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OBJECTIVELY HOMOGENEOUS REFERENCE CLASSES

The statistical-relevance (S-R) model of scientific explanation involves homogeneous references classes.[1] A reference class A is homogeneous with respect to an attribute B provided there is no set of properties C_i in terms of which A can be relevantly partitioned. A partition of A by means of C_i is relevant with respect to B if, for some value of i, $P(A.C_i,B) \neq P(A,B)$. To say that a reference class is homogeneous with respect to an attribute does not mean merely that we do not know how to effect a relevant partition, or that there are practical obstacles to carrying out the partition. To say that a reference class is homogeneous – *objectively homogeneous* for emphasis – means that there is no way, even in principle, to effect the relevant partition.

There are two cases in which homogeneity obtains trivially, namely, if all A are B or if no A are B. This follows from an obvious logical truism. We shall not be interested in trivial homogeneity.

In the non-trivial cases, some restrictions must be imposed upon the types of partitions that are to be admitted; otherwise, the concept of homogeneity becomes vacuous in all but the trivial cases. Suppose that $P(A,B) = ½$. Let $C_1 = B$ and $C_2 = B$. Then $P(A.C_1,B) = 1$ and $P(A.C_2,B) = 0$; thereby a relevant partition has been achieved.

The problem of ruling out unsuitable partitions is precisely the problem Richard von Mises faced in attempting to characterize his 'collectives.' ([15], chap. I; [16], chap. I) A collective, it will be recalled, is an infinite sequence $x_1, x_2, x_3, \ldots$ in which some attribute B occurs with a relative frequency which converges to a limiting value p. Furthermore, the sequence must be *random* in the sense that the limiting frequency of B in any subsequence selected from the main sequence by means of a 'place selection' must have the same value p. This is the principle of insensitivity of the probability to place selections; it is also the principle of the impossibility of a gambling system. Roughly speaking, a place selection must determine whether a member of the main sequence belongs to the subsequence without reference to whether the element in question has or lacks the attribute B. There are two types of place selections: (1) selections which determine membership in the

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subsequence entirely on the basis of the ordinal position of the element in the original sequence – e.g., every third element, or every element whose place corresponds with a prime number – and (2) selections which determine the membership of the subsequence at least partly on the basis of attributes of members of the main sequence which precede the element in question – e.g., every element that immediately follows two tails in succession in a sequence of coin tosses.

This definition has been challenged, on the ground that the concept of the collective, thus defined, is empty (except in the trivial cases). Given any sequence of elements A, each of which either has the attribute B or lacks it, there exists a real number between zero and one whose binary representation contains a '1' wherever the attribute B occurs and a '0' wherever B is absent in the sequence. This real number could thus furnish a place selection which would pick out every element in the sequence which has the attribute B and reject all which are not B. The original sequence would not be a collective, for we have shown that there exists a place selection with respect to which the limiting frequency of B is not insensitive. The fact that we have no way of knowing in advance which real number would furnish such a place selection – for a given sequence of coin tosses, for example – is irrelevant. As far as von Mises's original definition of the collective is concerned, all that matters is the existence of such a place selection.

An answer to this objection, which von Mises enthusiastically endorsed, was provided by Abraham Wald (see [16], pp. 39–43). It runs as follows, Given the obvious limitations of standard mathematical languages, at most a denumerable infinity of rules can be formulated in any particular language. If we limit the class of place selections to those which can be represented by explicit rules, then at most a small subset of the real numbers between zero and one can correspond to actually formulated place selections. If such a restriction to a denumerable set of place selections is imposed, the existence of collectives is demonstrable.

This resolution of the difficulty is open to serious objection. Just as we must carefully distinguish between numbers and numerals (names of numbers) – noting that there is a superdenumerable infinity of real numbers but only a denumerable infinity of numerals – so also must we distinguish between the superdenumerable infinity of place selections that exist abstractly and the denumerable infinity of linguistic entities (rules or recipes) that represent them. Thus, if the definition of 'collective' rests upon the invariance of the limiting frequency with respect to the set of all place selections that exist, regardless of whether they are represented by explicitly formulated rules or not, then the concept of the collective remains empty. If, on the other hand, the collective is defined by reference to a set of formulated (or formulable) rules for effecting place selections, then the associated definition of 'randomness' is relativized to a particular language in which the rules are to be formulated. The consequence is that a sequence which qualifies as a collective with respect to one language may fail to qualify as a collective with respect to another.

