# **REPLY TO PROFESSOR MARCUS\***

Professor Marcus struck the right note when she represented me as suggesting that modern modal logic was conceived in sin: the sin of confusing use and mention. She rightly did not represent me as holding that modal logic *requires* confusion of use and mention. My point was a historical one, having to do with Russell's confusion of 'if-then' with 'implies'.

Lewis founded modern modal logic, but Russell provoked him to it. For whereas there is much to be said for the material conditional as a version of 'if-then', there is nothing to be said for it as a version of 'implies'; and Russell called it implication, thus apparently leaving no place open for genuine deductive connections between sentences. Lewis moved to save the connections. But his way was not, as one could have wished, to sort out Russell's confusion of 'implies' with 'if-then'. Instead, preserving that confusion, he propounded a strict conditional and called *it* implication.

It is logically possible to like modal logic without confusing use and mention. You could like it because, apparently at least, you can quantify into a modal context by a quantifier outside the modal context, whereas you obviously cannot coherently quantify into a mentioned sentence from outside the mention of it. Still, man is a sensemaking animal, and as such he derives little comfort from quantifying into modal contexts that he does not think he understands. On this score, confusion of use and mention seems to have more than genetic significance for modal logic. It seems to be also a sustaining force, engendering an illusion of understanding.

I am speaking empirically. There was a period twenty-five years ago when I kept being drawn into arguments with C. I. Lewis and E. V. Huntington over interpretation of modal logic; and in those arguments I found it necessary to harp continually on the theme of use versus mention. And

<sup>\*</sup> Presented as Commentary at the meeting of the Boston Colloquium for the Philosophy of Science, February 8, 1962.

now points that Professor Marcus has urged this evening, in favor of modal logic, force me back to that same theme again. Thus consider her 'informal argument:

(12) If p is a tautology, and p eq q, then q is a tautology'.

Her adoption of the letters 'p' and 'q', rather than say 'S<sub>1</sub>' and 'S<sub>2</sub>', suggests that she intends them to occupy sentence positions. Also her 'eq' is perhaps intended as a sentence *connective*, despite her saying that it names some equivalence relation; for she says that it could be taken as ' $\equiv$ '. On the other hand her clauses 'p is a tautology' and 'q is a tautology' do not show 'p' and 'q' in sentence position. These clauses show 'p' and 'q' in name positions, as if they were replaceable not by sentences but by names of sentences.

Or try the opposite interpretation. Suppose that Professor Marcus, contrary to custom, is using 'p' and 'q' as variables whose values are sentences, and whose proper substitutes are therefore names of sentences. Then 'eq' is indeed to be seen as naming some equivalence *relation*, just as she says; and the mention of ' $\equiv$ ' must be overlooked as an inadvertency. On this interpretation, (12) is unexceptionable. But on this interpretation (12) is no part of modal logic; it is ordinary non-modal metalogic. For on this interpretation 'eq' is not a non-truth-functional sentence connective at all, but an ordinary non-truth-functional two-place sentence predicate, like 'implies'. I have no objection to these. In my logical writings early and late I have used them constantly.

Twenty-five years ago, in arguing much the same matter with Lewis and Huntington at vastly greater length, I was forced to recognize my inability to make people aware of confusing use and mention. Nor have the passing years brought me the ability; they have only vindicated my despair of it. By now perhaps I should have concluded that I must be the confused one, were it not for people who do turn out to see the distinction my way.

I have said that modal logic does not require confusion of use and mention. But there is no denying that confusion of use and mention engenders an irresistible case for modal logic, as witness (12).

I should not leave (12) without touching upon a third interpretation. Perhaps 'p' and 'q' are to be seen as propositional variables, whose values are propositions (or meanings of sentences) and whose appropriate substitutions are therefore names of propositions, hence names of

## REPLY TO PROFESSOR MARCUS

meanings of sentences. Then again (12) is in order, if we countenance these subtle entities. But, on this interpretation, 'eq' comes to name a relation between propositions; again it is no connective of sentences. To suppose it were would be to confuse meaning with reference, and thus to view sentences as names of their meanings.

