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EPISTEMOLOGY AND PROBABILITY

ABSTRACT. Probability is sometimes regarded as a universal panacea for epistemology. It has been supposed that the rationality of belief is almost entirely a matter of probabilities. Unfortunately, those philosophers who have thought about this most extensively have tended to be probability theorists first, and epistemologists only secondarily. In my estimation, this has tended to make them insensitive to the complexities exhibited by epistemic justification. In this paper I propose to turn the tables. I begin by laying out some rather simple and uncontroversial features of the structure of epistemic justification, and then go on to ask what we can conclude about the connection between epistemology and probability in the light of those features. My conclusion is that probability plays no central role in epistemology. This is not to say that probability plays no role at all. In the course of the investigation, I defend a pair of probabilistic acceptance rules which enable us, under some circumstances, to arrive at justified belief on the basis of high probability. But these rules are of quite limited scope. The effect of there being such rules is merely that probability provides one source for justified belief, on a par with perception, memory, etc. There is no way probability can provide a universal cure for all our epistemological ills.

There are two divergent strains in epistemology. The "traditional" approach constructs theories of epistemic justification which proceed in terms of "basic beliefs", reasoning, coherence, etc. The resulting theories are elaborate attempts to describe the complex structure which epistemic justification appears to exhibit. In contrast to this, a number of philosophers have proposed probability as a universal panacea for epistemology. They have maintained that the rationality of belief is almost entirely a matter of probabilities. The resulting theories have the appeal of elegance. They are much simpler than the contorted theories arising out of the traditional approach. For this reason alone, the probabilistic theories have justifiably attracted a considerable following. But elegance and simplicity are not enough by themselves to warrant the adoption of an epistemological theory. The theory must also be true to the epistemological facts. In my estimation, the probabilistic theories do not withstand this test. Traditional epistemological theories are complex because the facts they seek to describe have a complicated structure. Probabilistic epistemological theories fail through not attending sufficiently to the details of that structure. By concentrating on their pretty picture of the way things ought to be rather than attending to the

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way they really are, probability theorists have been led into epistemological absurdity. In this paper I will begin by laying out some rather simple and uncontroversial features of the structure of epistemic justification, and then go on to ask what we can conclude about the connection between epistemology and probability in the light of those features.

1. THE EPISTEMOLOGICAL FRAMEWORK

The basic assumption I will make about epistemology is that reasoning plays a fundamental role in epistemic justification. We begin with some propositions which we are somehow initially justified in believing. These propositions comprise the *epistemic basis*. Then by reasoning from the epistemic basis we can (1) become justified in believing some new propositions, and (2) become justified in rejecting some propositions which we were originally justified in believing. A concrete epistemological theory will consist of an account, first, of the nature of the epistemic basis, and second, of the reasoning involved in extending and modifying the basis.

It should be emphasized that in talking about the epistemic basis, I am not assuming some version of foundationalism. What I am assuming is so general that virtually any epistemological theory can be cast in this form. A foundationalist theory is one that takes the epistemic basis to consist of a set of privileged propositions which are self-justifying in the sense that whenever you believe them you are automatically at least prima facie justified in believing them. One alternative to this would be a coherence theory which takes *all* beliefs to be prima facie justified, and hence identifies the epistemic basis with the set of all the beliefs you hold at any given time. Another alternative would be a theory which proposes that certain beliefs (e.g., about perceptible properties of physical objects) receive prima facie justification, and hence are included in our epistemic basis, by virtue of our being in various mental states. Theories of this latter sort would not require us to *believe* that we are in those mental states in order to become justified in holding the beliefs about physical objects. These examples should suffice to illustrate how little I am assuming by assuming that justified belief begins with an epistemic basis and then modifies it by reasoning.[1] For the purposes of this paper, no assumptions need be made about the epistemic basis.

I shall make some important assumptions about reasoning, however.

There are two kinds of reasons – defeasible and nondefeasible. Nondefeasible reasons are always conclusive – they logically entail what they are reasons for. Defeasible reasons are what I have called 'prima facie reasons'.[2] These are reasons which provide justification only when unaccompanied by defeaters. For example, in induction, observing a number of A's that are B's provides a reason for believing that all A's are B's, but it is a defeasible reason. If we know of another A that is not a B, that defeats the reason. In general, if P is a prima facie reason for Q, there can be two kinds of defeaters for P. *Rebutting defeaters* are reasons for denying Q in the face of p. There being an A that is not a B is a rebutting defeater for the inductive reason for believing that all A's are B's. *Undercutting defeaters* are reasons for denying that P wouldn't be true unless Q were true. For example, a reason for believing that our inductive sample may not be a fair sample is a reason for denying that we wouldn't have that inductive evidence unless the generalization were true. Defeaters are defeaters by virtue of being reasons either for $\sim Q$ or for $\sim(\sim Q > \sim P)$. They may be only defeasible reasons for these propositions, in which case we can have defeater defeaters, and defeater defeater defeaters, etc.

We can regard an *argument* as a finite sequence of propositions each of which is either in the epistemic basis or such that earlier propositions in the sequence comprise a reason for it.[3] Arguments are patterns of reasoning leading from the epistemic basis, via reasons, to conclusions. As such, arguments can justify new beliefs. They can also lead to the rejection of beliefs already held, in either of the two ways. If a belief is held on the basis of an argument, a new argument might justify a defeater for one of the reasons employed in the first argument. Alternatively, members of the epistemic basis may themselves have various kinds of defeasible statuses,[4] and arguments can lead to their rejection by supporting defeaters for them.

