MARIE-PAULE LECOUTRE

# COGNITIVE MODELS AND PROBLEM SPACES IN "PURELY RANDOM" SITUATIONS

ABSTRACT. As part of a study on the natural interpretations of probability, experiments about elementary "purely random" situations (with dice or poker chips) were carried out using students of various backgrounds in the theory of probability. A prior study on cognitive models which analyzed the individual data of more than 600 subjects had shown that the most frequent model used is based on the following incorrect argument: the results to compare are equiprobable because it's a matter of chance; thus, random events are thought to be equiprobable "by nature". The present paper is divided into two parts. In the first, the findings of a series of experiments are summarized. In the second, the following two hypotheses are tested: (1) Despite their incorrect model, subjects are able to find the correct response. (2) They are more likely to do so when the "chance" aspect of the situation has been masked. An experiment testing 87 students showed, as expected, that there is a way to induce the utilization of an appropriate cognitive model. However, the transfer of this model to a classical random situation is not as frequent as one might expect.

## INTRODUCTION

Consider the following problem. Two dice are simultaneously thrown, and the following two results are obtained: R1 "a 5 and a 6 are obtained" and R2 "a 6 is obtained twice". The question asked is, "Do you think the chance of obtaining each of these results is equal? Or is there more chance of obtaining one of them, and if so, which, R1 or R2? Or is it impossible for you to give an answer, and if so, why?". Now consider another problem. There are three poker chips in a jar, two red and one white, and two chips are drawn together. Asking the same question as before, compare result R1 "a red chip and a white chip are obtained" and result R2 "two red chips are obtained". Subjects were asked to give "spontaneous answers" by choosing one of the four possibilities proposed. This set-up was used because our main purpose was to provide evidence of intuitions or "natural interpretations" in the sense used by Feyerabend (1975). (The study of *natural interpretations of probability* is one of the main topics of research in the Groupe Mathématiques et Psychologie where these experiments were carried out.)

In the two previous examples, R1 is twice as probable as R2, and so the correct response is "there is more chance of obtaining R1". It is interesting to systematically study the responses given to this type of problem, which will hereafter be called the "standard problem", because it is an example of a large class of problems that bring into play the notion of exchangeability, now recognized in the theory of probability as more fundamental than the notion of independence. (See de Finetti, 1974.)

Our motivation for conducting research in this area is twofold. There is a

*Educational Studies in Mathematics* 23: 557–568, 1992.

cognitive motivation — the formalization of various descriptive models of cognitive functioning in situations of uncertainty — and there is also a pedagogical motivation — the derivation, from experimental findings, of potential implications for the teaching of probability and statistics.[1]

In our experimental study, using either chips or dice to test more than 1000 students of various backgrounds in probability theory, we observed a bias that we have called *equiprobability bias*. We suggest that this bias be added to the list of biases observed in various situations of uncertainty, especially those described by Tversky and Kahneman: representativeness bias, availability bias, anchoring bias, and so on (see Kahneman, Slovic, and Tversky, 1982, or Nisbett and Ross, 1981). A systematic study of the available cognitive models shows that the combinatorial ones (involving the explicit enumeration of all possible cases) or the logical ones (involving hypothetico-deductive reasoning) which lead to the correct response are very rarely utilized spontaneously (only about 5 to 10 percent of the subjects).[2] The equiprobability bias is highly resistant to variations in the classification factors of the subjects or in the experimental context. These findings will be summarized in the first part of this paper.

Nevertheless, it seems reasonable to assume that combinatorial or logical models are available to most subjects, but are not spontaneously associated with situations in which "chance" is a factor which somehow inhibits by the equiprobability bias. In the second part of the paper, experiments testing the following two hypotheses are discussed: (1) Despite their incorrect model, subjects are able to find the correct response when the situation is conducive to the activation of the appropriate cognitive models. (2) In particular, in the situation of interest here, it is hypothesized that subjects are more likely to respond correctly when the "chance" aspect of the problem is masked.

Accordingly, we set up new situations which were isomorphous to the preceding ones, but in which the "chance" aspect was masked by formulating the problem in practical terms and bringing the subject's attention to a geometric figure. This was aimed at introducing a new problem space, in the sense defined by Newell and Simon (1972) (i.e., wherein the subjects' representations of the task lead to a choice of possible operations that can be applied to the problem), which might promote the activation of combinatorial or logical models.

