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Vacuum Energy in a Friedmann-Lemaître Cosmos

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The age of our Galaxy has been derived from two different sets of Th/U ratio measurements in meteorites to be 20.8 (+2, −4) or (17.6 ±4)·10^9 years [28, 27]. This implies a lower limit for the age of the universe of 14·10^9 years if we accept the method as reliable cosmic chronometer.

A consequence of this is that solutions of the Einstein-Friedmann equations with zero cosmological constant Λ are only possible if the present Hubble parameter $H_0 < 70$ km s^{-1} Mpc^{-1} (see Fig. 4 of Part I of our review [2]).

On the other hand derivations of the Hubble parameter from observations span a range from 50 (+10, −7) [24] to 100 (±10) km s^{-1} Mpc^{-1} [4]. At the Patras meeting of the International Astronomical Union the value of 75 was favoured.

The acceptance of this value would imply that solutions of the Friedmann equations require values of the cosmological constant Λ which correspond to an equivalent constant vacuum density $\rho_v > \rho_0$, where ρ_0 is the present matter density of the universe.

This situation has prompted us to reinvestigate solutions of the cosmological equations with nonzero Λ.

An average value of the present density of the baryonic matter $\rho_{0,B} = 0.5\cdot 10^{-30}$ g·cm^{-3} has been derived from the primordial ^{4}He and ^{2}H abundances. This value is independent of the special choice of the Hubble parameter.

Quantum field theory permits to interpret Λ as vacuum energy density ε_v:

$$\varepsilon_v = \rho_v c^2 = \frac{c^4}{8\pi G}\Lambda \tag{1}$$

where ρ_v is the equivalent vacuum density, c = speed of light, G = gravitational constant.

A nonzero vacuum energy contributes to the energy-momentum tensor in the most general Einstein equation [17, 30]:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^2} T_{\mu\nu} + \Lambda g_{\mu\nu}. \tag{2}$$

The identification of the $\Lambda g_{\mu\nu}$ term with the vacuum energy density was proposed by [18] and [31]. Gliner [13, 12] derived the equation of state of vacuum matter with pressure $p_v = -\rho_v c^2$ and calculated a model of a vacuum dominated universe with exponential inflation.

It is remarkable that Albert Einstein already in 1920 [6] anticipated the "vacuum state" in his discussion about the physical properties of empty space ("leerer Raum"). He named it the "new ether of General Relativity Theory":

"Der Äther der allgemeinen Relativitätstheorie ist ein Medium, welches selbst aller mechanischen und kinematischen Eigenschaften bar ist, aber das mechanische (und elektromagnetische) Geschehen mitbestimmt. Dieser Äther darf nicht mit den für ponderable Medien charakteristischen Eigenschaften ausgestattet gedacht werden, aus durch die Zeit verfolgbaren Teilen zu bestehen; der Bewegungsbegriff darf auf ihn selbst nicht angewandt werden."

With the definition of the vacuum density ρ_v in Eq. (1), one can write the Einstein equations for a homogeneous, isotropic universe with $k = -1$, 0 or +1 in the following form:

$$\left(\frac{\dot R}{R}\right)^2 = \frac{8\pi G}{3}(\rho(t) + \rho_v) - \frac{kc^2}{R^2} \tag{3}$$

and

$$\frac{\ddot R}{R} = -\frac{4\pi G}{3}(\rho(t) - 2\rho_v). \tag{4}$$

Here is $\rho(t)$ the density of pressure-free matter ("dust-model"). Models with $\rho_v > 0$ have a point of inflection (R_*, t_*) in the scale factor function $R(t)$. Equation (4) with $\ddot R = 0$ leads immediately to

$$\rho_* = \rho(t_*) = 2\rho_v. \tag{5}$$

The present value of the Hubble parameter

$H_0 = \frac{\dot R(t_0)}{R(t_0)}$ defines the critical density

$$\rho_{cr} = \frac{3H_0^2}{8\pi G}.$$

The deceleration parameter q_0 is given by

$$q_0 = -\frac{\ddot R(t_0)\cdot R(t_0)}{(\dot R(t_0))^2} = -\frac{1}{\rho_{cr}}\left(\rho_v - \frac{\rho_0}{2}\right) \tag{6}$$

and the present scale factor

$$R_0 = R(t_0) = \frac{c}{H_0}\sqrt{\frac{k}{\frac{\rho_0+\rho_v}{\rho_{cr}} - 1}}. \tag{7}$$

