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FORMAL SYSTEMS OF DIALOGUE RULES

ABSTRACT. Section 1 contains a survey of options in constructing a formal system of dialogue rules. The distinction between *material* and *formal* systems is discussed (section 1.1). It is stressed that the material systems are, in several senses, formal as well. In section 1.2 variants as to language form (choices of logical constants and logical rules) are pointed out. Section 1.3 is concerned with options as to initial positions and the permissibility of attacks on elementary statements. The problem of ending a dialogue, and of infinite dialogues, is treated in section 1.4. Other options, e.g., as to the number of attacks allowed with respect to each statement, are listed in section 1.5. Section 1.6 explains the concept of a 'chain of arguments'.

From section 2 onward four types of dialectic systems are picked out for closer study: *D*, *E*, *Di* and *Ei*. After a preliminary section on dialogue sequents and winning strategies, the equivalence of derivability in intuitionistic logic and the existence of a winning strategy (for the Proponent) on the strength of *Ei* is shown by simple inductive proofs.

Section 3 contains a – relatively quick – proof of the equivalence of the four systems. It follows that each of them yields intuitionistic logic.

Since dialogue theory re-entered modern logic,[1] quite a number of systems called 'dialogue games' or 'dialectic systems' have been published. I shall attempt to point out and briefly discuss some of the main points of divergence between extant types of system (section 1). This list will not be complete for several reasons. One, I shall restrict the survey to such systems as appear in, or are related to, the writings of German constructivists.[2] Accordingly, I shall leave out of account those dialectic systems (or, systems of formal dialectics) that do not operate with the concepts of 'winning' and 'losing'. Also, I shall, after section 1.1, discuss only those systems that are strictly formal in a sense to be explained below.

Next, I shall in particular discuss four systems that each yield a constructive (i.e., intuitionistic) logic: *D*, *E*, *Di* and *Ei*.[3] The equivalence of one of these systems, *Ei*, with other (nondialogical) systems for constructive logic will be demonstrated in section 2. Finally, in section 3, each of the other three systems will be shown to be equivalent to *Ei*.

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1. A SURVEY OF SOME OPTIONS IN CONSTRUCTING A FORMAL SYSTEM OF DIALOGUE RULES

In this survey I shall merely list, and briefly explain, a number of alternatives. Each theoretician engaged in the construction of a formal system of dialogue rules will have to deal with these alternatives. He must either select and defend one of them or, for good reasons, propose yet another. The reasons adduced for particular choices are quite diverse, since they depend on the particular ends that the author of a system of dialogue rules may have in view. Thus different philosophical foundations may obviously lead to different systems. Surprisingly, quite different foundations may also lead to the same system or to very similar systems. In the following I shall hardly touch upon foundational matters: the alternatives will be described, not motivated.[4]

In section 1.1 I shall discuss the differences between 'material' and 'formal' systems of dialogue rules. It will be stressed that the material systems are, in several senses, formal as well. Next, variants as to language form (choices of logical constants and logical rules) will be briefly pointed out (section 1.2). Section 1.3 is concerned with options as to initial positions and the permissibility of attacks on elementary statements, whereas the problem of ending a dialogue and of infinite dialogues is treated in section 1.4. Choices as to (other) structural rules, mainly concerned with repetitive behavior (the bugbear of dialogue theory), will be listed (but not motivated) in section 1.5. At the end it will be briefly explained how 'dialogues' can be incorporated in larger units of discourse as the 'chains of arguments' out of which these larger units consist (section 1.6).

1.1. *Material versus Formal Dialogues*

These are some senses in which all the systems of dialogue rules, whether they are called 'formal', 'semiformal' (*halbformal*) or 'material', are formal. First, they are all based upon P. Lorenzen's strip rules for logical constants (see section 1.2) and these strip rules are formal in the sense that attacks and defenses are described in terms of the syntactic forms of the sentences involved. Second, all systems are formal in the sense that they consist of rules for the regulation of debates, thus bestowing a definite structure or form upon them.[5] Third, the term 'formal' may refer to the normative (versus the descriptive) aspect of these systems.[6]

A dialogue game (dialectic system) is characterized as *material* by the inclusion of rules that authorize material moves. These are such moves as depend on the content of some nonlogical constant: e.g., pointing out the truth, or falsity of an elementary statement. Consequently, the language employed in material dialogues must be at least partially interpreted. The dialogue games that do not incorporate such rules and moves are called 'formal' (or, sometimes, 'semiformal', if their strategies contain infinite branchings). Note that the material dialogue games are just as formal as the 'formal' ones in any of the other senses of the word 'formal'.

In the writings of P. Lorenzen and K. Lorenz the material dialogues clearly have priority over the formal (i.e., nonmaterial) ones. Not only are the material dialogues introduced before the formal ones in most texts (if the latter are treated at all),[7] but they also constitute the locus where the logical constants are introduced. Systems of rules for formal dialogues are then used to reconstruct logical notions, such as 'validity' or 'logical truth'.

For the latter purpose, however, one need not have recourse to formal (nonmaterial) dialogues or dialogue games at all. For, instead of saying that a sentence is logically true iff it can be upheld by the Proponent in debates following the rules of a certain formal dialogue game, one may introduce the concept of a *formal winning strategy* in a material dialogue game. A formal strategy, for a party *N*, is simply a strategy according to which *N* never makes any material moves, except for those moves copied from *N*'s adversary. One may then, equivalently define the class of logical truths (of a given language) as the class of sentences such that there is a formal winning strategy, for the Proponent of each of them, in a certain material dialogue game. Since the expedient of first defining formal *dialogues* and formal *games* is thus easily bypassed, the role of these dialogues in the expositions by P. Lorenzen and K. Lorenz is clearly of secondary importance.

On the other hand, from the standpoint of theory of argumentation and verbal conflict resolution the formal dialect systems constitute the more fundamental case from which material systems can be derived.[8] For, it is clear that even if a certain company (seeking an instrument for the verbal resolution of conflicts) does not agree about the truth value of any elementary sentence nor upon any procedure for attaining such an agreement – it may nevertheless be able to agree upon a formal (nonmaterial) system of rules for rigorous debate. In this situation systematic debate is still possible.[9] In the reverse situation – with agreement about some elementary sentences but lack of agreement about the nonmaterial rules – debate is impossible.

In the sections that follow I shall disregard material moves, i.e., the discussion will be restricted to those systems of dialogue rules that are fully 'formal' in all senses of the word hitherto considered.

1.2. *The Choice of a Language: Rules for Logical Constants*

There is no debate without language. Each complete system of rules for dialogues, therefore, must include specifications as to the language(s) that debates may employ, e.g., a propositional language, a first order language, a modal language, etc. The choices here concern the logical constants and the logical rules by which they are 'defined'. The actual symbolism used by the debaters may be left unspecified (in a moderately abstract treatment), but the logical constants must be listed, and for each constant, it must be resolved in what ways statements employing this constant as their principal operator can be attacked and defended.

Systems often differ with respect to either of the following two issues:

(i) Whether to have a propositional constant for *absurdity*. (This constant I shall denote by "$\wedge$".)

(ii) Whether to permit free individual variables in the sentences that occur in a dialogue, or, rather, to employ a separate infinite set of parameters or "constants".

The second issue is a minor (but sometimes confusing) one, since systems that differ only in this respect are easily seen to be equivalent. From now on I shall assume that we are dealing with a first order language (or its propositional or implicational sublanguage) with sentences, U, V, W . . . , individual variables, x, y, z . . . , parameters, a, b, c . . . (enumerated as b_1, b_2, b_3 . . . .) and logical constants $\rightarrow$, $\sim$, $\vee$, &, $\forall$, $\exists$, and (perhaps) $\wedge$. These logical constants are characterized by the following Table I of *logical rules* (Lorenzen's strip rules), that form the basis of (almost) all extant dialogue games:[10]

TABLE I

	Statement by Speaker 1	Attack by the Critic (Speaker 2)	Defense by Speaker 1	
$\text{Rule}_{\rightarrow}$	$U \rightarrow V$	U	V	
$\text{Rule}_{\sim}$	$\sim U$	U	none,	if $\wedge$ is not a logical constant of the language
			$\wedge$,	if $\wedge$ is a logical constant of the language
$\text{Rule}_{\vee}$	$U \vee V$	?	U / V	(Speaker 1 may choose either one)
$\text{Rule}_{\&}$	$U \& V$	L? / R? (the Critic may choose either one)	U	if the attack was L?
			V	if the attack was R?
$\text{Rule}_{\forall}$	$\forall xU$	a? (the Critic may choose any parameter)	$[a/x]U$	
$\text{Rule}_{\exists}$	$\exists xU$	?	$[a/x]U$	(Speaker 1 may choose any parameter)

1.3. *Initial Positions. Attacks on Elementary Statements*

Most systems assume that dialogues start from a situation where only one of the parties (the Proponent: P) has uttered a declarative sentence (the initial thesis: Z). The first move within the dialogue consists of an attack by the other party (the Opponent: O) on P's statement of Z. This attack is to take the form prescribed by the one and only logical rule applicable to Z. Hence Z is, usually, assumed to be a logically complex (nonelementary) sentence. Under these circumstances initial positions may be denoted by:

$$\emptyset /_O Z \ (Z \text{ complex}).$$

In this notation sentences stated by $P(O)$ appear on the right (left) of the slash. The subscript indicates which party is to make the next move. ("$\emptyset$" denotes the empty set.)

