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A Review on Measurements of Particle Velocities and Diameters by Laser Techniques, with Emphasis on Thermal Plasmas

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An overview of measurements of particle velocities and diameters by laser techniques, with emphasis on thermal plasmas, is given. As far as velocities are concerned, laser-Doppler velocimetry is discussed as a well-established technique. Diameter measurements are much less developed. The state of the art is described and prospective considerations are stressed.

KEY WORDS: Laser velocimetry; optical sizing; plasma measurement techniques.

1. INTRODUCTION

Knowledge of fluid and particle velocities, and of particle sizes, under plasma conditions, is fundamental to the understanding of various phenomena and processes such as heat and mass transfer between the high-temperature flow and entrained particles with, for instance, the objective of better design and operation of plasma spraying equipment.

Several systems have been used for velocity measurements such as discussed in Refs. 1–7. In the past, photographic techniques have also been extensively used and many laboratories continue to use them. A very good classification and discussion of the different techniques is given by Lemoine,[8] and typical works are described in,[9–13] among others. Such techniques can be used to simultaneously measure particle sizes as is done in combustion systems.[14] Let us also suggest that holographic techniques may be useful in plasma flows, for both particle size analysis[15] and velocity measurements,[16] although we are not aware of such measurements. Attention will be focused in this paper on Laser-Doppler velocimetry (LDV), a

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well-established method in cold flows with great potential applications to thermal plasmas. Time of flight laser anemometry (TFLA) will also be discussed, since it could be considered as a variation of LDV.

Developments in LDV are not completely perfected, especially as far as plasma applications are concerned. Nevertheless, we share Stevenson's opinion[17] that "we are now somewhere near the point at which the instrument is fully developed and, barring major conceptual advances, only minor improvements in capabilities can be expected," although such a definite assessment is always somewhat dangerous. On the other hand, the situation is rather different as far as diameter measurements are concerned, particularly if interest is focused on simultaneous measurements of velocity and diameter of particles. The present state of the art will be given for systems derived from LDV or strongly related to them.

2. LASER-DOPPLER VELOCIMETRY

2.1. General

Laser-Doppler velocimetry provides us with a unique opportunity to study velocity fields in thermal plasmas. In such high-temperature flows, it is fair to say that the laser anemometers (LDV and TFLA) have no competitors.

Advantages are that the technique is nonintrusive, provides *in situ*, local, and rather instantaneous measurements, with a linear response and without calibration. Each of these statements would nevertheless need to be refined. For instance, in discussing only one, it is true that the technique is nonintrusive as far as the optical probe is concerned. But artificial seeding with scattering centers may be required, a possibly troublesome feature due to the usual sensitivity of the plasma to the presence of added particles. The method is versatile (LDV measurements can be achieved through various optical and electronic designs), making it possible to adapt it to a large range of situations. This fact should be regarded as an advantage. But it is closely connected to a disadvantage: although extremely simple in principle, correct LDV measurements usually require a lot of expertise. Furthermore, the medium under study must be transparent and optical access is required. Also, scattering centers are necessary, with possibly the need for artificial seeding. The seeding problem can be cumbersome in plasma applications.

Since the first experiments in water by Yeh and Cummins,[18] then by Foreman in gases,[19] interest in LDV rapidly increased with applications to turbulence, combustion, geophysical flows, etc. On the other hand, experiments in plasmas started rather recently, obviously because of the expected difficulties, such as high temperatures giving rise to noise problems due to plasma and seeding particle radiation, electromagnetic saturation of electronic devices when working with radiofrequency torches or electrical interference from the large current supplied to arcs, fast evaporation of the scattering centers, and the thermophoresis phenomenon producing possible errors in measurements and sometimes very slow rates of data acquisition.

As a result of such difficulties, LDV experiments in plasmas remain relatively scarce. This scarcity is obvious in a recent publication of LDV in plasmas.[20] The list of references[21-38] corresponds to the pioneering papers in this special field. In general, basic books on LDV are available[39-42] as well as workshop and symposium proceedings,[43-47] and the literature concerning LDV systems (whatever the application) is very large, preventing us to attempt here even to present a selected bibliography. The papers given in the list of references are typical, but a significant number of valuable works have been omitted because of lack of room.

2.2. BASIC PRINCIPLE: THE DOPPLER EFFECT

Although a laser source is not necessary in principle,[48] the basic idea in LDV is to measure velocities from the frequency shift due to the Doppler effect undergone by laser light that is scattered from particles carried by the flow under study (Fig. 1). Let U_k be the velocity at time t of a scattering center passing the optical probe in M, in a medium of refractive index of unity. It is illuminated by a laser beam of wave number vector $K_{i,k}$ (i for "incident"), with $|K_{i,k}| = 1/\lambda$, where λ is the wavelength of the incoming light. The scattered light of wave number $K_{s,k}$ (s for "scattered") is observed in the direction O_s. The vectors are drawn in the same plane for convenience. Due to the Doppler effect, the incident frequency f_i is not equal to the scattered frequency f_s. The beating frequency, or Doppler frequency, $f = |f_i - f_s|$, is found to be

$$f = \left| \sum_k (K_{i,k} - K_{s,k}) U_k \right| \tag{1}$$

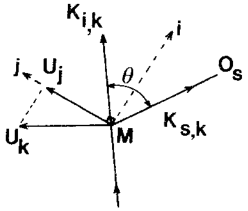

Fig. 1. The basic principle of LDV.