We can distinguish between a number of different concepts of epistemic justification. The basic concept is that of justification *simpliciter* – a person is justified in believing P just in case he has adequate reason to believe P (that is, it is in his epistemic basis, or he believes it on the basis of a good argument supporting it), and he does not have any defeaters for it at his immediate disposal. This of course is not a definition – only a rough characterization whose primary purpose is to distinguish between justification simpliciter and the other concepts I will now define. A person might be justified in believing P even though by reasoning from other propositions which he is justified in believing he could discover a defeater for P. Justification simpliciter requires only that the availability of such reasoning not be obvious. This suggests a stronger concept of justification – a person is *objectively justified* in believing P iff he is justified in believing P and his justification could not be defeated by any amount of reasoning proceeding exclusively from other propositions he is justified in believing. A third concept is that of a "justifiable" proposition, or as I will say, a *warranted* proposition. P is warranted for S iff S could become justified in believing P through reasoning proceeding exclusively from the propositions he is objectively justified in believing. A warranted proposition is one which S would become justified in believing if he were an ideal reasoner.

These three concepts of epistemic justification have importantly different logical properties. Let us say that a proposition P is a *deductive consequence* of a set Γ of propositions iff by assuming the members of Γ, deductive reasoning can lead one to the conclusion P. I will say that a set of propositions is *deductively consistent* iff it does not have an explicit contradiction as a deductive consequence. P is a *logical consequence* of Γ iff, necessarily, P is true if all the members of Γ are true. Logical consequence and deductive consequence may or may not coincide, depending upon whether all necessary truths are provable a priori. If they are not, there will be logical consequences that are not deductive consequences, and deductively consistent sets of propositions that are not logically consistent.

There is no reason to expect a person's set of justified beliefs to be either deductively consistent or closed under deductive consequence. For example, prior to the discovery of the set-theoretic antinomies, people were presumably justified in believing the (demonstrably inconsistent) axiom of comprehension in set theory. Every proposition is a deductive consequence of that axiom, but clearly people were not thereby justified in believing everything.

A person's set of objectively justified beliefs is deductively consistent. If a contradiction could be derived from it, then reasoning from some objectively justified beliefs would lead to the denial (and hence defeat) of other objectively justified beliefs, in which case they would not be objectively justified.[5] A person's objectively justified beliefs need not be closed under deductive consequence, however, for the simple reason that a person's beliefs need not be closed under deductive consequence.

The set of warranted propositions, on the other hand, is both deductively consistent and closed under deductive consequence. The set of warranted propositions is deductively consistent for the same reason the set of objectively justified propositions is deductively consistent. Turning to deductive consequence, suppose $P_1, \ldots, P_n$ are warranted for S and Q is a deductive consequence of them. Then an argument supporting Q can be constructed by combining arguments for $P_1, \ldots, P_n$ and adding onto the end an argument deducing for Q from $P_1, \ldots, P_n$. The last part of the argument consists only of deductive nondefeasible steps of reasoning. If Q is not warranted, it must be possible to reason from S's justified beliefs to some defeater for the argument supporting Q. There can be no defeaters for the final steps, which are nondefeasible, so such a defeater would have to be a defeater for an earlier step. But the earlier steps all occur in the arguments supporting $P_1, \ldots, P_n$, so one of those arguments would have to be defeated, which contradicts the assumption that $P_1, \ldots, P_n$ are warranted. Thus there can be no such defeater, and hence Q is warranted.

2. ACCEPTANCE RULES

Given this epistemological framework, let us turn to probability and inquire what relationship it might bear to epistemology. Rules telling us when it is rational to believe something on the basis of probability are called 'acceptance rules'. A number of philosophers are of the opinion that epistemological warrant is entirely a matter of satisfying probabilistic acceptance rules.[6] A more moderate view is that satisfaction of acceptance rules gives us some warranted propositions, but that there are other sources of warrant as well. I will defend such a moderate view here.

Let us begin by distinguishing between statistical and logical probability. By 'statistical probability' I mean that kind of probability about which we can learn by discovering relative frequencies, counting cases (cards, dice, etc.), and so on. I shall stipulate that statistical probabilities are (at least ordinarily) contingent. This is to distinguish them from logical probabilities. The logical probability of a proposition is supposed to be a measure of the proportion of possible worlds in which it is true. Logical probability assigns (nontrivial) a priori probabilities to propositions unconditionally (i.e., against a background of no knowledge).[7] Because I find logical probability pretty murky, I am not going to discuss it explicitly. I will confine my attention to statistical probability. It is worth noting, however, that most of the moves made below concerning statistical probability can also be made with only slight modifications for logical probability.

Confining our attention to statistical probability, there are roughly two kinds of theories of statistical probability – what I will call 'empirical' and 'subjective'. The empirical theories take their impetus from Reichenbach, von Mises, and others of that ilk, and base probability (or our knowledge of it) on relative frequencies in some fashion or other. According to empirical theories, we start with *indefinite probabilities*. An indefinite probability is the probability of an unspecified object of one sort being also of another sort. For example, we can talk about the probability of a horse being a palomino. This isn't about any particular horse. Indefinite probabilities are not probabilities of propositions. Instead, they relate classes or concepts. The probability of a proposition is a *definite probability*, e.g., the probability that some specific horse is a palomino. According to empirical theories of statistical probability, definite probabilities are inferred from indefinite probabilities by what is called 'direct inference', the details of which are problematic.[8]

Subjective theories, on the other hand, deal exclusively with definite probabilities, taking them to be measures of degree of rational belief, a quantity supposedly measured by something like the Ramsey method or the Jeffrey method. I will say more about subjective probability later in this paper.