In the course of many a dialogue O, too, will make statements (in virtue of $R_{\rightarrow}$ and $R_{\sim}$, to begin with). These statements may then, later on, be attacked by P, etc. Thus in the course of a dialogue one may encounter positions of the following type:

$$\Pi/_O Z$$

Here Π stands for the set of sentences stated by O (hypotheses, concessions). Some authors admit positions of this type as initial positions. In that case, the set Π is called the set of initial hypotheses or initial concessions.

As long as there is no logical rule for elementary statements, dialogues cannot start with an elementary thesis, at least not if we stick to the principle that O makes the first move. But as soon as a rule that permits attacks by O on elementary statements is adopted, there remains no reason to shrink away from an elementary initial thesis. The rule is shown in Table II:

TABLE II

	Statement by P	Attack by O	Defense by P
Rule $_{At}$	U (elementary)	?	You said so yourself!, or *Ipse dixisti!*[11] (This defense is permissible only if U in fact occurs among O's concessions)

Since *Ipse dixisti!* is a "winning remark" that ends (at least a part of a) dialogue, the decision to admit elementary initial theses connects harmoniously with a particular way to end dialogues (see section 1.4: Terminal Rule II, type II systems).

Another way to incorporate elementary initial theses would be to let not O but P make the first move within the dialogue. The initial position then contains a right on P's side to state the initial thesis, but this statement is made *within* the dialogue as one of P's moves. It needn't be made as P's *first* move either, for P may prefer first to attack a complex initial concession (provided there is one).[12] Initial positions of this type will be denoted by

$$\Pi/_P[Z].$$

Here the brackets around "Z" serve to indicate that Z is not a sentence stated but a sentence that, as far as the rules of the game go, may be stated in the very next move.

To sum up, there are three common ways in which formal systems of dialogue rules differ as to their initial positions, for it must be decided:

(i) Whether initial concessions are admitted.
(ii) Whether an elementary initial thesis is admitted (and some rule like Rule_{At} incorporated in the system).
(iii) Whether O or P is to make the first move (and, accordingly whether the initial thesis counts as stated or as statable).

The three most common types of initial position are characterized as:

$$\emptyset/_O Z \ (Z \text{ complex}),\ \Pi/_O Z, \text{ and, } \Pi/_P[Z].$$

1.4. *Ending a Dialogue. Winning and Losing*

All formal systems of dialogue rules proposed by German constructivists, and all related systems, include some possibilities for P to win dialogues (and, correspondingly, for O to lose them). They do not always include possibilities for O to win a dialogue.[13] Sometimes infinite dialogues are admitted by a system and said to be "won by O".

If one wants to guarantee that each dialogue ends after a finite number of moves, special care should be taken to avoid repetitive behavior (section 1.5). Let us call such a system, that does not admit any infinite dialogues, *finitary*.[14] In finitary dialectic systems each dialogue ends either with P as the winner and O as the loser, or the other way around. (I do not know of any such system that admits draws.) There is a choice between two rules for determining who has won:

TERMINAL RULE I: If it is party N's turn to make a move, and no move is permitted by the other rules of the system, then N has lost, and its adversary has won, the dialogue. Moreover, if the language contains $\wedge$ and $\wedge$ appears among O's concessions, P has won, and O has lost, the dialogue.

TERMINAL RULE II: (i) As soon as P makes a "winning remark", the dialogue ends, and P has won it. But a "winning remark" constitutes a defense that can only be made if the statement to be defended (by P) occurs among O's concessions, or, given that the language contains $\wedge$, whenever $\wedge$ occurs among O's concessions. (ii) If it is P's turn to make a move, and no move is permitted by the rules of the system, then P has lost, and O has won, the dialogue.

The choices with respect to initial positions, admissibility of attacks (by O) on elementary statements, and terminal rules are not independent. Systems that do not admit attacks on elementary statements (by O) may be expected not to admit elementary initial theses either and to sustain Terminal Rule I. Moreover, such a system must incorporate a rule like $D1$ (ii) in order to avoid awkward results (see section 1.5, below). Let us call such systems type I systems. On the other hand systems that include Rule_{At} may be expected to admit elementary theses and to sustain Terminal Rule II. These systems will be called type II systems.[15]

By a P-liberalized variant of a finitary system σ we shall understand a simplified system, that is no longer finitary, but that is equivalent to σ as far as the set of initial positions for which there exist P-winning strategies is concerned. Thus, some or all of those positions for which an O-winning strategy existed in σ, will be positions for which not more than a O-no-loss-strategy exists in the P-liberalized system. The rules that enforce finite dialogues are rather cumbersome, if one wants to study the possibilities of P-winning strategies or to compare dialogue games with other logic systems. For this purpose the P-liberalized systems come in handy, even if the original motivation and philosophical foundation demands a finitary system.[16]

The different choices as to how to end dialogues are displayed in Figure 1.

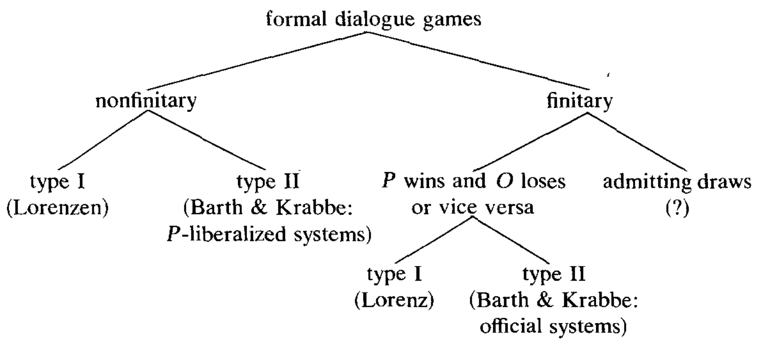

Fig. 1.

1.5. *How to Curb Repetitive Behavior: Structural Rules*

The most debated question in the theory of dialogues is the choice of structural rules (*Rahmenregeln*). How to justify a particular choice? On the one hand, rules are needed to limit unduly repetitive behavior (by either party if one wants to have a finitary system, but in any case repetitive behavior by O). On the other hand, the rules should be 'fair' and they should together constitute a well-founded dialectic system. I shall here do no more than list several alternatives, and briefly point out their more immediate effects. I shall not discuss foundational matters.[17] Rules pertaining to initial positions and ending a dialogue will be listed too. For convenience initial positions will be assumed to be of type $\Pi/_O Z$.

The following three rules seem to be common to (almost) all formal systems of dialogue rules:[18]

Do A dialogue consists of alternate moves by O and P.
Doo O makes the first move.
Dooo Each move is either an attack or a defense move according to the logical rules (including, in type II systems, Rule_{At} and winning remarks by P according to terminal Rule II).

The rules $D1$, and $D2$ characterize a system as being either of Type I or of Type II:

$D1$: (i) There are no attacks on elementary statements. Initial positions do not contain an elementary thesis. There are no winning remarks. Dialogues end according to Terminal Rule I.
(ii) P may state an elementary sentence only if it was stated before by O.

$D1,m$: Each of O's elementary statements may be copied by P at most m times

$D2$: O may attack elementary statements according to Rule_{At}. Initial positions may contain an elementary thesis. P may make winning remarks. Dialogues end according to Terminal Rule II.

The following rules serve to regulate the number of attacks on statements:[19]

*D*3,*n*,*m*: *O* may attack each statement of *P*'s at most *n* times; *P* may attack each statement of *O*'s at most *m* times.

*D*3: $(= D3, 1, \infty)$ *O* may attack each statement of *P*'s at most once. There are no restrictions of this kind for *P*.

There must also be rules to regulate the number of defenses answering an attack. Often the order of the defense moves is also prescribed:

*D*4,*n*,*m*: *O* may defend against an attack by *P* at most *n* times; *P* may defend against an attack by *O* at most *m* times.

*D*4: $(= D1, \infty)$ *O* may defend against an attack by *P* at most once. There are no restrictions of this kind for *P*.

*D*5: A defense move may answer only the latest attack by the adversary that has not yet been answered.

The following rule will be seen to be characteristic for *E*-systems:

*D*6: After the first move (*O*'s attack on the initial thesis), each further move by *O* consists of a reaction on the immediately preceding move by *P*.

In the next two sections I shall discuss the system types *E*, *D*, *Ei* and *Di*. These are types of nonfinitary systems that include the rules *D*3 and *D*5. *E* and *D*-systems are of type I and therefore, include rule *D*1. The initial positions according to these systems are of type $\Pi/_O Z$ (Z complex). (Often they are restricted to type $\emptyset/_O Z$, Z complex.) *Ei* and *Di*-systems are of type II and therefore include rule *D*2 (Initial positions: $\Pi/_O Z$). Finally *E* and *Ei*-systems are systems that include *D*6.

TABLE III

	type I	type II
not including *D*6	*D*	*Di*
including *D*6	*E*	*Ei*

All these systems include *D*3 and *D*5.

The logic these systems yield, i.e., the set of sequents Π/Z such that there is a winning strategy (for P) for the position $\Pi/_O Z$, is the same in all four cases: it is constructive (intuitionistic) logic, as will be shown presently. (Of course Π/Z with Z elementary must be disregarded if we are concerned with type I systems.)[20]

Other logics are obtained by varying the rules; thus to obtain a system that yields classical logic it suffices to omit *D*5 from the set of rules of *Ei* or *E*. (By virtue of *D*6, *D*4 then holds.) A system that yields minimal logic is obtained (from *Ei* or *E*) in either of two ways:[21]

(i) If the language contains a propositional *absurdum* constant ($\wedge$), omit the possibility of making a winning remark on account of $\wedge$ from the terminal rule.

