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Should Physicists Preach What They Practice?

Constructive Modeling in Doing and Learning Physics

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ABSTRACT: Does one need to think like a scientist to learn science? To what extent can examining the cognitive activities of scientists provide insights for developing effective pedagogical practices? The cognition and instruction literature has focused on providing a model of expert knowledge structures. To answer these questions, what is needed is a model of expert reasoning practices. This analysis is a step in that direction. It focuses on a tacit dimension of the thinking practices of expert physicists, "constructive modeling". Drawing on studies of historical cases and protocol accounts of expert reasoning in scientific problem solving, it is argued that having expertise in physics requires facility with the practice of "constructive modeling" that includes the ability to reason with models viewed generically. Issues pertaining to why and how this practice of experts might be incorporated into teaching are explored.

1. INTRODUCTION

To what extent can examining the thinking practices of scientists provide insights for developing effective pedagogical strategies? The assumption that to learn a science students must engage in active construction of their own representations of extant scientific knowledge is a major premise of much of the research in the field of cognition and instruction. On such a "constructivist" account of learning, to learn science requires engaging in authentic scientific practices, irrespective of whether the learner is in training to be a scientist or is simply satisfying our cultural desideratum that people have some knowledge of the scientific understanding of the world (See Duschl 1990 for an overview). From this perspective, then, the thinking practices a scientist engages in while constructing representations and other problem solving activities are directly relevant to learning. Thus, learning science should be facilitated by learning the problem solving practices of scientists. By developing an explicit and deep understanding of these practices, educators will be in a better position to devise explicit strategies for helping students to engage in them. Research on expert problem solving carried out by cognitive scientists can assist educators in forming this understanding.

From the outset, however, a significant problem faces those who wish to integrate what cognitive scientists have been learning about the thinking practices of scientists into teaching practices. By and large, the pertinent literature has focused, primarily, on providing models of the nature, structure, and characteristics of the knowledge an expert possesses. To meet pedagogical objectives, more attention needs to be given to the nature of the modes of thinking one has to acquire in gaining expertise and to the nature of the processes through which expertise is developed. I concentrate here on the modes of thinking used by expert physicists during problem solving. My answer to the question posed in the title to this paper is "yes". Developing pedagogical strategies that incorporate into the learning situation what physicists do in solving problems will go far in alleviating the disparities between the problem-solving practices of physics students and those of physicists that have been observed in the literatures discussed in Section 2. Further, although my focus is on physics, at the very least my more general remarks about incorporating realistic exemplars of practice into science teaching can be extended to all the sciences.

One plausible reason why physicists do not preach what they practice is that for the expert, knowledge of thinking practices is largely tacit. Discussions of "tacit knowledge" usually focus on content knowledge, but knowledge of processes is equally important for the expert reasoner. As with riding a bicycle, once these are practiced and facility has been developed, the processes become tacit. Based on my analyses of historical cases and of studies of expert reasoning in scientific problem solving carried out by cognitive scientists, I propose that having expertise in physics requires, in addition to domain knowledge, facility with the domain-independent practice of "constructive modeling". Constructive modeling is an integrative reasoning process that employs analogical and visual modeling and thought experimentation in creating and transforming informal representations of problems. Second, I propose that having the ability to reason with a special class of mental models, "generic models", in a specific domain is a significant component of constructive modeling. Generic mental modeling enables the expert to see, for instance, the specific spring in a new problem as belonging to the class of simple harmonic oscillators of which she already has some knowledge. Further, it underlies what the physicist calls "intuition" and contributes to judgments about the plausibility of the constructed models. That novices lack facility in generic modeling is a crucial component of what psychologists call "the transfer problem", i.e, the inability of novices to recognize, spontaneously, the relevance to new situations of work they have done in previous problems.

Presently, the primary means novices have for identifying and acquiring facility with expert constructive modeling practices is through osmosis. Those who are "good" at physics pick it up and then often go on to graduate school where participation in their mentors' research provides a kind of "cognitive apprenticeship" (Brown *et al.* 1989) for developing these practices. This knowledge then becomes tacit as a highly successful physics student becomes a physicist, and, in many cases, a teacher. However, for the vast majority of students, these physics thinking practices remain largely invisible. My hypothesis is that we will be more successful in training students to think scientifically if they are taught, explicitly, how to engage in the modeling practices of those with expertise in physics. One reason for this optimistic view is that similar modeling practices are widespread in human behavior in other venues (Ram *et al.*, in press). So, we can assume students do have the cognitive capacities to engage in constructive modeling. As a philosopher and historian of science I cannot speak with confidence about specific ways to translate the insights presented here about modes of expert thinking in physics into pedagogical strategies for developing expertise in students. I hope these insights will resonate with science educators, provide support for existing initiatives along these lines (See, e.g., Wiser 1992), and persuade others to take up that task.

2. THE NATURE OF EXPERTISE IN PHYSICS

The three literatures I take to be central to developing an account of modes of expert thinking comprise (1) "conceptual change", (also known as: "restructuring" or, in the case at hand, "naive physics"); (2) "expert/novice" problem solving; and (3) diverse literatures on what one might call "model-based" reasoning: analogy, case-based reasoning, mental modeling, qualitative reasoning, and heterogeneous or multi-modal reasoning. One problem confronting us at the outset is that there is little contact among these literatures; yet, the issues associated with learning cut across all three.