In view of this consideration, Carl G. Hempel[2] has rightly challenged my use of a concept of homogeneity explicated in terms of von Mises's notion of a place selection ([12], pp. 42–45). His account of inductive–statistical (I-S) explanation makes use, in effect, of a concept of homogeneity of reference classes which is relativized to a knowledge situation. It is worth noting, in passing, that certain theories of probability – e.g., Henry Kyburg's (see [6], chap. 8) – embody a concept of randomness that is likewise relativized to a knowledge situation.[3] The statistical-relevance model, as heretofore presented, involves a concept of homogeneity defined in terms of von Mises's place selections; as such, it is relativized to a particular language. While I think that relativization to a language – which might be construed as involving an entire conceptual framework – is preferable to relativization to a highly ephemeral knowledge situation, I am not really content with either type of relativization. We need, it seems to me, a reasonable concept of homogenity (or randomness) according to which a given reference class is *objectively homogeneous* with respect to the occurrence of a given attribute, quite independently of either the specific knowledge situation or any particular language.

It was evidently in response to the issue of language relativization, as well as concern about the Richard paradox, that Alonzo Church [2] offered a refinement of the concept of the collective. Instead of defining place selections in terms of the rules that can be explicitly formulated in a given language, Church proposed to restrict them to selections given in terms of 'effectively calculable' functions. As Church has defined this term, a function is effectively calculable if and only if it is λ-definable or (equivalently) it is general recursive. This concept has been shown by A. M. Turing [14] to be equivalent to computability on a Turing machine. Let us call such selections *Church place selections.*

As Church has pointed out, his definition of 'random sequence' has several advantages over various alternatives. (1) In contrast to von Mises's original definition, this one provides a concept that is demonstrably nonvacuous. It can be shown (see [2]) that, for each real number p $(0 \leqslant p \leqslant 1)$, there exists an uncountable infinity of random sequences in which the limit of the frequency of 1's has that value p. (2) Less stringent definitions, such as Copeland's admissible numbers [4] or Reichenbach's normal sequences [9], are too broad to serve as general concepts of randomness. According to these definitions, a sequence (in which the limit of the frequency of 0 does not vanish) could qualify as normal even if it had a 1 at each place corresponding to a prime number. Such concepts, though useful in certain contexts, are not sufficiently restrictive for the definition of 'homogeneity.' (3) The class of effectively calculable functions is well-defined independently of any arbitrary choice of language. Church's random sequences have the property of randomness objectively, without relativization to any language or any knowledge situation. (4) Quite apart from other considerations, it seems entirely reasonable to insist that place selections be defined in terms of effectively calculable functions. If someone directs us to select a subsequence from a probability sequence, it does not seem excessive to demand that there exist, in principle, some method (algorithm) by means of which it is possible to determine which elements of the original sequence belong in the selected subsequence and which ones do not. (5) It is worth adding a further consideration to those mentioned by Church. As long as we confine our attention to infinite sequences, Church's definition of randomness is equivalent to those recently formulated using the Kolmogorov-Chaitin concept of computational complexity ([5], p. 120).

Church's definition of 'random sequence' does, of course, involve 'Church's thesis' – the thesis that effective calculability coincides with the triad of (mutually equivalent) properties: λ-definability, general recursiveness, and Turing computability. While I realize that Church's thesis may be disputed, I am not aware of any reason for calling it into question in the present context.