(ii) If the language does not contain $\wedge$, add the following rule:

*D*7: P may attack a negation stated by O only if the last statement attacked by O was also a negation.

The "bounded systems", i.e., systems with rules *D*3,n,m, *D*4,n,m (for finite n, m) yield different logics too. We shall return to them briefly at the end of this paper.

1.6. *Chains of Arguments*

Up to now we have assumed that each tournament according to a dialectic system consists of one and only one dialogue. It is not hard to imagine a discussion that runs through several such dialogues, or *chains of arguments*, as we shall say.[22] This is what happens if we allow either party to retract a statement and to return to some earlier position in the chain of arguments. A discussion is then some complex structure consisting of several chains of arguments intertwined. Thus a more realistic dialogue theory is obtained.

For the existence of winning strategies it makes no difference whether we allow the debaters (within certain limits) to retract their statements. (If you possess a winning strategy for a game you may let your adversary have several tries!) Therefore I shall in the next two sections continue to equate dialogues with chains of arguments (rather than with more complex discussions).

2. A SYSTEM OF DIALOGUE RULES THAT YIELDS INTUITIONISTIC LOGIC

In this section I shall present a proof that the system *Ei*[23] yields constructive (intuitionistic) logic.[24] The proof comprises two parts. First, it must be shown that, if P has a winning strategy for an initial position $\Pi/_O Z$, the thesis, Z, is intuitionistically derivable from the set of concessions Π (section 2.2). This may be called, depending on one's point of view, either a completeness theorem for intuitionistic deductive logic, or a soundness theorem for *Ei*. Second, it must be shown that, conversely, if Z is intuitionistically derivable from Π, P has a winning strategy for $\Pi/_O Z$ (section 2.3). This may be called, either a soundness theorem for intuitionistic deductive logic, or a completeness theorem for *Ei*.

The proofs are inductive and split into many cases. Only those cases that pertain to the implicational fragment of the language will be fully described, but some indications will be given permitting the reader to supply the proofs for the other cases himself (including those cases that involve quantifiers).

2.1. *Preliminaries*

The positions in *Ei*-dialogues will be described by *dialogue sequents*:[25]

$$\Pi; \Delta / T /_N Z; \Gamma$$

Here N is the party whose turn it is to move (either P or O); T is the last sentence attacked by O (called, the *local thesis*); Π is the set of sentences conceded by O thus far (including the initial concessions); Z is the sentence (if any) that may be attacked by O in the next move; Δ and Γ are sets of sentences: Δ contains those sentences that may be stated, as a defense, by O in the next move, whereas Γ contains those sentences that may be stated, as a defense, by P in the next, or some later, move. Elements not present in a certain position will be simply omitted from the notation. Thus an initial position is always of the following type:

(type *OI*) $\Pi/_O Z$

(That is: $\Pi; \emptyset/\emptyset/_O Z; \emptyset$.) Only those features of positions are codified that are relevant to a party's chances of winning or losing a dialogue.

E.g., the order and frequency of statements, by O, of one and the same sentence are neglected.

If there is a declarative sentence that, according to the logical rules, must be stated in an attack (of the i-th kind) on U, it will be denoted by a_iU, otherwise a_iU will denote the empty set. Thus $a_iU \rightarrow V = a_i \sim U = U$. The set of sentences that may be used in the corresponding defense move will be denoted by d_iU. E.g., $d_1 U \& V = [U]$,[26] $d_iU \& V = [V] \quad (i > 1)$, $d_i\forall xU = [[b_i/x]U]$, $d_iU \vee V = [U, V]$, $d_i\exists xU = [[b_j/x]U]_{j\in\omega}$. The second position in each dialogue with an initial position $\Pi/_O T$ will, accordingly, be of the following type:

(type P) $\quad \Pi,\, a_iT/T/_P d_iT$

In a position of type P, the Proponent can either attack or defend, or perhaps make a winning remark. If P attacks, and if no statement (of a declarative sentence) need be made in the attack, P brings about a position of type OII:

(type OII) $\quad \Pi,\, a_iT;\Delta/\,T/_O d_iT \qquad (\Delta \neq \emptyset)$

On the other hand, if P attacks (say U) and makes a statement of Z ($= a_jU$) in the attack, and if $d_jU \neq \emptyset$, then the next position will be of type OIII:

(type OIII) $\quad \Pi,\, a_iT;[V]/\,T/_O Z;\, d_iT$

(Here $U \in \Pi$, $a_jU = Z$, $d_jU = [V]$. Either $U = Z \rightarrow V$, or else $U = \sim Z$ and the language contains $\wedge$ and $V = \wedge$.) Note that we have both $a_jU = Z$ and $d_jU = \emptyset$ iff $U = \sim Z$ and the language does not contain $\wedge$. In that case an attack by P on U will bring about a situation of type OI. Similarly, if P defends. (In these two cases the local thesis, T, may be dropped from the sequent, since in the next move O must attack Z and, therefore, T will be substituted by Z.) It is easily checked that O can, from any position of the types OI, OII or OIII, only move back to a position of type P. therefore *these are all the types there are* and the transitions between types may be diagrammed as follows:

$$\Pi/_O Z \Longleftrightarrow \Pi,\, a_iT/T/_P d_iT \begin{array}{l} \nearrow\!\!\swarrow \;\; \Pi,\, a_iT;\Delta/\,T/_O d_iT \quad (\Delta = \emptyset) \\ \searrow\!\!\nwarrow \;\; \Pi,\, a_iT;\,[V]/\,T/_O Z;\, d_iT \end{array}$$

Winning positions for P are those positions of type P in which P can make a winning remark, i.e., positions

$$\Pi,\ T,\ a_iTT/_P d_iT$$

or, if the language contains $\wedge$:

$$\Pi,\ a_iT,\ \wedge/\ T/_P d_iT$$

Winning strategies (for P) will be codified as trees labeled with dialogue sequents: $\mathbf{T} = \langle R, A, r\rangle$ is a *tree* (on A, with *root* r) iff, (1) R is a binary relation on A, (2) $r \in A$, (3) for no $e \in A$: eRr, (4) for every $e \in A$, $e \neq r$, there is exactly one $e' \in A$ such that $e'Re$ (e' is called the *predecessor* of e, whereas e is a *successor* of e'), (5) for every $e \in A$ there is a finite sequence $\langle e_1, \ldots, e_n\rangle$ $(n \geq 1)$ such that: $e_1 = r$, $e_n = e$, e_iRe_{i+1} $(1 \leq i < n)$.

The elements of A will be called *nodes* of the tree. A node without successors is called a *final node*. A sequence $\langle e_1, \ldots, e_n\rangle$ such that e_iRe_{i+1} $(1 \leq i < n)$ is called a *path* from e_1 to e_n. We say that e_1 *dominates* e_n, if there is a path from e_1 to e_n. A path from r to a final node e is called a (*finite*) *branch*.

A *tree diagram* (of dialogue or other sequents) is a pair $\langle \mathbf{T}, f\rangle$, such that $\mathbf{T} = \langle R, A, r\rangle$ is a tree and f is a function defined on A (and with dialogue or other sequents as values).

Let σ be any dialectic system with the structural rules Ei (σ is fixed as soon as a language has been selected). A *P-strategy diagram* (in σ) is a tree diagram $\langle\langle R, A, r\rangle, f\rangle$ (of dialogue sequents) with the following properties:

(i) If $f(e) = \Pi/_O Z$, then, for each mode of attack on Z (according to σ) there is a successor e_i of e such that $f(e_i) = \Pi,\ a_iZ/\ Z/_P d_iZ$; e has no other successors.

(ii) If $f(e) = \Pi;\ \Delta/T/_O\Gamma$, then for each $U \in \Delta$ there is a successor e_i of e such that $f(e_i) = \Pi,\ U/\ T/_P\Gamma$; e has no other successors.

(iii) If $f(e) = \Pi;\ [V]/\ T/_O Z;\ \Gamma$, then for each mode of attack on Z (according to σ) there is a successor e_i of e such that $f(e_i) = \Pi,\ a_iZ/\ Z/_P d_iZ$, moreover there is a successor e' of e such that $f(e') = \Pi,\ V/\ T/_P\Gamma$; e has no other successors.

(iv) If $f(e) = \Pi/\ T/_P\Gamma$, then, if $T \in \Pi$ or $\wedge \in \Pi$ or both $\Gamma = \emptyset$ and there is no complex formula in Π, e is final; otherwise, e has exactly one successor e', and either

(a) $f(e') = \Pi; d_j U / T /_O a_j U; \Gamma$ for some $U \in \Pi$ and for some j such that $d_j U \neq \emptyset$, or

(b) $f(e') = \Pi /_O Z$ for some $Z \in \Gamma$, or if the language does not contain $\wedge$, for some $\sim Z \in \Pi$.

A *P-winning strategy diagram*[27] (in σ) is a P-strategy diagram in σ such that (i) all branches are finite, (ii) for all final nodes e, $f(e)$ denotes a winning position for P.