Let me move now to Professor Marcus's discussion of her (13) and (14), viz. 'aIb' and 'aIa'. Suppose that aIb. Then, she argues, anything true of a is true of b. I agree. But, she says, 'aIa' is a tautology. Again I agree, not quarreling over the term. So, she concludes, 'aIb' must be a tautology too. Why? The reasoning is as follows. We are trying to prove this about b: not just that aIb, but that tautologously aIb. Now this thing that we are trying to prove about b, viz., that tautologously aIb, is true of a; so, since b is a, it is true of b.

Again our troubles condense about the distinction between use and mention. If we take 'tautologously' as a modal operator attachable directly to sentences, then the argument is all right, but pointless so long as the merits of modal logic are under debate. If on the other hand we accept only 'tautologous', as a predicate attributable to sentences and therefore attachable to quotations of sentences, then the argument breaks down. For, the property that was to be proved about b – viz., that tautologously alb – has to be seen now as a quotation-breaking pseudoproperty on which the substitutivity of identity has no bearing. What I mean by a quotation-breaking pseudo-property will be evident if we switch for a moment to the truth ''Cicero' has three syllables'. Obviously we cannot infer that ''Tully' has three syllables', even though Tully is Cicero. And from ''ala' is tautologous' there is no more reason to infer that ''alb' is tautologous', even granted that b is a.

Professor Marcus's reflections on identity led her to conclude that identity, substitutivity, and extensionality are things that come in grades. I have just now objected to some of the reasoning. I also do not accept the conclusion. My position is that we can settle objectively and absolutely what predicate of a theory to count as the identity predicate, if any, once we have settled what notations to count as quantifiers, variables, and the truth functions. Until we have found how to handle quantification in a given theory, of course we have no way even of telling what expressions of the theory to count as predicates and what signs to count as their subject variables; and, not being able to spot predicates, we cannot spot

the identity predicate. But show me the quantifiers and the variables and the truth functions, and I can show you when to read an open sentence ' $\phi xy$ ' as 'x = y'. The requirements are strong reflexivity and substitutivity, thus:

$$(\mathbf{x})\phi\mathbf{x}\mathbf{x}, \quad (\mathbf{x})(\mathbf{y})(\phi\mathbf{x}\mathbf{y}\cdot\ldots\mathbf{x}\ldots\cdot\supset\cdot\ldots\mathbf{y}\ldots).$$

If these requirements are met, then, as is well known, ' $\phi xy$ ' meets all the formal requirements of 'x = y'; and otherwise not.

The requirements fix identity uniquely. That is, if ' $\phi$ ' and ' $\psi$ ' both meet the requirements of strong reflexivity and substitutivity, then they are coextensive. Let me quickly prove this. By substitutivity of ' $\phi$ ',

$$(\mathbf{x})(\mathbf{y})(\phi \mathbf{x}\mathbf{y} \cdot \psi \mathbf{x}\mathbf{x} \cdot \supset \psi \mathbf{x}\mathbf{y}) \, .$$

But, by reflexivity of  $\psi$ , we can drop the ' $\psi xx$ '. So ' $\psi$ ' holds wherever ' $\phi$ ' does. By the same argument with ' $\phi$ ' and ' $\psi$ ' interchanged, ' $\phi$ ' holds wherever ' $\psi$ ' does.

There are a couple of tangents that I would just mention and not use. One is that there is no assurance, given a theory with recognized notations for quantification and the truth functions, that there is an identity predicate in it. It can happen that no open sentence in 'x' and 'y', however complex, is strongly reflexive and substitutive. But this is unusual.