In the main experiment, called the "House" experiment and described in detail in the second part of this paper, three cards were used instead of the three chips in the previous problem. A triangle was drawn on two of the cards (for the two red chips), and a square, on the third (for the white chip). The subjects were shown that it was possible to construct either a house or a rhombus. The findings will be compared to those obtained in our previous research.

## BRIEF RECALL OF PRIOR EXPERIMENTS

Over the past ten years, I have been working on an experimental project in collaboration with J. Cordier and J.-L. Durand to test the resistance of the equiprobability bias against systematic variations either in subject classification factors or experimental factors.

The four main experimental factors studied were the following:

*Combinatorial information.* According to Fischbein et al. (1971), who found similar results to ours in their pedagogical research, the high proportion of incorrect responses occurs because most subjects do not perceive the fact that result R1 is compound; that is, it can be obtained in two ways, unlike result R2, which can only be obtained in a single way. Various experimental conditions (either with chips or dice) were defined *a priori* favoring a better understanding of the structure of the standard problem, and especially its compound nature.

*Frequency information.* Subjects were shown (by presenting the results of 100 and 1000 dice throws) that with a large number of throws, result R1 is about twice as frequent as result R2.

*Modifications in the formulation of the results or of the questions asked.* It is now well known how important the wording and context are in various situations of uncertainty. (See especially Keren, 1984; Maury, 1984; 1985; and Zaleska, 1974.)

*Utilization of correct probabilistic intuitions.* Knowing that almost all subjects correctly solve problems which are more elementary than the standard problem (see Durand, 1989), the idea here was to study how subjects could utilize and transfer their correct intuitions to solving the standard problem.

The following classification factors were studied:

*Background in the theory of probability.* Three classes of subjects were considered: subjects without any background, subjects with a little background (one year of higher education in mathematics and probability), and subjects with a thorough background (four or five years of higher education in mathematics and probability).

*Type of studies (scientific vs literary).*

*Practice at games of chance.*

*Sex.*

Many of these experiments are reported in detail and discussed more fully in

Lecoutre (1984; 1985), and in Lecoutre, Durand, and Cordier (1990). Here we shall concentrate on the proportion of correct responses and incorrect equiprobability responses for the standard problem.

The results are reported in Table I for the experimental factors and in Table II for the classification factors.

TABLE I

Table I. Results for the experimental factors: proportion of correct responses (CR) and equiprobability responses (ER)

| | | *CR* | *ER* | |
|---|---|---|---|---|
| Combinatorial information[1] | dice of different colors | 0.37 | 0.51 | /293 |
| | successive throws of dice | 0.32 | 0.53 | /257 |
| Frequency information[2] | | 0.45 | 0.38 | /273 |
| Modifications in the formulation[3] | of the results | 0.43 | 0.45 | /273 |
| | of the questions | 0.20 | 0.75 | /220 |
| Utilization of correct intuition[4] | with dice | 0.19 | 0.75 | /88 |
| | with chips | 0.25 | 0.64 | /122 |

[1] Durand (1989) [2] Lecoutre (1985) [3] Lecoutre (1985) [4] Durand (1989)

TABLE II

Table II. Results for the classification factors: proportion of correct responses (CR) and equiprobability responses (ER)

| | | *CR* | *ER* | |
|---|---|---|---|---|
| Background in the theory of probabilities[1] | no background | 0.51 | 0.45 | /29 |
| | little background | 0.20 | 0.70 | /91 |
| | thorough background | 0.45 | 0.38 | /39 |
| Type of studies[2] | literary | 0.35 | 0.59 | /253 |
| | scientific | 0.34 | 0.54 | /293 |
| Practice at games of chance[3] | no practice | 0.32 | 0.60 | /37 |
| | little practice | 0.29 | 0.65 | /101 |
| | a lot of practice | 0.53 | 0.45 | /21 |
| Sex[4] | male | 0.42 | 0.49 | /174 |
| | female | 0.31 | 0.53 | /376 |

[1] Lecoutre (1984) [2] Durand (1989) [3] Lecoutre (1984) [4] Durand (1989)

The main findings can be summarized as follows:

1) None of the factors had a strong effect on the equiprobability bias. In almost all experiments, the proportion of equiprobability responses remains at least equal to 50 percent. It is interesting to point out here that even a thorough background in the theory of probability did not lead to a notable increase in the proportion of correct responses. These results show how highly resistant the

equiprobability bias is, and they are quite consistent with the idea recently brought up by Fischbein (1987), who claims that intuitions (correct as well as incorrect) are often very robust, "being deeply rooted in the person's basic mental organization".