The set of sequents, S, for which there is a P-winning strategy diagram (i.e., a P-winning strategy diagram $\langle\langle R, A, r\rangle, f\rangle$ with $f(r) = S$) will be denoted as $\mathbf{W}(\sigma)$. It is easily checked that, for the implicational fragment of a first order language, P-strategy diagrams are to be constructed according to the following rules:[28]

Rule $OL_{\rightarrow}$ If $f(e) = \Pi /_O U \rightarrow V$, e has exactly one successor e' and $f(e') = \Pi, U / U \rightarrow V /_P [V]$.

Rule OI_{At} If $f(e) = \Pi /_O Z_0$ (Z_0 elementary), e has exactly one successor e' and $f(e') = \Pi / Z_0 /_P \emptyset$.

Rule $OIII_{\leftarrow}$ If $f(e) = \Pi; [U] / T /_O V \rightarrow W$; Γ, e has exactly the successors e_1, e_2, and $f(e_1) = \Pi, V / V \rightarrow W /_P [W]$, and $f(e_2) = \Pi, U / T /_P \Gamma$.

Rule $OIII_{At}$ If $f(e) = \Pi; [U] / T /_O Z_0$; Γ (Z_0 elementary), e has exactly the successors e_1, e_2, and $f(e_1) = \Pi / Z_0 /_P \emptyset$ and $f(e_2) = \Pi, U / T /_P \Gamma$.

Rules $\left\{\begin{matrix} P_{\rightarrow} \\ Pd \end{matrix}\right.$ If $f(e) = \Pi / T /_P \Gamma$ then, if $T \in \Pi$ or both $\Gamma = \emptyset$ and there is no complex formula in Π, e is final; otherwise, e has exactly one successor e' and either (Rule $P_{\rightarrow}$) there is some $U \rightarrow V \in \Pi$ such that $f(e') = \Pi; [V] / T /_O U$; Γ, or (Rule Pd) there is some $Z \in \Gamma$ such that $f(e') = \Pi /_O Z$.

2.2. *From Strategies to Deductions*

We shall associate with each dialogue sequent that occurs in a P-winning strategy diagram a "deductive" sequent Π / Z (Π a set of formulas, Z a formula) such that $\Pi \vdash Z$. Here "$\vdash$" stands for intuitionistic (first-order) derivability. For the purposes of this section it is not important what system (axiom system, natural deduction system, sequent system, tableau system) the reader has in mind as defining intuitionistic logic.

DEFINITION 1. For each formula T and set of formulas Γ, let

$$\rho(T,\Gamma)\begin{cases} = T \text{ if } \Gamma=\emptyset \text{ or } T = U \vee V \text{ or } T = \exists xU; \\ = U \text{ if } \Gamma=[U] \text{ and } T \neq U \vee V \text{ and } T \neq \exists xU. \end{cases}$$

DEFINITION 2. φ is the function from dialogue sequents, of any type, to deductive sequents defined as follows:

for type OI: $\varphi(\Pi/_O Z) = \Pi/Z$
for type OII: $\varphi(\Pi;\Delta/T/_O\Gamma) = \Pi/\rho(T,\Gamma)$
for type $OIII$: $\varphi(\Pi;[V]/T/_O Z;\Gamma) = \Pi/Z$
for type P: $\varphi(\Pi/T/_P\Gamma) = \Pi/\rho(T,\Gamma)$.

LEMMA 1. Let σ be a dialectic system with the structural rules of Ei. Let $\langle \mathbf{T}, f\rangle$ be a P-winning strategy diagram in σ ($\mathbf{T} = \langle R, A, r\rangle$). For each $e \in A$ let $\varphi(f(e)) = \Pi e/Ze$. Then $\Pi e \vdash Ze$, for each $e \in A$.

Proof. This can be shown by a straightforward tree induction as follows.

Basis: Let e be a final node, $f(e) = \Pi, T, a_i T/T/_P d_i T$.

Case 1: T is elementary, $f(e) = \Pi, T/T/_P\emptyset$, hence $\varphi(f(e)) = \Pi, T/T$. $\Pi, T \vdash T$ holds.

Case 2: $T = U \to V$, $f(e) = \Pi, U \to V, U/U \to V/_P[V]$, hence $\varphi(f(e)) = \Pi, U \to V, U/V$. $\Pi, U \to V, U \vdash V$ holds (by Modus Ponens).

Etc.

Induction hypothesis: Suppose that for all e' dominated by e: $\Pi e' \vdash Ze'$.

Induction step: Proof by cases according to the rule applied at e, e.g.:

Case $OI_\to$: $f(e) = \Pi/_O U \to V$. Let e' be the successor of e, $f(e') = \Pi, U/U \to V/_P[V]$.

$\varphi(f(e')) = \Pi, U/V$. By the ind. hyp., $\Pi, U \vdash V$, hence by the deduction theorem $\Pi \vdash U \to V$. But $\varphi(f(e)) = \Pi/U \to V$, so we are through.

Case OI_{At}: $f(e) = \Pi/_O Z_0$ (Z_0 elementary). Let e' be the successor of e, $f(e') = \Pi/Z_0/_P \emptyset$.

$\varphi(f(e')) = \Pi/Z_0$. By the ind. hyp., $\Pi \vdash Z_0$. But $\varphi(\Pi/_O Z_0) = \Pi/Z_0$, so we are through.

Case $OIIL_{\rightarrow}$: $f(e) = \Pi; [U]/T/_O V \rightarrow W; \Gamma$. There is an $e' \in A$ such that e' is a successor of e and such that $f(e') = \Pi, V/V \rightarrow W/_P[W]$.

$\varphi(f(e')) = \Pi, V/W$. By the ind. hyp., $\Pi, V \vdash W$, hence by the deduction theorem $\Pi \vdash V \rightarrow W$. But $\varphi(f(e)) = \Pi/V \rightarrow W$, so we are through.

Case $OIII_{At}$: $f(e) = \Pi; [U]/T/_O Z_0; \Gamma$ (Z_0 elementary). There is an $e' \in A$ such that e' is a successor of e and such that $f(e') = \Pi/Z_0/_P \emptyset$.

$\varphi(f(e')) = \Pi/Z_0$. By the ind. hyp., $\Pi \vdash Z_0$. But $\varphi(f(e)) = \Pi/Z_0$, so we are through.

Case $P_{\rightarrow}$: $f(e) = \Pi, U \rightarrow V/T/_P \Gamma$. Let e' be the successor of e, $f(e') = \Pi, U \rightarrow V; [V]/T/_O U; \Gamma$. There is an $e'' \in A$ such that e'' is a successor of e' and such that $f(e'') = \Pi, U \rightarrow V, V/T/_P \Gamma$.

$\varphi(f(e')) = \Pi, U \rightarrow V/U$.
$\varphi(f(e'')) = \Pi, U \rightarrow V, V/\rho(T, \Gamma)$.

By the ind. hyp., since e dominates both e' and e'': $\Pi, U \rightarrow V \vdash U$ and $\Pi, U \rightarrow V, V \vdash \rho(T, \Gamma)$, hence (by Modus Ponens) $\Pi, U \rightarrow V \vdash \rho(T, \Gamma)$. But $\varphi(f(e)) = \Pi, U \rightarrow V/\rho(T, \Gamma)$, so we are through.

Case Pd (implicational fragment): $f(e) = \Pi, U/U \rightarrow V/_P[V]$. Let e' be the successor of e, $f(e') = \Pi, U/_O V$.

$\varphi(f(e')) = \Pi, U/V$. By the ind. hyp., $\Pi, U \vdash V$. But $\varphi(f(e)) = \Pi, U/V$, so we are through.

Etc., etc.

This proof holds good for quantifiers, too. If the language contains quantifiers the P-winning strategy diagram may contain infinite branchings, e.g., if $f(e) = \Pi/_O \forall x U$, and x free in U, then e has a successor e_i for each parameter b_i and $f(e_i) = \Pi/\forall x U/_P[[b_i/x]U]$. Now take any b_k that does not occur in any formula of Π nor in U.

$\varphi(f(e_k)) = \Pi/[b_k/x]U$. By the induction hypothesis, $\Pi \vdash [b_k/x]U$, hence $\Pi \vdash \forall xU$ (by Universal Generalization). But $\varphi(f(e)) = \Pi/\forall xU$, so we are through. Similarly for the other cases of infinite branching, viz., $f(e) = \Pi, \exists xU; [[b_i/x]U]_{i\in\omega}/T/_O\Gamma$; and $f(e) = \Pi; [V]/T/_O\forall xU; \Gamma$. ■

The completeness of intuitionistic deductive logic relative to *Ei* is an immediate consequence of Lemma 1:

THEOREM 1. Let σ be a dialectic system (for a first order language) with the structural rules *Ei*:

$$\text{If } \Pi/_O Z \in \mathbf{W}(\sigma) \text{ then } \Pi \vdash Z.$$

2.3. *From Deductions to Strategies*

The following lemma is needed only when quantifiers are present in the language:

LEMMA 2. If S is a dialogue sequent such that there is a P-winning strategy diagram for S, then there is a P-winning strategy diagram for any S' obtained from S by a simultaneous substitution of (individual) parameters for parameters.