The problem of how best to help students learn the conceptual structure of a science has generated investigations into the nature and content of novice ("naive" or "untutored") models of specific domains. These investigations have produced a vast and persuasive literature demonstrating that the conceptual understandings novices have of phenomena such as motion prior to and often post instruction differ significantly from those of physicists (See, e.g., Chi *et al.* 1981; Driver & Easley 1978; Halloun & Hestenes 1985; McDermott 1984). The findings show, clearly, that students are not empty vessels into which teachers pour knowledge and they raise the issue of how prior knowledge affects learning new material. Take, for example, one finding that I have discussed in some detail in previous work (Nersessian 1989; Nersessian & Resnick 1989). In learning Newtonian mechanics, students need to change their belief that "all motion implies force" and come to understand that "accelerated motion implies force". Further complicating this learning task, student protocols reveal that their concepts of "motion" and "force" are quite different from the Newtonian concepts. Thus, learning that "accelerated motion implies force" requires constructing new representations for "motion" and "force". Changing novice representations requires more than rearranging existing elements and more than fitting new facts to existing frameworks. It requires constructing new concepts and working them into a new framework. As argued in my earlier work, this is a problem-solving process.

The second literature has been investigating salient differences between expert and novice problem solving. These investigations show that the ways expert and novices approach solving problems differ in significant respects. Chief among the findings are that, unlike novices, during problem solving, experts (1) recall knowledge in large patterns or "chunks" of information; (2) work with principled representations of problems, emphasizing structural relationships over superficial features; (3) first construct "qualitative" models – often a series of intermediate models that approximately represent the target problem – and then derive equations, and (4) exercise strong "metacognitive" control over their reasoning (See, e.g., Chi & Glaser 1988; Ericsson & Smith 1991).

Collectively, what these findings show is that even though expert physicists have a deeper, more structured, understanding of the laws and formulas, their approach to a problem is not usually to subsume it under known laws as expressed in equations. Rather, that knowledge often remains in the background during the initial phases of the problem-solving process. In striking contrast to this, novices try immediately to find the appropriate equations. Although experts do not explicitly invoke equations while qualitatively modeling a problem, there is usually a great deal of mathematics implicit in their representations and reasoning. Because of this, I prefer the term "informal" (non-formulaic) rather than "qualitative" to characterize expert modeling. What is most significant for our purposes is that informal modeling is an important part of the process by which an expert creates a scientific understanding or explanation of a problem for herself, or, in the historical cases of conceptual change, is part of the process by which new understanding is created for the scientific community. On my view, these processes are not different in kind. They lie on a continuum of complexity and creativity.

The conceptual change and the expert/novice literatures, viewed together, produce a picture of the expert as one whose domain knowledge contains different representations of entities and processes and whose knowledge structures are richer, more integrated, and more abstract than those of the novice. How these differences in knowledge might be related to differences in problem-solving practices (3 and 4 above) has been addressed to some extent in the expert/novice literature. Specifically, researchers have proposed that there is a significant relationship between the differences in knowledge content and structure and why experts construct informal models to represent a problem while novices rush to find equations. For example, knowledge differences are thought to account for why the expert approaches a problem by classifying it with respect to underlying principles, rather than by focussing on surface features (Chi *et al.* 1981) and to enable the expert to apply a "working forward" strategy in solving a problem rather than the "working backward" strategy used by novices (Simon & Simon 1978; Larkin *et al.* 1980). However, surprisingly little attention has been paid in the expert/novice literature to developing an account of the nature of the processes through which experts construct the informal problem representations. What exists on this topic is primarily descriptive, e.g., it may be noted that "experts draw more diagrams," but an explanation of how the visual representation may contribute to generating the models has been lacking.

This issue brings us to see the significance of the third literature, which I will characterize as that on "model-based reasoning", for our problem. Whereas the expert/novice and restructuring literatures focus on knowledge and practices that are domain specific, the reasoning literature focusses on domain independent processes. By and large, findings in this literature have not been put to the service of analyzing and explaining the informal modeling practices of experts. But, to understand how these practices are productive of problem solving, this needs to be done. Given that model-based reasoning is investigated in too many literatures to be surveyed here, the best route is to point out the areas in which these literatures are lacking from the perspective of our problem. Before noting these deficiencies I want to qualify what I claim by admitting that I am, of course, not familiar with everything that has been written in these literatures and may have overlooked some relevant exceptions.

The first serious deficiency is that the analyses assume the source analogies and cases used in modeling are already available, whereas in much of the informal modeling experts practice, these are created and undergo significant revision during the problem-solving process. Of course, there are cases in which a direct analog representation for a problem exists, and substitution and mapping is all that is required. But, more often, substantive changes need to be made to the analogical source in order to fit the constraints of the target problem. Second, generic mental modeling has scarcely been addressed (See, Bhatta & Goel 1993; Stroulia & Goel 1992 for recent exceptions). However, this facility would seem to underlie many of the observed differences in approach, such as categorizing by means of principles rather than features of specific objects. Finally, although there has been a recent shift of attention in this direction, these literatures have been functioning largely as separate areas of investigation. Integration is necessary if we are to construct rich accounts of the complex intellectual work exhibited in expert problem-solving practices. For example, visual modeling is often used in conjunction with analogy and mental modeling and reasoning often requires simulating dynamical phenomena to see what happens when things change in the model. We will be examining two cases exhibiting all these features later in the paper. First some introductory explanation of the nature of constructive modeling is called for.

3. CONSTRUCTIVE MODELING

"Constructive modeling" is a dynamic reasoning process involving analogical and visual modeling and mental simulation to create models of the target problem where no direct analogy exists. I first identified constructive modeling in a quite exceptional case of problem solving: James Clerk Maxwell's derivation of the electromagnetic field equations (Maxwell 1861–2; Nersessian 1984, 1989, 1992, 1993, in press). As we will see below, Maxwell derived the field equations by constructing a series of models embodying the pertinent physical and mathematical constraints. In the process he used multiple knowledge domains and informational formats, such as equations, linguistic representations, diagrammatic representations. Most philosophical and historical accounts accorded the models Maxwell created in deriving the field equations only ancillary status. On my reading of the historical records, this looked implausible. The models seemed to be central in his reasoning process. I believed the difference in interpretation to lie in my willingness to view modeling as a form of reasoning, while the other accounts derive largely from a position in which reasoning is taken to comprise applying formal rules of inference (deductive or inductive) to systems of propositions. I sought support in the model-based reasoning literatures in cognitive science for my explanation of how Maxwell derived the field equations via the models he generated in a dynamic reasoning process and discovered processes of that level of complexity to be absent from these literatures. What these literatures did provide is important evidence that analogical models employed in empirical studies of problem solving are generative (See, e.g., Gentner & Gentner 1983). That is, reasoning with them provides information about the target problem they represent that goes beyond what is available directly from the problem.

