REFERENTIAL AND NONREFERENTIAL SUBSTITUTIONAL QUANTIFIERS*

It is common to find philosophers claiming that it is possible to free the quantifiers – especially the particular (or so-called existential) quantifier – from questions of reference, existence, and ontology, by having recourse to what is now referred to as the substitutional interpretation of the quantifiers. Although there may be ontologically neutral uses of the substitutional interpretation, it is one of the goals of this paper to point out where this feature has been misconceived and to give equal due to uses of the substitutional account that have ontological import. I will examine the relation of the substitutional treatment of the quantifiers to the most important nonsubstitutional one, that of Tarski, with reference to recent claims that the difference between the two kinds is that the former lacks ontological significance.

Uncritical use of the somewhat entrenched Quinian terminology of "substitutional" versus "referential" quantification is most likely at the root of these mistaken views.¹ In his debates with Ruth Barcan-Marcus and his comments on Lešniewski, Quine has put forward the view that substitutional quantification has no referential force and thus lacks any ontological import.

Quantification ordinarily so-called is purely and simply the logical idiom of objective reference. When we reconstrue it in terms of substituted expressions rather than real values, we waive reference. We preserve distinctions between the true and false, as in truth-function logic itself, but we cease to depict the referential dimension. (Quine, 1966, p. 181)

Such is the course that has been favoured by Lešniewski and by Ruth Marcus. Its nonreferential orientation is seen in the fact that it makes no essential use of namehood. That is, additional quantifications could be explained whose variables are place-holders for words of any syntactical category. Substitutional quantification, as I call it, thus brings no way of distinguishing names from other vocabulary, nor any way of distinguishing between genuinely referential or value-taking variables and other place-holders. Ontology is thus meaningless for a theory whose only quantification is substituinally construed; meaningless, that is, insofar as the theory is considered in and of itself. The question of its ontology makes sense only relative to some translation of the theory into a background theory in which we use referential quantification. (Quine, 1969, pp. 63–64)

Given these views it is not surprising that some would claim to solve

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philosophical problems pertaining to quantifiers, reference, and ontology by simply switching to the substitutional approach. Thus, for example, Susan Haack in her book *Deviant Logic* suggests that a way of *tout court* taking care of the problem of particular-existential generalizations being theorems of logic is simply to switch to a substitutional interpretation on which, if Quine were right, no problems would arise.

The origin of these views can probably be traced back to a shallow analysis of the different interpretations of the quantifiers, i.e., a substitutional interpretation and a Tarskian account. The following is a somewhat oversimplified statement of these semantic conditions, which will nonetheless do for the points to be discussed in this paper. (I limit the discussion here to monadic predications.)

Tarskian Condition: $(\exists x)Fx'$ is true iff 'Fx' is satisfied by some object

Substitutional Condition S: $(\exists x)Fx'$ is true iff some substitution instance of $(\exists x)Fx'$ is true

(An instance of $(\exists x)Fx$ ' results from replacing 'x' in 'Fx' by a constant of the category of singular terms, e.g., proper names.)

By restricting oneself to these conditions, one might come to believe that substitutional quantification has no ontological force because the explicans part of Condition S does not appeal to any word-object relation, e.g., satisfaction, assignments in domains, or reference. By contrast, the Tarskian condition appeals straightforwardly to objects (actually to sequences of objects) in virtue of the word-object (i.e., open sentence – sequence) relation of satisfaction. On the strength of this relation, Tarskian quantifications are correctly said to have referential force and ontological import.

However, one might proceed mistakenly on the basis of this comparison and regard the substitutional and referential distinction as providing a mutually exclusive classification; the error is reinforced if one equates the referential and the Tarskian.

Though such opinions are not uncommon, a little reflection will suffice to show that they are incorrect. An informal account of substitutional generalizations is that they are true if some (or all) of their instances are. Thus 'Something is a Siamese cat' would be true if 'Bouncer (my cat) is a Siamese cat' is true. A quantifier should be called substitutional when its truth conditions appeal to the truth of its instances. This, however,

leaves open the question of how the instances themselves get to be true or false. On a natural account of the truth of instances – or the atomic sentences they depend upon – 'Bouncer is a Siamese cat', is true if and only if the object referred to by the subject is one of the objects to which the predicate applies. On such an explication of the truth of atomic sentences, the substitutionally interpreted generalization based on it would have referential force. The referential force and ontological import of ' $(\exists x)$ (x is a Siamese cat)' would not be due to Substitutional Condition S but to the referential aspect of the truth condition for the instances it depends upon.