Proof. By tree induction this may be seen to hold for all sequents in a given P-winning strategy diagram. For instance if $f(e) = \Pi/_O\forall xU$, select a successor e_i of e such that the variable b_i substituted for x in U at e_i is "fresh" (i.e., does not occur in any formula of Π nor in U), $f(e_i) = \Pi/\forall xU/_P[[b_i/x]U]$. By the induction hypothesis, and extending the substitution for parameters in $f(e)$ by a substitution of b_j for b_i, we find P-winning strategy diagrams, for each j, for $\Pi'/\forall xU'/_P[[b_j/x]U']$ (the prime denotes the substitution), and hence there is one for $\Pi'/_O\forall xU'$, etc. ■

To prove the converse of Theorem 1 we select as our system for intuitionistic logic a system for the construction of deductive tableaux,[29] with the following reduction rules:

Left Rules	Right Rules
$\rightarrow l$ $\Pi, U\rightarrow/Z$ reduces to $\begin{cases}\Pi, U\rightarrow V/U\\ \Pi, U\rightarrow V, V/Z\end{cases}$	$\rightarrow r$ $\Pi/U\rightarrow V$ reduces to $\Pi, U/V$
$\&l_1$ $\Pi, U\& V,/Z$ reduces to $\Pi, U\& V, U/Z$ $\&l_2$ $\Pi, U\& V/Z$ reduces to $\Pi, U\& V, V/Z$	$\&r$ $\Pi/U\& V$ reduces to $\begin{cases}\Pi/U\\ \Pi/V\end{cases}$
$\vee l$ $\Pi, U\vee V/Z$ reduces to $\begin{cases}\Pi, U\vee V, U/Z\\ \Pi, U\vee V, V/Z\end{cases}$	$\vee r_1$ $\Pi/U\vee V$ reduces to Π/U $\vee r_2$ $\Pi/U\vee V$ reduces to Π/V
$\sim l$ $\Pi, \sim U/Z$ reduces to $\Pi, \sim U/U$	$\sim r$ $\Pi/\sim U$ reduces to $\Pi, U/\sim U$
$\sim l_\wedge \Pi \sim U/Z$ reduces to $\begin{cases}\Pi, \sim U/U\\ \Pi, \sim U, \wedge/Z\end{cases}$	$\sim r_\wedge$ $\Pi/\sim U$ reduces to $\Pi, U/\wedge$
($\sim l$ for languages that contain no $\wedge$; $\sim l_\wedge$ for languages that contain $\wedge$)	($\sim r$ for languages that contain no $\wedge$; $\sim r_\wedge$ for languages that contain $\wedge$)
$\forall l$ $\Pi, \forall xU/Z$ reduces to $\Pi, \forall xU, [a/x]U/Z$ (for any parameter a)	$\forall r$ $\Pi/\forall xU$ reduces to $\Pi/[b/x]U$ (b "fresh", i.e., not occurring in the given sequent $\Pi/\forall xU$)
$\exists l$ $\Pi, \exists xU/Z$ reduces to $\Pi, \exists xU, [b/x]U/Z$ (b "fresh")	$\exists r$ $\Pi/\exists xU$ reduces to $\Pi/[a/x]U$ (for any parameter a)

Closure rules (c and $\wedge c$): Sequents Π, Z/Z and Π, $\wedge/Z$ are called closed.

A *deductive tableau* is a tree diagram of sequents $\langle \mathbf{T}, f\rangle$, constructed according to the reduction rules (i.e., each node e in the tree, such that $f(e)$ is not closed, has exactly one successor e_i for each sequent S_i to which $f(e)$ reduces, and $f(e_i) = S_i$). It is said to be *closed* if all the branches are finite and, for each final node e, $f(e)$ is closed. It is well known that $\Pi \vdash Z$ iff there is a closed deductive tableau for Π/Z.

THEOREM 2. Let σ be a dialectic system (for a first order language) with the structural rules *Ei*:

$$\text{If } \Pi \vdash Z \text{ then } \Pi/_O Z \in \mathbf{W}(\sigma).$$

Proof. It suffices to show that, if there is a closed deductive tableau for Π/Z, then $\Pi/_O Z \in \mathbf{W}(\sigma)$. We shall write $\mathbf{W}$ for $\mathbf{W}(\sigma)$. Let a closed deductive tableau τ for Π/Z be given, let τ contain n nodes. Assume the *induction hypothesis*: for each sequent Π'/Z' such that there is a closed deductive tableau for Π'/Z' containing less than n nodes $\Pi'/_O Z' \in \mathbf{W}$ (i.e., there is a P-winning strategy diagram for $\Pi'/_O Z'$).

We must show that $\Pi/_O Z \in \mathbf{W}$. Proof by cases according to the first rule applied in τ.

Case c: Π/Z is closed, i.e., $Z \in \Pi$. We may write $\Pi, Z/Z = \Pi/Z$. Obviously Π, $Z/_O Z \in \mathbf{W}$ (since all Π, Z, $a_i Z/Z/_P d_i Z \in \mathbf{W}$).

Case $\rightarrow r$: $Z = U \rightarrow V$. There is a closed deductive tableau, contained in τ, with $n-1$ nodes, for Π, U/V. By the ind. hyp., Π, $U/_O V \in \mathbf{W}$ Hence Π, $U/U \rightarrow V/_P[V] \in \mathbf{W}$ (P can state V) and therefore $\Pi/_O U \rightarrow V \in \mathbf{W}$ (O must attack).

Case $\rightarrow$1: $\Pi/Z = \Pi$, $U \rightarrow V/Z$. There are closed deductive tableaux, contained in τ, with less than n nodes each, for Π, $U \rightarrow V/U$ and Π, $U \rightarrow V$, V/Z. By the ind. hyp., Π, $U \rightarrow V/_O U \in \mathbf{W}$ and Π, $U \rightarrow V$, $V/_O Z$. Also, Π, $a_i Z$, $U \rightarrow V/_O U \in \mathbf{W}$. Hence for all $i, j \in \omega$:

(1) Π, $a_i Z$, $U \rightarrow V$, $a_j U/U/_P d_j U \in \mathbf{W}$ and
(2) Π, $U \rightarrow V$, V, $a_i Z/Z/_P d_i Z \in \mathbf{W}$ (O must attack U, resp. Z)

Hence for all $i \in \omega$:

(3) Π, $a_i Z$, $U \rightarrow V$; $[V]/Z/_O U$; $d_i Z \in \mathbf{W}$ (O can realize only positions (1) and (2)).

Hence for all $i \in \omega$:

(4) Π, $a_i Z$, $U \rightarrow V/Z/_P d_i Z \in \mathbf{W}$ (P can attack $U \rightarrow V$ and bring about situation (3)).

Hence Π, $U \rightarrow V/_O Z \in \mathbf{W}$ (O can realize only positions of type (4)). Etc., etc.

This proof holds good for quantifier rules as well, but of course Lemma 2 is needed, e.g., to infer (Case $\forall r$)

$$\Pi/\forall x U/_P[[b_i/x]U] \in \mathbf{W} \text{ for each parameter } b_i, \text{ from}$$
$$\Pi/\forall x U/_P[[b/x]U] \in \mathbf{W} \text{ for some “fresh” } b. \qquad \blacksquare$$

3. EQUIVALENCE OF *Ei* AND OTHER TYPES OF SYSTEM

In order to prove that each of D, E and Di is equivalent to Ei, I shall first show that if there is a P-winning strategy diagram according to Ei for an initial position $\Pi/_O Z$ (Z complex), there is also a P-winning strategy for this position according to E (section 3.1). Next, it will be shown how a Proponent can use a P-winning strategy according to E, for $\Pi/_O Z$ (Z complex), to win each possible dialogue, for $\Pi/_O Z$, in the corresponding D-game (section 3.2).

The other steps are trivial: If the Proponent has a winning strategy in the D-game (for a certain initial position), it may use the same strategy in the Di-game. As soon as the dialogue would be won according to Terminal Rule I (but not on account of a concession $\wedge$), O has no choice but to attack the statement Z P made in it's last move (there must be such a statement) and this statement must be elementary. Otherwise, if there were other options for O, Terminal Rule I would not apply. Since $D1$ (ii) was observed in the course of the dialogue, Z also occurs as one of O's concessions. therefore, after O's attack on Z, P can make a winning remark (*Ipse dixisti*!) and win the dialogue according to Terminal rule II.

Finally, if P has a winning strategy (for a certain initial position) in the Di-game, the same strategy works for the Ei-game. For, it is only O's options that are limited if we include $D6$ among the rules. Thus, going around clockwise in Table III (section 1.5), we shall have established the equivalence of the four systems.[30]

3.1. *From Ei to E*

In order to transform a P-winning strategy according to Ei into one according to E, it suffices

(i) to restrict winning positions to those involving $\wedge$ and those in which the local thesis is elementary:

$$\Pi,\ Z_0/Z_0/_P\emptyset \quad (Z_0 \text{ elementary})$$

(ii) to have P defer each statement of an elementary sentence Z_0, until Z_0 has appeared among O's concessions ($D1$ (ii)).

Assume that we can modify the strategy so as to conform to (i) and (ii). A proponent employing the modified strategy will make moves that observed $D1$ (ii). Therefore, attacks (by O) on elementary statements will immediately result in a winning position followed by a winning remark. (Assume also that P never postpones the making of a winning remark, whenever such a remark can be made.) All winning positions (that do not involve $\wedge$) will be preceded by an attack (by O) on an elementary statement.

Consider a P-winning strategy diagram $\langle\langle R, A, r\rangle, f\rangle$ that depicts such a reformed strategy. For each final node e, either $f(e) = \Pi,\ \wedge/T/_P\Gamma$ or $f(e) = \Pi,\ Z_0/Z_0/_P\emptyset$ (Z_0 elementary). If we simply delete the final nodes of the second type (i.e., the attacks by O on elementary statements), the diagram will depict a winning strategy according to E, i.e., such that $D1$ is observed.