2) From the systematic analysis of about 600 individual protocols aimed at characterizing the possible cognitive models used, it appears that the most frequent one is based on the following type of argument: "The two results to compare are equiprobable because it's a matter of chance". According to this model, which accounts by itself for more than 65 percent of the equiprobability responses, random events should be equiprobable by nature. All the data and findings related to the definition and characterization of the various cognitive models are reported in detail and discussed more fully in Lecoutre and Durand (1988).

## NEW EXPERIMENTAL PARADIGM: HOUSE EXPERIMENT

In this House experiment, subjects were asked three successive questions. The first dealt with a problem in which the "chance" aspect of the situation was masked by explicitly showing the subjects that a geometric figure could be constructed from the cards drawn. According to our hypothesis, this should cause them to "forget" the random nature of the situation. The second dealt with the "classical" standard problem, and the third concerned a comparison of the two problems.

### *Procedure*

Three cards and three pieces of candy were used. A triangle was drawn on two of the cards (instead of the two red chips) and a square (instead of the white chip) was drawn on the third (see Fig. 1). Two of the pieces of candy were orange flavored and the third was lemon flavored.

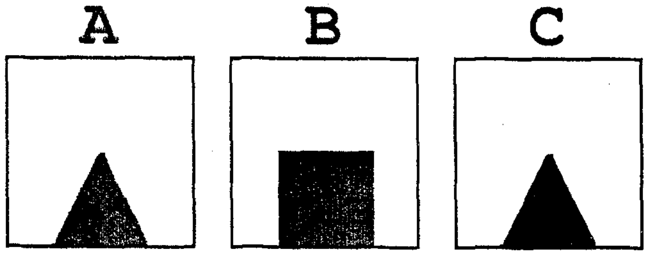


Fig. 1 The three cards used in the experiment.

The subjects were shown[3] that it is possible to construct a house if pair AB or pair BC was drawn, or a rhombus if AC was drawn.

After the experimenter read the instructions which asked the subjects to give their responses by choosing one of the four proposed possibilities, and to justify their response,[4] the following three questions were asked:

*Question 1* (Q1). I put the three cards in this box and I am going to draw two cards of them. With the two cards drawn, I will be able to construct either a house or a rhombus as you have just seen.

Do you think there is:

- an equal chance of obtaining a house and a rhombus?
- more chance of obtaining a house than a rhombus?
- more chance of obtaining a rhombus than a house?
- If it is impossible for you to give an answer, why?

Justify your response.

*Question 2* (Q2) (Standard Problem). In this box, there are three pieces of candy; two are orange flavored and one is lemon flavored. I am going to draw two pieces.

Do you think:

- the chance of obtaining one lemon-flavored piece and one orange-flavored piece is equal to the chance of obtaining two orange-flavored pieces?
- there is more chance of obtaining one orange-flavored piece and one lemon-flavored piece?
- there is more chance of obtaining two orange-flavored pieces?
- If it is impossible for you to give an answer, why?

Justify your response.

*Question 3* (Q3). Now compare the situations considered in each of the two questions.

If you think the two situations are alike, explain why.
If you think the two situations are different, explain why.
If you think there is a contradiction in the two responses you gave, explain why, and how you could clear up that contradiction.
Be careful not to change the responses you gave to the first two questions.

*Subjects*

Eighty-seven students, about 15 to 17 years old without any background in the theory of probability, answered the questionnaire in notebooks distributed for that purpose during a mathematics class.

*Results*

*Questions 1 and 2* (Q1 and Q2)
The distribution of the responses to questions Q1 and Q2 is reported in Table III.

TABLE III

Table III. Results for the two questions Q1 ("House") and Q2 ("Standard problem")

*Response**

| | +R1 | = | +R2 | ? | |
|---|---|---|---|---|---|
| Q1 | 65 (0.75) | 20 (0.23) | 2 (0.02) | 0 (0) | (/87) |
| Q2 | 39 (0.45) | 38 (0.44) | 6 (0.06) | 4 (0.04) | (/87) |

* +R1 There is more chance of obtaining result R1
= There is as much chance of obtaining each of the two results, R1 and R2
+R2 There is more chance of obtaining result R2
? It is impossible to give an answer

Two main points should be noted.

1) For Question 1, the proportion of equiprobability responses decreased substantially, to the benefit of the correct response, which was given in 75 percent of the cases. A non-informative Bayesian analysis[5] showed that we obtain a guarantee of 0.95 that the parent frequency $\varphi$ of equiprobability responses was lower than 0.31 ($P(\varphi<0.31)=0.95$).[6] So, masking the "chance" aspect of the problem, thus modifying the space in which the solution is searched (Richard, 1984), had a strong positive effect.

