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MATHEMATICS HAS A FRONT AND A BACK

ABSTRACT. It is explained that, in the sense of the sociologist Erving Goffman, mathematics has a front and a back. Four pervasive myths about mathematics are stated. Acceptance of these myths is related to whether one is located in the front or the back.

In the famous book, *The Presentation of Self in Everyday Life*, by American sociologist Erving Goffman, there is a chapter called 'Regions and Region Behavior'. There Goffman introduces the concept of the "front" and the "back": regions to which the public is admitted, and from which it is excluded. In a restaurant, for example, the serving area is the "front"; the kitchen is the "back". In a theater, of course, the front of the stage is for the audience; backstage is for the actors, stagehands, props, and costumes. In front, the actors (waiters) wear costumes (uniforms); in back, they change clothes or rest in their casual dress. In general, the front is the region to which the public is admitted, where service is performed; the back is a region restricted to professionals, where preparations are made to provide a service.

Goffman's contribution was to extend this concept of the "front region" and the "back region" from restaurants and theaters to all, or almost all, institutions of modern life. In universities, the classrooms and certain parts of the library are the "fronts" where the "public" (the students) are served. The Chairman's or the Dean's offices are the "backs", where the products (classes and courses) are prepared "behind the scenes".

There is nothing sinister in this separation; it is a practical necessity. Goffman gives examples of the distress that can arise from blurring the line between "front" and "back"; for instance, a gasoline (petrol) station whose customers feel free to wander into the parts department and help themselves to wrenches and hammers.

Goffman quotes Orwell:

It is an instructive sight to see a waiter going into a hotel dining room. As he passes the door a sudden change comes over him. The set of his shoulders alters; all the dirt and hurry and irritation have dropped off in an instant. He glides over the carpet, with a

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solemn, priest-like air . . . he entered the dining room and sailed across it, dish in hand, graceful as a swan. (Goffman, p. 121)

The waiter who does this is performing automatically. If his split persona were brought to his attention, he would acknowledge its existence. But in the ordinary course of things, he just waits on tables. He is not conscious of putting on an act, or fooling anybody.

My purpose here is to point out that, like other social institutions, mathematics too has its "front" and its "back", and to identify and describe them. It should be clear that now we are not speaking of "regions" in the literal, physical sense, as in a dining room and kitchen: mathematics is not necessarily associated with any particular physical setting; it is just a certain sort of activity. So its "front" and "back" will be particular kinds or aspects of mathematical activity, the public and private, or the part offered to "outsiders" (down front) versus the part normally restricted to "insiders" (backstage).

In this sense of the term, the "front" of mathematics is mathematics in "finished" form, as it is presented to the public in classrooms, textbooks, and journals. The "back" would be mathematics as it appears among working mathematicians, in informal settings, told to one another in an office behind closed doors.

Compared to "backstage" mathematics, "front" mathematics is formal, precise, ordered and abstract. It is separated clearly into definitions, theorems, and remarks. To every question there is an answer, or at least, a conspicuous label: "open question". The goal is stated at the beginning of each chapter, and attained at the end.

Compared to "front" mathematics, mathematics "in back" is fragmentary, informal, intuitive, tentative. We try this or that, we say "maybe" or "it looks like".

Observe that in all our examples the front is divided into subregions, of first, second, and even third class. A restaurant, for example, may include both a banquet hall and a snack bar. A theater has box, orchestra, and balcony seats. And the public for mathematics includes, among others, professional mathematicians themselves, graduate students, and undergraduates.

The back is also divided, for efficiency and convenience, into subregions. In a restaurant, there are the domains of the salad chef, pastry chef, dishwasher, and so on. The reader can fill in the analogous divisions among working mathematicians.

The purpose of a separation between front and back is not just to keep the customers from interfering with the cooking; it is also to keep the customers from knowing too much about the cooking.

Everybody down front knows that the heroine of the melodrama is wearing rouge. They probably don't quite know what she looks like without it. The diners know what's supposed to go into the ragout, but they don't know for sure what *does* go into it.