Let us take care of (i) first: we want to show that it makes no difference for P if the first kind of winning remark ("You said so yourself!", or "*Ipse dixisti!*") can only be made with respect to an elementary statement.

LEMMA 3.[31] Let σ be a dialectic system with the structural rules Ei. Let σ' be the corresponding system with the restriction that *Ipse dixisti!*-remarks can only be made with respect to an elementary statement. Then, if P has a winning strategy for $\Pi/_O Z$ in σ, it has one in σ'.

Proof. It suffices to show that for all T, and i:Π, T, $a_iT/T/_P d_iT \in \mathbf{W}(\sigma')$. For then the P-winning strategy diagrams constructed according to σ can be extended, from the final nodes e with $f(e) = \Pi$, T, $a_iT/T/_P d_iT$ (T complex) onward, so as to obtain a P-winning strategy diagram constructed according to σ'. Using induction on (the logical complexity of) T, we may assume that for all V of lesser complexity than T and for all Π', and i:

$$\Pi',\ V,\ a_iV/V/_P d_iV \in \mathbf{W}(\sigma').$$

Let $d_iT = [U_1, U_2 \ldots]$. By the induction hypothesis (if $d_iT \neq \emptyset$):

(1) Π, T, a_iT, U_j, $a_kU_j/U_j/_P d_kU_j \in \mathbf{W}(\sigma')$ for all k, j.

Hence,

(2) Π, T, a_iT, $U_j/_O U_j \in \mathbf{W}(\sigma')$ (for all j, if $d_iT \neq \emptyset$).

For O can, in position (2), realize only one of the positions (1).

It follows that

(3) Π, T, a_iT, $U_j/T/_P d_iT \in \mathbf{W}(\sigma')$ (for all j, if $d_iT \neq \emptyset$).

For P may, in position (3), realize position (2) by a defense move. On the other hand, the induction hypothesis warrants that

(4) Π, T, a_iT, $a_ha_iT/a_iT/_P d_ha_iT \in \mathbf{W}(\sigma')$ (for all h, if $a_iT \neq \emptyset$).

From (3) and (4) we conclude, since not both a_iT and d_iT are empty:

(5) Π, T, a_iT; $d_iT/T/_O a_iT$; $d_iT \in \mathbf{W}(\sigma')$.

(If $d_iT = \emptyset$, write: Π, T, $a_iT/_O a_iT$.) For, obviously, O must, in position

(5), either attack and realize some position of type (4), or defend and realize some position of type (3). Finally, it follows that

(6) $\Pi, T, a_i T/T/_P d_i T \in \mathbf{W}(\sigma')$.

For, P may attack T in the i-th manner and thus realize position (5). ■

Let us now turn to condition (ii).

LEMMA 4. If there is a P-winning strategy diagram, according to *Ei*, for $\Pi/_O Z$ (satisfying the restriction (i) on *Ipse dixisti*!-remarks), Z complex, then there is a P-winning strategy diagram, according to *Ei*, for $\Pi/_O Z$ that conforms to $D1$ (ii) (and also satisfying the restriction).

Proof. Let τ be the given P-winning strategy. I shall, in informal terms, describe how P can use τ to win and at the same time observe ($D1$ (ii)) (even though τ itself doesn't observe this condition on P's moves).

P may move according to τ until a position

(1) $\Pi/T/_P\Gamma$

of type P occurs and τ tells P to state an elementary sentence Z, whereas $Z \notin \Pi$. The next position, according to τ, is

(2) $\Pi; [V]/T/_O Z; \Gamma$ (Z elementary)

if P is told to attack $Z \rightarrow V \in \Pi$, or (if the language contains $\wedge$) $\sim Z \in \Pi$. Otherwise the next position is

(3) $\Pi/_O Z$ (Z elementary)

Let us first assume that it is (3). This position must be followed by

(4) $\Pi/Z/_P\emptyset$

Let P skip the move prescribed by τ for position (1) and, instead, make the move τ prescribes for position (4), i.e., some attack on a $U \in \Pi$. Thus P "ignores" the fact that the local thesis is not Z but T. Indeed as long as O defends and does not attack any new statement made by P, P should behave in each position $\Pi'/T/_P\Gamma$ as if it were $\Pi'/Z/_P\emptyset$.

If P manages to put this through until O at some time attacks some new statement W made by P, P will get away with this. For, after this attack the position will be $\Pi', aW/W/_P dW$. Whether the preceding local thesis was T or Z does not matter (nor does it matter whether the preceding set of statable sentences on P's side was Γ or $\emptyset$). Consequently, P may, from that position on, simply use τ again. There is, then, only one way these tactics may fail: if, before any attack by O, the strategy described by τ directs P to make a winning remark on account of Z in a position $\Pi'/Z/_P\emptyset(Z \in \Pi')$. For, actually, the position is $\Pi'/T/_P\Gamma$. P should now execute the postponed move and state Z. The next position is

(5) $\Pi'/_O Z \quad (Z \in \Pi')$

O must attack Z and thus realize the winning position

(6) $\Pi'/Z/_P\emptyset \quad (Z \in \Pi')$

If we have to deal with (2) instead of (3) the argument is similar. Position (2) must be followed, both by (4) and

(7) $\Pi, V/T/_P\Gamma$

P should, again, skip the move prescribed by τ for position (1) and, instead, make the move τ prescribes for (4). Following the same tactics as before, P should execute the postponed move as soon as (and only if) the strategy described by τ directs P to make a winning remark on account of Z. The next position will be:

(8) $\Pi'; [V]/T/_O Z; \Gamma \quad (Z \in \Pi')$

After O's reaction on (8) the position will be, either a winning position (6), or

(9) $\Pi', V/T/_P\Gamma$

In the latter case P can use the strategy τ prescribes for (8), for $\Pi \subseteq \Pi'$ and P may ignore the additional concessions. ■

THEOREM 3. Let σ be a dialectic system with the structural rules Ei. If there is a P-winning strategy, according to σ, for $\Pi/_O Z$ (Z complex), then there is one according to the corresponding system with the structural rules E.

Proof. By Lemmas 3 and 4 and the discussion preceding Lemma 3. ■

3.2. *From E to D*

For simplicity, we shall assume that the initial positions are of type $\emptyset/_O Z$

(Z complex). The proof presented below[32] is easily extended to include initial positions with a nonempty set of initial concessions.

For the purpose of this section, revise the definition of a_jU as follows: $a_1 U \to V = U$, $a_1 \sim U = U$, $a_1 U \vee V = ?$, $a_1 U$ & $V = L?$, $a_2 U$ & $V = R?$, $a_i \forall x U = b_i?$, $a_1 \exists x U = ?$.

Positions will henceforward be codified as sequences, $p = \langle r_0, \ldots, r_n \rangle$ of *rounds*.[33] Each nonempty round r_i is an ordered triple $r_i = \langle r_{i1}, r_{i2}, r_{i3} \rangle$; it is said to be either *open* or *closed*. The first round, r_0, is *open*; $r_0 = \langle -1, \emptyset, Z \rangle$, with Z the initial thesis. Assume that i is odd. In that case either $r_i = \emptyset$[34] or r_i is said to be opened by O and $r_{i2} = a_jU$ (an attack on U of some kind j), where U appears as a statement of P's in round r_{i1} (i.e., if $r_{i1} = k$, then k is even, $0 \leq k < i$ and $r_{k3} = U$). The component r_{i3} is either a sentence or a set of sentences; if it is a set of sentences $r_{i3} = d_jU$ and r_i is said to be *open*; otherwise, $r_{i3} = V$, for some $V \in d_jU$, and r_i is said to be *closed*. If i is even, either $r_i = \emptyset$ or r_i is said to be opened by P. If $i \neq 0$, $r_{i3} = a_jU$, where $U = r_{k2}$, for $k = r_{i1}$ (k odd, $0 < k < i$) and for some mode of attack j. Again, either $r_{i2} = d_jU$ and r_i is said to be open, or $r_{i2} = V$ for some $V \in d_jU$ and r_i is said to be closed.

Sequences of rounds represent, in an obvious way, the relevant features of a dialogue up to a certain moment. In the rounds $r_0, r_1, r_2 \ldots r_n$ one will find the initial thesis and the attacks (in chronological order) and also information as to what statements have been attacked and how often. Thus it can be seen which moves are excluded by $D3$ (or $D3,n,m$). Each defense appears in the same round as the attack to which it reacts (it is said to *close* that round). Hence, if $D5$ is observed, the only permissible defense move by O (by P) would be to close the last open and even (and odd) round. In E-dialogues O can only defend by closing the preceding round, and it can only attack P's statement of that round ($D6$).

LEMMA 5. If p is a position in a D-dialogue the open rounds are alternately even and odd. If the last open round of p is even (odd) it is O's turn (P's turn) to make the next move.

Proof. By induction of the length of the dialogue, i.e., the number of moves in the dialogue.

In the next proof I shall make use of indices: An *index* will be a finite sequence of natural numbers (we shall only need 1 and 2), denoted α, β,

γ, etc. If $\alpha = \langle \alpha_1, \ldots, \alpha_i \rangle$ and $\beta = \langle \beta_1, \ldots, \beta_j \rangle$, $k \in \omega$, then $\alpha k = \langle \alpha_1, \ldots, \alpha_i, k \rangle$ and $\alpha\beta = \langle \alpha_1, \ldots, \alpha_i, \beta_1, \ldots, \beta_j \rangle$. Further, $\alpha \leqslant \beta$ iff there is a γ such that $\alpha\gamma = \beta$.[35]

THEOREM 4. Let σ be a system with the structural rules E. If there is a P-winning strategy, according to σ, for $\emptyset /_O Z$ (Z complex), then there is one according to the corresponding system with the structural rules D.