There are two relevant possibilities for interpreting atomic sentences. The first would be to follow the more natural course mentioned in the preceding paragraph and construe the atomic sentences as true if and only if the object denoted by the subject term were one of the objects to which the predicate applies. On this interpretation of the atomic sentence - plus the standard interpretation of truth-functional connectives - the instances would be such that substitutional generalizations would be per accidens a vehicle of reference. The reference achieved by asserting 'Some thing is a cat' would not be due directly to the substitutional truth conditions, but to the referential force of the instances appealed to in those substitutional conditions. If, as many, though not all, philosophers would assume, the referents of the terms in the atomic sentences are existents, the above particular generalization could and should be read existentially and as having ontological significance.² It is thus incorrect to say that what makes substitutional quantification different is its lack of referential, existential, or ontological force. Although, as we shall see, the substitutional account can be construed as ontologically neutral, whether or not it is so construed is not of the essence of the substitutional interpretation.

The second relevant possibility for giving truth conditions to our atomic sentences would be to do so without appealing to any such concepts as reference, designation, domains, etc. (i.e., without appealing to any word-object relations). One way of doing this is to say that an atomic sentence is true or holds if and only if it is a member of a given model or truth set of sentences. (The privileged given model or truth set would be the one that conforms to the actual world. This feature, however, is not appealed to in model or truth set approaches, which merely appeal to membership in a set of sentences.) The key point is that such an instance without referential force bestows no referential-

ontological significance upon a substitutional generalization based on that instance. So where the instances are not construed referentially, the substitutional quantification has no such force.

To illustrate this matter. let us consider two substitutional theorists. Hugues Leblanc and Ruth Barcan Marcus. Let me begin by contrasting these two theorists on the example 'Pegasus is a flying horse'. In a number of his papers as well as his book Truth-Value Semantics. Leblanc propounds a substitutional interpretation of the quantifiers. However, on singular terms he has at times held a referential view that is in effect very much like those of Ouine and Russell, and what we above called natural (Leblanc, and Wisdom 1976, p. 148). According to Leblanc, in Leblanc and Wisdom, singular sentences such as the above are true only if the subject designates an existing object. A particular generalization derived from such a sentence is hence true only if something exists, so that his substitutionally construed first-order particular quantifier has the same existential force Ouine and Russell would accord it. In contrast to Leblanc, consider Ruth Barcan Marcus's position in her 1962 paper 'Interpreting Quantification' (pp. 256–257). There she considered the Pegasus sentence above as true, presumably by divorcing its truth from any questions of reference. She sanctioned the particular generalization from that Pegasus sentence to the substitutionally construed: 'There is a true substitution instance of 'x is a winged horse". The generalization is considered true in virtue of the truth of its instances and, unlike Leblanc's substitutional generalization. has no existential force.

There is need for caution in using the terms 'substitutional' and 'referential' with regard to quantifiers and their interpretations. The use I am proposing is that the two are not exclusive of each other.³ Leblanc's quantifiers have both substitutional and referential force. Quantifiers construed in a Tarskian fashion are nonsubstitutional and referential. Barcan Marcus's ' $(\exists x)$ (x is a flying horse)' is substitutional and nonreferential. Let us regard an interpretation and an interpreted quantifier as substitutional when the explicans portion of the truth condition for the quantifier employs the relevant notion of a substitution instance. In constrast to a quantifier's being substitutional, a quantifier is referential (Tarski) or the instances on which the generalization is based are interpreted referentially (Leblanc). The phrase

'referential interpretation' applies to either semantic conditions for quantifiers (Tarski) or conditions for atomic sentences (Leblanc). Of course in a trivial sense, the substitutional versus nonsubstitutional distinction is mutually exclusive.

There are at least three areas of problems where the substitutional interpretation has been applied in order to achieve ontological neutrality and thereby furnish solutions: (1) topics in free logic, (2) opaque constructions, and (3) quantifiers for grammatical categories other than singular terms.

In the first of these areas – free logic – the requirement is laid down that logic should be free of existence assumptions. In one sense, this means that logic should allow for individual constants that do not refer to any existents, i.e., vacuous singular terms, and that existential generalizations should not be logical truths, i.e., theorems.

To find a case where adopting the substitutional interpretation plus supplementing it with a nonexistential English reading of the particular quantifier was offered as a solution to a problem concerning vacuous singular terms, recall the Barcan Marcus proposal mentioned earlier. She maintained that if 'Pegasus is a flying horse' is true, then

 $(\exists x)$ (is a flying horse)' is true as well, even though neither Pegasus nor any other flying horse exists. While there is nothing formally, i.e., purely logically, wrong with such a position, it is philosophically suspect. The position involves taking the instance as true with little if anything said as to how it gets to be true. This would be sanctioned formally on a substitutional semantics like Kripke's by taking the truth value of the atomic sentences for granted (Kripke, 1976, pp. 329–331). Another possibility is to say that the sentence holds in a given model set.