We can describe this state of affairs by saying that the front/back separation makes possible the preservation of a myth – whether it be the flavoring of the food or the beauty of the actress.

By a myth we shall mean simply taking the performance seen from up front at face value; failing to be aware that the performance seen "up front" is created or concocted "behind the scenes" in back. This myth, in many cases, adds to the customer's enjoyment of the performance; it may even be essential.

More generally, a myth is a story that possesses a certain allegorical or metaphorical power; it is not literally true, but it survives while the generations pass by. Such, for instance, was the myth of the divine right of kings. Such are the myths of Christmas and Easter, and of course, the corresponding myths of other religions.

Mathematics, too, has its myths. One of the unwritten criteria separating the professional from the amateur, the insider from the outsider, is that the outsiders are taken in (deceived), the insiders are not taken in.

It would be straining patience to try to compile a complete dictionary of myths in mathematics. We list a few; enough to illustrate our point, and to enable the reader (as an exercise) to extend the list at pleasure.

To present, describe and refute all of these myths would generate a thick volume. We content ourselves with some provocative comments; the reader can follow them up with the readings listed in the bibliography.

First, the myth of Euclid. This is discussed on pages 322–30 of *The Mathematical Experience* (see Davis and Hersh). The Euclid myth is defined there as the belief that the books of Euclid contain truths about the universe which are clear and indubitable. In view of the general availability of *The Mathematical Experience*, we need not go into a detailed discussion of the Euclid myth here. We merely point out that advanced students of geometry, and certainly professional mathematicians, are well aware that Euclid's axioms are unintelligible, his proofs incomplete, and his results limited to very restricted and special cases. Nevertheless, in secondary schools, in watered-down versions that fail even to mention his impressive achievements in solid geometry, Euclid continues to be upheld as the ideal model of pure mathematics and rigorous proof.

In a similar way, the plaster Newton created in the eighteenth century ("God said, Let Newton be – and all was light") is intact as a myth; the complex historical reality of Newton is almost unknown, even among the mathematically literate.

The myths of Russell, Brouwer and Bourbaki – of logicism, intuitionism, and formalism – have also been treated in *The Mathematical Experience*. Formalism is the subject (object?) of a beautiful diatribe in the preface to Lakatos' *Proofs and Refutations*. Therefore, in the hope of encouraging the circulation of *The Mathematical Experience*, we pass on to the more general myths on our list.

(1) *Unity*: There is only one mathematics, indivisible, now and forever. Mathematics is a single/inseparable whole.

(2) *Objectivity*. Mathematical truth or knowledge is the same for everyone. It does not depend on who in particular discovers it; in fact, it is true whether or not anybody ever discovers it.

(3) *Universality*: Mathematics as we know it is the only mathematics that there can be. If the little green men (and women?) from Quasar X9 sent us their math textbooks, we would find again $A = \pi r^2$.

(4) *Certainty*: Mathematics possesses a method, called "proof" or sometimes "rigorous proof", by which one attains absolute certainty of the conclusions, given the truth of the premises.

It would not be hard to find quotations to show that these beliefs are indeed widely held. Fortunately *Eureka* strives for entertainment, not pedantry, so we dispense with references.

By calling these beliefs myths, I am not declaring them to be false. A myth need not be false to be a myth. The point is that it serves to support or validate some social institution; its truth is irrelevant, and most likely not determinable.

Who can say, for example, that the doctrine of the divine right of kings is false? In the absence of a clear channel to the mind of God, this dogma can never be absolutely proved or disproved. But it was a useful belief, which in its time was credible, and served a purpose.

In a similar way, the unity, universality, objectivity, and certainty of mathematics are beliefs that support and justify the institution of mathematics. (For mathematics, which is an art and a science, is also an institution, with budgets, administrations, publications, conferences, rank, status, awards, grants, etc.)

Part of the job of preparing mathematics for public presentation – in print or in person – is to get rid of all the loose ends. If there is disagreement whether a theorem has really been proved, then that theorem will not be included in the text or the lecture course. The standard style of expounding mathematics purges it of the personal, the controversial, and the tentative, producing a work that acknowledges little trace of humanity, either in the creators or the consumers. This style is the mathematical version of "the front".