If we restrict the discussion at first to flat models ($k=0$), an analytical solution can be obtained by elementary integration:

$$\left(\frac{R(t)}{R_*}\right)^2 = \operatorname{Cosh}\left(\frac{t}{\tau}\right) - 1 = \frac{2\rho_v}{\rho(t)}. \tag{8}$$

The time scale τ depends on the constant vacuum density only:

$$\tau = \left(\sqrt{24\pi G\rho_v}\right)^{-1} = \frac{1{,}413\cdot 10^{-5}}{\sqrt{\rho_v}}\ \text{years} \tag{9}$$

(ρ_v in g·cm^{-3}).

The time t_* of the point of inflection is given by

$$t_* = \tau \cdot \mathrm{ArCosh}\, 2 = 1.317 \cdot \tau. \qquad (10)$$

In the flat models the vacuum density ρ_v follows from Eq. (3):

$$\rho_v = \rho_{cr} - \rho_0. \qquad (11)$$

ρ_0 is the present value of the matter density. The present age of the universe t_0 is obtained from Eq. (8):

$$t_0 = \tau\, \mathrm{ArCosh}\left(1 + \frac{2\rho_v}{\rho_0}\right). \qquad (12)$$

From this set of equations the parameters for ($k=0$)-models are derived for four selected values of the Hubble parameter H_0 and for $\rho_0 = 0.5 \cdot 10^{-30}\ \mathrm{g \cdot cm^{-3}}$. The results are given in Table 1.

For the value $H_0 = 75\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ the scale factor $R(t)/R_0$ as function of time corresponding to $\rho_v = 10.1 \cdot 10^{-30}\ \mathrm{g \cdot cm^{-3}}$ is given in Fig. 1. For the same values of Hubble parameter and present matter density the acceptable range of cosmological models with spherical ($k = +1$) and hyperbolic ($k = -1$)-metric has also been considered. The parameters for two selected models are given in Table 2. The corresponding scale factor curves are given in Fig. 1.

The ($k = -1$)-model with $q_0 = 0$ was selected because it has such a low vacuum density $\rho_v = \frac{1}{2}\rho_0$ that its scale factor function is barely distinguishable from the $\Lambda = 0$-model with the same present matter density. In this model t_* coincides with t_0. In our context this model can be considered as the case with the lowest acceptable age if we compare it with the meteoritic age.

As an example of a ($k = +1$)-model we have chosen $q_0 = -1.0$. Its age is $t_0 = 22 \cdot 10^9$ years. It is a closed model with perpetual expansion. This model and the ($k=0$)-model agree very well with the average cosmic age values from the Th/U analysis.

For comparison, the Friedmann models with $\Lambda = 0$ are given on the left-hand side in Fig. 1 for four selected values of the present matter density from $\rho_0 = 1.0$ to $10.6 \cdot 10^{-30}\ \mathrm{g \cdot cm^{-3}}$. They would correspond to the case that for instance neutrinos could provide a significant amount of "missing mass". In the lower right corner the age range of our Galaxy from the Th/U ratio is marked.

Solutions of the Friedmann equations with $\rho_v > \rho_0$ are required if $H_0 > 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and if the age of the universe $t_0 > 14 \cdot 10^9$ years. Models with $\rho_v \ll \rho_0$ cannot be distinguished observationally from ($\Lambda = 0$)-models today.

In Fig. 2 we present the run of model parameters for six selected values of the Hubble parameter between 40 and 100 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and a range of the values of the present total matter density ρ_0 between 0.02 and

