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Dynamic Modelling of a Four-legged Robot

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Abstract. A unified dynamic modelling approach of closed and/or open kinematic chain mechanisms is established. It is based on the use of the Newton–Euler formalism and the explicit formulation of kinematic holonomic constraints for the closed loop mechanisms. The approach is then applied to derive the dynamic modelling of a four-legged robot adopting a walking gait. The different movement sequences of the gait are analysed in order to calculate the all necessary terms in the dynamic equations of the quadruped robot.

Key words: Newton–Euler formalism, quadruped robot, dynamic modelling.

1. Nomenclature

g	acceleration vector due to the gravity
m_i	mass of body S_i
G_i	mass centre of body S_i
I_i	inertia tensor of the body S_i about its mass centre
f	coefficient of viscous friction
V_{G_i}	3-dim absolute velocity vector
Ω_i	3-dim absolute angular velocity vector
t_i	6-dim twist vector of the body S_i defined as $t_i = [V_{G_i}^T, \Omega_i^T]^T$
n, l	number of bodies, joints
d_i	number of degree of freedom of the joint i
I_d	3×3 identity matrix
T_i^r	6-dim wrench vector acting on the body S_i in which r stands for: l (constraint wrench), g (gravity wrench), f (friction wrench), e (external wrench), m (driving wrench)
θ_i	joint angle of the body S_i
$0_{(3)}$	3-dim zero vector

2. Introduction

Recently, legged robot have generated considerable interest, because of their high performances in various robotic tasks [1] (nuclear power stations maintenance, underwater works, space, etc.). They are also distinguishable from the wheeled robots by their great mobility and adaptability on the rough terrain (ditch, slope, stairs, etc.).

Few works are addressed to the dynamic modelling of the legged robots. This is due to the complexity of both the mechanical structure which generally contains an important number of bodies and joints and the locomotion planning on various terrains called the gaits. The lack of thorough understanding of the legged robots gaits often leads to adopt the inverse dynamic approach [2, 3]. It is based on the use of the force sensors and the integration of the dynamic equations to derive the motion of the legged robot. So it is limited by the material cost and the difficulties of dealing with the high coupled and non-linear dynamic equations [4]. We propose in the second part of this paper a direct dynamic approach of a four-legged robot in a walking gait. The analysis of the cinematographic recording [5], of the horse walking allows to represent the walking gait of the quadrupled robot by a set of the legs movement sequences. The legs Cartesian trajectories can then be proposed and transformed by a kinematic modelling to the joint trajectories [6]. The different sequences in the walking gait deal with closed and/or kinematic chain mechanisms, enhancing therefore the unified dynamic modelling approach developed in the first part of the paper. Although the common points with the formulation of dynamic equations of holonomic mechanical systems presented by Jerkovsky [7] and Angeles [8], the said method differs from the latter in the explicit formulation of the kinematic holonomic constraints in the case of the closed loop mechanisms as described next.

3. Dynamic Modelling

Considering a mechanical system composed of n bodies and l joints and structured in closed and/or open kinematic chains. Different forces and moments are acting on each body of the system due to the friction, the gravity, the actuators, the external environment and the constraint efforts. The Newton–Euler equations of one body S_i can then be written as follows:

$$T_i^l + T_i^e + T_i^f + T_i^m + T_i^g = M_i\dot{t}_i + R_i M_i t_i. \tag{1}$$

Let T_i^r be the resultant wrench defined as the sum of T_i^e,T_i^f,T_i^m and T_i^g wrenches. Thus, expression (1) becomes:

$$T_i^l + T_i^r = M_i\dot{t}_i + R_i M_i t_i. \tag{2}$$

With M_i and R_i are the 6×6 matrices denoting respectively the extended mass and the extended angular velocity and defined as:

$$M_i = \begin{pmatrix} m_i I_d & 0_{(3)} \\ 0_{(3)} & I_i \end{pmatrix} \quad \text{and} \quad R_i = \begin{pmatrix} \tilde{w}_i & 0_{(3)} \\ 0_{(3)} & 0_{(3)} \end{pmatrix}, \tag{3}$$

and $\tilde{w}_i$ is the 3×3 antisymmetric matrix associated to the angular velocity and defined as:

$$\tilde{w} = \begin{pmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{pmatrix} \tag{4}$$