Usually $f \ll f_i \sim f_s$. Then (1) becomes

$$f = 2U_j \sin(\theta/2)/\lambda \qquad (2)$$

where θ is the scattering angle and U_j the velocity component of the scattering center in the direction j perpendicular to the bisectrix M_i of the scattering angle.

Therefore, the measurement of f is a measurement of a component of the velocity U_k. Two- or three-dimensional information is nevertheless obtainable.[49] For large enough velocities, the Doppler frequency can be measured by optical means.[50–52] But, usually, the incident and scattered light is superimposed on a quadratic photodetector to produce an electronic signal which is modulated at the frequency f and further processed.

2.3. Optical Systems

The above-discussed basic principle has led to various optical systems which can be classified into three main categories:

(i) the reference system, first used in Ref. 18, where beating is achieved as in Section 2.2 between the incident light frequency and the scattered light frequency.
(ii) the interference system where two laser beams are focused on the flow under study and beating is achieved between the light scattered in a given direction from each incident beam.
(iii) the dual scatter system, due to Durst and Whitelaw,[53] where again a single laser beam is focused but the beating is achieved between scattered light received from two directions simultaneously.

The interference system is the most widely used and the only one we shall further discuss. The setup is shown in Fig. 2. Adapting and applying the relation (1), we find that the Doppler frequency is given by the relation (2), θ being now the angle between the converging beams, and U_j the velocity component perpendicular to the bisectrix of this angle.

This setup presents decisive advantages when compared to the other ones. It is very easy to adjust and align, and rather insensitive to vibrations,

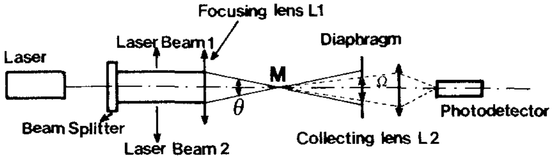

Fig. 2. Basic optical interference setup.

in particular due to the fact that the two scattered beams are geometrically perfectly superimposed. Thus, this system must be preferred in industrial and hostile environments. Furthermore, the solid angle of collection Ω can be made large without producing any broadening of the scattered light spectrum, a key advantage when velocity fluctuation measurements are needed. Large solid angles of collection also mean that small single particles can be more easily detected, a particular advantage for measurements in plasmas.

2.4. Fringe Concept

The two incident beams produce a three-dimensional set of fringes in the common crossover region. The structure of the fringe zone is indeed very complex but, as a first approximation, all the fringes are parallel to the bisector plane of the angle θ. The fringe spacing is equal to

$$d_f = \lambda/[2\sin(\theta/2)] \tag{3}$$

A particle passing through the fringe system will then scatter light modulated with a frequency $f = U_j/d_f$ which is identical to the beating frequency of relation (2). This is the reason why the setup may be called an interference system.

It is often claimed that the Doppler and the fringe concepts are equivalent. Rudd even generalized this equivalence and has shown that it could be used whatever the optical category by introducing the notion of virtual fringes.(54) However, the fringe model is mainly a heuristic one since a large particle passing outside the crossover region can also produce a Doppler signal.(55)

The fringe model nevertheless also enables us to emphasize the coherence properties of the laser beams. The existence of several longitudinal modes in the laser output will affect the efficiency with which the set of fringes can be produced, depending on the pathlength difference between the incident beams.(56,57) Furthermore, this multiaxial mode produces parasitic frequencies in the electronic signal, in the case of broadband detection, due to intermode beating. This can lead to erroneous measurements,(58) particulary for high velocity flows.

2.5. Collecting Optics

The collecting optics determines numerous characteristics of the velocimeter, such as the spatial resolution (later discussed) and the signal/noise ratio (SNR) of the Doppler signals. The influence on SNR is mainly due to the fact that scattering diagrams of the scatter centers exhibit very strong asymmetries. Forward scattering is typically two or three orders of magnitude larger than backward scattering. Thus forward collection is more efficient.

But the choices are often limited by practical considerations (room available, optical access, etc.), and a backward scattering configuration is sometimes unavoidable, leading to dramatic reductions in the SNR. Higher laser powers are then needed, and processing can require the use of a photon correlation technique (Section 2.10). Off-axis collection is also used, as close as possible near forward directions, in order to obtain reasonably good SNR while improving the spatial resolution.[29]

The SNR also depends on the solid angle of collection Ω through the visibility concept (Section 3). Furthermore, in plasma situations, the collecting optics must usually contain a spectroscopic device (interference filter or monochromator) to get rid of parasitic emitted light.

2.6. Spatial Resolution

The shape and characteristic dimensions of the control volume are determined by both the focusing and the collecting optics, and also to some extent by the electronic systems and adjustments. Only optical aspects are discussed here.