However, several features distinguish Maxwell's reasoning process from the cognitive accounts. First, the cognitive accounts examine cases where the source models are ready to hand and no reconstructing of the source during problem solving takes place. Second, little notice is taken of the importance of visual representation in some instances of analogical modeling (See, Thagard *et al.* 1993 for a recent attempt). Third, although the evidence is linguistic and sketch-like in nature, there appears to be a simulative dimension to Maxwell's reasoning that leads to the hypothesis that constructive modeling involves a mental modeling process that affords simulations for the static representations in the sketches and linguistic utterances. Indeed, although more traditional explanations of constructive modeling as a reasoning process might be possible, that the practice facilitates reasoning by means of mental modeling provides a plausible explanation of why the practice is so effective. I have been developing this hypothesis in work with James Greeno on mental modeling in creating scientific understanding (Nersessian & Greeno, in process). On our account, analogies provide constraints for building models and visual representations (external and internal) and mental simulation facilitate the reasoning process.

The contemporary notion that mental modeling plays a significant role in human reasoning was formulated, initially, by Kenneth Craik (1943). Craik contended that to have an explanation of a phenomenon requires having a well-understood model of it. He proposed that people reason, in general, by carrying out thought experiments on internal models. Though not uncontroversial, the centrality of mental modeling to cognition is a hypothesis under investigation in many domains. Experimental results that demonstrate the effect of semantic information on reasoning provided the main impetus for the resurgence of the hypothesis (See, Johnson-Laird 1983, for an extensive discussion). Mental modeling has been investigated in a wide range of phenomena from thinking about causality in physical systems (See, e.g., deKleer & Brown 1983) to reasoning with representations of domain knowledge (See, e.g., Gentner & Stevens 1983) to analogical reasoning (See, e.g. Gentner & Gentner 1983) to deductive inferencing (See, e.g., Johnson-Laird 1983) to comprehending narratives (See, e.g., Perrig & Kintsch 1985).

There are several distinct theoretical accounts of mental models that tend to be conflated in the literature. The most significant distinction for our purposes is between those investigations that treat mental models as structures stored in long term memory and are later called upon in reasoning and those that treat them as temporary structures constructed in working memory for a specific reasoning task. Greeno & Nersessian (in process) postulate that the mental modeling component of the constructive modeling process takes place in working memory. Of course, the process must be interactive with long-term memory, since much of the information drawn upon is in background knowledge, and with external text, visual renderings, and objects. Philip Johnson-Laird's (1983) account is the best articulated of those analyses that focus on the temporary reasoning structure. In his terms, a mental model is a structural analog of a real-world or imaginary situation, event, or process that a person constructs to reason with in the mind. What it means for a mental model to be a structural analog is that it embodies a representation of the spatial and temporal relations among and the causal structure connecting the events and entities depicted. I take this notion to include more abstract cases that just provide functional analogs.

Once identified in the Maxwell case, it became apparent that although this is an extraordinary application, the practice of constructive modeling is much more widespread in physics – especially when working on novel problems – and is a feature of the informal modeling exhibited by expert physicists in less exceptional circumstances. Further, instances similar to Maxwell's reasoning have been identified by researchers investigating problem solving using cased-based reasoning in engineering and design domains (Wills & Kolodner 1994) and in studies of technological invention (Gorman & Carlson 1990). So, constructive modeling has the potential for producing an integrated analysis of analogy, case based reasoning, visual reasoning, and mental modeling in problem solving and "discovery" (which I regard as a problem-solving process).

Constructive modeling is a semantic process. The models produced are interpretations satisfying constraints derived from text, equations, diagrams, and any other salient informational sources in the external environment and the mental representations of the problem solver. Equations and text represent a physical structure or process by making propositional claims about it. A model represents a physical structure or process by having surrogate objects with relations and/or functions that are in correspondence with it. To engage in constructive modeling one calls on knowledge of the generative principles and constraints for physical models in a domain. These constraints and principles may be represented mentally in different informational formats and knowledge structures that act as tacit assumptions employed in constructing and transforming models while problem solving. Retrieved analogous cases, either inter- or intra-domain, serve as sources of constraints to be used in interaction with those provided by the target problem. The process involves constructing analogous cases until the constraints fit the target problem. The models thus constructed are proposed interpretations of the target problem. Further, the ability to construct and reason with generic models is a significant dimension of the constructive modeling process.

3.1 *Generic mental modeling*

In viewing a model generically, one takes it as representing features common to a class of phenomena. This way of viewing the model can, of course, only take place in the mind. Again, although accounts of reasoning generically via linguistic representations can be constructed, the simulative dimension of constructive modeling leads to the hypothesis that the generic representations are model-like in nature. In reasoning, e.g., about a triangle, one often draws or imagines a concrete representation. However, to consider what it has in common with all triangles, we need to imagine it as lacking specificity in the angles and the sides. That is, the reasoning context demands that the interpretation of the concrete figure is as generic. Generic modeling requires idealization and abstraction. What I mean can be more easily conveyed by looking at a simple example taken from Polya (1954).