We can also define another kind of probability. The *epistemic probability* of a proposition is the degree to which it is warranted. It is not immediately obvious whether epistemic probability can be quantified, and even if it can it is far from obvious that it will satisfy the probability calculus. Nevertheless, the concept proves suggestive. The question of what the relationship is between epistemology and probability can be reformulated as the question what the relationship is between epistemic probability and statistical probability. The simplest and initially most appealing answer to this question is that epistemic and statistical probability are identical, i.e., a proposition is warranted iff its statistical probability is sufficiently high.[9] I will call this *The Simple Rule*. Obvious though it may seem, it is now generally recognized that the simple rule immediately runs afoul of the lottery paradox.[10] Suppose you hold one ticket in a fair lottery consisting of one million tickets. Suppose it is known that one and only one ticket will win. Observing that the probability that your own ticket will win is only .000001, it seems reasonable to accept the conclusion that your ticket will not win.[11] But by the same reasoning, it will be reasonable to believe, for each ticket, that it will not win. This will make the set of warranted propositions deductively inconsistent, and we have already observed that that is impossible. Thus we cannot be objectively justified in believing of each ticket that it will not win. Hence high statistical probability cannot be a sufficient condition for warrant. It follows that statistical and epistemic probability must be distinguished from one another, and the simple rule rejected.

In the face of the preceding reasoning, Kyburg bites the bullet and concludes that the set of warranted propositions is not closed under deductive consequence. Specifically, he denies that the conjunction of two warranted propositions must be warranted.[12] This is an example of what I called 'the epistemological absurdity' into which probability theorists have been lured. As we have seen, if we begin with some very elementary epistemological observations, we are led inexorably to the conclusion that the set of warranted propositions is deductively closed and deductively consistent. Any attempt to ground epistemology on probability which conflicts with this must simply be rejected.

High statistical probability cannot automatically provide epistemological warrant for a proposition, but it seems inescapable that we sometimes take propositions to be warranted on statistical grounds. What goes awry in the lottery paradox is that there are statistical grounds both for accepting and for rejecting the conclusion that any given ticket will lose. The grounds for rejecting the conclusion consist of the grounds for accepting the conclusions that the other tickets will lose, because those conclusions jointly entail that the given ticket will not lose. This suggests that high statistical probability provides warrant *if there are no conflicting considerations*. In the lottery paradox there are conflicting considerations. To say that high statistical probability provides warrant in the absence of conflicting considerations is just to say that high statistical probability is a prima facie reason for believing a proposition. This suggestion seems plausible, and it handles the lottery paradox very nicely. It provides prima facie reasons for all propositions of the form 'Ticket *n* will not win', but these propositions jointly defeat one another and hence none of them are warranted.

In light of its ability to handle the lottery paradox, the proposal that a proposition's having high statistical probability is a prima facie reason for believing it seems initially quite appealing. But the very fact that it can handle the lottery paradox shows that it cannot work. This is best seen by considering a variant of the lottery paradox due to Keith

Lehrer.[13] Lehrer has us consider a race in which there are five horses, numbered 1 through 5, and we are told that one of the following twenty five patterns will prevail in the next race:

winner	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
second place	1	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	5	5	5	5	5	5
third place	3	3	3	4	4	5	5	2	2	4	4	5	5	2	2	3	3	5	5	2	2	3	3	4	4
fourth place	4	4	5	3	5	3	4	4	5	2	5	2	4	3	4	2	5	2	3	3	4	2	4	2	3
fifth place	5	5	4	5	3	4	3	5	4	5	2	4	2	5	3	5	2	3	2	4	3	4	2	3	2

Lehrer writes, "we should, in this example, accept that the number 1 horse will win the next race, for he virtually always does, but we should not accept anything about how the other horses will place since they place virtually randomly in the patterns supplied."[14] The difficulty is now that, given our background information, the proposition that the number one horse will win is equivalent to the proposition that one of the last twenty-four patterns will prevail, and hence has the same probability as the latter proposition. Supposing that each of the twenty-five patterns is equally probable, it follows that each disjunction of twenty-four of them is equally probable. But the set of all such twenty-four member disjunctions is inconsistent. We have a situation analogous to the lottery paradox here. But then, as we have seen, given that a proposition's being highly probable is a prima facie reason for believing it, it follows that the proposition that the number one horse will win is not warranted.

If this difficulty just concerned horse races, perhaps we could conclude that our intuitions concerning this particular case are in error and the proposition that the number 1 horse will win is not warranted after all. Perhaps all that is warranted in that case is that the number 1 horse will *probably* win. What is disturbing is that, given certain assumptions, Lehrer's race horse paradox can be generalized to show that no proposition can ever be made warranted by having any probability less than 1. This is because every case of a highly probable proposition can be turned into a case of the lottery paradox using the model of Lehrer's race horse paradox. To see this, suppose P is highly probable. Pick the smallest integer n such that $1/2^n \leq 1 - \text{prob}(P)$. Now let us suppose that there are n propositions $Q_1, \ldots, Q_n$ statistically independent of consistent truth functional compounds of each other and of P and each having probability 1/2. These might be propositions about coin tosses, for example. A *Boolean conjunction* of the $Q_1, \ldots, Q_n$ is a conjunction of n conjuncts such that for each i, the ith conjunct is either Q_i or $\sim Q_i$. There are 2^n Boolean conjunctions, each with probability $1/2^n$. Let us enumerate the Boolean conjunctions as $B_1, \ldots, B_{2^n}$. Each disjunction of the form

$$\sim P \vee \sim B_i$$

is at least as probable as P, and the set of all such disjunctions entails $\sim P$. Thus we have an instance of the lottery paradox just like the one involved in Lehrer's race horse example. Therefore, the proposal that a proposition's being highly probable gives us a prima facie reason for believing it has the consequence that if P has any probability less that 1, it is not warranted by virtue of having high probability.