The shape, characteristic dimensions, and general features of the crossover region of the incident beams can be deduced from the theory of Gaussian beams[59] as discussed in Refs. 60 and 61. Boundaries at $(1/e)$ and $(1/e^2)$, corresponding to generated Doppler signals having an amplitude

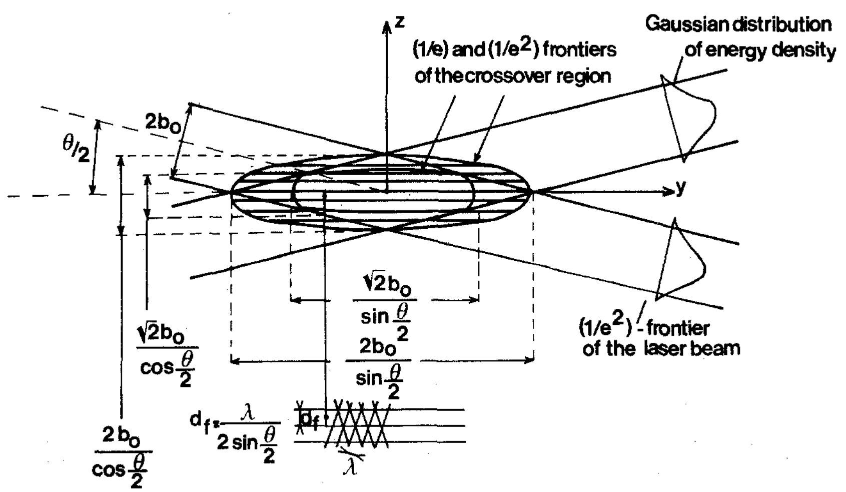

Fig. 3. The crossover region.

of $(1/e)$ and $(1/e^2)$ of the maximal amplitude, are ellipsoids given by

$$x^2+y^2\sin^2(\theta/2)+z^2\cos^2(\theta/2)=b_0^2/\alpha \tag{4}$$

where α is 2 and 1 for $(1/e)$ and $(1/e^2)$ boundaries, respectively, x, y, z are defined in Fig. 3, and b_0 is the $(1/e^2)$ waist radius estimated by

$$b_0=(2\lambda f_L/\pi d_0) \tag{5}$$

where f_L is the focal length of the focusing lens and d_0 the diameter of the laser beam before focusing. This is indeed a very rough picture, but nevertheless it is useful as a guide. According to relation (5), spatial resolution is better when decreasing f_L or increasing d_0, justifying sometimes the use of a beam expander in the focusing optics. Changing f_L affects the angle θ, unless compensated by simultaneously changing the beam separation distance after the beam splitter. The angle θ usually does not influence significantly the transverse dimensions of the probe when $\cos(\theta/2)$ is of the order of 1, but longitudinal dimensions are much more sensitive.

The control volume is the intersection of the focusing domain and of a viewing volume defined by the collecting optics. Off-axis collection enables one to considerably enhance the longitudinal spatial resolution.

2.7. Pedestal Removal

A particle crossing the control volume produces a Doppler burst as shown in Fig. 4. The high-frequency modulation is the beating frequency associated with the Doppler effect, and the low-frequency modulation is the pedestal associated with the Gaussian distribution of the energy density in a TEM_{00} laser beam. In order to measure the Doppler frequency by using a counter (Section 2.10), the pedestal must be removed to avoid false counting.

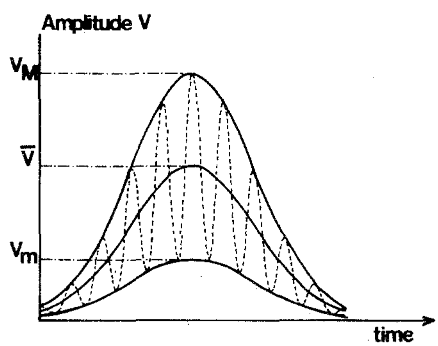

Fig. 4. Typical Doppler signal.

The most common way to remove the pedestal is by high-frequency band pass electronic filtering. The low frequency cutoff F is about equal to the high frequency f over the number of fringes; thus $F \sim f d_f/(2b_0)$. In turbulent flows, care must be taken to not simultaneously remove low-frequency parts of the signal frequency spectrum coming from low-velocity contributions of the velocity probability density function. Frequency shifting (later discussed) helps by increasing the ratio F/f. An alternative to electronic removal is optical removal.[62]

2.8. Signal/Noise Ratio

The SNR of a Doppler signal may completely determine the kind of signal processing to choose and will play a determining role in the accuracy of the measurements.

Neglecting the dark current with respect to the signal current, we can roughly estimate the SNR associated with a single particle from the following relation[42,63]:

$$\mathrm{SNR} \sim \frac{\eta_q P}{4 h f_s \Delta f} \tag{6}$$

where η_q is the quantum efficiency of the photocathode, P the collected light power, f_s the frequency of the scattered light, Δf the bandwidth of the electronic system, and h the Planck constant.