While this may be above reproach from the standpoint of formal logic and formal semantics, it appears to me more questionable than the philosophically and intuitively compelling view that regards such simple sentences as having a logical form that makes them true when the subject refers to an existing object of which the predicate is true, and false otherwise. Adopting a semantics that leaves the instances – atomic sentences – as true or false without informing us how, or by saying that they are members of a model set, i.e., that they are merely consistent with certain other sentences, should be disquieting. To follow such a course is to absolve oneself from answering the difficult question of what makes such sentences true or false. Indeed, to take the sentence as true simpliciter resembles truth by convention, and to make consistency the nature of truth smacks of the coherence theory of truth.

It is worthwhile to say a word here about the relation of interpretations of the quantifiers to natural-language readings for them. The semantical conditions (satisfaction or truth conditions) provide an account of how well-formed formulas governed by the '(x)' or the ' $(\exists x)$ ' operations get to be true or false. Related to these conditions is the matter of precisely which English locutions are most appropriate to the

different interpretations. Thus, where a Tarskian account of $((\exists x))$ is given, especially by such authors as Quine, it is quite natural to read

 $(\exists x)$ ' existentially in English as 'there exists'. It is also appropriate to

give a special English reading to the substitutionally interpreted ' $(\exists x)$ ' quantifier, viz., 'sometimes true' or 'for some x'. The substitutional reading provides no specific connotation of existence and thus has been used uncritically as the basis for spurious claims that substitutional quantification is ontologically neutral.

Like the substitutional interpretation – Condition S – the substitutional reading does not by itself guarantee that ' $(\exists x)$ ' is nonreferential and ontologically neutral. A substitutional reading of a quantifier can depend on instances that are referential. In such a situation there is little difference between the 'there exists' and the 'sometimes true' locutions. Indeed in Russell's writings, we find examples of the two readings used hand in hand (Russell, 1956, p. 46, and Whitehead and Russell, 1962, p. 15).

Susan Haack's suggested solution for the problem of what to do about existence theorems in logic is to adopt a substitutional reading so that the theorems in question will have no existential import (Haack, 1974, p. 143). It would then be unnecessary to revise a logical system so as to exclude particular generalizations being theorems. However, as noted in the preceding paragraph, merely switching to a substitutional

'sometimes true' reading of sentences such as ' $(\exists x)$ ($Fx \lor \sim Fx$)' does

not guarantee that $(\exists x)$ has no referential-existential significance. The crucial question, granted that $(\exists x)$ is interpreted substitutionally according to a condition such as S, is how the instances appealed to in S get their truth values. There are two possibilities. In the first, the

instances are accounted for in the usual referential way, in which case $\int_{T}^{T} (\exists x) (Fx \lor \sim Fx)$ has as much existential import as $(\exists x) (Fx \lor \sim Fx)$ and should not be a theorem. The other possibility is that the truth values of the instances are accounted for without appealing to reference (word-object relations). There are at least two reasons why this will not do as a solution to the free logician's problem. In the first place, it is largely beside the point, since a system of logic where all the instances are construed nonreferentially is not the standard interpretation of quantificational logic even when it is construed substitutionally.

The second reason why this will not solve the problem even if we interpret all the instances nonreferentially involves a deeper parallel between substitutional and referential interpretations of the quantifiers. A more precise way of putting the free logician's point that particular-existential generalizations should not be theorems of logic is that a logical truth is a sentence that is true under every interpretation including the one involving the empty domain. Looking at the semantic condition below for a referentially interpreted quantifier, we see that a universal generalization will be true under the interpretation with the empty domain, i.e., the antecedent ' $d \in D$ ' is false in the truth condition for the universal quantifier; however, an existential generalization will be false, i.e., the conjunct ' $d \in D$ ' is false. The following is somewhat oversimplified, but will nonetheless do to highlight the points to be discussed here.

Referential Truth Conditions

val ((x)A) = T iff (d) $(d \in D \supset$ every relevant sequence of objects d with respect to 'x' leaves 'Ax' satisfied).

val $((\exists x)A) = T$ iff $(\exists d)$ $(d \in D$ and at least one relevant sequence of objects d with respect to 'x' leaves 'Ax' satisfied).