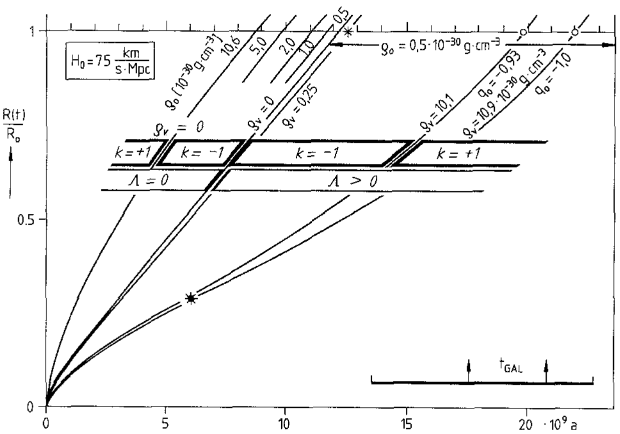

Fig. 1. The scale-factor R/R_0 versus cosmic time t for models with positive vacuum densities $\rho_v(\Lambda > 0)$ and $k = +1$, 0 and -1 for the present matter density $\rho_0 = 0.5 \cdot 10^{-30}\ \mathrm{g \cdot cm^{-3}}$ and $H_0 = 75\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. The age of our Galaxy from Th/U is marked in the lower right corner. For comparison the ($\Lambda = 0$)-models are given in the left part of the figure for five selected values of present matter density

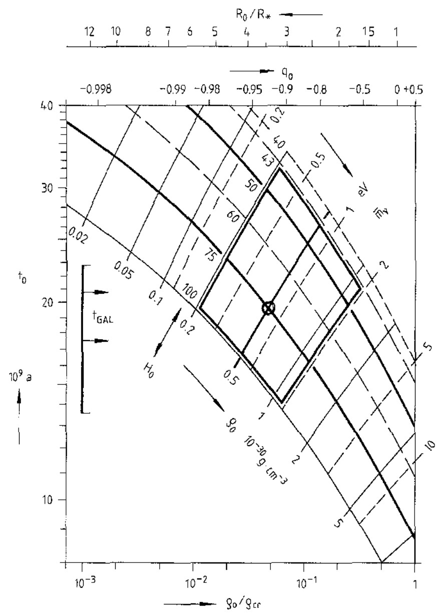

Fig. 2. The cosmic age t_0 as function of the present matter density and the present Hubble parameter for models with euclidean metric ($k=0$). On the left-hand side the age of the galaxy, as derived from Th/U, is inserted. Also given is the range of possible neutrino masses from 0.2 to 10 eV

$5 \cdot 10^{-30}\ \mathrm{g \cdot cm^{-3}}$ for $k = 0$. The circle marks $H_0 = 75$ and $\rho_0 = 0.5 \cdot 10^{-30}\ \mathrm{g \cdot cm^{-3}}$. The double lines indicate the limit of the observational uncertainties. The value 1 on the right hand side represents the ($\Lambda = 0$, $k = 0$) standard Friedmann model with euclidean space structure.

The deceleration parameter q_0 and the ratio R_0/R_* are given on the two upper abscissae.

It will be revealing to compare this diagram with Fig. 4 of part I of our review. That figure gives the corresponding data for Friedmann models ($\Lambda = 0$).

The age of our Galaxy as derived from the thorium/uranium ratio in meteorites is given on the left-hand side of Fig. 2. In order to obtain an estimate of the age of the universe, one has to add a time for the formation of galaxies. For this time, $1 \cdot 10^9$ years is usually adopted. Because of the slower expansion rate near the point of inflection, the process of galaxy formation is alleviated in Friedmann-Lemaître models as compared to the standard ($\Lambda = 0$) models. This statement is still valid even if the bulk of galaxy formation takes place already about 10^9 years after the big bang.

For the first few minutes, however, i.e. during the epoch of the nucleosynthesis of deuterium and helium, the models provide very closely the same conditions as the $(\Lambda=0)$-models. Thus the densities of the baryonic matter derived by [22, 29] and others [20] can be used without alteration. For our choice of $\rho_0 = 0.5 \cdot 10^{-30}\ \mathrm{g \cdot cm^{-3}}$ for the baryonic density of the universe today we refer to part I of our review.

The ages of the universe t_0 as given in Tables 1 and 2 are significantly longer than the corresponding Hubble ages, which represent upper limits for the standard $(\Lambda=0)$-models. The ages of the new models can more easily be reconciled with the rather large age of our Galaxy derived from meteorites and also with the ages of some globular clusters. For these one can find evolution ages of up to $25 \cdot 10^9$ years in the literature [19]. For critical remarks, however, see [23] and references therein.

An observational determination of q_0 is plagued by uncertainties in the long term evolution of the luminosity of galaxies. From their models of stellar evolution in galaxies, Gunn and Tinsley [15, 14] argue in favor of negative values of q_0 down to $-1{,}27$. This would imply $\Lambda > 0$ or $\rho_v > 0$. On the other hand, calculations of galactic evolution with accretion of small galaxies onto large ones yield corrections toward positive values of q_0 [21]. This problem was discussed by [5].