The dynamic equation of the whole system is obtained by combining the equations for all bodies:

$$T^l + T^r = M\dot{t} + RMt. \tag{5}$$

With T^l is the $6l$-dim vector of generalised constraint wrench, T^r and t are the $6n$-dim vectors denoting respectively the generalised resultant wrench and the generalised twist:

$$\begin{aligned} T^l &= (T_1^{lT} \dots T_l^{lT})^T, \\ T^r &= (T_1^{rT} \dots T_n^{rT})^T. \end{aligned} \tag{6}$$

The $6n \times 6n$ matrices of generalised mass M, and generalised angular velocity R are defined as:

$$\begin{aligned} M &\equiv \operatorname{diag}(M_1 \dots M_n), \\ R &\equiv \operatorname{diag}(R_1 \dots R_n). \end{aligned} \tag{7}$$

3.1. KINEMATIC EQUATION OF HOLONOMIC CONSTRAINTS

The writing of the kinematic equations which concern the system joints leads to a set of $6l$ equations and $6n$ unknown variables (absolute linear and angular velocities of the body S_i). Among the $6l$ equations, there are $6l - \sum_{i=1}^{l} d_i$ null terms. One thus has the following kinematic constraints relation:

$$At = 0. \tag{8}$$

With A is the $6n \times 6n$ matrix.

The rank of Equation (8) is defined for the isostatic mechanism as follows:

$$r = 6l - \sum_{i=1}^{l} d_i. \tag{9}$$

From the Equation (8), the vector of generalised constrained wrenches is derived in function of the vector of Lagrange Multipliers:

$$T^1 = A^t\lambda. \tag{10}$$

With λ is a d-dim vector and d is the number of the motorised joints.

3.2. KINEMATIC TRANSFORMATIONS IN THE JOINT SPACE

The dynamic model (5) has to be determined explicitly in the joint space. Furthermore, it must contain only the motorised joint variables θ^I. So it is necessary to perform a kinematic transformation in the joint space as follows:

$$t = T\dot{\theta}^I. \tag{11}$$

With $\dot{\theta}^I$ is the d-dim vector of the active joint velocities.

The relation (11) can be easily found for the open kinematic chain mechanisms by applying some kinematic modelling methods [6]. Otherwise there is some problems with the closed kinematic chain mechanisms because of the presence of the $(l--d)$ passive joint variables θ^D in the relation (11). To determine these variable, we use the following constraint equation:

$$f(\theta) = 0. \tag{12}$$

Where θ is the l-dim vector regrouping the active and passive joint variables:

$$\theta = (\theta^I,\ \theta^D)^T. \tag{13}$$

By derivation of relation (12) with respect the time, we obtain:

$$(J_i\ J_d)\begin{pmatrix}\dot{\theta}^I\\ \dot{\theta}^D\end{pmatrix} = 0. \tag{14}$$

The expression of $\dot{\theta}^D$ is derived from (14) as follows:

$$\dot{\theta}^D = -J_d^{-1}J_i\dot{\theta}^I = H\dot{\theta}^I. \tag{15}$$

By combining the relation $t = Q\dot{\theta}^I$ and (15), we obtain the desired equation:

$$t = T\dot{\theta}^I, \tag{16}$$

With

$$T = (Q_i + Q_d H)\dot{\theta}^I \quad \text{and} \quad Q = (Q_i\ Q_d)^T. \tag{17}$$

At second hand, the derivation of (11) gives:

$$\dot{t} = \dot{T}\dot{\theta}^I + T\ddot{\theta}^I. \tag{18}$$

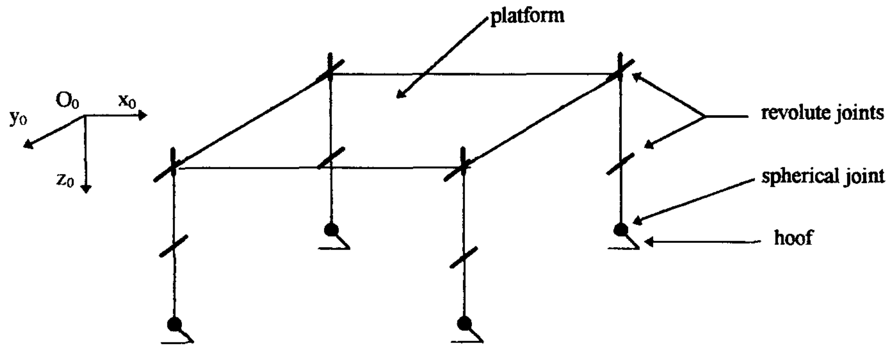

Figure 1. Description of the four-legged robot.