Polya considers two cases, abstracting from an equilateral triangle to a triangle-in-general and from it to a polygon-in-general (Figure 1). Loss of specificity is the central aspect of this kind of abstraction process. Polya calls this process "generalization" in mathematics, but I prefer to call it "generic modeling" in order to distinguish it from the process of "generalization" in logic. The abstracted geometrical figures are "generic". The generic triangle represents those features that all kinds of triangles have in common. In generic modeling, specificity of salient dimensions is lost; in this case, the equality of the lengths of the sides and of the degrees of the angles. In contrast, a logical generalization from one equilateral triangle to all equilateral triangles does not involve loss of specificity of these salient aspects of "equilateral". In moving from the generic triangle to the generic

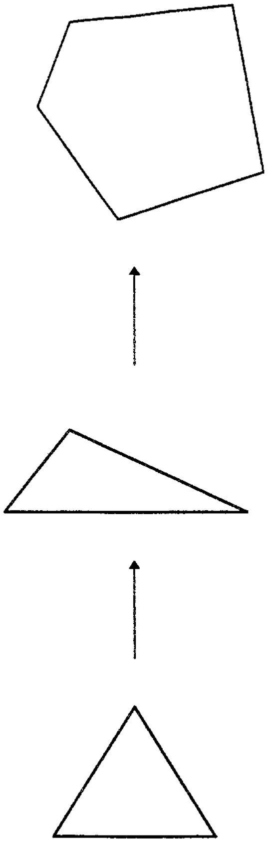

Figure 1. Abstraction via generic modeling.

polygon, there is an additional loss of specificity of the number of sides and the number of angles of the figure.

Generic modeling is a strategy that is commonly employed in solving physics problems. Ronald Giere's (1988) analysis of how the linear oscillator is presented in graduate physics textbooks provides a good example. In modeling a problem about a pendulum by means of a spring, the scientist understands the spring model as generic, that is, as representing the class of simple harmonic oscillators of which the pendulum is a member. Studies by cognitive psychologists of problem-solving practices employed by expert physicists can be interpreted as exhibiting the same strategy of generic modeling (See, e.g., Chi *et al.* 1981; Clement 1989). Generic modeling facilitates transfer among problem solutions. The ability to represent and view information generically would seem to be central to analogical retrieval as well as mapping.

Devising the practice of generic modeling was of great significance for the development of modern science. It was only by generic modeling, e.g., that Newton could see the commonalities among the motions of planets and of projectiles which enabled his formulating a unified mathematical representation of motion. The generic model represents what is common among the members of specific classes of physical systems, viewed with respect to a problem context. Newton's inverse-square law of gravitation abstracts what a projectile and a planet have in common in the context of determining motion. The inverse-square-law model served as a generic model of action-at-a-distance forces for those who tried to bring all forces into the scope of Newtonian mechanics.

At this point I want to illustrate "constructive modeling" and "generic mental modeling" by discussing briefly two cases: Maxwell's derivation of the electromagnetic field equations (1890) and John Clement's S2 expert protocol of the solution of a spring problem.

4. CASE 1: MAXWELL'S DERIVATION OF THE ELECTROMAGNETIC FIELD EQUATIONS

This case provides a particularly salient case study for our purposes because the standard physics textbook accounts at both the undergraduate and graduate levels present Maxwell as starting from a set of equations (Coulomb's Law, Ampère's Law and Faraday's Law) for closed circuits plus the equation for continuity of charge (Feynman *et al.* 1964; Jackson 1962; Panofsky & Phillips 1962). Most of the accounts in the philosophical literature also employ this understanding in their analyses. Maxwell's problem is portrayed as that of reconciling these equations for the case of open circuits. Through considering how to make the equations formally consistent, as the account goes, he saw that a term needed to be added to Ampère's Law to represent the contribution of electrostatic polarization to current. Adding this term introduced the critical time delay in the propagation of electromagnetic actions.

My analysis reveals a quite different story. Maxwell derived the field equations through a constructive modeling process that involved synthesizing multiple constraints drawn from the physics of elastic fluids and of machine mechanisms, experimental data on electricity and magnetism, Faraday's hypotheses about the lines of force and his models, William Thomson's hypothesis of rotational motion of magnetism and his analogies, and mathematical equations. There are many salient dimensions of this case that we are not able to consider in this brief overview. For example, the social context is crucial to understanding Maxwell's approach to the problem and the conceptual and analytical resources he was able to draw upon. Maxwell's location in Cambridge led to his training as mathematical physicist, proficient in the emerging field of continuum mechanics. This shaped the nature of the theoretical, experimental, and mathematical knowledge and the methodological practices with which he formulated the problem and approached it's solution. The work of Michael Faraday and William Thomson (later, Lord Kelvin) contributed signifi-

Figure 2. (a) Actual pattern of lines of force surrounding a bar magnet (from Faraday (1839–55), vol. 3); (b) Schematic representation of lines of force surrounding a bar magnet.

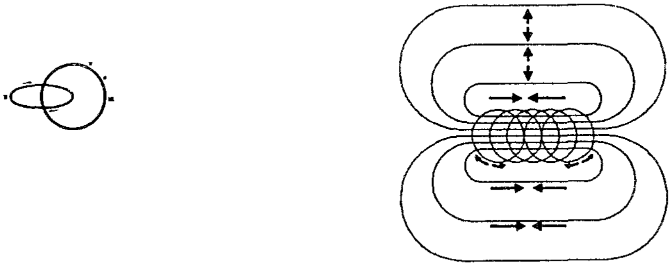

Figure 3. (a) Faraday's represention of the interconnectedness of electric currents and magnetic force (from Faraday (1839–55), vol. 3); (b) Schematic representation of the reciprocal relationship between magnetic lines of force and electric current lines.

cantly to these. Continental physicists working on electromagnetism at the same time employed quite different practices and drew from fundamentally different mathematical and physical representational structures.