At this point it is tempting to conclude that the traditional view is just wrong and that high statistical probability never gives us a reason for believing a proposition. Perhaps all we are justified in believing on probabilistic grounds is that various propositions are *probable*. But it seems to me that there is at least one clear case in which we must be warranted in believing propositions on probabilistic grounds. A great deal of what we believe and regard ourselves as warranted in believing comes to us through the testimony of others. We do not regard such testimony as infallible. That is, we do not believe the contents of such testimony on the basis of inductive generalizations of the form 'All testimony of such-and-such a kind is true'. Rather, we believe that *most* testimony of certain kinds is true. This seems an unmistakeable case in which we reason from 'Most A's are B's' to 'This A is a B'. Such reasoning must sometimes be permissible. And presumably to say that most A's are B's is to say that prob(Bx/Ax) is high.

Although 'Most A's are B's' is sometimes a reason for 'This A is a B', it is clearly a *defeasible* reason. Thus we are led to the conclusion that 'Most A's are B's' is sometimes a prima facie reason for 'This A is a B', but not always. There must be some restrictions on its applicability. Discovering the nature of those restrictions is the central problem for constructing a correct acceptance rule. At this point I want to make a tentative suggestion regarding those restrictions. My suggestion is that 'Ac and most A's are B's' is a prima facie reason for believing 'Bc' provided B is *projectible with respect to* A.[15] On the assumption that 'Most A's are B's' can be rewritten as 'prob$(Bx/Ax) > r$' for suitable r, I actually want to propose two related acceptance rules:

(A1) If B is projectible with respect to A, then 'Ac & $\text{prob}(Bx/Ax) > r$' is a prima facie reason for believing 'Bc'.

(A2) If B is projectible with respect to A, then 'Ac & $\text{prob}(Bc/Ac) > r$' is a prima facie reason for believing 'Bc'.

The difference between these two rules is that (A1) proceeds in terms of indefinite probabilities, and (A2) in terms of definite probabilities. Because I am not a subjectivist, I believe that our basis for evaluating definite probabilities is ordinarily direct inference from indefinite probabilities, in which case (A1) and (A2) go more or less hand in hand.

(A1) and (A2) handle the racehorse paradox, and distinguish between it and the original version of the lottery paradox, in a manner congenial to intuition. In the lottery paradox, we do not want to be able to conclude of any particular ticket that it will not win, but in the racehorse example we do want to be able to conclude that horse number 1 will win. It seems to me that the difference between these two cases has to do with the potential defeaters for each conclusion. In the lottery, the conclusion 'Ticket *n* will lose' conflicts with other propositions of the form 'Ticket *m* will lose', and the predicate involved in each, viz., 'will lose', is projectible with respect to 'is a ticket in this lottery'. Thus we have a prima facie reason for believing each of these propositions, and so they defeat one another.

Turning to the racehorse example, 'Horse number 1 will win' conflicts with 24-member disjunctions. We observe that the probability of each 24-member disjunction is .96, which is very high. But (A2) only gives us a prima facie reason for believing the disjunction if the disjunctive predicate describing the disjunction of patterns is projectible, and there is no reason to think that it is. Disjunctive predicates are not generally projectible. More precisely, the class of predicates projectible with respect to a given predicate is not closed under disjunction.[16] Thus we are blocked from acquiring prima facie reasons for believing the 24-member disjunctions. On the other hand, although the proposition that the number 1 horse will win is equivalent to a 24-member disjunction, it need not be expressed in that way. The predicate 'will win' is projectible, so by (A2) we have a prima facie reason for believing that the number 1 horse will win. Unlike the case of the lottery, we have no reasons to believe the propositions with which this proposition conflicts, so we are justified in believing it.

Thus far, the only reason I have given for endorsing (A1) and (A2) is that they seem to work. But it is a bit surprising that the required constraint should be one of projectibility. Projectibility has to do with induction. Why should it have anything to do with acceptance rules? To my surprise, I have recently discovered what seems to be strong confirmation that this is the correct constraint. I have discovered that it is possible to derive (and thereby justify) principles of enumerative induction from this acceptance rule together with some related acceptance rules and a theory of direct inference.[17] The derivation is such that any constraints on the acceptance rules are carried over to the principles of induction. The constraint on the principles of induction is precisely that of projectibility, so that must also be the constraint on the acceptance rules. In effect, the projectibility constraint of induction is *explained* by observing that it follows from the projectibility constraint on these acceptance rules. Projectibility has first and foremost to do with acceptance rules, and plays only a derivative role in induction.

Although the discovery of a correct acceptance rule is of considerable importance, what is really of more interest is how little role probability has in epistemology according to the foregoing account. It has no central role at all. Statistical probability is just one source of prima facie reasons for beliefs, on a par with perception, memory, etc. This conclusion will be unsatisfying for many philosophers, particularly subjectivists, so it is worth considering just why philosophers have felt that probability should play a more profound role in epistemology.

3. EPISTEMIC PROBABILITY

We defined the epistemic probability of a proposition to be the degree to which it is warranted. Of course, this is relative to a particular person. It is clear that comparative judgments of epistemic probability are often possible, even though it is not so clear whether epistemic probability can be measured in terms of real numbers, and whether it will satisfy the probability calculus if it is real-measurable. Despite these unclarities, I think it is often epistemic probability that we are talking about when we talk about how probable something is. Because this is a common meaning of 'probable', it is natural to suppose that probability plays a central role in epistemology. Epistemic probability does play a central role, but statistical probability does not. It was by confusing epistemic and statistical probability that philosophers were led to suppose statistical probability to play a central role.