2) For Question 2, the proportion of correct responses was not as high as for Question 1, although it was noticeably higher than in all of our previous experiments. We obtained the Bayesian guarantee of 0.95 that the parent frequency $\varphi$ of correct answers was greater than 0.36 ($P(\varphi>0.36)=0.95$) while all previously observed correct responses frequencies were approximately 0.30. We can also see that the proportion of equiprobability responses was higher than for the first question, but not as high as in our previous experiments.

The distribution of the pairs of responses to questions Q1 and Q2 is reported in Table IV.

TABLE IV

Table IV. Distribution of the pairs of responses for questions Q1 ("House") and Q2 ("Standard problem")

| Q1 | Q2 | |
|---|---|---|
| +R1 | +R1 | 38 (0.44) |
| +R1 | = | 20 (0.22) |
| +R1 | +R2 | 3 (0.03) |
| +R1 | ? | 4 (0.04) |
| = | = | 18 (0.22) |
| = | +R1 | 1 (0.01) |
| = | +R2 | 1 (0.01) |
| +R2 | +R2 | 2 (0.02) |
| ? | ? | 0 |

We can see here that 58 subjects gave the same answer to both questions.

Out of the 65 subjects who responded correctly to Question 1, 38 of them (60 percent) again responded correctly to Question 2, whereas almost all other subjects gave the equiprobability response. So, the correct response was transferred to the subsequent standard problem in only 60 percent of the cases. All subjects (except two) who gave the incorrect equiprobability response to the first question repeated this response for the second question.

*Question 3*

The responses given to Question 3 were distributed as follows:

- For 70 percent of the subjects, the two situations were alike.
- For 24 percent of them, they were different.
- Six percent of the subjects give a vague, unintelligible response.

In these results, our attention is mainly attracted by the high proportion of subjects (24 percent) who thought the two situations were different. Considering the sequence of three responses, the following remarks may be made. For all subjects (except one) who gave the same response to Questions 1 and 2, the two situations were alike. For the subjects who gave two different responses to Questions 1 and 2, 60 percent thought that the two situations were different. The arguments brought up in this case will be analyzed in the next section ("Some Models"). For the other subjects, the two situations were alike. These subjects either perceived an incoherence in their responses and modified one of them (this was especially the case for three subjects who answered +R1/=: they thought they had made a mistake on Question 1 and that the two results were equiprobable after all), or did not perceive an incoherence and confirmed their responses without giving any further justification.

*Some Models*

The above analysis of the sequences of responses and justifications points to several possible cognitive models, including the following five most important ones.

1) *"Correct model"*. For both questions, subjects utilize the same logical or combinatorial model and think that the two situations are alike. This model, which of course leads to correct responses, was used by 24 percent of the subjects. Compared with the proportion obtained in our previous experiments (about 6 to 7 percent), this proportion is high.

2) *"Construction vs chance model"*. Here, subjects say that the two situations are different; the first one appeals to "reasoning", to "logic", or to the "construction of a figure or an object", and so on (these are examples of the justifications given), whereas the second is a problem of "pure chance", "probability", or "randomness", and so on. This model was used by all subjects who answered +R1/=, that is, 23 percent of the subjects. Here, it clearly appears that these subjects spontaneously defined a different problem space in each of the situations.

3) *"Conditional model"*. For both questions, the subjects utilize an interesting representation of the following type: knowing that in the pair of drawn elements, one of the two identical ones will inevitably be obtained (a triangle or an orange-flavored piece of candy), then one element of each kind remains, and so there is an even chance for the second element drawn. Consequently, they conclude there is equiprobability of the two results. This model was utilized by 16 percent of the subjects who gave two equiprobability responses.

4) *"Chance model"*. This model leads to equiprobability responses with the following argument: it is equiprobable "because it is by chance". This model was used by 5 percent of the subjects. Compared with the proportion obtained in all our previous experiments (about 50 percent, see Lecoutre and Durand, 1988), this proportion is quite small.

5) *"Numbers model"*. Subjects answer that in both situations, there is more chance of obtaining a pair with two identical elements (+R2 response) because there is a greater number of them: there are more triangles than rectangles, so there is more chance of drawing two triangles. This model, which is based on the number of elements, is valid when only one element is drawn (it is the only model spontaneously utilized by subjects in this case, see Lecoutre, 1984), but generalization to the case of two elements is erroneous. Two percent of the subjects applied this model.