This formula is a guide to design details of a laser velocimeter, but it is in practice very optimistic. The effective SNR can be decreased by a factor of ten[42] or more due to nonperfect adjustments, parasitic lights, particles passing outside the control volume which do not produce useful signals but contribute to the noise level, and poor visibility of large particles. When radiofrequency torches are studied, additional noise can exist due to parasite electronic oscillations from rf waves invading the system. This is particularly troublesome when the torch frequency is of the same order of magnitude as the beating frequency.[28,29]

2.9. Photon-Resolved Signals

When the collected power decreases, the signal will appear more and more noisy according to relation (6) up to a stage where, instead of a modulated wave, it looks like individual random peaks (Fig. 5) where isolated photons are resolved, due to the fact that light scattering is actually a quantum stochastic process. The relevant information is then contained in the mean rate of photon arrival and in the statistics of the time intervals between arrivals. The photon arrival is a Poisson process. Applications of

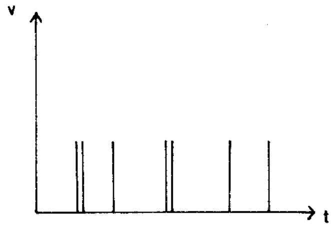

Fig. 5. Sketch of a photon-resolved signal.

these statistics to LDV, including background noise and many particle events, are discussed by Lading.[64] A photon-resolved signal can lead to rather accurate velocity measurements, depending on the number of detected photons. A total of 100 detected photons is indeed a strong signal, and signals corresponding to an averaged number of three photons per Doppler cycle produce a 1% accuracy in velocity, according to Pike.[65]

2.10. Signal Processing

The basic information is the Doppler frequency from which we want to derive at least mean velocities, but there may also be interest in velocity fluctuations, higher moments, velocity correlations, or spectral densities.

Attention being focused on electronic signals, the kind of processing system to choose and its complexity depend on the information needed, the required accuracy, and the specific situation under study, the latter determining the nature of the signal (classical with more or less noise, or photon resolved).

We can distinguish between processing in the frequency or in the time domains, and correlation methods.[39] The spectrum analyzer, in the frequency domain, which was the first kind of processor used from a historical point of view,[18] has mostly been replaced in favor of more sophisticated and accurate devices.

In the time domain, we can mention frequency trackers, counters, and transient recorders. In frequency trackers, the frequency is tracked and converted to analogous voltage which can be processed, for instance, with the same electronic systems that are used in hot-wire anemometry. Edwards made a comparison between trackers and counters[66] and concluded that trackers are superior to counters in many respects. Nevertheless, roughly speaking, trackers should rather be used for nearly continuous signals corresponding to highly seeded flows. They do not seem well adapted to plasma conditions where the seeding is usually rather low.

Counters are very well suited to classical signals and mostly employed nowadays. They have been successfully used in plasmas.[27–29,34,37] The present available commercial counters work on zero-crossing detections, with possibly dual-counter validations. In dual-counter systems, a first counting device measures the time t_1 corresponding to n_1 cycles, and a second counting device the time t_2 corresponding to n_2 cycles. Measurements are only retained if both period measurements agree within a preset accuracy. The first burst processor used an $(n_1=4)/(n_2=8)$ comparison. Then, it was suggested that it was advantageous to compare an odd and even number of cycles, and the 5/8 comparison became very popular. Some systems continue to work on this concept. Nevertheless, it is now rather accepted that the foregoing idea has no definite basis, and present counters usually offer a large range of comparisons plus possibly a total burst mode employing end of burst detection for signal validation. An interesting comparison between single and dual counters has been performed by Pfeifer.(67)

To end with the time domain, let us mention again the use of a high-speed transient recorder with digital computer processing of the stored signal.(68,69) This approach seems to be very promising.

Photon-resolved signals cannot be processed by the aforementioned systems. In this case, correlation methods are necessary, although they can also be used for processing classical signals. The photomultiplier is used as a photon-counter system. The output of the photomultiplier is a sequence of pulses corresponding to the stochastic Poisson process of photon detection. The correlation function exhibits the same periodicity as the Doppler signal and its decrease versus the time lag is closely linked to velocity fluctuations. Photon correlators have been used in plasma situations.(30,35,36,38)

2.11. Sampling Characteristics

From now on, interest will be focused on dual-counter systems previously described which appear at the present time more versatile, elaborate, and able to produce more information than other systems (in the author's opinion). Then, a validated single Doppler burst gives rise to a value $U_{(i)}$ of the velocity component under study. Assuming the ideal case of periodic sampling without bias effects, the basic formulas for the mean velocity $\bar{U}$ and the relative standard deviation are

$$\bar{U}=\frac{1}{N}\sum_{i=1}^{N} U_{(i)} \tag{7}$$

$$\sigma/\bar{U}=\frac{1}{\bar{U}}\left[\frac{1}{N-1}\sum_{i=1}^{N}(U_{(i)}-\bar{U})^2\right]^{1/2} \tag{8}$$

Using confidence intervals of 10% and accepting statistical uncertainties ε_1 and ε_2 for $\bar{U}$ and $(\sigma/\bar{U})$, respectively, the required numbers of individual realizations N_1 and N_2 can be estimated by the following relations[70]:

$$N_1 \sim \frac{3}{\varepsilon_1^2}\left(\frac{\sigma}{\bar{U}}\right)^2 \tag{9}$$

$$N_2 \sim 5/\varepsilon_2^2 \tag{10}$$

Similar expressions have also been proposed for higher moments.[71] For spectra measurements, the Shannon theorem states that the acquisition frequency must be at least twice the highest frequency present in the process.[72] The situation is different when the sampling is not periodic as is usually the case in LDV. Norsworthy pointed out that randomization of the sample interval can circumvent the Shannon theorem.[73]

2.12. Directional Ambiguity Removal

In the basic LDV systems previously considered, information on the direction of the measured velocity component is lost. This information may be necessary in highly turbulent flows or reversing and recirculating situations. Thus the directional ambiguity must be removed in some cases.