Maxwell inherited several visual models from Faraday. Faraday had hypothesized that the lines of force that form when iron filings are sprinkled around magnets and charged matter indicate that some real physical process is going on in the space surrounding these objects and that this process is part of the transmission of the actions (Faraday 1835–55). That visual display of lines of force in geometrical configuration had a profound influence on Faraday's understanding of electric and magnetic actions, and of forces and matter in general. Figure 2a shows the actual lines as they form around a magnet. Figure 2b shows Faraday's sketch of the model that represents them in geometrical and dynamical form on his analysis. That is, although the visual representation is of a static geometrical pattern, Faraday reasoned with it as representing dynamic processes (Gooding 1980, 1990; Nersessian 1984, 1985; Tweney & Gooding, in process). So, he envisioned the various forces of nature as motions in the lines, such as waving, bending, stretching, and vibrating. Near the end of his research, Faraday constructed another model to represent the dynamical balance between electricity and magnetism – that of interlocking curves shown in Figure 3a. The interlocking curves model is abstracted from the earlier lines of force model. As illustrated by Figure 3b, a lateral repulsion

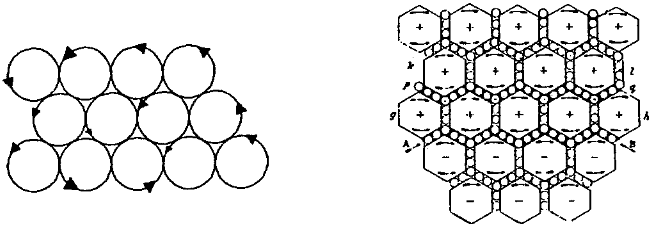

Figure 4. (a) Schematic representation of inital crude source retrieved by Maxwell; (b) Maxwell's representation of his fully elaborated "physical analogy" (from Maxwell (1861–2)).

of the magnetic lines (outer lines) has the same effect as a longitudinal expansion of the current lines (inner lines).

Maxwell made use of these models in his kinematical analysis of electric and magnetic forces (Maxwell 1855–6). There he replaced Faraday's relationship between the number of lines cut and the intensity of the induced force with a continuous measure by constructing models of the lines of force on analogy with the flow of an incompressible fluid through a fine tube of variable section, filling all space. Although the kinematical analysis is significant in itself, we will focus on the dynamical analysis, because it is there that he used constructive modeling to introduce a fundamentally new mathematical representation for force into physics: the field equations.

To carry out a dynamical analysis of the underlying forces that could produce the lines of force required constructing a model that could embody the dynamical relationships between electric and magnetic forces. That is, the model needed to account for how electricity and magnetism are produced, interact, and are transmitted. The complete model is an imaginary hybrid construction that draws physical and mathematical constraints from two analogical source domains: continuum mechanics (fluids, elastic media, etc.) and machine mechanics. In the analysis, constraints from the source and the target domains interact to enable creating and modifying a series of models that are the objects with which Maxwell reasoned. Further, reasoning with the models appears to require that they provide simulations and thus are animated in a manner similar to thought experiments (Nersessian 1993). In the paper itself, Maxwell provided an extensive set of instructions for how his readers should visualize and animate the models in their own reasoning.

The processes of model construction are as follows. Maxwell first constructed a primitive model (Figure 4a) consistent with an initial set of constraints from the electromagnetic domain. That model was a fluid medium composed of elastic vortices and under stress. With this form of the model he derived a mathematical representation for various magnetic phenomena. Analyzing the relationships between current and magnetism led to alteration of the model. If we animate Figure 4a we see that all the vortices are rotating in the same direction and, since they touch, friction is produced so they will eventually stop. Mechanical consistency, thus, requires the introduction of "idle wheels", such as those used in machine gears, in this case to surround the vortices. He argued that their translational motion could be taken to represent electricity. Figure 4b shows a cross section of the constructed model. To carry out the calculation, Maxwell had now to modify the elastic vortices, considering them as rigid pseudospheres. We can see how the imaginary system provides a mechanical interpretation for electromagnetism: motion of the particles creates motion of the vortices and vice versa. Thus, as is known experimentally electric current produces magnetic effects and changes in magnetic effects produce current. Using the model he was able to derive mathematical equations to represent these causal relationships.

It then took him nine months to figure out how to represent the final, critical, piece of the problem, electrostatic actions. By making the magnetic vortices elastic and identifying electrostatic polarization with elastic displacement, he was able to calculate the wave of distortion that propagates through the medium during polarization. That is, adding elasticity to the model enabled him to show that electromagnetic actions are propagated with a time delay, i.e., they are field actions and not Newtonian actions at a distance. At this point he had derived a full mathematical representation of the electromagnetic field.

4.1. *Discussion*

As we have seen, Maxwell's constructive modeling process involved integrating common physical and mathematical constraints abstracted from continuum-mechanical systems, certain machine mechanisms, and electromagnetic systems into a series of models. These models are taken to represent the production and transmission of electric and magnetic forces in a mechanical aether. In their mathematical treatment, these common dynamical properties and relationships are separated from the specific systems by means of which they had been made concrete. During the modeling process, Maxwell continually evaluated the models and the inferences he drew from them, and integrated the solutions to the sub-problems into a consistent mathematical representation. As was to be discovered only later, Maxwell's mathematical representation is of a non-Newtonian dynamical system. That is, the mathematical structure will not map back onto the Newtonian source domains. This is an extraordinary instance of constructive modeling in that a fundamentally new representational system was introduced into physics. Constructive modeling enabled Maxwell to derive the laws of a non-Newtonian system, using Newtonian systems as sources. To explain how this could happen, we need to see how this case of constructive modeling employs generic modeling of the salient properties, relationships, and processes.

A key feature of Maxwell's mechanical models is that to reason with them as representing the electromagnetic phenomena, they must lose specificity of the mechanisms creating the stresses in the medium. He supplies concrete mechanisms, but in the context intends for them to represent generic processes. Take, e.g., the analysis of electromagnetic induction. Here the causal relational structure between the vortices and the idle wheel particles is maintained but not the specific causal mechanisms. The idle wheel – vortex mechanism is not the cause of electromagnetic induction; it represents only the causal structure of that unspecified process. Any mechanism that could represent the causal structure would suffice.