The von Mises definition of randomness, even when modified so as to employ Church place selections, is not altogether without its problems. These have been reviewed by P. Martin-Löf in [8]. Additional work by Martin-Löf [7] and C. P. Schnorr [13] appears to have overcome the difficulties. The subsequent developments are within the spirit of von Mises and Church; the requirement of effectiveness, with its language-independence, is retained throughout. In his survey article [3], J. A. Coffa sums it up: "The end-result of this process seems to be a successful explication of a concept [of randomness] useful to the theoretically-minded statistician" (p. 107). If that goal has not been fully achieved, we can at least take comfort in the fact that the job is in very good hands.

The task of defining homogeneity is not yet finished, for this concept has an empirical as well as a mathematical aspect. As Coffa remarks in his very next sentence, "The question remains whether it [the new concept of randomness] relates in any interesting way to that of physical randomness" (*ibid.*). While I am satisfied that Church's earlier work – augmented by the more recent work of Martin-Löf, Schnorr, and others – has provided the means to deal with the mathematical aspect, we must say something about the empirical side. For this purpose, let us begin by considering a fanciful example.

> Example (1): Suppose that I possess a 'magic penny.' Whenever I toss it immediately before the turn of a particular roulette wheel, it enables me to predict the result of the play – if the penny lands heads up, the outcome of the play is red; if the penny lands tails up, the outcome is black.

Under these circumstances, the class of turns of that particular roulette wheel would not constitute a homogeneous reference class with respect to the attribute red/black. In order to define homogeneity, we must rule out the existence of devices like the magic penny, as well as more commonplace objects which achieve the same type of result.

Suppose we are given a reference class A consisting of an infinite sequence of events $x_1, x_2, \ldots$. Any other class D consisting of an infinite sequence of events $y_1, y_2, \ldots$ will be called an *associated sequence* provided only that each $y_i \in D$ occurs in the absolute past (in the past light cone) of the corresponding $x_i \in A$. While this requirement rules out the possibility that x_i and y_i are one and the same event, it does not exclude such possibilities as that $y_i = x_{i-1}$.

As usual, let B be an attribute class whose probability within sequence A concerns us. Let C be an attribute class to which the members of D may be meaningfully assigned – i.e., '$y_i \in C$' and '$y_i \notin C$' are meaningful expressions. We want to use C to define a *selection by an associated sequence.* We shall define a selection S by means of the associated sequence D by stipulating that

$$x_i \in S \quad \textit{iff} \quad y_i \in C.$$

We shall then say that A is not homogeneous with respect to B if there exists a selection by an associated sequence such that the probability of B within $A.S$ is not equal to the probability of B within A – in other words, the reference class A is homogeneous with respect to B only if the occurrence of C within D is statistically irrelevant to the occurrence of B within A. This, in turn, is tantamount to the requirement that the sequence of B's within A be statistically independent of the sequence of C's within D. In order to avoid making the concept of homogeneity vacuous (except in the trivial cases), it is obviously necessary, however, to impose certain restrictions upon the class C, or equivalently, upon the properties which determine the membership of C. Coffa calls attention to this problem [above] in his discussion of 'physical properties.'

Two restrictions, one on the sequences A and D, the other on the attributes B and C, seem obvious. First, the sequences A and D must not be identical – i.e., we must satisfy the condition (already imposed) that $x_i \neq y_i$, which is assured by the requirement that y_i be located in the absolute past of x_i. Perhaps this requirement seems stronger than needed; it might seem sufficient to stipulate that the events x_i and y_i be spatio-temporally disjoint. But we shall find reasons to stick with the requirement of temporal priority of y_i to x_i. Second, B and C must be logically independent of one another. These two requirements, together or separately, rule out several types of undesirable cases.

Example (2): As in the example mentioned at the outset of this paper, let A and D be precisely the same sequence of coin tosses ($x_i = y_i$), and let C be identical with the attribute B (heads).

Example (3): A is a sequence of coin tosses and D is the same as A except that, for each $i, y_i = x_{i-1}$.[4] B is the attribute of landing heads up; C is the attribute of being a toss preceding a toss which lands heads up.