In the interference set up, directional ambiguity removal is usually achieved by frequency shifting one of the two incident beams with respect to the other, by means of acousto-optic Bragg cells[74] or a rotating grating[75,76] with the result that the interference fringes now move through the observation volume. Let us also mention a system using electro-optic materials, in which the application of an electric field produces a birefringence phenomenon,[77] or removal without frequency shifting by using two parallel control volumes, each of them being built with a different color, and by determining which fringe system is first crossed by the scatter center.[78]

2.13. Broadening Sources

In practice, the signal spectrum always exhibits a finite width even in laminar flows. This spectrum broadening will be called here "Doppler ambiguity," whatever its causes. The Doppler ambiguity is very troublesome when velocity fluctuations are to be measured, especially in low turbulent intensity flows. Corrections can be made when the standard deviation associated with the Doppler ambiguity is small with respect to the one associated with turbulence.

Doppler ambiguity broadening sources are dependent on the electronic processing system and also on the optical setup. When processing the signal with a spectrum analyzer, a signal spectrum broadening appears which was attributed to an uncertainty in the transmitting optics[79] or, equivalently, to the finite lifetime of the bursts.[80] In the case of an interference setup with zero-crossing detections, there is an insensitivity of the system to this kind of Doppler ambiguity.[81] Furthermore, in an interference setup, there is no broadening of the signal spectrum when the solid angle of collection is increased, contrary to what happens in a reference device.

Other typical broadening sources are (i) noise broadening due to the noise superimposed on the ideal signals,[67] (ii) velocity gradient broadening due to the fact that the fluid velocity is not constant in the control volume, (iii) Brownian broadening due to the Brownian motion of the scattering centers, usually neglected, although it is the basis for another system used for sizing (and possibly velocimetry) when homodyne detection is performed,[82,83] (iv) nonsteady flow broadening occurring when the time of acquisition is not short with respect to another time characterizing the evolution of the mean flow, in nonsteady situations (v) strioscopic broadening due to the fringe disturbances produced by refractive index effects, (vi) thrermophoresis broadening due to thermophoresis acting differently on particles having different sizes, and (vii) fringe broadening due to the fact that the fringes are actually not parallel in the control volume,[84] a problem which can be very nicely visualized with the aid of Moiré fringes.[85]

2.14. Bias Errors

Sample characteristics have been discussed assuming that the signal sampling was a nonbiased process (Section 2.11). Unfortunately, this is in general not the case.[86–89] Ten reasons for bias have been discussed by Thompson and Flack,[90] probably a rather optimistic review. In plasmas, the thermophoresis phenomenon is also of relevance.[29] Furthermore, the literature on bias effects is controversial, as illustrated by Refs. 91–94. Durão and Whitelaw emphasized a supplementary source of bias due to correlations between velocity and signal amplitude.[95]

Due to the complexity and controversial character of the matter, it appears rather difficult to give (in a restricted space) a definite assessment of bias effects here. So the reader is referred to the specialized literature to form his own opinion. Caution should be taken according to specific experiments since bias effects are very dependent on specific situations.

Note that the error in mean velocities associated with bias effects may become very important in some cases. An example is given[96] where the error may reach (5%) for a turbulence intensity of 8% in a combustion system. For systems without combustion, the maximum bias error can be estimated by the following expression[91]:

$$U_M / U = 1 + \overline{u'^2} / U^2 \tag{11}$$

where U and U_M are the real and measured velocities, respectively, and $\overline{u'^2}$ is the variance of velocity fluctuations. The relation (11) can be used as a guide to estimate the risk of errors encountered if no caution is taken for bias effects. Let us finally mention that very little work has been done concerning the influence of bias effects on fluctuation measurements. This should be considered as a disturbing state of affairs.

2.15. Particle Behavior in Plasma Flows

The scattering centers must be small enough to follow the velocity field when fluid velocity measurements are needed. But they are then likely to vaporize quickly in plasma flows, possibly before reaching the control volume. Thus seeding is usually a difficult problem in such high-temperature situations. As a consequence it is interesting to try to compare several refractory materials. The complete theoretical study of heat and mass transfer between a particle and the surrounding plasma is a very difficult one,[97,98] so a simple criterion to decide which kind of particles must be chosen could be useful. Such a criterion is suggested here, according to Gerdeman and Hecht[99] and Engelke.[100] The quantity S_p defined below should be maximized:

$$S_p = L_q^2 / \rho_p \tag{12}$$

where ρ_p is the specific mass of the particle material, and L_q is the heat which is contained per unit volume of the particle at its melting point:

$$L_q = \rho_p [c_p \Delta T_m + H_f] \tag{13}$$

where c_p is the specific heat of the solid, ΔT_m the temperature increment necessary to melt the particle, and H_f the latent heat of fusion. Using this criterion, seven materials are classified below,[33] with an obvious decay, from the best one (high S_p) to the worst:

$$Al_2O_3 > ZrO_2 > SiZrO_4 > NiCr > SrTiO_3 > NiAl \sim Ni \tag{14}$$

Obviously, other considerations can help to choose the best material, such as efficiency in scattering the light. A better criterion could be based on vaporization rather than melting. Let us also mention that coal has been successfully used under plasma conditions (Prof. Boulos, Sherbrooke University, private communication).