Proof. Let S be a P-winning strategy for $\emptyset /_O Z$ according to σ. We shall first see how to conceive of a P-no-loss strategy s^* (for $\emptyset /_O Z$) according to the structural rules D. Eventually, it will be shown that s^* is actually a P-winning strategy.

Intuitively, one should think of a dialogue in which P employs s^* as composed of several subdialogues in which P employs s. Each of several O-reactions on one and the same utterance by P (O may now both attack and defend!) is assigned to a different subdialogue. P is to employ indices in order to keep track of these assignments.

Assignment Rules

(i) The initial thesis is to be assigned $\emptyset$ (the empty index).

(ii) If an utterance by O constitutes the k-th reaction on an utterance by P ($k = 1$ or $k = 2$), and if this later utterance was assigned α, then P will assign αk to the new utterance by O.

(iii) P assigns to its own utterances (other than the initial one) the same index as to the chronologically preceding utterance by O.

At each stage of the D-dialogue P is engaged simultaneously in several E-dialogues, one for each assigned index of maximal length. Let α be an assigned index of maximal length, then the utterances belonging to the α-dialogue are precisely those with an index γ such that $\gamma \leqslant \alpha$.

It is not hard to check that the following conditions hold, if P applies the Assignment Rules:

(a) Each assigned index is assigned once to an utterance of O's and once to an utterance of P's, with the following two exceptions:

1. $\emptyset$ is assigned only once.
2. If it is P's turn to move, the index assigned to O's last utterance has not yet been assigned to a P-utterance.

(b) If α is assigned to an attack by P in round r_i, no indices $\gamma \leqslant \alpha$ are assigned to utterances in rounds r_j with $i < j$.

Let p be an indexed position.[36] The *α-erasure* of p is to be the sequence of rounds obtained by erasing all the (representations of) utterances in p, except those to which an index γ such that $\gamma \leqslant \alpha$ has been assigned. In those cases where no $\gamma \leqslant \alpha$ is assigned to any statement in a round the whole round is replaced by $\emptyset$. If an index $\gamma \leqslant \alpha$ is assigned to the attack in an open round, the whole round is preserved. If, in a closed round, an index is assigned to the attack $a_j U$ in that round but not to the defense $V \in d_j U$, V is to be replaced, in that round, by $d_j U$.

There is no guarantee that the α-erasure of an indexed position, will again be a position in a dialogue. However, we shall see that the α-erasures have very near properties, provided that P employs the strategy s^* defined as follows:

Description of s^*. Let p be an indexed position in a D-dialogue.[37] Assume that p has employed s^* until position p was reached, assigning indices according to the assignment rules. Let it be P's turn to make a move. We further assume that the following two conditions hold (otherwise s^* will be undefined for p):

(Assumption I) For all indices β, the β-erasure of p is a position that can occur in an E-dialogue (for $\emptyset /_O Z$) in which P employs the strategy s (an s-position, for short).

(Assumption II) Let r_i be an open round in p, opened by O using an utterance with the index γ_1. Let r_j be the next open round in p, opened by P (Lemma 5) using an utterance with the index γ_2. Then $\gamma_1 \leqslant \gamma_2$ and the rounds r_i and r_j will also be *consecutive* open rounds in the γ_2-erasure of p.

Let O's last utterance be indexed α. P should then, in order to determine its next move, consider the α-erasure of p. This α-erasure is an s-position p_α (Assumption I). In p_α s assigns a move and P should make the same move in position p. The Assumptions I and II guarantee that P can do so:

Case 1: If P is to attack (I) suffices.

Case 2: If P is to execute a defense move, but O's chronologically last move (in p and in p_α) was an attack, P's defense must answer this very attack. So P must close the bottommost open round: again (I) suffices.

Case 3: Let O's chronologically last move (in p and in p_α) be a defense and let s (for p_α) assign a defense move to P. We now need Assumption II (besides Assumption I). Let p_0 be the position that preceded p in the D-dialogue. O's last move in this dialogue, then, effected a closure of the bottommost open round r_j in p_0. Let the attack by P in r_j be indexed γ_2. O's last utterance will be indexed $\gamma_2 k = \alpha$ ($k = 1$ or $k = 2$). Let r_i be the bottommost open round in p and let the attack by O in that round be indexed γ_1. Clearly, r_i and r_j are consecutive open rounds in p_0. According to Assumption II $\gamma_1 \leqslant \gamma_2$, so both r_i and r_j are in the γ_2-erasure of p_0, and moreover r_i and r_j are consecutive open rounds in this γ_2-erasure. According to (b), r_j is the bottommost (nonempty) round in it. Hence, in the α-erasure (i.e., the $\gamma_2 k$-erasure) p_α of p, r_i is the bottommost open round. Strategy s now tells P to make a defense move in position p_α. According to $D5$ this means that P is to close round r_i. Since r_i is also the bottommost open round in p, the same move can be executed in p as well.

It remains to be shown that s^* always defines a move for P (as long as P employs this strategy), i.e., it must be shown that (I) and (II) hold as long as P employs s^*. Clearly these assumptions hold for the initial position $\langle -1, \emptyset, Z\rangle$, with $\emptyset$ assigned to Z.

Induction hypothesis. Assume that, in a D-dialogue, (I) and (II) hold for an (indexed) position p and for all positions that precede p. Assume further that P employs s^* (whenever this strategy defines a move for P). Let the next (indexed) position be p'. It must be shown that (I) and (II) hold in p'.

Case 1. *O makes a move.* O's move does not interfere with Assumption II, since no new pairs of consecutive open rounds come up for consideration. Let β be the index assigned in p', to O's chronologically last utterance, $\beta = \alpha k$. We have to check whether (I) holds for β. The α-erasure of p is, by hypothesis, an s-position p_α. P's utterance indexed α occurs in that position, but *no reaction on this utterance occurs in p_α*. Let P's utterance indexed α be U_α. If U_α is (the initial thesis or) an attack it must (according to (b)) appear in the bottommost round of p_α, so O can in p_α both attack or defend in a reaction to U_α conforming to $D3$ and $D6$. If U_α was a defense move, O's reaction on it in the D-dialogue must be an attack, and this attack may also be executed in p_α without violation of $D3$ or $D6$. (Since in p_α, U_α is the only statement by P put forward as a defense and not yet attacked, it must be P's last utterance in p_α.) Hence O can in position p_α, and without violation of the E-rules, make the same move it made in the D-dialogue in position p. The result of this move is exactly the αk-erasure (i.e., the β-erasure) p_β of p. Hence p_β is an s-position, too.

Case 2. *P moves employing* s^*. Let O's last utterance be indexed α. P's move does not infere with Assumption (I), precisely because P copies its move from what s prescribes for the α-erasure, p_α, of p. We must check whether (II) holds for p. If P defends there are no new pairs of consecutive open rounds to consider, so let us assume that P attacks.

Case 2.1. O's last move was an attack. There is now one new pair of consecutive open rounds to consider: the last two rounds of p'. Since the utterances in these rounds are both indexed α, the pair obviously complies with Assumption II.

Case 2.2. O's last move was a defense. Let the position that preceded p in the D-dialogue be p_0. O's last move, then, consisted of closing the last open round r_j. Let P's utterance in that round be indexed γ_2. So $\alpha = \gamma_2 k$ for some k. Let r_i be the last open round in position p and let O's utterance in it be indexed γ_1. Finally, let r_h be the bottommost (open) round in p'. P's utterance in r_h is also indexed α.

There is, in p', one new pair of consecutive open rounds to consider, viz., the pair r_i, r_h. Let us apply the induction hypothesis to situation p_0. Clearly, $\gamma_1 \leqslant \gamma_2$ and r_i and r_j are consecutive open rounds in the γ_2-erasure of p_0. According to (b), r_j is the bottommost round in this erasure.

As to r_i, r_h, it immediately follows that $\gamma_1 \leqslant \gamma_2 \leqslant \gamma_2 k = \alpha$, so $\gamma_1 \leqslant \alpha$. Further it is easily seen that r_i and r_h are consecutive open rounds in the α-erasure of p'. For the only differences between the γ_2-erasure of p_0 and the α-erasure of p' are (i) that the bottommost round r_j is closed and

(ii) that round r_h is added. So r_h immediately follows r_j in the α-erasure of p' (i.e., all rounds in between are empty) and, since there are no open rounds in the erasure between r_i and r_j, there are no open rounds between r_i and r_h either.

This concludes the proof of the feasibility of strategy s^*. Clearly, if P employs s^*, and P is to make a move, there is a move that P can make: the one prescribed by s^*. Otherwise, s would not be a winning strategy for E-dialogues. So s^* is, at least, a no-loss-strategy for P in D-dialogues (with initial position $\langle -1, \emptyset, Z\rangle$).