Maxwell's idle wheel – vortex mechanism is highly implausible as a real fluid-dynamical system. It does not need to be realistic since what it represents is a causal structure. That is, the specific mechanism is treated generically, in the way the spring is treated generically when it represents the class of simple harmonic oscillators. The causal structure is to be viewed as separated from the specific physical systems by means of which it has been made concrete. In representing generic mechanisms by concrete processes, the mind has an embodied form to reason about. But the reasoning context demands that no specific physical hypotheses belonging to the domain in which the analogy originates be imported to the analysis of the target problem.

There are several ways of seeing that the modeling process is generative. For example, in the initial analysis of magnetic phenomena, Maxwell focused on a single rotating vortex. In the next phase of the analysis he considered the behavior of a medium full of rotating vortices, which led directly to the introduction of the idle wheel particles as a way of representing the causal structure of electromagnetic induction. Also, there are significant sign "errors" in this part of Maxwell's analysis, but as I have argued in previous work (1984, in press), all but one (a minor substitution error) can be seen not to be errors when we view him as reasoning via the constructed models. Additionally, it was only through the models that he came to understand how to represent the energy of the electromagnetic system, which was necessary for rederiving the mathematical representation using generalized dynamics, as he did in the next paper (1864).

The 1864 analysis assumes only that the electromagnetic medium is a generic "connected system", possessing elasticity and thus having energy. The connected system needs to be elastic to provide for the time delay. Elastic systems can receive and store energy. The energy of such a system has two forms: "energy of motion", or kinetic energy, and "energy of tension", or potential energy. Maxwell identified kinetic energy with magnetic polarization and potential energy with electric polarization. We can see how the constructive modeling process we examined showed him how to do this. Schematically, in those models kinetic energy is associated with the rotation of vortices, which when considered generically becomes rotational motion, which then in the general dynamical analysis becomes the motion in the medium associated with magnetic effects. Potential

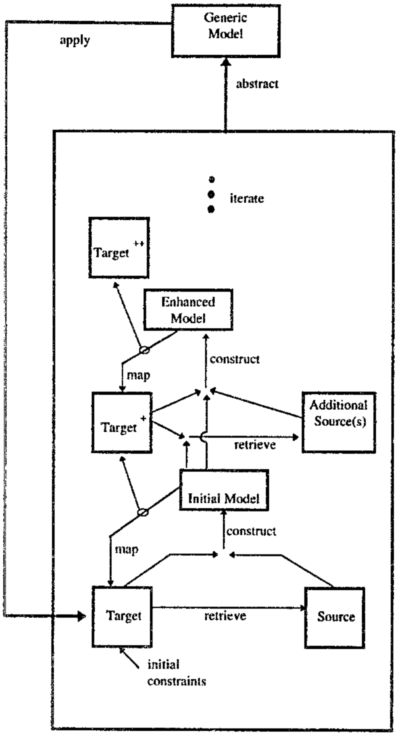

Figure 5. Maxwell's Constructive Modeling Process.

energy is associated with elastic tension between the vortices and the particles, which, generically, becomes elastic stress, which in the general dynamical analysis is elastic tension due to electrostatic effects. Once he had abstracted the appropriate set of general dynamical relations, he could then apply these back to the source domain without the need for any specific model. Figure 5 provides a schematic representation of Maxwell's constructive modeling process.

5. CASE 2: S2'S SOLUTION OF THE SPRING PROBLEM

Our second case study derives from problem solving protocols taken from an expert subject during an experiment by John Clement (1989). Clement's analysis of his subject's informal modeling during problem solving is the only detailed analysis of this practice I have found in the literature on analogical reasoning. Clement's own analysis discusses a process he calls modeling via "bridging analogies". He characterizes this process as one in which the subject "produces models via a successive refinement process of hypothesis generation, evaluation, and modification or rejection" (p.358). I take this to be a form of constructive modeling. Clement provides several short examples from other studies in the paper, but focuses mainly on the subject S2's novel solution of the spring problem he had posed. He focuses on this case because it led to "the invention of a new model of hidden mechanisms in the spring that [the subject] had not observed" (p. 378). So, although this is a more ordinary example of constructive modeling of a problem that, unlike the Maxwell case, can be solved within the existing scientific framework, for S2 it was an instance of highly creative problem solving. For S2 to find a satisfactory explanatory model for his solution, he had to construct a novel representation for himself of how a spring works. A case more closely parallel to the Maxwell case might be the discovery of Hooke's law.

The problem to be solved is "a weight is hung from a spring. The original spring is replaced with a spring made of the same kind of wire; with the same number of coils; but with coils that are twice as wide in diameter. Will the spring stretch from its natural length more, less, or the same amount under the same weight? (Assume the mass of the spring is negligible compared to the mass of the weight.) Why do you think so?" (Figure 6a, b) Clement notes that a number of subjects constructed an analogy with a bent rod (Figure 6c, d), leading them to the correct conclusion that the wide spring would stretch farther. This model captures the constraints of variable length and springiness. That is, unwinding the coil, produces a flexible straight rod. Rotating the rod ninety degrees in the vertical plane reveals that increasing the length of the rod would increase the amount it would bend under the same weight.

Subject S2 was dissatisfied with this model because he took his understanding that the slope of a stretched spring remains constant to be a salient constraint from the problem domain that the rod model violates. That is, visually inspecting the rod model leads to the evaluation that, unlike a spring, the slope of a rod becomes greater as the rod increases in length. S2 first constructed two hybrid modifications to the spring-as-rod model in interaction with the slope constraint from the target problem (Figure 6e, f) and with the constraint that a spring is coiled. Noting that the spring could be coiled in squares led to the zigzag model (Figure 6e). But, since the bending is located at the joint, the problem of increasing slope remains, as it does with the modification that connects the flexible segments with small rigid rods (Figure 6f).