2.16. Kinematic Particle Behavior

We intend to give here some basic elements of discussion to decide whether the scattering centers are able or not to follow (i) the mean fluid velocity and (ii) velocity fluctuations.

For mean fluid velocity, let us consider the behavior of a spherical particle having a velocity U_p equal to zero at the time $t = 0$ while the uniform and steady plasma velocity is U_f. Writing a simplified equation of motion of the particle where the only force is the Stokes drag force,[101] we find

$$U_p = U_f(1 - e^{-t/\tau_p}) \tag{15}$$

where the time of relaxation τ_p is

$$\tau_p = \frac{m_p}{3\pi\mu d_p} \tag{16}$$

where m_p is the mass of the particle of diameter d_p and μ the kinematic viscosity of the surrounding fluid. Relations (15) and (16) form an obvious basis to discuss the problem and to decide whether U_p and U_f are nearly equal at the control volume location.

Nevertheless, the above discussion is only valid when the Knudsen number $\text{Kn} = \lambda_{mf}/(d_p/2)$ is much smaller than one, where λ_{mf} is the mean free path of molecules. When $\text{Kn} \gg 1$, the molecular theory of fluids should be used. Unfortunately, for small particles in plasmas, the Knudsen number is often of the order of magnitude of 1.[33] No rigorous theory has been developed in that range. The above formalism can nevertheless be corrected by writing the drag coefficient C_D as[102]

$$C_D = \frac{C_{DS}}{C_c} \tag{17}$$

where C_{DS} is the drag coefficient for the Stokes law and C_c the Cunningham coefficient:

$$C_c = 1 + \text{Kn}[1.25 + 0.44\, e^{-1.08/\text{Kn}}] \tag{18}$$

A precise description should be more complete, including possibly gravity and thermophoresis, and situations where the fluid is nonuniform and/or non-steady. The influence of recirculation zones is sometimes important.

The situation is much more complex when the flow is turbulent, and can hardly be discussed here due to lack of space. The basic problem is to estimate the variance of the particle velocity fluctuations over the variance of the fluid velocity fluctuations. Extensive discussions are given in Refs. 103–105.

2.17. Two-Point Measurements

Laser two-point measurements, or time-of-flight laser anemometry (TFLA), are based on the simple idea of measuring the transit time for a moving object, here a scattering center, to go from point A to point B, the distance AB being known.[106] The TFLA is sometimes considered as an interference LDV system with only one fringe spacing. The analogy between the two systems is also well illustrated by Lading[64] with the aid of a Fourier transform.

Many particles cross the two-point optical probe with the result that double pulse signals are usually superimposed. Furthermore, some signals contain only one pulse if the scattering center passes through only one of the beams. The complete signal is thus rather complicated, and the most efficient way to process it is to autocorrelate te output of the detector. Other systems involve two detectors, producing a two-beam cross-correlation anemometer.[107]

The TFLA is considered to be superior in situations where the amount of scattered light is so small that the SNR for a classical LDV is too low. The theoretical improvement is up to 10^5, but factors of 10^2 to 10^3 are classicallly easy to reach.[108] Smart gave an example of measurements in an unseeded afterburning exhaust from an aeroengine. The typical velocities and temperatures are 1000 m s^{-1} and 2000 K, respectively, and steep refractive index gradients are present. Thus the TFLA is expected to be useful in plasmas. Lading[64] confirms the superiority of the TFLA in some situations.

An important disadvantage of the TFLA is the reduced data rate, especially in turbulent flows since not so many particles are able to cross the two control volumes of a TFLA as they could cross a fringe system. Furthermore, a specific discussion of bias effects should be developed.

3. LDV-BASED OPTICAL SIZING

3.1. General

We are here concerned with a developing field where some fundamental challenges have not yet been met completely. The main challenge is to provide the researcher and the engineer with a single instrument able to measure the size and the velocity of a single particle carried away in a flow simultaneously, and not to measure simultaneously size and velocity distributions as some systems do. The interest in such a measuring technique is sufficient to justify the effort. Let us mention some of the potential applications.

One of the specific problems concerning the application of LDV to plasma velocity measurements is to make sure that the scattering centers are able to follow the kinematics of the fluid. Roughly speaking, they must be small enough to meet this requirement. But if the seeding is made with small particles, it is likely that these scattering centers will vaporize before reaching the optical probe.

Therefore, it would be desirable to know the diameter of the scattering center when it passes through the set of fringes. It is one of the reasons why there is so much interest in the development of the aforementioned technique.

Let us also discuss again the problem of heat and mass transfer between a plasma and a transported solid phase. The fundamental study of this problem requires one to know the relative velocities between the fluid and the particles, and the diameters of the particles and how they evolve along the flow. Simultaneous measurements of sizes and velocities of single particles could thus greatly increase our understanding of the phenomena involved, with application, for instance, to the study of plasma spraying processes.