The transformation of the dynamic model in the joint space is realised by considering relation (11) and (18). Moreover, the multiplication of the two terms of (5) by T^T allows to eliminate the constraint actions ($(TA)^T\lambda = 0$), we thus obtain:

$$\tau^m = I(\theta^I)\ddot{\theta}^I + C(\theta^I,\ \dot{\theta}^I)\dot{\theta}^I - \tau^f - \tau^g, \tag{19}$$

with

I	$d \times d$ matrix of generalised inertia $I = T^T MT$	(20)
$C(\theta^I, \dot{\theta}^I)$	$d \times d$ matrix of centrifugal and coriolis terms	
	$C(\theta^I, \dot{\theta}^I) = T^T M\dot{T} + T^T RMT$	(21)
τ^m	d-dim vector of generalised driving action $\tau^m = T^T T^m$	(22)
τ^f	d-dim vector of generalised friction action $\tau^f = T^T T^f$	(23)
τ^g	d-dim vector of generalised action due to the gravity	
	$\tau^g = T^T T^g$	(24)

4. Application

4.1. MECHANICAL DESCRIPTION

The mechanism shown in Figure 1 comprises a rectangular platform connected to four legs, each leg being composed of two links and one foot (called a hoof by analogy with the horse foot). The joints used are: a spherical joint between a hoof and the lower link in which the y axis is motorised, a motorised revolute joint (y axis) between the two links and two motorised revolute joints (y and z axes) between the platform and the upper link.

4.2. ROBOT LOCOMOTION

The adopted strategy of locomotion of the four-legged robot is the walking. This gait is almost used by all the quadrupled animals when moving with moderate

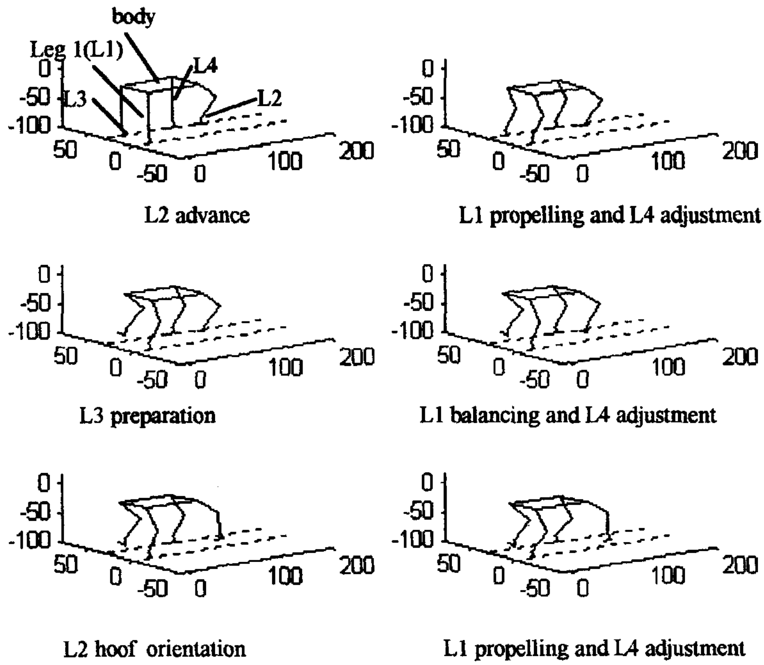

Figure 2. Movement sequences of the first step.

speeds. The walking movements are periodic and composed of four steps. In the first and the third step, one leg advances in the walking direction and orientates its hoof to be parallel to the ground. A second leg is lifting to prepare its advance movement. The support of the whole mechanism is realised by the two other legs. One support leg propels all the robot links in order to put the raising leg in the ground while the other support leg submits the propelling action. The same movements are observable in the second and the fourth step only that the support is realised by the lateral legs and the propelling leg adjusts its motion to place the centre mass projection within the support surface of the legs putting in the ground. We then distinguish, in the different steps, the following movement sequences:

advancing, propelling, preparing, hoof orientation, adjustment and *balancing sequences.*

4.3. DYNAMIC MODELLING OF THE FOUR-LEGGED ROBOT

For each step j ($j = 1 \dots 4$), the different movement sequences can be dynamically modelled by the following equations:

$$\tau_i^j = h_i^j(\theta_i^j)\ddot{\theta}_i^j + H_i^j(\theta_i^j, \dot{\theta}_i^j) \quad (i = 1 \dots q_j), \tag{25}$$

with

q_j number of generalised variables in the sequence j,

h_i^j $q_j \times q_j$ inertial matrix of the robot,

H_i^j q_j-dim vector of coriolis, centrifugal and gravity terms,

τ_i^j q_j-dim vector of generalised joint actions.