The simplest way to see that epistemic and statistical probability must be distinct is by observing that it follows trivially from the definition of epistemic probability that the simple rule is satisfied, i.e., a proposition is warranted iff its epistemic probability is sufficiently high. But the simple rule fails for statistical probability. What is going on here that makes it possible to endorse the simple rule for epistemic probability but not for statistical probability? Why can't the arguments we employed in connection with statistical probability be repeated for epistemic probability? Those arguments assumed the probability calculus, so it is natural to suppose that the reason they cannot be reproduced for epistemic probability is that epistemic probability does not satisfy the probability calculus. But when those arguments are applied to epistemic probability, they fail for a different reason which has little to do with the probability calculus. They turn essentially on the lottery paradox and presuppose that there can be a set of mutually inconsistent propositions of arbitrarily high statistical probability. Such sets of propositions arise from lotteries and other chance occurrences. But it is not possible for there to be a set of mutually inconsistent propositions of arbitrarily high epistemic probability. If a set of propositions is demonstrably inconsistent, then they cannot all be highly warranted, and hence cannot all have high epistemic probability. Thus it seems that the reason the simple rule can be endorsed for epistemic probability is not that the probability calculus fails, but rather that it is not possible to construct an analogue of the lottery paradox for epistemic probabilities.

Nevertheless, it is of some interest to inquire whether epistemic probabilities do satisfy the probability calculus. Let us suppose, for the sake of argument, that epistemic probabilities are real-measurable. It is quite easy to see that there is one respect in which they do not satisfy the probability calculus. It is a theorem of the probability calculus that if P and Q are logically equivalent then they have the same probability. But all that is true of epistemic probabilities is that if P and Q are deductively equivalent (i.e., each is a deductive consequence of the other) then they are equally warranted and hence have the same epistemic probability. If there can be propositions which are logically equivalent but not deductively equivalent, then they can have unequal warrant and the probability calculus will fail in this respect when applied to epistemic probabilities. But this is a minor failing. We can easily weaken the probability calculus in this respect without affecting much of anything else.

Given one reasonable assumption, it is nevertheless possible to show that epistemic probability does not satisfy even the weakened probability calculus (i.e., the probability calculus with the weakened equivalence axiom). To say that epistemic probability does satisfy the weakened probability calculus is to say that, necessarily, there is a function prob which satisfies three conditions: (1) prob satisfies the weakened probability calculus. (2) Given any propositions P and Q, P is at least as highly warranted as Q iff $\text{prob}(P) \geq \text{prob}(Q)$. (3) A proposition is warranted (symbolized '$\mathbf{W}(P)$') iff it has a sufficiently high probability. This third condition can be made precise in either of two ways: (3a) there is an $r \leq 1$ such that $\mathbf{W}(P)$ iff $\text{prob}(P) \geq r$; (3b) there is an $r < 1$ such that $\mathbf{W}(P)$ iff $\text{prob}(P) > r$.

The assumption needed to establish that epistemic probability does not satisfy the weakened probability calculus is developed as follows. Suppose P and Q are two propositions which are such that each is precisely as warranted as its negation. In other words, for each we have no reason for believing either that it is true or that it is false. If P and Q are suitably independent of one another, and we do not know of any contingent relations between them, then each of $(P \& Q)$, $(P \& \sim Q)$, $(\sim P \& Q)$, and $(\sim P \& \sim Q)$ should be equally warranted. What I assume is that it is possible to have an arbitrarily large finite set of propositions satisfying these conditions. More precisely, I assume the following 'multiplicity' condition:

(mult) For each $n > 0$, it is possible that there are propositions Q, $P_1, \ldots, P_n$ such that $\sim\mathbf{W}(Q) \& \sim\mathbf{W}(\sim Q)$ and if B_i and B_j are any two Boolean conjunctions of the P_i's, $(Q \& B_i)$ and $(Q \& B_j)$ are equally warranted.

I have already defended adjunctivity:

(adj) $\Box(\forall P)(\forall Q)[\mathbf{W}(P) \& \mathbf{W}(Q). \rightarrow \mathbf{W}(P \& Q)]$.

Now suppose epistemic probabilities satisfy the probability calculus, and let r be the number involved in (3). I will employ the (3a) formulation of (3), but the argument for the (3b) formulation is similar. Let N be the least n such that $1 - r \geq 1/2^n$. Choose $Q, P_1, \ldots, P_N$ in accordance with (mult). Suppose that $0 < i \leq N$ and $\mathbf{W}(\sim(Q \& B_i))$. Then by (3a), $\text{prob}(\sim(Q \& B_i)) \geq r$, and so by (2), for each j, $\text{prob}(\sim(Q \& B_j)) \geq r$. Hence by (3a), for each j, $\mathbf{W}(\sim(Q \& B_j))$. Then by (adj), $\mathbf{W}(\prod_j \sim$

$(Q \ \& \ B_j))$.[18] So $\text{prob}(\prod_j \sim(Q \ \& \ B_j)) \geq r$. But $\prod_j \sim(Q \ \& \ B_j)$ is demonstrably equivalent to $\sim Q$, so $\text{prob}(\sim Q) \geq r$, and hence $\mathbf{W}(\sim Q)$, which contradicts the choice of Q. Hence for each i, $\sim\mathbf{W}(\sim(Q \ \& \ B_i))$. By (3a), $\text{prob}(\sim(Q \ \& \ B_i)) < r$, so $\text{prob}(Q \ \& \ B_i) \geq r$. Q is demonstrably equivalent to $\sum_j (Q \ \& \ B_j)$, so

$$\text{prob}(Q) = \sum_{j=1}^{2^N} \text{prob}(Q \ \& \ B_j) \geq 2^N(1-r) \geq 1.$$

So $\text{prob}(Q) \geq r$, and hence $\mathbf{W}(Q)$, contrary to assumption. Therefore, epistemic probability cannot satisfy the weakened probability calculus.

I have just argued that epistemic probabilities do not satisfy the probability calculus, but there is a well-known argument which purports to establish that they do. That is the Dutch book argument. According to the Dutch book argument, if when presented with a set of propositions and required to bet on the likelihood of their being true a person did not compute his odds in accordance with the probability calculus, then a wily opponent could place a set of bets which the person would be bound to lose no matter what. According to the argument, it would be irrational to place oneself in such a situation, so one's estimates of the likelihoods of various propositions should conform to the probability calculus.