On a nonreferential substitutional account, a parallel problem arises. It is not a question of existence or of the empty domain, if the instances do not appeal to existent object or domains. It is a question of whether the classes of substituted constants can be empty. If there were no true instances, the substitutional universal generalization would be true, i.e., when the antecedent ' $s \in L$ ' is false in the condition for '(x)' and the particular generalization will be false, i.e., the conjunct ' $s \in L$ ' is false in the condition for ' $(\exists x)$ '.

Substitutional Truth Conditions

val (x)A = T iff $(s) (s \in L \supset val (s/xA) = T)$ wherever x is free in A.

val $((\exists x)A) = T$ iff $(\exists s)$ $(s \in L \text{ and val } (s/xA) = T)$ [where s is a substituend in the language L].

When the substitution class is empty, we have an analogous problem to that posed by the empty domain for referential quantifiers. A possible example of this would arise for Quine's canonic notation, which contains individual variables but has no individual constants (names), if one interpreted the quantifiers for that notation substitutionally.

The second area where substitutional quantification has been put forward as a way of solving problems concerns quantifying into opaque constructions. Here as in the case of free logic, the problems are of two sorts. The first concerns applying such rules as existential generalization to opaque sentences. The second problem bears on whether certain sentences should be theorems of modal logic.

It has been argued by some that there is nothing problematic about inferences such as the following when the conclusion is construed substitutionally (Barcan Marcus, 1962, pp. 258–259, and Geach, 1972, pp. 139–146).

 $\frac{\Box \text{ (the evening star = the evening star)}}{(\exists x) \Box (x = \text{the evening star})}$

If one takes the premise as true without further explanation and uses it as an instance that is the base for the substitutional generalization, then there is nothing formally wrong with such a move. It is probably faultless from the standpoint of formal logic and the formal semantics involved. But, as with the Pegasus example, the problem is philosophical. The question of the logical form of the instance and what determines its truth value have been sidestepped.

There has also been discussion as to whether the Barcan formula, $\diamondsuit(\exists x)Fx \supset (\exists x)\diamondsuit Fx$, is a truth of modal logic and ought to be a theor-

em. Barcan Marcus has taken the view that adopting a substitutional approach to the quantifiers will nullify the problem associated with the formula's being a logical truth. In "Interpreting Quantification" (1962, pp. 257–258), she suggested that the substitutional reading 'sometimes true' avoids the counterexample associated with reading the quantifiers in the formula existentially, i.e., if it is possible that there exists an F, then there exists something that is possibly an F. We noted earlier that merely switching readings or even interpretations of the quantifier does not guarantee that the substitutional reading is not also referential and existential; i.e., if the instances appealed to in the substitutional interpretation of the Barcan formula were construed referentially, then there might be no difference between the substitutional and the existential versions of the formula.

At a deeper level, there is an instructive similarity between the questions of whether existence sentences should be truths of first-order logic and whether the Barcan formula should be a truth of modal logic. The validity of a formula depends to a large extent on what the truth conditions for that formula are. To the extent that substitutional and nonsubstitutional truth conditions parallel each other (and a large amount of parallelism is to be expected - since they are two construals of the same formalism), sentences that are or are not logical truths should have their status preserved under the change in interpretation. Thus it is the semantics of modal logic, and not a superfical, informal reading of the quantifiers, that determines whether the Barcan formula is false under some interpretation. Kripke showed that it is false for what can be taken as a nonsubstitutional view of the quantifiers (Kripke, 1971, p. 67). Using parallel considerations but within a substitutional framework, Dunn has provided an analogous counterexample (Dunn, 1973, pp. 87-100).

The last area to be considered where the substitutional interpretation's purported ontological neutrality has been appealed to is in connection with quantifiers for diverse grammatical categories such as predicates, $(\exists \phi) \phi a'$, sentences, $(\exists p) \ (p \supset p)'$, sentence operators, $(\exists f) \ (p f p)'$, etc.

In an earlier paper (Orenstein, 1983), I argued that on a broad use of the concept of reference (which is in keeping with ordinary usage), we could speak of various parts of speech as referring. Thus both 'is human' and 'human' might be said to refer to men. This broad use of 'refers' is generic for various word-object relations, e.g., predicates applying, names denoting singularly, common nouns denoting multiply, etc. In this paper I will abide by the strict sense of referring, according to which only singular terms refer.

What then of the claim that substitutionally interpreted quantification for grammatical categories other than singular terms is ontologically neutral? For simplicity's sake I will confine the discussion to quantifying into predicate positions.