The constant slope constraint led him to consider how a spring whose length was doubled would behave and to focus on what salient differences there might be between doubling the length of a spring and doubling the width of its coils. The solution to the problem came from focusing on additional constraints of the spring: it's coils are circular and they lie in the horizontal plane (Figure 6g). S2 concluded that doubling the size of the circle should lead to no salient differences with the target problem. However, the rotation into the horizontal plane does provide the key insight that there is twisting in addition to bending and this is what accounts for the constant slope of the spring. Clement does not discuss this

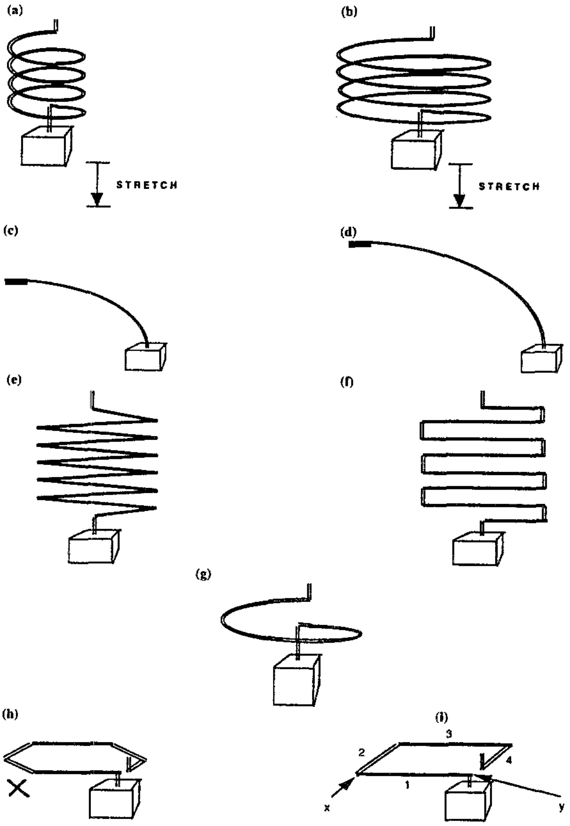

Figure 6. Clement S2 Protocol (Figures taken from Clement 1989).

move explicitly, but it seems crucial. Unfolding the coil in the horizontal plane stretches it bit by bit, as though it had joints, but with even distribution, which brought him back to the earlier idea of modeling a spring by means of wound square coils. Squares, considered generically are polygons and polygons approximate circles in the limit.

S2 focused first on an hexagonal model (Figure 6h) to stand for a generic polygon. The hexagon rotated in the horizontal plane provides a new model for how the constraints would interact. In the horizontal plane, the force from the weight would create twisting at the joints – torsion – as the hexagon unfolds. Torsion can be evenly distributed, unlike the force at the joints in the zigzag model. He then settled on the final model as being a square coil (Figure 6i) in order to accentuate the torsion effect. Unlike the rod model, this one satisfies the constant slope constraint. If the width of a coil is doubled, the increase in bending would also increase the torsion. Although the answer remains unchanged, i.e., the spring with wider coils will stretch further, the understanding S2 had of a spring is considerably altered. With the spring-as-rod model, "springiness" is equated with bending. With the spring-as-horizontal-square model (or spring-as-horizontal-polygon), "springiness" is bending plus torsion.

Both the hexagon and the square models incorporate features of the rod because straight lines can bend. But with the torsion localized at the corners, the motion in stretching becomes that of twisting rather than bending at the joints. So, the torsion that is localized at the corners spreads itself out in such a way that it becomes a uniform property of the spring. The key difference between the polygonal models (Figure 6h,i) and the zigzag models (Figure 6e,f) is that the thing that bends has to change directions in the latter so the bend cannot be spread out so as to occur continuously in the wire. When the wire is rotated into the horizontal plane, the bend is in the same relation to each piece, satisfying the constraint of distributing the springiness. That the distribution of the twist would be even is seen by extrapolating the polygon to the limit of a circle. Unlike the Maxwell case, no equations were derived, but if they were to be, the final square model would provide the equation that the amount of the torsion force and bending are functions of the length of the segments.

5.1 *Discussion*

Clement's protocols afford more information on the kinds of things going on during the reasoning process than the historical records, which, of course, were not provided for the same purpose. S2 often speaks of visualizing models, draws several schematic representations, and uses gestures to simulate dynamical processes to be performed with the models and to provide a three dimensional perspective. I believe protocol records of this sort, can provide supplementary support for claims about reasoning processes made in the historical cases. We no longer have access to Maxwell, but the traces he left in the historical record parallel those in protocol records.

S2's constructive modeling process consisted of integrating constraints drawn from geometry and from the domains of springs and bending rods into a series of models. Throughout the process, as Clement has argued, he critically evaluated the plausibility of the models and the inferences he drew from them. This process enabled S2 to derive a new understanding that a spring maintains constant slope when stretched through both twisting and bending.

We can see that generic modeling played a significant role in this process. For example, S2 recognized that a spring of any size and shape would have constant slope on stretching under a weight, and, most importantly, that there are not any salient differences when considering the behavior of a coiled spring between a polygonal shaped coil and a circular one.

Clearly, as with the Maxwell case, S2's constructive modeling process is generative. The visually evident non-constant slope of the bending rod model cued S2 about the additional constraint that the stretched spring has a constant slope. The two zigzag models directed him to focus on the circular nature of the spring's coils and that they lie in the horizontal plane. And, most critically, simulating bending in the horizontally rotated segmented hexagonal coil model led him to recognize that there is an invisible twist distributed along the coils of a spring, keeping the slope constant when it is stretched.