Finally, to see that s^* is actually a winning strategy, it must be excluded that the D-dialogue in which P employs s^* be infinite. Suppose it is. Consider the tree of indices assigned in this infinite dialogue: $\mathbf{T} = \langle R, A, r\rangle$ (A = the set of assigned indices, $r = \emptyset$, and for $\alpha, \beta \in A$: $\alpha R \beta$ iff $\beta = \alpha k$ for some k). Since each index occurs twice A must be infinite. Since each $\alpha \in A$ has at most two successors in A, $\mathbf{T}$ is a finitely branching tree. By König's Lemma there must be an infinite branch: $\alpha_0, \alpha_1, \alpha_2 \ldots$ Let p_i be the α_i-erasure of the infinite dialogue, the sequence $p_0, p_1 \ldots$ then shows how there can be an infinite E-dialogue in which P employs strategy s. But this is impossible, since s is a winning strategy. ∎

FINAL REMARKS

It is well known that P-winning strategy diagrams (for E-dialogues) can be reduced to bounded ones.[38] I.e., if P has a winning strategy (for an initial position) in an E-system, there is a bound m, so that P has a winning strategy (for that position) in the corresponding system with $D3,1,m$ instead of $D3$. So in the last part of the proof the use of König's Lemma can be replaced by the calculation of the bound beyond which the dialogue cannot extend.

Let the D,n,m-systems be those that differ from (corresponding) D-systems in having $D3,n,m$ instead of $D3$ (and $D,1,m$ besides). It can then be shown, by the method of the proof of Theorem 4, that P has an E-winning strategy (for an initial position) iff for each n there is an m so that P has a D,n,m-winning strategy (for that position).[39]

The methods of section 2 and section 3 can also be applied to minimal and classical D and E-systems, both with and without a $\wedge$ in the underlying language.

NOTES

[1] 1958. Cf. Lorenzen (1960).

[2] The so-called *Erlanger* and *Konstanzer Schule*. The term *Deutscher Konstruktivismus* was proposed by G. Mayer. Cf. Mayer (1981), p. 1.

[3] The terms '*D*-dialogue' and '*E*-dialogue' I owe to W. Felscher. Cf. Felscher (1982) and (1983). I am in many, often subtle, ways indebted to these two papers.

[4] The following texts contain motivations, or philosophical foundations, for dialogue rules: Lorenzen and Schwemmer (1975), Lorenz (1973) Barth and Krabbe (1982), Ch. III, IV, Felscher (1982), Krabbe (1982a), (1982c) and (1984), Cf. Haas (1980), Thiel (1980), Gethmann (ed.) (1982).

[5] Distinctions between different senses of the term 'formal' were pointed out to me by E. M. Barth, Cf. Barth and Krabbe (1982), I.3.

[6] Cf. Hamblin (1970), p. 256.

[7] Cf. Kamlah and Lorenzen (1973), Lorenzen and Schwemmer (1975), Lorenz (1961), (1968).

[8] Cf. Barth and Krabbe (1982), Ch. III, IV, esp. IV. 5.

[9] Cf. Krabbe (1982c).

[10] In $\text{Rule}_\forall$ and $\text{Rule}_\exists$, '$[a/x]U$' stands for the formula obtained by substituting the parameter a for each free occurrence of x in U. Other *logical rules* are found in Kindt (1972). Cf. also the alternative sets of rules in Barth and Krabbe (1982) p. 89, p. 102. In Stegmüller and Varga von Kibéd (1984), a different rule for $\rightarrow$ occurs. In their system $U \rightarrow V$ abbreviates, one might say, $\sim U \vee V$. Hence, the system does not have a proper dialogical $\rightarrow$.

[11] In the terminology of Barth and Krabbe (1982), the defense moves of Figure 1 are 'structural', i.e., they depend upon the grammatical structures of the sentences that are to be defended and of the sentences used in the attacks. *Ipse dixisti*!-remarks, on the other hand, constitute a 'general' type of defense that does not depend on grammatical structures. There is no reason to restrict the use of such remarks to the defense of elementary statements.

[12] Cf. Haas (1980).

[13] Cf. Kamlah and Lorenzen (1973), p. 221.

[14] All those dialectic systems are finitary, in which, for each dialogue, either the total number of moves, or the number of attacks as well as the number of defenses pertaining to any one utterance, is limited beforehand. The bounds may be fixed either by a rule of the system itself (bounded systems), or by choices made by the parties as a part of the dialogue. Cf. Lorenz (1968), (1973). Cf. the D,n,m-systems mentioned at the end of this paper. In Barth and Krabbe (1982) the "official" systems are finitary, but the *P*-liberalized systems of Ch. V.1 are not.

[15] Type I systems are most common. Type II systems occur in the publications by Barth, Krabbe and Mayer.

[16] Cf. Barth and Krabbe (1982), Ch. V.1.

[17] Cf. Note 4.

[18] Again, Kindt (1972) is an exception. In Haas (1980), initial positions are of type $\text{II}/_P[Z]$, so P makes the first move.

[19] Cf. Lorenz (1968). I omit the rather complicated rules, used to limit attacks and defenses in the 'official systems' in Barth and Krabbe (1982) (*FD D*6, *FD D*8, *FD K*).

[20] For suitable choices of languages, *Ei* yields the *P*-liberalized systems *CND* and $C \wedge D$ in Barth and Krabbe (1982) (with logical rules for quantifiers added). Cf. Krabbe (1982b) for the quantifier rules. (There are some misformulations in (1982b) on p. 253: in Rule $O I \forall$ read '$\Pi / \forall x U(x) /_P [U(a)]$', instead of '$\Pi /_P [U(a)]$', similarly in Rule $O I \exists$ read '$\Pi / \exists x U(x) /_P [U(x)]$' instead of '$\Pi /_P [U(x)]$'; in Rule Pd^x read '$\Pi /_O U(a)$' instead of '$\Pi / T /_O U(a)$'; on p. 255, line 13, omit '$+\exists r$'.) P. Lorenzen's systems are mostly of type *E*, whereas the systems proposed by K. Lorenz are generally of type *D*.
[21] For classical systems, cf. Lorenzen and Lorenz (1978), Barth and Krabbe (1982), Stegmüller and Varga von Kibéd (1984). For minimal systems, cf. Barth and Krabbe (1982), esp. IV. 2 and V.5.
[22] This term was introduced by E. M. Barth. Cf. Barth and Krabbe (1982), III.6.
[23] Actually, *Ei* is not a dialectic system, but a type of system. To obtain a dialectic system based on the *Ei* rules it is sufficient to fix a language. I shall, however, sometimes speak loosely of *Ei* as if it were a dialectic system.
[24] The first 'completeness' or 'equivalence' proofs for dialectic systems were presented by K. Lorenz in (1961). A number of proofs and sketches of proofs have been published since, widely diverging in methods and as to the type of system to which they pertain. Cf. Kindt (1972) (§10); Thiel (1978); Haas (1980) (for initial positions of type $\Pi /_P [Z]$); Mayer (1981) (constructive and classical logic); Stegmüller and Varga von Kibéd (1984) (classical logic); Felscher (1983) (constructive logic, a very detailed exposition); Barth and Krabbe (1982) Ch. VII, XI (propositional logic: minimal, constructive and classical); Krabbe (1982b) (additions needed for predicate logic). The proofs in this section, however, are self-contained.
[25] Cf. Barth and Krabbe (1982), V.1.
[26] Following K. Lorenz I shall write $[U_1, U_2, \ldots]$, rather than $\{U_1, U_2 \ldots\}$ to denote a set that codifies a defense obligation.
[27] Cf. Barth and Krabbe (1982), V.2. In propositional logic these diagrams are called *dialogical tableaux*.
[28] Cf. Barth and Krabbe (1982), V.2.2.
[29] Cf. E. W. Beth (1962); Barth and Krabbe (1982), Ch. VII. A deductive tableau is like a derivation in a sequent system turned upside down. Note that the sequents always have exactly one sentence on the right.
[30] The equivalence of *Ei* and *Di* holds for initial positions $\Pi /_O Z$ with *Z* elementary, too. For, the proof of Theorem 4 holds for the transition from *Ei* to *Di* as well.
[31] Cf. Lorenz (1961) Lemmas 5 and 8; Barth and Krabbe (1982), V.4, Theorem 3.
[32] The equivalence of *E* and *D* systems is the subject of Kindt (1970). G. Haas sketches a proof in his (1980), section 1.4; a detailed proof is contained in Felscher (1983). The present – relatively quick – proof connects with K. Lorenz's ideas in (1968), p. 86, 87 (Lorenzen and Lorenz (1978), pp. 143–145). It is self-contained.
[33] Following Lorenz (1961), (1968).
[34] Empty rounds are admitted for technical reasons. For the intuitive interpretation of a sequence of rounds (as a codification of a position in a dialogue) they must be disregarded.
[35] Cf. Schütte (1968), p. 22.
[36] Formally, an indexed position can be defined as a pair $\langle p, f \rangle$, such that *p* is a position, $p = \langle r_1, \ldots, r_n \rangle$ and such that *f* is a index-valued function defined for each *sentence* r_{ij} $(1 \leq i \leq n, j = 2$ or $j = 3)$. (Questions are sentences, too.)

[37] More accurately: a dialogue executed according to a dialectic system σ' that is the D-system corresponding to the E-system σ.
[38] In fact, P-winning strategies can be represented by finitely branching diagrams (*dialogical tableaux*). Cf. Felscher (1983) (skeletons); Krabbe (1982b) (troublesome-but-not-finicky Opponents).
[39] Cf. Lorenz (1968), *SO*6 and *SO*8, pp. 86, 87 (Lorenzen and Lorenz (1978), pp. 143, 144).

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