It seems to me that there is a fatal flaw in the Dutch book argument. That argument confuses epistemic and statistical probabilities. A person's estimate of the likelihood of a proposition's being true is his estimate of its statistical probability, not its epistemic probability. We have seen that the two notions cannot be the same, and it seems clear that the notion that is relevant to betting is the chance notion of statistical probability. Another way of putting basically the same point is that the Dutch book argument confuses epistemic and prudential rationality. This way of putting it is due to Chihara and Kennedy.[19] The question of how good are one's epistemic reasons for believing something is quite different from the question whether it is to one's advantage to believe it, and that in turn is different from the question whether it is to one's advantage to bet on it, but the Dutch book argument seems to run all three of these questions together. Thus I do not think that the Dutch book argument can be regarded as establishing that epistemic probabilities must conform to the probability calculus.

4. SUBJECTIVE PROBABILITY

I have argued that we must distinguish between epistemic and statistical probability. That conclusion is going to be anathema to subjectivists, because subjective probability is explicitly intended to play both roles. Subjectivists say that by 'probability' they mean either 'degree of belief' or 'degree of rational belief'. The latter is ambiguous between 'degree of rationality of belief', i.e., epistemic probability, and 'degree to which one ought to believe', which may or may not be the same thing. Subjectivists go on to tell us how to measure the subjective probability of a proposition in terms of the odds one would accept on a bet on its truth. One of the greatest apparent virtues of subjective probability, and no doubt one of the main sources of its great popularity, has been its apparent ability to give us actual numerical values for probabilities. Other approaches to probability tend to flounder on precisely this point. Subjective probability seems justifiably popular, but received opinion regarding it is incompatible with our conclusions about the distinction between statistical and epistemic probability, so let us examine it more closely.

The subjectivist begins by observing that people can believe a proposition more or less firmly – that they can have different degrees of belief in a proposition. The subjectivist proposes that a person's degree of belief in a proposition can be measured in terms of how much he is willing to risk on a bet that that proposition is true. More precisely, the subjectivist proposes that a person's degree of belief in a proposition P can be identified with the largest number r such that he would accept odds of r to $1 - r$ on a bet that P is true. Some subjectivists go on to identify subjective probability with degree of belief, but most have not on the grounds that ordinary people's actual degrees of belief will not normally satisfy the probability calculus. Instead, they define the subjective probability of a proposition to be the degree of belief one rationally *ought* to have in the proposition.

I cannot resist interpolating one objection here. If subjective probability is identified with the degree of belief one ought to have rather than the degree of belief one actually has, then ascertaining the betting quotients that people would accept on various propositions does not straightforwardly give us the subjective probabilities of those propositions. Unless the betting quotients that people would actually accept uniquely determine the betting quotients they ought to accept, subjective probability is not determined by the actual betting quotients. I see no good reason to think that the betting quotients people ought to accept are determined by the betting quotients they actually accept, but even if they are, the subjectivist has given us no recipe for getting the one out of the other. And without such a recipe, it just isn't true that the subjectivist can actually give us values for probabilities where other approaches to probability fail to do that. This supposed virtue of subjectivism is a fraud. Subjectivism has no easier a time ascertaining the values of probabilities than does any other theory of probability.

Returning to our main point, the definition of subjective probability as the degree to which one ought to believe a proposition is subject to an immediate difficulty.[20] In talking about degree of belief, we start with the notion of belief and then observe that belief comes in degrees. Talk of belief simpliciter is talk of believing something to a certain minimal degree. Let us call this degree k. Trivially:

> S should believe P to at least degree k iff S should believe P to at least degree k.

The left side of this biconditional is equivalent to saying that S should believe P. But by the definition of subjective probability, the right side is equivalent to saying that $\text{prob}(P) \geq k$. Thus the whole biconditional is equivalent to:

> S should believe P iff $\text{prob}(P) \geq k$.

This is just the simple rule. The simple rule becomes trivially analytic, but we have agreed that it is false. Even most subjectivists do not want to endorse the simple rule.

There are three possible responses to this argument. One could deny that belief simpliciter makes sense, one could give an alternative account of the relationship between degrees of belief and belief simpliciter, or one could give up on subjective probability. A few philosophers have pursued the first alternative, maintaining that we can never talk about simply believing something, but only about believing it to various degrees. If this is really meant to imply that we don't believe things, it is outrageous, but it is also unhelpful. We can generate precisely the same problem just talking about believing something firmly. Thus the only real way out for the subjectivist is to deny that believing something to at least degree k is the same thing as believing it simpliciter. If we understand 'degree of belief' in terms of our ordinary notion of how firmly we believe something, there is no way to avoid this identification. But Carnap and Jeffery both profess to be using 'degree of belief' as a technical term only loosely connected with our ordinary notion.[21] 'Degree of belief' is taken to refer to a person's disposition to take risks associated with a proposition's being true. Degree of belief becomes, *by definition*, what is measured by betting quotients, i.e., it is the largest r such that a person would accept odds of r to $1-r$ on a bet that the proposition is true.

Given this reconstrual of degrees of belief, the preceding problem dissolves, but so does any apparent connection between subjective probability and epistemic probability. *Perhaps* the degree to which one is warranted in believing a proposition can be identified with how firmly he ought to believe it, but it definitely cannot be identified with the betting quotient he ought to accept on the proposition. Betting has to do with statistical probability, not epistemic probability, and as such subjective probability can only be a candidate for statistical probability, not epistemic probability. There is no more reason to think that subjective probability is going to play a fundamental role in epistemology than there is for thinking that any other concept of statistical probability will play a fundamental role.