The other is that if an open sentence in 'x' and 'y' does meet these two requirements, we may still find it to be broader than true identity when we interpret it in the light of prior interpretations of the primitive predicates of the theory. But this sort of discrepancy is always traceable to some gratuitous distinctions in those prior interpretations of the primitive predicates. The effect of our general rule for singling out an identity predicate is a mild kind of identification of indiscernibles.<sup>1</sup>)

Tangents aside, my point is that we have an objective and unequivocal criterion whereby to spot the identity predicate of a given theory, if such there be. The criterion is independent of what the author of the theory may do with '=' or 'I' or the word 'identity'. What it does depend on is recognition of the notations of quantification and the truth functions. The absoluteness of this criterion is important, as giving a fixed point of reference in the comparison of theories. Questions of universe, and

1) See my Word and Object (New York, 1960), p. 230.

individuation, take on a modicum of inter-systematic significance that they would otherwise lack.

In particular the criterion makes no doubt of Professor Marcus's law for modal logic:

$$(x)(y)(x = y \cdot \supset \cdot \text{ necessarily } x = y)$$
.

It follows from 'necessarily x = x' by substitutivity.

Notice that my substitutivity condition was absolute. There was no question what special positions to exempt from substitutivity, and no question what special names or descriptions to exempt in special positions. Hence there was no scope for gradations of identity or substitutivity. What enabled me to cut so clean was that I talked in terms not of names or descriptions but of 'x' and 'y': variables of quantification. The great philosophical value of the eliminability of singular terms other than variables is that we can sometimes thus spare ourselves false leads and lost motion.

In her own continuing discussion, Professor Marcus developed a contrast between proper names and descriptions. Her purpose was, I gather, to shed further light on supposed grades or alternatives in the matter of identity and substitutivity. I have urged just now that we can cut through all this by focusing on the bindable variable. And I am glad, for I think I see trouble anyway in the contrast between proper names and descriptions as Professor Marcus draws it. Her paradigm of the assigning of proper names is tagging. We may tag the planet Venus, some fine evening, with the proper name 'Hesperus'. We may tag the same planet again, some day before sunrise, with the proper name 'Phosphorus'. When at last we discover that we have tagged the same planet twice, our discovery is empirical. And not because the proper names were descriptions.

In any event, this is by the way. The contrast between description and name needs not concern us if we take rather the variables of quantification as our ultimate singular terms. Already for the second time we note the philosophical value of the eliminability of singular terms other than variables: again it spares us false leads and lost motion.

Let us look then to Professor Marcus's next move. Alarmingly, her next move was to challenge quantification itself, or my object-oriented interpretation of it. Here she talks of values of variables in a sense that I

must sharply separate from my own. For me the values e.g. of number variables in algebra are not the numerals that you can substitute, but the numbers that you talk about. For Professor Marcus, the values are the expressions you can substitute. I think my usage has the better history, but hers has a history too. Ryle objected somewhere to my dictum that to be is to be the value of a variable, arguing that the values of variables are expressions and hence that my dictum repudiates all things except expressions. Clearly, then, we have to distinguish between values of variables in the *real* sense and values of variables in the *Ryle* sense. To confuse these is, again, to confuse use and mention. Professor Marcus is not, so far as I observe, confusing them. She simply speaks of values of variables in the Ryle sense. But to forestall confusion I should like to say 'substitutes for variables' rather than 'values of variables' in this sense, thus reserving 'values of variables' for values of variables in the real sense.

Thus paraphrased, Professor Marcus's proposed reinterpretation of existential quantification is this: the quantification is to be true if and only if the open sentence after the quantifier is true for some substitute for the variable of quantification. Now this is, I grant, an intelligible reinterpretation, and one that does not require objects, in any sense, as values, in the real sense, of the variables of quantification. Note only that it deviates from the ordinary interpretation of quantification in ways that can matter. For one thing, there is a question of unspecifiable objects. Thus take the real numbers. On the classical theory, at any rate, they are indenumerable, whereas the expressions, simple and complex, available to us in any given language are denumerable. There are therefore, among the real numbers, infinitely many none of which can be separately specified by any expression, simple or complex. Consequently an existential quantification can come out true when construed in the ordinary sense, thanks to the existence of appropriate real numbers, and yet be false when construed in Professor Marcus's sense, if by chance those appropriate real numbers all happen to be severally unspecifiable.