These five models account for the answers of 70 percent of the subjects. For the other subjects, it is impossible to define any characteristic model because their answers were vague, and no justifications were given.

These findings are consistent with other experiments in which only the material differed. (See Lecoutre and Cordier, 1990b.) In one experiment, for example, two head sides of a coin and one tail side of a coin were drawn on the three cards instead of two triangles and a square. Thus, it was possible to construct either a "normal" coin or a two-headed coin. The same three questions as in the House experiment were asked to 37 subjects. For Question 1, the proportion of equiprobability responses was low, as in the preceding experiment (11 percent), and for Question 2 the proportion of correct responses was equal to 67 percent. We again found the same five main models, in proportions very close to the ones obtained here.

Thus, as expected, there is a critical difference between the "classical" situation and situations in which the "chance" aspect is masked by bringing the subject's attention to a construction or to logical reasoning. But one remark is called for here. In both experiments, the correct response was associated with a "good" figure (a house, more salient than a rhombus, or a "normal" coin), and one might propose a "gestaltist" interpretation something like: the proportion of correct responses increases, not because the "chance" aspect is masked, but because correct responses are associated with a "good" form. To test this hypothesis, an additional experiment was conducted in which the incorrect response was associated with the "good" form (a circle), while the correct response was associated with an ambiguous figure. Similar questions were asked to 65 subjects. The findings are clearly incompatible with the "gestaltist" interpretation, since only seven subjects gave the incorrect response associated with the circle for Question 1. Furthermore, all the findings of this experiment were very close to those obtained for the other two experiments (see Lecoutre and Cordier, 1990a).

## CONCLUSION

Two main results will be examined.

1) Our findings show that it is possible, using experimental tricks and especially here by masking the "chance" aspect of the situation, to induce the utilization of appropriate cognitive models. Thus, we can see that correct models are often available, but are not spontaneously associated with the situations at hand. Such a result should be compared to the one found by Escarabajal and Richard (1986) for arithmetic problem-solving tasks, where subjects who did not spontaneously develop correct reasoning could still do so when prompted.

More generally, when a bias is observed in spontaneously devised models, correct models may in most cases be available, but experimental tricks are required to activate them. We have just seen an illustration of such an approach. This finding appears to have significant implications for teaching in general, and in particular, for the teaching of certain mathematical concepts. It indeed suggests the value, if not the necessity, of attempting to act upon the cognitive models used by subjects, by determining the conditions under which the appropriate models are activated (provided, however, given the current state of research, that the second finding reported above be taken into account).

2) It also appears that the transfer of a relevant model to the subsequent "standard problem" is not as frequent as one might expect. Transfer of the correct response only occurred in about 60 percent of the cases. For more than 20 percent of the subjects, the two situations were quite different. Thus, one of the questions on this matter is whether the number of transfers can be increased, for example, by insisting even more in the similarity of the two situations, or whether the perception of a clear-cut difference between the two situations will always exist in some subjects. Furthermore, a final question can be raised: when experimental tricks are used to trigger the activation of an appropriate model, and transfer to the standard problem occurs, is such an acquisition stable? New experiments designed to answer these questions are in progress at the present time.

## NOTES

[1] For an illustration of such an approach, see Lecoutre and Lecoutre (1979) and Rouanet et al. (1987, 1990); the latter two books are recommended for the teaching of statistics in French universities, especially to students in psychology.

[2] For a detailed description of these models, see Lecoutre and Durand (1988).

[3] Exactly the same experimental condition in which the possible cases were shown to the subjects with chips was studied in Lecoutre (1985).

[4] There is no difference between situations in which subjects are asked to answer the questionnaire first and justify their responses later, and situations such as the one considered here in which subjects are asked to justify each response as they go. This point is discussed in Durand (1989).

[5] The analyses were carried out at the C.I.R.C.E. (the Computing Center of the C.N.R.S., Orsay).

[6] For more details about Bayesian inferences on frequencies for small samples, see Poitevineau and Bernard (1986) and Bernard (1991).

## REFERENCES

Bernard, J.-M.: 1991, "Inférence bayésienne et prédictive sur les fréquences". In H. Rouanet, M.-P. Lecoutre, M.-C. Bert, B. Lecoutre, and J.-M. Bernard (eds.), *L'Inférence Statistique dans la Démarche du Chercheur*, Peter Lang, Bern, pp. 121–153.

de Finetti, B.: 1974, *Theory of Probability* (two volumes), Wiley, New York.