6. CONCLUSION: IMPLICATIONS FOR LEARNING

What relevance does the constructive modeling practice of experts have for teaching students – most of whom will never become physicists – some physics? My general philosophical stance in science education is "constructivist". On a constructivist view of learning in order to learn physics, students need to be able to do physics. This does not mean that they must be able, e.g., to derive Maxwell's laws for themselves or that they will need to approach the level of expertise of a physicist. But, even to master basic Newtonian mechanics, they need to be able to engage in the real theoretical and experimental practices of physicists. By and large students get the message that solving physics problems is a process of searching for formulas. Yet, we have just seen that both in conceptual change and in problem solving within an existing framework, constructive modeling is what physicists do in order to understand a problem well enough to get to the formulas that represent the problem solution. The expert's understanding of the requisite mathematical relationships derives from their embodiment in the models constructed to represent the target problem situation. Though not all cases of informal modeling to be found in the expert/novice literature are of the constructive modeling variety, when viewed with a reinterpretive eye many cases will be seen to fall into that category. This is a significant problem-solving practice that experts possess and novices lack. Thus it is well worth exploring how this thinking practice can be made explicit and incorporated into teaching practices.

Why don't teachers of physics and writers of physics texts preach what they practice? This is a complicated issue and I will discuss only two aspects of it here. First, as noted already, the constructive practices of a scientist become tacit as she becomes expert. It will take considerable effort to render practice explicit and even more to figure out how to develop effective pedagogical strategies coupling it with the content to be learned in a domain. In rendering practice explicit, we must be careful not to follow in the footsteps of famous scientific methodologists, such as Bacon and Descartes, whose preachings were notoriously at variance with practice.

Second, the implicit – and often explicit – metatheoretical conception of the nature of a scientific theory and of scientific method informing teaching practices continues to derive from positivist philosophers' conceptions of these. On the positivist view, reasoning is a syntactical process, i.e., the application of a set of rules. On this view of a scientific theory, the equations and definitions constitute the complete description. Presentations of theories and of scientific method provided in physics texts are strongly influenced by these conceptions. A conceptual structure is a set of definitions, the proper formulation of a theory is as an axiomatic structure and reasoning with the theory is deductive, reasoning to the theory is inductive. One learns the concepts by rote and practices solving exemplary problems to learn the set of rules. Teachers often do present models to convey the content but do not show modeling as a scientific reasoning strategy.

What is wrong with the positivist formulations from the perspective of learning is that even if the precise formulation of a theory were linguistic/formulaic in format, this does not imply that the kinds of representations humans employ in thinking with the theory are primarily of this format or that reasoning during problem solving is solely carrying out logical operations on linguistic/formulaic objects. Both the historical and the cognitive science literatures indicate that the semantic process we have been discussing, constructive modeling, is highly productive of solving problems and widely employed by experts in physics. On the account provided here this is a process of abstracting and integrating constraints into successive models of the target problem.

One problem for pedagogy arises immediately. Success at constructive modeling requires sufficient domain knowledge. Experts understand the physical and mathematical constraints of a domain sufficiently well for them to function as recipes for constructing models. Initially, students do not have requisite knowledge of the constraints of the domain to construct workable models. And they do not know how to view the exemplars they are presented generically. They do, however, possess the basic cognitive capacities employed in constructive modeling: to make analogies, to create mental simulations, to perform idealization and generic abstraction, and this fact can be taken advantage of and cultivated in the domain of science.

Although the constructive modeling process cannot be formalized fully as a procedure, as the cases we examined show, the techniques it employs can be made explicit and specific applications can be evaluated as good or bad. Further, I fully realize that my analysis rests on material drawn from classical physics. There are further issues about how constructive modeling might function in problem solving in quantum physics. Constructive modeling does take place in that domain (See, e.g., Cartwright 1989) but there may be added difficulties in coming to understand how to view the informal problem representations that would make it a hard domain in which to first learn the practice. For example, the models employed as problem representations in this domain would be more functional than structural analogs. Such difficulties actually may provide a good argument for why classical physics should continue to be taught first even though we no longer believe it to provide the best representation of the world.

In concluding, I want to introduce for the purposes of furthering discussion some issues I believe need to be considered in developing a way of teaching constructive modeling and other physics practices.

- There is enough evidence that constructive modeling is practiced in other domains to support the claim that it is a domain independent process. However, it would be mistaken to approach teaching it in a decontextualized fashion, e.g., as logic is often taught. Rather, the practice should be introduced and skill in it developed in the context of solving real domain problems.
- The most difficult problems facing physics teachers are how to start engaging students in this practice, how to determine what scaffolding might need to be developed, and how to go about developing it. A successful approach will need to develop expertise in content and in processes in tandem. Clearly, models conveying the content will need to be devised at the same time modeling skills are being developed. This is a multidimensional problem, critical aspects of which are:
  - What is the knowledge students have when they arrive in the classroom?
  - What knowledge is presupposed for students to begin engaging in the practice? Specifically, what background knowledge and what skill knowledge is required?
  - How can we build this into the initial problem contexts (including laboratory) we provide them? How can we use their preexisting knowledge in this process?
- We can be encouraged in attacking these problems in that significant resources are available already. The literatures on conceptual change and on expert/novice reasoning provide information, in general terms, about the base line of students arriving in physics classes. The field of "qualitative physics" has been developing models based on untutored physical intuitions to reason about classical physical systems (See, e.g., Bobrow 1985). There is a body of research on situated reasoning and on group problem solving that can help in building meaningful problem contexts and problem-solving situations. The history of science provides a resource for salient analogies and thought experiments used both in constructing scientific representations and in conveying these to the community (a form of instructing). Some of these could be directly imported into problem models; others could serve as a basis from which to develop more appropriate present-day models.
- The potential of computer simulations to help students develop facility with generic modeling needs exploration. Computer simulations, such as the "thinker tools" system developed by Barbara White (in press), might help students "encounter" the phenomena at a level of abstraction sufficient for understanding the generic nature of the models and, thus, how to transfer an understanding from one problem to another. Hands-on laboratory experiences could be supplemented by computer laboratories simulating the same phenomena at a level of abstraction interme-

diate between the real-world object and the scientific object. For example, unlike a commonsense object, a Newtonian object is a point mass moving through an idealized Euclidean space. Computer simulations of Newtonian objects have the potential to facilitate students recognizing both how a point mass can represent both a projectile and a planet and how the mathematical formulas represent point masses and real-world phenomena.

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