In the $(\Lambda > 0)$-models a "missing mass" problem in the usual sense does not exist. If, however, it should turn out that the neutrinos (ν_e, ν_μ, ν_τ) have indeed an average mass of more than 0.2 eV one could simply add them to ρ_0, since their present number density can be considered as known. For this purpose we have given the possible contributions of massive neutrinos in Fig. 2 in the upper right corner for the mass range from 0.2 to 10 eV. An average mass of more than 10 eV is not compatible with these models.

A closed Friedmann-Lemaître model with $q_0 = -1.2$ and $\Lambda = 5 \cdot 10^{-56}\ \mathrm{cm^{-2}}$ was obtained by [7, 8] from other evidence. They used primarily the observed distribution of quasars. Their parameters are very similar to those in our $(k=+1)$-model given in Table 2.

It is an unfortunate situation for cosmology that for the energy density of the vacuum independent information from quantum field theory is not yet available. Moreover, even the definition of vacuum states in curved space-time is plagued by conceptional difficulties ([1] and references therein).

In the Grand Unified Theories the main contribution to ρ_v comes from virtual Higgs particles. They can constitute different states of vacuum energy (Fig. 3). For these so-called "false vacuum" states high energy values have been discussed (A.H. Guth, A.D. Linde, P.J. Steinhardt et al. in [11]). This energy could cause an exponential increase of the cosmos at very early times ($t \approx 10^{-35}$ s). Compare also Fig. 3 of Part II of our review. During this period of inflation a phase transition (② in our schematic diagram) would take place when the vacuum energy density $\rho_v c^2$ equals the energy density $\rho_s c^2$ of the relativistic plasma at $T = 10^{28}$ K. At that point ρ_v must drop very quickly until it finally reaches the present level of ρ_v.

It is still controversial whether there is a significant level of ρ_v connected with the Weinberg-Salam phase transition at point ③ in Fig. 3 ($T = 10^{15}$ K, $t = 10^{-10}$ s). The behaviour of the Weinberg-Salam phase transition depends on how the

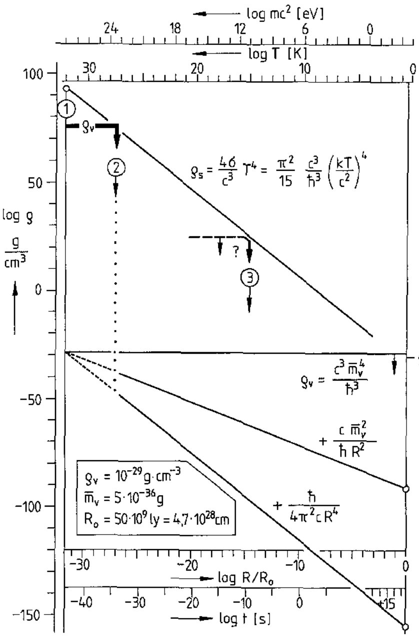

Fig. 3. Comparison of the "vacuum density" ρ_v with the density ρ_s of the relativistic particles ("radiation cosmos"). "Inflation" occurs when ρ_v passes through the diagonal line (ρ_s). The symmetry-breakings are marked by the numbers ①, ② and ③. Streeruwitz's results for the vacuum density are given by the three lines in the lower part of the Figure. The horizontal line marks the admissible upper limit for the constant cosmological term. The abscissae give the cosmic time t, the scale factor R/R_0, the temperature T of the radiation cosmos and its mass equivalent mc^2

Table 1. Model parameters of a flat Friedmann-Lemaître cosmos ($k=0$) for 4 values of the Hubble parameter and a present matter density $\rho_0 = 0.5 \cdot 10^{-30}\ \mathrm{g/cm^3}$

H_0 $\left[\frac{\mathrm{km}}{\mathrm{s \cdot Mpc}}\right]$	t_0 $[10^9\ \mathrm{a}]$	q_0	ρ_v $[10^{-30}\ \mathrm{g \cdot cm^{-3}}]$	Λ $[10^{-56}\ \mathrm{cm^{-2}}]$	t_* $[10^9\ \mathrm{a}]$	R_0/R_*
50	24.5	−0.84	4.2	0.8	9.1	2.56
60	22.3	−0.89	6.3	1.2	7.4	2.93
75	19.7	−0.93	10.1	1.9	5.9	3.44
100	16.4	−0.96	18.4	3.4	4.3	4.19