5. RECONSIDERATION OF THE ROLE OF ARGUMENTS

The principal conclusion that I have defended so far is that statistical probability really doesn't have much to do with epistemology. That conclusion has been based upon the epistemological framework that I defended in section one. The only way to avoid the conclusion is to reject the framework. The main epistemological assumption from which everything else follows concerns the role of arguments in epistemological warrant. I have assumed that reasoning is a step-by-step process proceeding in terms of arguments and transmitting warrant from one step to the next in the argument. From this it follows that warrant is closed under deductive consequence, and that is the main thing we need in order to show that epistemic and statistical probability must diverge. An unrepentant probabilist might dig in his heels at this point and insist that arguments do not transmit warrant without attenuation. He could maintain that each step of reasoning diminishes warrant to some extent, in accordance with the probability calculus, and if the reasoning gets too long (e.g., if it conjoins too many propositions) then the degree of warrant will drop below the critical threshold required for warrant. This position would have the consequence that warrant is not closed under deductive consequence, and might make it possible to hold that statistical and epistemic probability are the same thing.

My view is that an argument transmits warrant without attenuation. This is the view that an argument is as good as its weakest link. The probabilist wants to deny this. The probabilist picture of arguments would be that the initial premises have a certain probability, and associated with each step of reasoning is a function which, when applied to the probability of the premises of that step, yields a somewhat diminished probability for the conclusion. For example, if the argument infers $(P \& Q)$ from P and Q separately, and P and Q are statistically independent, then the probability assigned $(P \& Q)$ will be $\text{prob}(P) \cdot \text{prob}(Q)$. Although this seems like an initially plausible way of looking at arguments, it has consequences which, I think, are totally implausible.

One implausible consequence is that the probabilities would diminish so rapidly as we went through an argument that we could not employ any argument more than two or three lines long. In general, if Q is a deductive consequence of $P_1, \ldots, P_n$, the probability of Q can be anything from 0 to 1. But if $P_1, \ldots, P_n$ are statistically independent of one another, and they are all actually involved in getting Q, then it seems reasonable to expect that $\text{prob}(Q)$ will be approximately $\text{prob}(P_1) \cdot \ldots \cdot \text{prob}(P_n)$ "on the average". But then starting with premises having probabilities of .9, and employing only reasons which proceed from two premises at a time, we could expect the conclusion of a four line argument to have a probability no greater than .65, and the conclusion of a seven line argument would have a probability less than .5. Arguments would be of virtually no use for acquiring warranted beliefs. But that is absurd.

It seems to me that the probabilist misconstrues the way in which different propositions come to have different degrees of warrant attached to them. This comes about in two ways. First, members of the epistemic basis may be believed with different degrees of justification.[22] Second, some reasons are better than other reasons. If we have only a rather weak reason for believing a proposition, then we regard the degree of warrant of that proposition as correspondingly low. But when we assess the degree of warrant of the conclusion of an argument, we do not somehow combine all of these considerations in some fancy mathematical calculation to arrive at a computed degree of warrant for the conclusion. Rather, we regard an argument as being as good as its weakest link. We take the degree of warrant of the conclusion to be the minimum of the warrant of the initial premises and the "goodness" of the individual reasons. Making the argument longer does not diminish the warrant of the conclusion as long as we employ equally good reasons throughout. This seems to me to be an accurate description of the way we actually regard arguments, but we can go further and present considerations which strongly suggest that this *must* be the way we regard arguments. These considerations are due mainly to Gilbert Harman.[23] The essential observation is that if we consider reasons which proceed from multiple premises to a conclusion, the probability of that conclusion is not a function merely of the probabilities of the premises. It also depends upon a "mixing factor" which has to do with how the premises are related to one another. For example, consider adjunction. In computing the value of prob(P & Q), it is not sufficient to know the values of prob(P) and prob(Q). We must also know the value of the conditonal probability prob(Q/P). If we were to reason in accordance with the probabilistic model, in order to compute the probability of the conclusion at each step of the reasoning we would have to somehow have all of these mixing factors at our disposal. For example, for any two propositions in our "inferential repertoire", we would have to know the conditonal probability of one on the other. There is no way to compute these conditional probabilities from the unconditional probabilities, so it seems that we would have to have them innately stored in some way which would make them accessible to us a priori. This means that for n propositions, we must store 2^n conditional probabilities. As Harman observes, given just 300 propositions, which is a very small number, this would require storing 10^{90} probabilities. It has been estimated that there are just 10^{78} atoms in the entire universe. It is clear that this cannot be the way we assess warrant.

Let me close by considering two possible responses to the argument. First, I have heard it protested that the subjectivist is immune to this argument, because he does not have to store the conditional probabilities at all. He can, in effect, just make them up as he goes along because they measure whatever his degree of belief actually is. Subjectivists often play fast and loose with the distinction between actual degree of belief and rational degree of belief, and that is what is going on here. Subjective probability cannot reasonably be defined as actual degree of belief, because that will not normally satisfy the probability calculus. It must be defined as rational degree of belief, but as we have seen, the connection between rational degree of belief and actual degree of belief is not simple. It is true, I guess, that the subjectivist is faced with no storage problem for actual degrees of belief, but that does not provide him with any simple solution to our problem. Rational degrees of belief are another matter altogether, and there is no reason to think that the problem of computing the rational degree of belief in a conclusion from the rational degrees of belief in the premises is going to be any easier for the subjectivist than it is for anyone else.