The dynamic model of the four-legged robot, at each step is expressed as:

$$\tau^j = h^j(\theta^j)\ddot{\theta}^j + H^j(\theta^j, \dot{\theta}^j), \tag{26}$$

with h^j, θ^j and H^j are defined as:

$$\begin{aligned} h^j &\equiv \mathrm{diag}(h_1^j \ldots h_{q_j}), \\ \theta^j &= (\theta_1^j \ldots \theta_{q_j})^T, \\ H^j &= (H_1^j \ldots H_{q_j})^T. \end{aligned} \tag{27}$$

The dynamic model (26) requires the determination of all the matrices in the relation (19). The computation is then performed for each walking movement.

4.4. DETERMINATION OF T MATRICES

4.4.1. *Advance and Preparation Sequences*

The vectors of generalised twist t and generalised joint variables θ, as shown in Figure 3, are defined as:

$$t = (t_1\ t_2\ t_3)^T \quad \text{and} \quad \theta = (\theta_2\ \theta_3\ \theta_4)^T,$$

with

$$t_1 = \begin{pmatrix} 0 \\ \dot{\theta}_2 \end{pmatrix}, \qquad t_2 = \begin{pmatrix} V_{G_2} \\ \dot{\theta}_2 + \dot{\theta}_3 \end{pmatrix}, \qquad t_3 = \begin{pmatrix} V_{G_3} \\ \dot{\theta}_3 + \dot{\theta}_4 + \dot{\theta}_2 \end{pmatrix}. \tag{28}$$

Let d_i and i_{ij} be respectively the distances $d(O_iG_i)$ and $d(O_iO_j)$, then the kinematic modelling yields:

$$\begin{aligned} x_{G_2} &= d_2 S3 \\ y_{G_2} &= -d_2 S2C3 \\ z_{G_2} &= d_2 C2C3 \end{aligned} \quad \text{and} \quad \begin{aligned} x_{G_3} &= l_{12} S3 + S34 d_3 \\ y_{G_3} &= -l_{12} S2C3 - S2C34 d_3 \\ z_{G_3} &= l_{12} C2C3 + C2C34 d_3 \end{aligned} \tag{29}$$

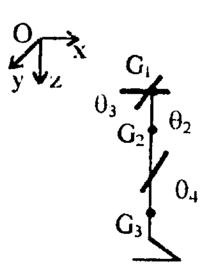

Figure 3. Scheme of one robot's leg.

Deriving the relations (29), the expression of the matrix $T(18\times 3)$ is therewith:

$$\begin{aligned} T = (\; & 0\,0\,0,\ 0\,0\,0,\ 0\,0\,0,\ 0\,0\,0,\ 1\,0\,0,\ 0\,0\,0,\ 0\,0\,0,\ 0\ b_{12}\ 0, \\ & a_{22}\ b_{22}\ 0,\ 1\,0\,0,\ 0\,1\,0,\ 0\ b_{13}\ c_{13},\ a_{23}\ b_{23}\ c_{23},\ a_{33}\ b_{33}\ c_{33}, \\ & 1\,0\,0,\ 1\,1\,1,\ 0\,1\,1,\ 0\,0\,0)^T. \end{aligned} \tag{30}$$

With

$$\begin{cases} b_{12} = d_2C3 \\ a_{22} = -d_2C3C2 \\ b_{22} = d_2S3S2 \\ b_{13} = l_{12}C3 + d_3C34 \\ c_{13} = d_3C34 \\ a_{23} = -l_{12}C3C2 - d_3C34C2 \\ b_{23} = l_{12}S3S2 + d_3S34S2 \\ c_{23} = d_3S34S2 \\ a_{33} = -l_{12}C3S2 - d_3C34S2 \\ b_{33} = -l_{12}S3C2 - d_3S34C2 \\ c_{33} = -d_3S34C2 \end{cases} \tag{31}$$

4.4.2. *Orientation Hoof Sequence*

This is a simple case where the twist vector t_4, the joint variable θ, and the T matrix are defined as follows:

$$t_4 = (V_{G_4}\ \dot{\theta}_4)^T, \qquad \dot{\theta} = \dot{\theta}_4, \qquad T = (-d_{4z}\ 0\ d_{4x}\ 0\ 1\ 0)^T. \tag{32}$$