There are different ways in which these generalizations can be interpreted substitutionally when the relevant instances are interpreted

referentially in the narrow or strict sense. Thus ' $(\exists \phi)\phi a$ ', e.g., 'something is true of Alfred', is true if at least one instance of ' ϕa ' is true (where the instance is obtained by substituting a predicate constant for the predicate variable ' ϕ '). This provides the substitutional interpretation of $(\exists \phi)$. What kind of truth condition would we provide for the instance 'Alfred is human' which could serve as a basis for the generalization. At least two alternatives with different ontological import come to mind. The ontologically more modest condition is that the sentence is true just in case the subject term refers to an object (individual) to which the predicate applies. The second interpretation requires treating all variables as having values which are referred to in the strict sense by the substituends for the variables. Hence 'is human', the substituend for ' ϕ ', would refer here to the set of humans and purportedly in the strict and narrow sense. The instance is true because the object (individual) referred to by the subject term is a member of the set referred to by the predicate. Unlike the ontologically more modest account, this condition requires sets (or on an alternative view, properties), as well as the objects (individuals) referred to by the subject term. Following Quine, we could call this the "set theory in sheep's clothing" interpretation of higher-order logic. Though substitutional, such an account is also referential and has no ontological neutrality. It can be seen as a substitutional adaptation of Quine's view that to be is to be the value of a variable, and that different styles of quantification (e.g., as in higher-order logic) ontologically commit us to different types of entities. A similar exposition could be provided showing that substitutional and referential sentential quantification, e.g., (p) $(p \supset p)$ ', would commit us to truth values or propositions.

Of course in Quine's own canonic notation, there is no place for any quantifiers save those for singular terms (all quantification is first-order

quantification), and so only Tarskian satisfaction conditions are needed (Orenstein, 1977, pp. 67–68, 95–102). For Quine, higher-order logic (predicate variables and quantifiers) is set theory in disguise and $s'(\exists \phi)\phi a'$ is misleading as to its logical form. He would find $(\exists \phi)\phi a'$ written more appropriately as $(\exists x) \ (a \in x)'$ and the instance as 'Alfred $\in \{x | x \text{ is human}\}'$. For Quinians, a misleading feature of the interpretation above is that it mistakenly treats predicates as though they were singular terms. In other words predicate quantifications treat nonreferring (in the strict sense) positions as if they were referring (in the strict sense) positions. In fact if 'referential' in 'referential quantification' is taken in the narrow and strict sense (a relation between singular terms and their referents), then by definition there can be no referential quantification for nonreferring grammatical categories such as predicates, nouns, sentences, etc.

However, some – Prior, Williams, followers of Lešniewski, et al. – have found uses for ' $(\exists \phi)\phi a$ ' and argue that it can be understood without appealing to sets of properties (Prior, 1971, and Williams, 1981). The only way I can see of doing this is to interpret the quantifiers substitutionally and then appeal to the ontologically more modest interpretation of the sentence 'Alfred is human', viz., Alfred is one of the objects to which 'is human' applies. Substitutional quantification on this view is not completely neutral, since it commits us ontologically to whatever objects that the predicate constants substituted for the

variable are true of i.e., apply to. $(\exists \phi)\phi a'$ does have existential import, but only for objects of the ontological category to which the singular term strictly refers, and of which the predicate constant is true. Thus, if the predicate constant were 'is human', commitment would be to a concrete object, while if the predicate constant were 'is odd', commitment would be to an abstract object.

Thus the claim that providing a substitutional interpretation for predicate quantifiers is ontologically neutral runs into difficulties. A truly ontologically neutral interpretation of $(\exists \phi) \phi a$ would interpret the generalization substitutionally, and then interpret the relevant instances without appealing to any word-object relation. However, such an interpretation would be nonstandard. One is left here with the difficulty discussed earlier of saying how an instance such as 'Alfred is human' that could be the basis for a substitutional generalization has a truth

value without appealing to any word-object relation.

NOTES

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¹ Quine also uses the term "objectual quantification", and as interchangable with "referential quantification". I have avoided the former locution in this paper, and in Orenstein 1983 I have suggested different uses for the two terms.

² Among the philosophers excluded here are Meinongians who appeal to referents which subsist but don't exist.

³ Professor Quine clarified his own position in correspondence with me: "You write that 'Quine's introduction of the terms "referential" and "substitutional" quantification suggests that the two kinds are mutually exclusive.' It shouldn't suggest that. They overlap. Quantification in elementary number theory is referential and substitutional; there is no difference when everything quantified over has a designator. See *Roots of Reference*, p. 114, lines 21–24; *Ways of Paradox*, enlarged edition, pp. 318–320; *Philosophy of Logic*, page 92, fourth line from bottom. Actually my view is the one you propose." (Quine is here referring to the combination of the substitutional and referential views for first-order cases discussed in this essay.)

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