3.2. Theoretical Background

The basic theory is the Lorenz–Mie theory which describes the light scattered by a spherical, homogeneous, isotropic, nonmagnetic particle illuminated by a plane light wave with rectilinear polarization. The original papers are by Lorenz,[109] Mie,[110] and Debye,[111] and basic books on this subject are available.[112–115] At the present time, the Lorenz–Mie theory can also be considered as a special case of a generalized theory where the illuminating light consists of a Gaussian beam.[116,117]

Due to the complexity of the theory, extensive numerical calculations were only possible with the advent of powerful computers associated with progress in computational algorithms, leading to computer programs such as discussed in Refs. 118–120, among others. Lentz[121] published in 1976 a new algorithm permitting one to compute, without recurrence formulas, the Bessel function terms appearing in the theory. A new computer program for rigorous Lorenz–Mie theory (the so-called Supermidi) has then been built on the basis of the Lentz algorithm.[122,123]

In particle sizing, the aim is to link the diameter to be measured and a scattering property of the light through a monotonic relationship. Such monotonic relationships have been found to exist between diameters and collected scattered powers. Examples have been published.[124,125] These relationships will form the basis for the top-hat corrected laser beam technique to be discussed later.

Monotonic relationships can also be studied using a limiting theory of the Lorenz–Mie concept, namely geometrical optics, extensively discussed in.[113,126] Applications to particle sizing are found in Refs. 127–129, among others. Nevertheless, a complete geometrical optics theory is very complicated and would indeed become more involved than the pure Lorenz–Mie approach. However, for transparent particles in the near forward scattering direction, a simplified version can be designed and used. Systematic comparisons between this simplified version and the Lorenz–Mie theory have been published,[130] involving a discussion of nonspherical particles.

3.3. Visibility Technique

Figure 4 shows the shape of a Doppler signal, including the pedestal, of a single particle that passes through the center domain of the fringe system. Let us define the visibility as

$$\mathcal{V} = \frac{V_M - V_m}{V_M + V_m} \tag{19}$$

In some pioneering work, Farmer has shown that a simple relation between the visibility and the diameter of the scattering center (only spherical particles will be discussed here) exists,[131] when certain assumptions are satisfied. The simple relation reads

$$\mathcal{V} = 2|J_1(\pi d_p/d_f)/(\pi d_p/d_f)| \tag{20}$$

where J_1 is the first-order Bessel function and d_p is the particle diameter. Since this function is not monotonic, the size cannot be determined unambiguously when $\mathcal{V} < {\sim}0.15$. Notwithstanding this point, measurement of $\mathcal{V}$ and knowledge of the fringe spacing d_f enable one to determine the diameter of the scattering center, simultaneously with its velocity.

Unfortunately, we have to consider 13 assumptions, a rather large list discussed elsewhere.[20] Let us only discuss the following:

A1. The wavefronts are planar.
A2. Only paraxial front or backscattering is considered.
A3. The scattered intensity is calculated by assuming that the scattering center receives a mean illuminating intensity equal to a simple average of the illuminating intensity over the cross-sectional area of the scattering center.
A4. The fringe spacing is smaller than the radius b_0 of the focused laser beams in the control volume.
A5. The diameter of the particle is smaller than b_0.

A6. Only the location at the geometric center of the probe volume is considered.

Assumption A1 is not correct if we need to measure large particles. In this case, a complete analysis of the visibility theory should include the generalized Lorenz–Mie theory,[116,117] an impossible task at the present time since extensive numerical calculations are not yet available. Assumption A2 is also an important limitation since a lot of LDV systems use off-axis collection. The exact influence of A3 seems to be difficult to appreciate, but it should be pointed out that progress in the generalized Lorenz–Mie theory could also overcome this problem, which is certainly important for large particles. Assumption A6 asks the fundamental question: Is some information on the particle trajectories necessary to correctly interpret visibility data?

Let us finally consider Assumptions A4 and A5. We must then have $d_f \ll b_0$ and $d_p \ll b_0$, and also, to avoid ambiguity problems linked with the relation (20), we need $d_p < d_f$. It may be difficult to simultaneously comply with the conditions $d_p < d_f$ and $d_f \ll b_0$, particularly when large particles ($d_p \sim 50\ \mu$m) are under study. Furthermore, a small angle of convergence is required to obtain $d_p < d_f$, producing possibly SNR and spatial-resolution problems.

Following this work, other papers have been published in order to refine the analysis, to overcome the aforementioned limitations, and to provide effective measurements. Farmer[132] reported experimental observations for sizes in the range 10–120 μm. The influence of the solid angle of collection and of the particle trajectory are discussed. Reasonable agreement between theory and experiments is obtained when the assumptions of[131] are obeyed, but large deviations are observed when $d_p \sim b_0$. It is also found that nonparaxial observation of large particles can lead to ambiguous measurements. Orloff *et al.*[133] studied aerosol sizes by using the relation (20) with an on-axis backscattering system. Significant differences between theory and experiments are found, and the conclusion is that the relationship involving $\mathscr{V}$, d_p, the refractive index, and details of the experimental setup must be much more complex than the relation (20). Robinson and Chu performed an analysis similar to Farmer's but using scalar diffraction theory.[134] Computations of visibility have been carried out on the basis of rigorous Lorenz–Mie theory by Hong and Jones[135] and by Adrian and Orloff.[136] In these papers, the complexity of a complete rigorous theoretical visibility analysis is well exemplified. Chu and Robinson reported improvements of their previous analysis by avoiding diffraction analysis and going instead to an exact solution of Maxwell's equations.[137] Roberds used an analysis based on Fraunhofer diffraction and introduced the Gaussian profile of the illuminating beams.[138] He showed that the signal is dependent on the position of the particle in the scattering volume, a dependence also discussed by Hong and Jones[139] who furthermore compared predictions from geometrical optics, diffraction theory, and scattering theory. Owen and Bachalo reported measurements in spray flames using off-axis collection.[140]

