings – should be a part of the middle school curriculum. In addition, in order to acquire a more comprehensive view of students' spatial ability and individual differences related to this domain, it is necessary to design measures that include both figural and non-figural tasks.

NOTES

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REFERENCES

Baldy, R.: 1988, 'From drawing to object space. Analysis of activities connecting objects with their perspective drawings', *Educational Studies in Mathematics* **19**, 43–57.

Ben-Chaim, D., G. Lappan, and R. T. Houang: 1985, 'Visualizing rectangular solids made of small cubes: Analyzing and effecting students' performance', *Educational Studies in Mathematics* **69**, 389–409.

Ben-Chaim, D., G. Lappan, and R. T. Houang: 1986, 'Development and analysis of a spatial visualization test for middle school boys and girls', *Perceptual and Motor Skills* **63**, 659–669.

Ben-Chaim, D., G. Lappan, and R. T. Houang: 1988, 'The effect of instruction on spatial visualization skills of middle school boys and girls', *American Educational Research Journal* **25**(1), 51–71.

Ben-Chaim, D., G. Lappan, and R. T. Houang: 1989, 'The role of visualization in the middle school mathematics curriculum', *FOCUS: Learning Problems in Mathematics.*

Bishop, A. J.: 1973, 'Use of structural apparatus and spatial ability: A possible relationship', *Research in Education* **9**, 43–49.

Bishop, A. J.: 1979, 'Visualizing and mathematics in a pre-technological culture', *Educational Studies in Mathematics* **10**, 135–146.

Bishop, A. J.: 1983, 'Space and geometry', in R. Lesh and M. Landau (eds.), *Acquisition of Mathematics Concepts and Processes*, Academic Press, New York, pp. 175–203.

Blade, M. and W. S. Watson: 1955, 'Increase in spatial visualization test scores during engineering study', *Psychological Monographs* **69**(12).

Brinkman, E. H.: 1966, 'Programmed instruction as a technique for improving spatial visualization', *Journal of Applied Psychology* **50**, 179–184.

Burden, L. D. and S. A. Coulson: 1981, *Processing of Spatial Tasks*, Unpublished master thesis, Monash University.

Burton, L., M. Cooper, and G. Leder: 1986, 'Representations of three-dimensional figures by mathematics teachers – in-training', in L. Burton and C. Hoyles (eds.), *Proceedings of the Tenth International Conference for the PME*, London, pp. 81–86.

Connors, J. M. and L. A. Serbin: 1985, 'Visual spatial skill: Is it important for mathematics? Can it be taught?', in S. F. Chipman, L. K. Brush and D. M. Wilson (eds.), *Women and Mathematics: Balancing the Equation*, Erlbaum Associates Inc., Hillsdale, New Jersey, pp. 151–174.

Connors, J. M., M. Schakman, and L. A. Serbin: 1978, Sex-related differences in response to practice on a visual-spatial test and generalization to a related test', *Child Development* **49**, 24–29.

Eliot, J. and J. S. Fralley: 1976, 'Sex differences in spatial ability, *Young Children* **31**, 487–498.

Fennema, E.: 1977, 'Influences of selected cognitive, affective and educational variables on sex-related differences in mathematics learning and studying', in L. H. Fox, E. Fennema and J. Sherman (eds.), *Women and Mathematics: Research Perspectives for Change*, National Institute of Education, Washington, DC, pp. 79–135.

Fennema, E. and J. Sherman: 1977, 'Sex-related differences in mathematics achievement, spatial visualization and affective factors', *American Educational Research Journal* **14**, 51–71.

Gaulin, C.: 1985, 'The need for emphasizing various graphical representations of 3-dimensional shapes and relations', in L. Streefland (ed.), *Proceedings of the Ninth International Conference for the PME*, Noordwijkehout, The Netherlands, Volume II, pp. 53–71.

Goodnow, J.: 1977, *Children's Drawing*, Fontana, London.

Guay, R. B.: 1980, 'Spatial ability measurement: A critique and an alternative', Paper presented at the annual meeting of the American Educational Research Association, Boston.

Harris, L. J.: 1978, 'Sex differences in spatial ability: Possible environmental, genetic and neurological factors', in M. Kinsborne (ed.), *Asymmetrical Functions of the Brain*, Cambridge University Press, Cambridge, pp. 405–522.