Example (4): A is a sequence of draws of balls from an urn, and D is precisely the same sequence of draws – again, $x_i = y_i$. B if the attribute of being red; C is the attribute of having a color at the opposite end of the visible spectrum from violet. In this example, I am assuming that it is *not* a logical truth – but rather, a contingent regularity – that red is at the opposite end of the visible spectrum from violet.

Example (5): A is a sequence of weather conditions on successive days in a particular city, while D is a sequence of forecasts of weather, made on the preceding day, for that city. B is the occurrence of a storm; C is a reliable prediction of a storm. For purposes of this example, a *reliable* prediction is defined as a prediction which comes true. B and C are, consequently, not logically independent.

It is obvious that in each of the foregoing examples C must be disqualified as an attribute defining a selection by an associated sequence, for in none of these cases does the relevance of C to B have any genuine bearing on the question of whether A is homogeneous with respect to B.

The foregoing restrictions are not, however, sufficient. Consider a further example:

Example (6): Let A be the sequence of days, and let B be days in which there is a fatal accident in a certain town, Centerville. Let us assume that Centerville has a daily paper, the *Centerville Gazette*, which reports with reasonable accuracy on the fatal accidents which occur in that town on the preceding day. Note that in this example – in contrast to (5) – the concept *report of a fatal accident* is logically independent of the concept *occurrence of a fatal accident*, for neither logically entails the other. Let D be the sequence of daily editions of the paper, and let the attribute C be the property of *carrying a dateline two days earlier than that of an issue containing a report of a fatal accident.*

In this example the sequences A and D are distinct, and for every value of i, y_i is in the absolute past of x_i. Moreover the attributes B and C are logically independent of one another. Nevertheless, we would not want to say that a selection S based upon property C has any bearing on the homogeneity of A with respect to B. The reason is that, although strictly speaking C applies to events which occur before the associated members of A, it is essentially defined in terms of events which occur after the fact – namely, newspaper reports of fatal accidents.

If we knew which members of D had the attribute C we would be able to use it to make a relevant partition in A with respect to fatal accidents in Centerville. We cannot, of course, use this partition for purposes of prediction, for we cannot know which members of D do possess attribute C until it is too late to make predictions. But knowing and predicting are epistemic concepts, and we are attempting to provide an objective, non-epistemic concept of homogeneity. We must therefore find non-epistemic grounds on which to block such attributes from providing selections by associated sequences.

It is worth recalling that an important philosophical issue hinges on these considerations. One of the basic reasons for worrying about objectively homogeneous reference classes is to try to make sense of the concept of indeterminism. If indeterminism is true it seems hard to avoid the consequence that there exist, in fact, objectively homogeneous reference classes. If, conversely, there are objectively homogeneous reference classes of an appropriate sort, they would seem to provide a reasonable concept to employ is giving an explication of indeterminism.

In view of the importance of the relation between objective homogeneity and indeterminism, let us consider a further example:

Example (7): Suppose that we have a sample of some radioactive material; this sample consists of atoms of one particular isotope of one particular element. A Geiger counter is so situated as to detect any radioactive decay that occurs in that sample. Assume, further, that this detector is connected with a tape recorder which records a 'click' on a magnetic tape whenever the counter detects a decay-event. Assume also that the speed of the tape across the recording head is one cm/sec. Now let A be the sequence of seconds during which this set-up is in operation, and let B be the class of seconds during which a decay-event occurs. Let us stipulate that the sample is small enough and the nuclei are stable enough to have a non-vanishing probability of seconds during which no decay occurs in the sample. Let D be the sequence of centimeter-long segments on the tape; some of the members of D will have 'clicks' recorded on them and others will not. We do not need to assume that the detector is perfectly reliable, or that the recorder is faultless; it is sufficient to assume a fair degree of reliability in each case. Now, using a definition quite parallel to that employed in example (6), we define C as the class of centimeter-long segments which immediately precede those segments which contain a recorded 'click' (see Figure 1). While there is some delay between the occurrence of a decay and its recording on the tape, we can assume that it is negligible in comparison with the second-long durations we are considering. In this example, as in (6), each y_i is in the absolute past the corresponding x_i, and the attributes B and C are logically independent of one another.