In conclusion, although visibility techniques have been successfully used for simultaneous measurements of sizes and velocities in some cases, it is thought that the number of parameters involved in the data interpretation is too large and their respective influence not sufficiently well understood for these techniques to generally be accurately and easily used. As a matter of fact, a complete rigorous theory of visibility is not yet available. A first necessary step to build such a theory is to provide numerical calculations in the framework of the generalized Lorenz-Mie theory. While we wait for such a complete theory, visibility techniques should be used with caution.

3.4. Pedestal Calibration Methods

Due to the difficulties involved in the visibility techniques, other authors developed another approach where the diameters are correlated with the pedestal amplitude $\bar{V}$ (Fig. 4). The theoretical analysis has been based on geometrical optics previously described, including possibly the Gaussian character of the incident beams.[127-129]

But a large particle passing through the edge of the control volume can lead to a mean amplitude equal to the one produced by a small particle passing through the control volume center. This effect is the trajectory ambiguity. A gate photomultiplier has then been introduced at right angles to the control volume in order to select only those particles passing through the central region.[141] This system does not really solve the problem however, since dependence on transverse directions remains. Thus, that approach has been abandoned. A second approach is to compute actual size distributions from measured distributions with the aid of a mathematical inversion based on the assumption that particles have an equal probability of passing through any element of the cross section of the control volume. The corresponding mathematical algorithm is known as a trajectory dependence-inversion technique and was carefully described by Holve and Self.[142]

These pedestal calibration methods have been successfully used in combustion systems,[143] and adaptation to plasma experiments seems straightforward. Nevertheless, the trajectory dependence is a shortcoming of the technique. Because of it, simultaneous measurements of size and velocity distributions are achieved, but not simultaneous measurements of the size and velocity of single particles.

3.5. Top-Hat Laser Beam Technique

In order to reach the aim of measuring simultaneously the size and the velocity of single particles, another method is under development at Rouen University. This approach relies on three principles:

(i) In contrast to the techniques discussed in Sections 3.3 and 3.4 where velocimetry and sizing are achieved on the same signals, separate optical probes for LDV and sizing are used in order to get more flexibility. The two probes are spatially superimposed but use different colours. For instance, the laser source is an Ar-ion laser, the 514.5 nm green line being used to form the LDV probe and the 488.0 nm blue line being used to form the sizing probe.

(ii) Monotonic relationships between diameters and collected scattered powers are used to achieve the measurements, as discussed in Section 3.2.

(iii) In order to avoid the trajectory ambiguity, the sizing Gaussian beam is corrected to a top-hat beam which will produce top-hat signals. The height of these signals is linked to the diameters through the aforementioned monotonic relationships.

Measurements based on these ideas have been reported, the correction of the Gaussian beam being achieved using either an anti-Gaussian metal-coated filter or a holographic filter.(144-146)

A complete system has been realized in collaboration with Karlsruhe University and the company OEI(147,148) and is now currently undergoing scientific tests.

3.6. Miscellaneous Methods

Lee and Srinivasan have shown that it is possible to obtain statistics on the size number density distribution and, for each size range, the velocity distribution of the particles and of the fluid phase by using a scheme of discrimination of the amplitude, residence time, and frequency of the laser-Doppler bursts.(149) Durst and Umhauer used a separate white light source to measure the particle size, whereas a LDV system was used to measure the velocity.(150) Chou and Waterson used the ratio between the forward and backward scattered light to measure diameters of small particles.(151) Ratioing methods have also been used by Jovin *et al.*,(152) Gravatt,(153,154) and Born and Waldie,(155) as was first suggested by Hodkinson.(156)

Jin Wu used dual parallel laser beams crossed by particles embedded in the flow to simultaneously measure their size and velocity.(157) Chabay and Bright used only one laser beam, the size being measured through the amplitude of signals detected from particles crossing the beam.[158] Wigley determined sizes of large particles crossing a LDV-control volume from observations of the glancing angle reflections.[159] Hirleman suggested the use of parallel beams to completely specify the particle trajectory, and simultaneously measure particle size and velocity.[160] The diffraction pattern at infinity of a coherent beam passing through a sample has also been used for sizing.[161,162] Let us finally mention Laug's work permitting sizing of particle clouds by recording scattering diagrams and comparing them with theoretical predictions.[163]

4. CONCLUSION

The practice of laser-Doppler velocimetry has been reviewed with emphasis on applications to thermal plasmas. In this situation, the seeding problem appears the most difficult one, although LDV is now a well-established technique in cold flows. A discussion is also given concerning methods of laser particle sizing, especially with the aim of measuring simultaneously the velocity and size of particles carried by the fluid. Visibility, pedestal, and top-hat laser beam techniques are described. A list of miscellaneous methods has also been provided.

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Concerning bias errors (Section 2.14), one referee pointed out to me that some recent papers have resolved much of the controversy I mentioned. References 164–167 have been added to the list.

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