Harris, L. J.: 1981, 'Sex-related variations in spatial skill', in L. S. Liben, A. H. Patterson, and N. Newcombe (eds.), *Spatial Representation and Behavior Across the Life Span: Theory and Application*, Academic Press, New York, pp. 83–125.

Lappan, G., E. D. Phillips, and M. J. Winter: 1984, 'Spatial visualization', *Mathematics Teacher* **77**, 618–623.

Liben, L. S.: 1981, 'Spatial representation and behavior: Multiple perspective', in L. S. Liben, A. H. Patterson, and N. Newcombe (eds.), *Spatial Representation and Behavior Across the Life Span: Theory and Application*, Academic Press, New York, pp. 3–36.

Linn, M. C. and A. C. Petersen: 1985, 'Emergence and characterization of sex differences in spatial ability: A meta-analysis', *Child Development* **56**, 1479–1498.

Maccoby, E. E. and C. N. Jacklin: 1974, *The Psychology of Sex Differences*, Stanford University Press, Stanford, California.

Mayer, R. E.: 1974, 'Acquisition processes and resilience under varying testing conditions for structurally different problem-solving procedures', *Journal of Educational Psychology* **66**, 644–654.

Mendicino, L.: 1958, 'Mechanical reasoning and space perception: Native capacity of experience', *Personality Guidance Journal* **36**, 335–338.

McGee, M. G.: 1978, 'Effects of training and practice on sex differences in Mental Test scores', *Journal of Psychology* **100**, 87–90.

McGee, M. G.: 1979, 'Human spatial abilities: Psychometric studies and environmental, genetic, hormonal and neurological influences', *Psychological Bulletin* **5**, 889–918.

Mitchelmore, M. C.: 1975, 'The perceptual development of Jamaican students, with special reference to visualization and drawing of three-dimensional geometrical figures and effects of spatial training', (Doctoral Dissertation, Ohio State University, 1974), *Dissertation Abstracts International* **35**, 7310A.

Mitchelmore, M. C.: 1983, '3D drawings and the parallel concept', in R. Hershkowitz (ed.), *Proceedings of the Seventh International Conference for the PME*, Jerusalem, Israel, pp. 40–44.

Mundy, J. F.: 1987, 'Spatial training for calculus students: Sex differences in achievement and in visualization ability', *Journal for Research in Mathematics Education* **18**, 126–140.

Myers, C. T.: 1958, *The Effects of Training in Mechanical Drawing on Spatial Relations Test Scores as Predictors of Engineering Drawing Grades*, ETS Research Bulletin 58-4, Educational Testing Service, March, 1958, Princeton, N.J.

Parzysz, B.: 1988, '"Knowing" vs. "Seeing". Problems of the plane representation of space geometry figures', *Educational Studies in Mathematics* **19**, 79–92.

Richardson, A.: 1977, 'Verbalizer, visualizer, a cognitive style dimension', *Journal of Mental Imagery* **1**, 109–126.

Sedgwick, L. K.: 1961, *The Effect of Spatial Perception of Instruction in Descriptive Geometry*, Masters thesis, Southern Illinois University.

Sherman, J. A.: 1967, 'Problems of sex differences in space perception and aspects of intellectual functioning', *Psychological Review* **741**, 290–299.

Sherman, J. A.: 1979, *Women and Mathematics: Summary of Research from 1977–1979*, NIE grant, Unpublished manuscript.

Smith, I. M.: 1964, *Spatial Ability: Its Educational and Social Significance*, University of London, London.

Smith, W. S. and C. K. Schroeder: 1979, 'Instruction of fourth grade girls and boys on spatial visualization', *Science Education* **63**, 61–66.

Wattanawaha, N. and M. A. Clements: 1982, 'Qualitative aspects of sex-related differences in performance on pencil-and-paper spatial questions, grades 7–9', *Journal of Educational Psychology* **74**, 878–887.

Willis, S.: 1980, 'Spatial ability: Some neurological research', in K. Milton (ed.), *Research in Mathematics Education in Australia*, University of Tasmania, Hobart, Tasmania, pp. 183–197.

Winter, M. J., G. Lappan, E. Phillips, and W. Fitzgerald: 1986, *Spatial Visualization – Middle Grades Mathematics Project*, Addison-Wesley Publishing Company, Reading, Massachusetts.

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