4.4.3. *Balancing Sequence*

We notice three principle elements in this sequence: a balanced support leg (bodies S_i,S_{i+1},S_{i+2}), a passive support leg (S_j,S_{j+1},S_{j+2}) and the system composed of the upper link S_p,S_{i+1},S_{i+2}, S_k,S_{k+1},S_{k+2}, S_l,S_{l+1} and S_{l+2} (Figure 4). The

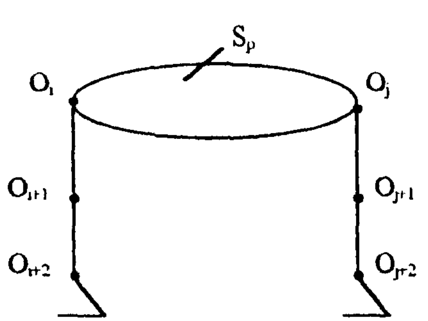

Figure 4. Scheme of the mechanism in the balanced sequence.

two support legs are on the diagonal or on the lateral. The indices j and k denote the two other legs.

The kinematic analysis gives the relation between the twist vector t_s of the system S and the balanced angle θ:

$$\begin{aligned} \dot{x}_{G_s} &= 0 \\ y_{G_s} &= \dot{\theta}(-aS\theta - bC\theta) \\ z_{G_s} &= \dot{\theta}(aC\theta - bS\theta) \end{aligned} \tag{33}$$

From relation (33), we can derive the matrix T as follows:

$$T = (0 \ \ -aS\theta \ \ -bC\theta \ aC\theta \ \ -bS\theta \ 1 \ 0 \ 0)^T, \tag{34}$$

with $a = y_{G_s} - y_{O_{i+2}}$ and $b = z_{G_s} - z_{O_{i+2}}$.

4.4.4. *Propelling Sequence*

The same analysis as in the balanced sequence, with respect to Figure 5, allows to establish the T matrix as follows:

$$\begin{aligned} T = (\ & d_{i+1}C(\theta_{i+1} + \theta_s) \ d_{i+1}C(\theta_{i+1} + \theta_s), \ 0 \ 0, \\ & d_{i+1}S(\theta_{i+1} + \theta_s) \ d_{i+1}S(\theta_{i+1} + \theta_s), \ 0 \ 0, \\ & 1 \ 0, \ 0 \ 0, \ 0 \ -llC\theta_s, \ 0 \ 0, \ 0 \ llSC\theta_s \ 0 \ 0, \ 0 \ 1, \ 0 \ 0)^T, \end{aligned} \tag{35}$$

with

$$ll = \sqrt{(x_{i+2} - x_i)^2 + (z_{i+2} - z_i)^2}. \tag{36}$$

4.5. GENERAL EXPRESSIONS OF τ^f, τ^g AND τ^m

The general expressions of the wrench vectors of friction, driving actions and forces due to the gravity are defined, for each robot's link, as follows:

$$\tau_i^f = \begin{pmatrix} 0_{(3)} \\ -f\dot{\theta}_i \end{pmatrix}, \quad \tau_i^m = \begin{pmatrix} 0_{(3)} \\ C_i^m \end{pmatrix}, \quad \tau_i^g = \begin{pmatrix} 0_{(3)} \\ -m_i g \end{pmatrix}. \tag{37}$$

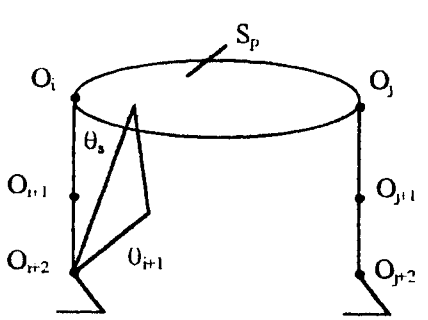

Figure 5. Scheme of the mechanism in the propelling sequence.

5. Conclusion

The movement sequences of a quadrupled robot in a walking gait can be represented by the closed and/or loop mechanisms. This is justified to the use of the unified dynamic modelling approach proposed in this paper. It is characterized by the explicit formulation of the kinematic holonomic constraints for the closed loop mechanism. Moreover, it avoids to compute the constraint efforts increasing thus the efficiency in the control process. The approach can be either applied to the dynamic modelling of other quadrupled gaits such as the amble, the trot and all the well planning gaits. The principle is always to determine the gait's movement sequences, the legs Cartesian and joint trajectories and the dynamic equations.

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