Table 2. Parameters for two selected Friedmann-Lemaître models with $(k=+1)$ and $(k=-1)$ for $H_0 = 75\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\rho_0 = 0.5 \cdot 10^{-30}\ \mathrm{g \cdot cm^{-3}}$

k	t_0 $[10^9\ \mathrm{a}]$	q_0	ρ_v $[10^{-30}\ \mathrm{g \cdot cm^{-3}}]$	Λ $[10^{-56}\ \mathrm{cm^{-2}}]$	t_* $[10^9\ \mathrm{a}]$	R_0 $[10^{28}\ \mathrm{cm}]$
+1	22	−1.0	10.9	2.1	6.2	4.7
−1	12.7	0	0.25	0.05	12.7	1.3

mass of the Higgs boson compares to the Coleman-Weinberg mass [3]. At $t = 10^{-10}$ s cosmology yields an upper limit of $\rho_v = 10^{25}$ g/cm^3 to avoid a second inflationary phase. If ρ_v were to become larger than ρ_s, the vacuum density would dominate the dynamical evolution of the universe. This limit is compatible with values derived by [16] from particle theory.

Zeldovich and Streeruwitz [31, 25, 26], for instance, have attempted to interpret Λ as vacuum expectation value of a quantum field theoretical stress-energy tensor. Taking into account a massive scalar field in a closed universe, Streeruwitz obtained three terms for the vacuum energy density $\varepsilon_v = \rho_v c^2$ which can be expressed by

$$\rho_v = \sum_{n=0}^{2} f(n) \frac{\bar{m}_v}{L^3} \left(\frac{L}{R}\right)^{2n} \quad (13)$$

with $f(0) = 1$.

Here $L = \frac{\hbar}{\bar{m}_v c}$ is the Compton length, R the curvature radius of the closed universe and $\bar{m}_v$ a characteristic mass of the virtual particles. The first term (= constant cosmological term) can be compared with the vacuum densities derived above (Tables 1 and 2). These densities can be considered as upper limits still compatible with astronomical observations. If we take $\rho_v = 10^{-29}$ g · cm^{-3} we obtain $\bar{m}_v = 5 \cdot 10^{-36}$ g. With this value and $R_0 = 4.7 \cdot 10^{28}$ cm taken from our ($k = +1$)-model we get the lines in Fig. 3 labeled $\frac{c \cdot \bar{m}_v^2}{\hbar R^2}$ and $\frac{\hbar}{4 \cdot \pi^2 c \cdot R^4}$. We adopted $f(1) = 1$. For $f(2)$ we used the value derived by [9, 10]: $f(2) = \frac{1}{4\pi^2}$.

In the concept of an early inflation the curvature radius R is extremely small for times shorter than 10^{-35} s as compared to the standard models. Under these conditions the third term of Eq. (13) by far dominates the other two. It would yield an energy density comparable to the "false vacuum" state of the Higgs particles, if R were as small as 10^{-29} cm at $t \approx 10^{-36}$ s. Note the rapid drop of the third term during inflation ("dotted line" in Fig. 3).

Within this short communication it was not possible to discuss further aspects of the cosmological term and its relation to the vacuum energy. A cosmological constant in the order of 10^{-56} cm^{-2} leads to Friedmann-Lemaître models in which the vacuum energy could already have dominated the dynamical evolution of the universe for 10^{10} years. These models are in acceptable agreement with the astronomical and astrophysical data.

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Microvesicles in Meteorites, a Model of Pre-biotic Evolution

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In the transmission electron microscope, a considerable portion of the meteorite carbonaceous matter appears as structured particles ranging from 10 to 500 nm in size [1]. *Tubular vesicles* ca. 200–400 nm in length and 12–15 nm wide occur in dense crowds (Fig. 1a). A coaxial channel hollow is developed within the filament. The surrounding wall is about 2 nm thick and exhibits a reticulate sculpture of helicoid arrangement. *Spherical vesicles* are mostly in the size range of 50–200 nm diameter (Fig. 2), sometimes larger. The specimens occur singly, in pairs or united in clusters.

The exterior wall of the vesicles is underlain by a delicate membrane which in cross section shows a bilayer structure (Fig. 3). The surface of the mem-