Without it, the myths would lose much of their aura. If mathematics were presented in the same style in which it is created, few would believe in its universality, unity, certainty, or objectivity.

Beliefs (1) through (4) are not self-evident or self-proving; they can be questioned, doubted, or rejected. Indeed, by some people they *are* rejected. Standard and "official" as these doctrines are, they are not taken so literally, so naively, by the backstage people. (A busboy or a stagehand is likely to be skeptical about the contents of the stew or the complexion of the ingenue.) Let us examine them critically, in order from (1) to (4), to justify our calling them myths.

From a backstage point of view, then, what about myth (1), unity? We see pure and applied mathematicians cooperating sometimes, but more often unaware of each other's work, and usually working to quite different standards and criteria. The pure may even declare that applied mathematics is not mathematics at all ("Where are the definitions? Where are the theorems?"). Or even worse, it is bad mathematics. (See Halmos. This article is a landmark piece for having the courage to express an attitude, common but unspoken, among "pure" mathematicians.) And even within pure mathematics, it is plainly visible at meetings of the American Mathematical Society that any contributed talk is understood by only a small fraction of those present at the meeting. The "unity" claimed in principle does not exist in practice.

As to myth (2), objectivity – yes, there is an amazingly high consensus in mathematics as to what is "correct" or "accepted". But beside this, and equally important, is the issue of what is "interesting" or "important" or "deep" or "elegant". These aesthetic or artistic criteria vary widely, from person to person, specialty to specialty, decade to decade. They are perhaps no more objective than aesthetic judgments in art or music.

And universality (myth (3)) – who is to say? If there is "intelligent life" in Quasar X9, whatever we should mean by that, it might not be little green women and men. It might be blobs of plasma which we could not even recognize as intelligent beings. What would it mean to talk about their literature, or art, or mathematics? The very notion of comparing presupposes beings enough like us to make communication conceivable. But then the possibility of comparison is not universal; it's conditional on their being "enough like us".

And last of all, myth (4), certainty. Most of us are certain that $2 + 2 = 4$, though we probably would find we don't all mean exactly the same thing by that equation. But it's quite another matter to claim equal certainty for the theorems of contemporary mathematics.

Many of them have proofs which fill dozens of pages, which rely on other theorems whose proofs have not been rigorously checked by their users. These proofs do not pretend to be complete but often contain such phrases as, "it is easily seen" or "a standard argument then yields" or "a short calculation gives" and so on. Moreover, more and more often, the paper will have several co-authors, not one of whom has carefully read the whole paper; and very possibly it will use the result of some calculations of a computing machine that none of the authors, and possibly no living human being, completely understands. Certainty, like unity, can be claimed only "in principle", not in practice.

Myths, of course, need not be true; they need to be useful. Whatever the reason, it is clear that mathematicians want to believe in unity, objectivity, universality, and certainty, somewhat as Americans want to believe in the Constitution and free enterprise, or other nations, in their Queen or their Revolution. But even while they believe, they know better.

An important part of becoming a professional, in mathematics or anywhere else, is to move from the "front" to the "back". And part of this transition is to develop a less naive, more sophisticated attitude toward the myths of the profession. The leading lady needs her rouge; the stagehands know that she is the same actress they see behind the scenes with an ordinary, everyday face.

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REFERENCES

Davis, P. J.: 1972, 'Fidelity in Mathematical Discourse: Is 1 + 1 Really 2?', *American Mathematical Monthly* **78**, 252–63.

Davis, P. J. and R. Hersh: 1981, *The Mathematical Experience*, Houghton Mifflin Company, Boston.

Goffman, E.: 1959, *The Presentation of Self in Everyday Life*, Doubleday Anchor Books, New York.

Halmos, P.: 1981, 'Applied Mathematics is Bad Mathematics', in L. Steen (ed.), *Mathematics Tomorrow*, Birkhäuser, Boston.

Lakatos, J.: 1978, *Proofs and Refutations*, J. Worrall and E. Zahar (eds.), Cambridge University Press, Cambridge.

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