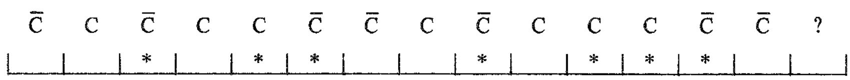

Fig. 1. (An asterisk in a segment indicates a 'click'.)

Let us now ask whether it makes sense to say that the spontaneous radioactive disintegrations are genuinely undetermined events (as many contemporary physicists and philosophers would maintain). It is evident, I believe, that the existence of fairly reliable records of the times of the decays, which can be examined subsequent to the occurrences, has no bearing upon the determinism-indeterminism issue. The attribute C is *defined* on the basis of the subsequent records, but it *applies* to segments of the type which passed through the recorder prior to the decay-events in question. Nevertheless, the fact that it effects a relevant partition of the reference class A does not show either that indeterminism is false or that A is actually inhomogeneous in any sense which is pertinent to the issues we are discussing.

How can such attributes be blocked? In a somewhat similar context Reichenbach says that the classes we use must be 'codefined.' where "we say that class A is *codefined* if it is possible to classify an event x as belonging to A coincidentally with the occurrence of x .... observing x we must be able to say whether x belongs to A, and it must be unnecessary to know, for purposes of this classification, whether certain other events $y, z, \ldots$ occurred earlier or later, or simultaneously at distant places" ([10], p. 187). Informally, Reichenbach's intent seems plain enough. He elaborates: "Logically speaking, a codefined class term is a one-place predicate which is not contracted from many term predicates" (*ibid.*).

Reichenbach's remarks about codefined classes can hardly be taken to provide a precise characterization of that concept. First, those who have worried about Goodman's predicates, 'grue' and 'bleen,' will naturally wonder whether they enjoy the status of uncontracted one-place predicates. It seems to me that a negative answer to that question can be established, for it follows from a resolution of Goodman's puzzle which I have offered elsewhere [11]. I shall not attempt to reargue that issue here. Second, Reichenbach seems to suggest that codefined classes must be determined by directly observational properties. Many philosophers, nowadays, question the very existence of properties which are purely observational, and many deny the viability of any sharp distinction between the observational and the theoretical. It seems to me that, whatever stance one adopts on these issues, there is no reason to exclude from the realm of codefined classes those which are determined by theoretical properties. The class of spontaneous radioactive decays is highly theoretical, but it should qualify, I believe, as codefined. What seems essential to the concept of a codefined class is that the inclusion or exclusion of events should be determined by the spatio-temporally local characteristics of the events involved. It may not be totally clear, however, just how the locality of *characteristics* is to be defined.

I shall assume that the concept of the location of an *event* in space-time is clear enough. We know how to delineate approximately the space-time region in which a toss of a coin, a thunderstorm, a radiocative decay, or a supernova explosion occurs. Thus, the events x and y which constitute the membership of classes A and D are taken to have definite space-time locations (at least to a reasonable approximation). Reichenbach then goes on to stipulate that the *classes* A, B, C, D be codefined. Roughly speaking, this means that it is possible in principle to ascertain whether a given event x belongs to one of these classes by examining the space-time region in which x occurs. This rough characterization suffers, however, from the fact that it is framed in epistemic terms. We want to say, non-epistemically, that the membership of x in any of these classes is objectively determined by facts which obtain within the space-time region in which x occurs. But this statement does not help us much, since it seems, for example, to be a fact about the issue of the *Centerville Gazette* which appears on a particular day that it carries a dateline two days earlier than the dateline on an issue containing the report of a fatal accident.