But the fact remains that quantification can indeed be thus reinterpreted, if not altogether *salva veritate*, so as to dissociate it from objective reference and real values of variables. Why should this be seen as desirable? As an answer, perhaps, to the charge that quantified modal logic can tolerate only intensions and not classes or individuals as values of its variables?

#### **REPLY TO PROFESSOR MARCUS**

But it is a puzzling answer. For, it abstracts from reference altogether. Quantification ordinarily so-called is purely and simply the logical idiom of objective reference. When we reconstrue it in terms of substituted expressions rather than real values, we waive reference. We preserve distinctions between true and false, as in truth-function logic itself, but we cease to depict the referential dimension. Now anyone who is willing to abstract thus from questions of universe of discourse cannot have cared much whether there were classes and individuals or only intensions in the universe of discourse. But then why the contortions? In short, if reference matters, we cannot afford to waive it as a category; and if it does not, we do not need to.

As a matter of fact, the worrisome charge that quantified modal logic can tolerate only intensions and not classes or individuals was a mistake to begin with. It goes back to 1943; my 'Notes on existence and necessity'1) and Church's review of it.<sup>2</sup>) To illustrate my misgivings over quantifying into modal contexts I used, in that article, the example of 9 and the number of the planets. They are the same thing, yet 9 necessarily exceeds 7 whereas the number of the planets only contingently exceeds 7. So, I argued, necessarily exceeding 7 is no trait of the neutral thing itself, the number, which is the number of the planets as well as 9. And so it is nonsense to say neutrally that there is *something*, x, that necessarily exceeds 7. Church countered that my argument worked only for things like numbers, bodies, classes, that we could specify in contingently coincident ways: thus 9 is what succeeds 8, and is what numbers the planets, and these two specifications only contingently coincide. If we limit our objects to intensions, Church urged, this will not happen.

Now on this latter point Church was wrong. I have been slow to see it, but the proof is simple. Anything x, even an intension, is specifiable in contingently coincident ways if specifiable at all. For, suppose x is determined uniquely by the condition ' $\phi x$ '. Then it is also determined uniquely by the conjunctive condition ' $p \cdot \phi x$ ' where 'p' is any truth, however irrelevant. Take 'p' as an arbitrary truth not implied by ' $\phi x$ ', and these two specifications of x are seen to be contingently coincident: ' $\phi x$ ' and 'p  $\cdot \phi x$ '.

Contrary to what Church thought, therefore, my 1943 strictures were

<sup>1)</sup> Jounal of Philosophy, Vol. XL, pp. 113-127.

<sup>&</sup>lt;sup>2</sup>) Journal of Symboliclogic, Vol. VIII, pp. 45-47.

cogent against quantification over any sorts of objects if cogent at all; nothing is gained by limiting the universe to intensions. The only course open to the champion of quantified modal logic is to meet my strictures head on: to argue in the case of 9 and the number of the planets that this number is, of itself and independently of mode of specification, something that necessarily, not contingently, exceeds 7. This means adopting a frankly inequalitarian attitude toward the various ways of specifying the number. One of the determining traits, the succeeding of 8, is counted as a necessary trait of the number. So are any traits that follow from that one, notably the exceeding of 7. Other uniquely determining traits of the number, notably its numbering the planets, are discounted as contingent traits of the number and held not to belie the fact that the number does still necessarily exceed 7.

This is how essentialism comes in: the invidious distinction between some traits of an object as essential to *it* (by whatever name) and other traits of it as accidental. I do not say that such essentialism, however uncongenial to me, should be uncongenial to the champion of quantified modal logic. On the contrary, it should be every bit as congenial as quantified modal logic itself.<sup>1</sup>)

<sup>1</sup>) For more in the vein of these last few paragraphs see my *From a Logical Point of View*, Revised Edition (Cambridge, Mass., 1961), pp. 148–157.