Durand, J.-L.: 1989, *Jugements Probabilistes dans une Situation Aléatoire: Etude Expérimentale de leur Stabilité*, unpublished doctoral thesis, Paris V.

Escarabajal, M.-C. and Richard, J.-F.: 1986, "Le transfert analogique de procédures dans l'interprétation et la résolution de problèmes d'inclusion de classes chez des adultes", *Archives de Psychologie* **54**, 39–64.

Feyerabend, P.: 1975, *Against Method*, New York Books, London.

Fischbein, E.: 1987, *Intuition in Science and Mathematics: An Educational Approach*, D. Reidel, Cambridge.

Fischbein, E., Barbat, I. and Minzat, I.: 1971, "Intuitions primaires et intuitions secondaires dans l'initiation aux probabilités", *Educational Studies in Mathematics* **4**, 264–280.

Kahneman, D., Slovic, P., and Tversky, A.: 1982, *Judgment Under Uncertainty: Heuristics and Biases*, Cambridge University Press, New York.

Keren, G.: 1984, "On the importance of identifying the correct 'problem space'", *Cognition* **16**, 121–128.

Lecoutre, M.-P.: 1984, "Jugements probabilistes chez des adultes: pratique des jeux de hasard et formation en théorie des probabilités", *Bulletin de Psychologie* **38**, 891–899.

Lecoutre, M.-P.: 1985, "Effet d'informations de nature combinatoire et de nature fréquentielle sur les jugements probabilistes", *Recherches en Didactique des Mathématiques* **6**, 193–213.

Lecoutre, M.-P., and Cordier, J.: 1990a, *"Réponses et bonnes formes", Groupe Mathématiques et Psychologie*, technical report, Paris V.

Lecoutre, M.-P., and Cordier, J.: 1990b, "Effet du mode de présentation d'un problème aléatoire sur les modèles développés par les élèves", *Bulletin de l'A.P.M.E.P.* **372**, 9–22.

Lecoutre, M.-P., and Durand, J.-L.: 1988, "Jugements probabilistes et modèles cognitifs: étude d'une situation aléatoire", *Educational Studies in Mathematics* **19**, 357–368.

Lecoutre, M.-P., Durand, J.-L., and Cordier J.: 1990, "A study of two biases in probabilistic judgments: Representativeness and equiprobability", in J.-P. Caverni, J.-M. Fabre and M. Gonzalès (eds.), *Cognitive Biases*, Elsevier Science Publishers B.V., North Holland, pp. 563–575.

Lecoutre, M.-P., and Lecoutre, B.: 1979, *Enseignement Programmé sur l'Utilisation d'une Table de la Distribution Normale*, C.D.U.-S.E.D.E.S., Paris.

Maury, S.: 1984, "La quantification des probabilités: analyse des arguments utilisés par les élèves de classe de seconde", *Recherches en Didactique des Mathématiques* **6**, 198–213.

Maury, S.: 1985, "Influence de la question dans une épreuve relative à la notion d'indépendance", *Educational Studies in Mathematics* **16**, 283–301.

Nisbett, R., and Ross, L.: 1981, *Human Inference: Strategies and Shortcomings of Social Judgments*, Englewood Cliffs, NJ, Prentice Hall.

Newell, A., and Simon, H.A.: 1972, *Human Problem Solving*, Englewood Cliffs, NJ, Prentice Hall.

Poitevineau, J., and Bernard, J.-M.: 1986, "La série des programmes IBF", *Informatique et Sciences Humaines* **68–69**, 135–137.

Richard, J.-F.: 1984, "Mechanisms of problem solving behavior", In G. D'Ydevalle (ed.), *Cognition, Information Processing and Motivation*, North-Holland, Amsterdam, pp. 425–444.

Rouanet, H., Le Roux, B., and Bert M.-C.: 1987, *Statistiques en Sciences Humaines: Procédures Naturelles*, Dunod, Paris.

Rouanet, H., Bernard, J.-M., and Le Roux B.: 1990, *Statistiques en Sciences Humaines: Analyse Inductive des Données*, Dunod, Paris.

Zaleska, M.: 1974, "Influence du mode de présentation des séries aléatoires sur le choix d'une stratégie. 1 - Situations hypothétiques", *Année Psychologique* **74**, 125–144.

*Groupe Mathématiques et Psychologie*
*C.N.R.S. and Université René Descartes*
*75005 Paris*
*France*