There is an approach which may help to clarify the situation. In examples (6) and (7) above, we were dealing with pairs of events x_i, y_i where y_i is in the past light cone of x_i. In both examples, a third event z_i in the future light cone of x_i was invoked to define the class C Reference to the event z_i was the source of all the trouble. It is tempting to say that membership of y_i in C must not depend upon any characteristics of z_i, but following this approach will only get us back into trouble again over the need to specify the *kind* or characteristic of z_i to which such a restriction is supposed to appeal. Let us rather attempt to frame our restriction in terms of the occurrence or non-occurrence of z_i. To begin, let us agree to construe a statement of the form '$y_i \in C$ if and only if $z_i \in F$' along the following lines:

> '$y_i \in C$' is true if z_i occurs and $z_i \in F$;
> '$y_i \in C$' is false if z_i occurs and $z_i \notin F$;
> '$y_i \in C$' is indeterminate with respect to truth value if z_i does not occur.

With this understanding in mind, we can proceed to a definition of *selection by an associated sequence.*[5] First, we recall our earlier definition:

DEFINITION 1. Let A be a reference class consisting of an infinite sequence of events $x_1, x_2, \ldots$. Any other infinite sequence D consisting of events y_1, $y_2, \ldots$ will be called an *associated sequence* if each event y_i occurs in the absolute past (the past light cone) of the corresponding event x_i.

Then, we propose the following definition:

DEFINITION 2. A *selection by an associated sequence* is any selection S within A defined by the rule,

$$x_i \in S \text{ iff } y_i \in C,$$

where the class C is defined in such a way that the classification of y_i with respect to C would remain unambiguous even if the event x_i failed to occur, or if any event z_i in the future light cone of x_i should fail to occur.

The motivation for this definition is the need to impose, as a condition on the homogeneity of a reference class, a requirement of invariance of the limiting frequency of the attribute under any such selection by an associated sequence. Notice that this definition implies both of the obvious restrictions mentioned earlier. Since y_i must be in the past light cone of x_i, x_i cannot be identical to y_i. Since y_i must retain its unambiguous classification with respect to C even if x_i were to fail to occur, the properties defining C must be logically independent of those defining B.

Let us see how this definition applies to some examples. In the fanciful example (1) of the magic penny, the members of the sequence $x_1, x_2, \ldots$ (members of A) are the turns of the roulette wheel, and the attribute B is the outcome black. The members of the sequence $y_1, y_2, \ldots$ (members of D) are the tosses of the 'magic penny,' C is the outcome tails. The probability of B within A is, let us assume (ignoring the 0 and 00 sectors of the wheel), one-half. If, however, we select a subsequence S of turns of the wheel immediately following tosses of the coin resulting in tails, the probability of B within $A.S$ is one. S qualifies as a selection by an associated sequence and A is patently inhomogeneous with respect to B.

In example (6) above, the sequence $x_1, x_2, \ldots$ consisted of a sequence of days (beginning, let us say, with the day after the first day of publication of the *Gazette*); the associated sequence $y_1, y_2, \ldots$ consisted of the daily issues of the paper. The attribute class B was the class of days on which a fatal accident occurred in Centerville. The attribute C was the attribute of carrying a dateline two days earlier than the dateline of an issue in which a fatal accident was reported. If C is taken as a basis for a selection S, then the probability of B within $A.S$ (days immediately preceding issues of the *Gazette* in which fatal accidents in Centerville are reported) is much higher than the probability of B within A; the probability of B is not invariant with respect to the selection S. The selection S does not, however, qualify as a selection by an associated sequence as defined above, for membership in C would become indeterminate if the *Gazette* suspended publication the following day. The existence of a selection such as S in this case does not render the reference class A inhomogeneous with respect to B (although we presume that there are other grounds for regarding A as inhomogeneous with respect to the occurrence of fatal accidents).

Drawing all of the foregoing considerations together, let us now attempt to formulate an adequate definition of *objectively homogeneous reference class.*[6]

DEFINITION 3. A reference class A is *objectively homogeneous* with respect to an attribute B iff the sequence of B's within A is mathematically random, and the probability of B within A is invariant under selections by associated sequences.

In characterizing a sequence as mathematically random, I mean roughly that it is invariant under Church place selections, or more precisely, that it is random in the technical sense developed by Martin-Löf [7] or Schnorr [13].[7]

This definition can be illustrated by further examples; consider one which resembles the magic penny (1) but which is less frivolous. It is, in principle, similar to instances treated in scientific contexts.