An alternative response to our problem would be to agree that we cannot employ an argument unless we have the requisite mixing factors at our disposal, and acknowledge that we do not generally have them, but then go on to conclude that when we do not have them we just cannot use the argument. For example, I can imagine someone who holds views similar to Kyburg's saying this. But surely the conclusion is absurd. Suppose we are warranted in believing six contingent propositions, and we have a purely deductive argument showing that another proposition follows from them. Suppose the validity of the argument is not in doubt. Everyone agrees that it is valid. Under these circumstances we would not hesitate in drawing the conclusion. Our not knowing the conditional probabilities of the premises on one another would not be regarded as a relevant objection. Thus this does not seem to be a satisfactory way of avoiding the difficulty.

It seems that any attempt to compute the degree of warrant of a conclusion in a more complicated way than by identifying it with the strength of the weakest link of the argument is doomed to fail for reasons of the general sort I have been considering. That is why we regard an argument as being as good as its weakest link, and it explains why any attempt to ground epistemology on probability is bound to fail. There is no way that probability can provide us with a universal cure for all our epistemological ills.

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NOTES

[1] For further discussion of the variety of possible epistemological theories and how they can be cast in this form, see my 'A Plethora of Epistemological Theories', in *Justification*

and Knowledge, ed. George Pappas, D. Reidel, Dordrecht, 1979.
[2] In *Knowledge and Justification*, Princeton, 1974.
[3] In most cases, reasons can be regarded as individual propositions, but to accommodate the general case we must regard reasons as sets of propositions. For example, in modus ponens our reason consists of the set $\{P, (P \rightarrow Q)\}$ of propositions.
[4] For example, the members of the epistemic basis might be *prima facie justified* in either of at least two senses:

(PF_1) Necessarily, if S believes P then S is justified in believing P unless he has a reason for believing $\sim P$.

(PF_2) Necessarily, if S believes P then he is justified in believing P unless he has a reason for denying that he wouldn't believe P unless it were true.

[5] I am assuming here that the set of *S*'s nondefeasibly justified propositions is deductively consistent. *P* is nondefeasibly justified for *S* iff *P* is supported by an argument for which (1) all the reasons employed are conclusive reasons, and (2) all members of the epistemic basis to which the argument appeals are nondefeasibly justified. Thus this assumption is equivalent to assuming that the set of nondefeasibly justified propositions in *S*'s epistemic basis is deductively consistent. If there are any such propositions, it seems clear that they will be extremely weak. E.g., they might be about how one is appeared to. On any reasonable epistemological theory, the set of such propositions could not be inconsistent.
[6] E.g., Henry Kyburg, jr., Isaac Levi, Keith Lehrer, Richard Jeffrey.
[7] The point of the "nontrivial" qualification is to ensure that theories like Kyburg's which take probabilities to be intervals are regarded as theories of statistical probability.
[8] But see my 'A Theory of Direct Inference', *Theory and Decision* **15** (1983), 29–96.
[9] This traditional view was endorsed, for example, in Chisholm, *Perceiving*, Cornell, 1957, 155; and Hempel, 'Deductive-Nomological and Statistical Explanation', *Minnesota Studies in the Philosophy of Science*, vol. III, ed. Herbert Feigl and Grover Maxwell, University of Minnesota Press, 1962, 155. More recently it has been endorsed by Kyburg in *The Logical Foundations of Statistical Inference*, D. Reidel, Dordrecht, 1974.
[10] This is due to Kyburg.
[11] This assumes that a probability of .999999 is sufficient for warrant. If not, just increase the number of tickets.
[12] 'Conjunctivitis', in *Induction, Acceptance, and Rational Belief*, ed. Marshall Swain, Reidel, 1970.
[13] 'Coherence and the Racehorse Paradox', in *Midwest Studies in Philosophy*, vol. V, ed. Peter French, Theodore Uehling, Jr., and Howard Wettstein, 183–92.
[14] *Ibid*, 186.
[15] *B* is projectible with respect to *A* iff knowing of *A*'s that are *B*'s gives us a prima facie reason for beliving that any *A* would be a *B*.
[16] This is pointed out in *Knowledge and Justification*, 233. The reason, briefly, is as follows. Suppose we have a test to determine whether an *A* is a *B*, and another test to determine whether an *A* is a *C*, but performing either test precludes performing the other. Thus we cannot test the same *A* for both *B*-hood and *C*-hood. On the basis, we observe that a number of *A*'s are *B*'s, and that a number are not. We also observe that a number of *A*'s are *C*'s, and a number are not. Clearly, these observations give us no reason to believe that any *A* would be either a *B* or a *C*. But we have observed a number of *A*'s that are $(B \vee C)$, and

none that are not, so if $(B \vee C)$ were projectible with respect to A, this would confirm that any A would be a $(B \vee C)$.

[17] See 'A Solution to the Problem of Induction', *Noûs*, forthcoming.

[18] 'Π' symbolizes conjunction. Similarly, in logical contexts, 'Σ' symbolizes disjunction, and in arithmetical contexts 'Σ' symbolizes addition.

[19] Kennedy and Chihara, 'The Dutch Book Argument: Its Logical Flaws, Its Subjective Sources', *Philosophical Studies* **36** (1979), 19–34.

[20] The following argument was suggested to me by an argument due to Mark Kaplan, 'Rational Acceptance', *Philosophical Studies* **40** (1981), 129–47, although his argument is somewhat different and has a different conclusion.

[21] Carnap, 'The Aim of Inductive Logic', *Logic, Methodology, and the Philosophy of Science*, ed. Ernest Nagel, Patrick Suppes, and Alfred Tarski, Stanford University Press, 1962, 303–318; Richard Jeffery, 'Dracula Meets Wolfman: Acceptance vs. Partial Belief', in *Induction, Acceptance, and Rational Belief*, 161.

[22] Precisely how that works depends upon one's specific view of how the epistemic basis works, and I am trying to remain neutral on that.

[23] 'Reasoning and Explanatory Coherence', *American Philosophical Quarterly* **17** (1980), 151–58.