> Example (8): Two marksmen fire at the same target; one of them T is a tyro, the other E is an expert. A large percentage of E's shots hit the bull's-eye, while a large fraction of T's shots are wide of the mark. Assume, moreover, that they do not fire in regular alternation, but quite irregularly. The total class of shots striking the target (from T or E) is not homogeneous with respect to the attribute B of hitting the bull's-eye. The class D of shots fired (they are fired before they hit the target) can be divided into those fired by T and those fired by E. If the latter attribute E is used to effect a selection S in the class A of shots striking the target, the probability of hits on the bull's-eye will be different in $A.S$ than it is in the entire class A.

We may assume, because of the irregularity with which T and E fire, that the sequence of hits on the bull's eye is *mathematically random*. The sequence of firings obviously fulfills the conditions for an *associated sequence* with respect to the sequence of hits on the target. The selection S defined by E is a bona fide *selection by an associated sequence*. Since the probability of a hit on the bull's-eye is not invariant under this selection, the reference class A of shots striking the target is not *objectively homogeneous* with respect to the attribute B.

It is easy to think up commonplace examples of reference classes which fail to be genuinely homogeneous because of the possibility of making selections on the basis of some sort of associated sequence. The class of tosses of a coin is seen to be inhomogeneous if we take note of the possibility of using the state of the coin immediately prior to its landing for purposes of making such a selection. The same kind of consideration applies to the roulette wheel; as the wheel slows down and the ball is just about to fall it is possible to predict the outcome with some reliability. Most of us believe it is possible in principle, though perhaps technically impossible at present, to predict with high reliability which victims of latent untreated syphilis will develop paresis and which ones will not.

Are there, in fact, any objectively homogeneous (non-trivial) reference classes? No one knows for sure, but there seems to be a strong possibility that cases similar to example (7), which exist in the quantum domain, embody objective homogeneity. Given a collection of heavy atoms, we can, in principle, sort them into different elements, and into different isotopes of these elements. Some of the isotopes are stable; others have half-lives ranging from billions of years down to tiny fractions of a second. Thus, the original collection is highly inhomogeneous with respect to the occurrence of spontaneous radioactive decay within a specified time span. If, however, we select only those atoms which belong to one isotope, say U^{238}, there is, to the best of our physical knowledge, no further partition which is relevant to the occurrence of spontaneous decay. There are, moreover, theoretical reasons for supposing that no as-yet-unknown property possessed by the nuclei prior to decay is relevant to spontaneous decay. If this is true, the physical world does, indeed, contain objectively homogeneous reference classes. It is my hope that we can at least assign a reasonable meaning to such a statement, whether it happens to be true or false.

The assertion that there are objectively homogeneous reference classes in the physical world is sometimes expressed by saying that certain types of physical laws are irreducibly statistical. I hope that the explication of 'objective homogeneity' given above helps to clarify the phrase 'irreducibly statistical law.' These concepts may, in turn, help us to say precisely what we mean by the terms 'determinism' and 'indeterminism.'

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NOTES

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[1] This model is discussed most fully in [12].

[2] Private correspondence.

[3] This seems to be a common feature of theories of logical probability. Cf. Carnap [1], pp. 493–495.

[4] The fact that D contains no element y_1 is of no real consequence.

[5] I shall not be using Reichenbach's concept of a codefined class in the present context, although it may well be useful in other contexts. On the basis of the foregoing stipulation it could now be reformulated to read, "... we say that class C is codefined if it is possible in principle to classify an event x unambiguously with respect to membership in C regardless of the occurrence or non-occurrence of any events $y, z, \ldots$ at any places or times outside of the space-time region of the occurrence of x itself."

[6] In this definition we are regarding reference classes as *ordered* sequences of members. In most physical applications some natural ordering (e.g., temporal) is given, but when that is not the case, some arbitrary fixed order can be imposed in advance.

[7] (Added in proof) Mr. Glenn Ross has pointed out that the attribute C must occur in a mathematically random manner in D if C is to be used to produce a selection by an associated sequence.

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