TERENCE PARSONS

ASSERTION, DENIAL, AND THE LIAR PARADOX

0. INTRODUCTION

In 1919, in the essay "Negation" (Frege, 1919), Frege addresses the issue of whether there are two distinct ways of judging – affirmative judging and negative judging – or only one. His answer, which is widely accepted nowadays, is that there is only one kind of judgment – affirmative judgment – and that the rejection of a proposition is always accomplished by accepting or affirming some *other* proposition, namely, the negation of the proposition being rejected. For example, we reject the claim that the accused was in Rome at the time of the murder by accepting the different claim that the accused was *not* in Rome at the time of the murder (Frege, 1919, pp. 129–130). Frege argues further that negation itself is always part of the content of a judgment, and never forms part of the judgmental act or attitude.

Frege's views are very different from certain earlier views, such as those of the *Port Royal Logic* (Arnauld, 1662). In that work, a judgment that the accused was *not* in Rome at the time of the murder would have the very same content as the corresponding positive judgment. The difference between the positive and negative judgments would lie in the judging activities themselves, not in their contents. The negative aspect of a negative judgment would be part of the act, not part of the content.

There is a parallel issue in the philosophy of language (also discussed by Frege): Are there two distinct types of illocutionary acts – assertion and denial – or only one? The Fregean answer is that there is only one – assertion – and that a linguistic denial of a proposition is always effected by means of asserting the negation of that proposition. Further, in Austinian terms, negation is always part of the locutionary content of a speech act, and never part of the illocutionary force (Austin, 1962). In Hare's terms, negation is always to be located in the phrastic, never in the neustic (Hare, 1952).

Journal of Philosophical Logic **13** (1984) 137–152. 0022–3611/84/0132–0137$01.60.

In Section 1 of this paper I will articulate a view that challenges these well-entrenched Fregean doctrines. In the remainder of the paper I will apply the ideas developed in Section 1 to two problems that grow out of certain standard solutions to the Liar Paradox.

1. ASSERTION AND DENIAL

In what follows it will be helpful to have available a bit of notation to distinguish various speech acts in terms of their force and content. I will use the familiar notation of the predicate calculus to symbolize contents of speech acts. I assume without argument that negation often forms part of such contents. (There are some relatively good arguments for this in Frege's essay cited above.) I will follow the tradition of using Frege's turnstile symbol '$\vdash$' to indicate that the speech act in question is an assertion. For example, if someone asserts that some cows are purple, the relevant speech act would be exhibited by

$$\vdash (\exists x)(Cx \,\&\, Px).$$

Assuming that there is such a thing as illocutionary denial, I will use the symbol '$\models^{*}$' for it, so that a denial that some cows are purple would be exhibited by

$$\models^{*} (\exists x)(Cx \,\&\, Px).$$

Regarding this denial sign there appear to be three relevant views. The first, which I identify with Frege, is that its use is simply illegitimate, since it incorporates negation into the force of the utterance, and incorrectly assumes that there is more than one kind of "indicative" force. I am simply going to ignore this view here, since the purpose of this paper is to explore certain other options. A second (almost Fregean) view might be that there is such a thing as illocutionary denial, but that every denial is equivalent to a certain assertion, and vice versa. The equivalence in question is given by the principle that by denying S one automatically asserts $\sim S$, and vice versa. That is, there is a principle of speech act theory according to which any speech act that is correctly analyzed as

$$\models^{*} S$$

is also automatically correctly analyzed as

$$\vdash \sim S$$

and vice versa. I will call this the *Equivalence Thesis*; it can be formulated more simply as

E.T.: Denying S is always the same as asserting $\sim S$.[1]

My intent in this paper is to cast doubt on E.T. That is, there may be cases in which we deny S without thereby asserting $\sim S$, and perhaps there are even cases in which we assert $\sim S$ without thereby denying S.

This formulation may seem paradoxical; after all, doesn't 'denial' just *mean* 'assertion of the negation'? Perhaps it does; that is a fair paraphrase of *one* of the dictionary entries under 'deny'. If so, perhaps I should be speaking instead of *rejection*. The important point is that sometimes when we "say something negative" we should not be thereby committed to an assertion of a negative claim, for we are not asserting at all, we are only rejecting something. I might say "Paul Bunyan is *not* bald" without thereby committing myself to the truth of the sentence 'Paul Bunyan is not bald', for I might think (as many people do think) that this sentence lacks truth-value. I may only want to *reject* the sentence 'Paul Bunyan is bald' (or perhaps the proposition that it expresses),[2] for I may think that *both* claims – "Paul Bunyan is bald" and "Paul Bunyan is not bald" – lack truth-value. If I am told that I can only deny the former by asserting the latter, then that is a language game I will not want to play. And I do not think that this is how ordinary English works. Sometimes saying a sentence with a negative word in a certain tone of voice just counts as a rejection of the corresponding positive version.

As a scholarly point, it is interesting to note that Frege's argument for his own view (that there is no such thing as negative judgment or force) contains a loophole that is crucial for the cases under consideration. His argument went as follows. First, he argued that negation must *sometimes* form part of the contents of our utterances. He then took for granted that if there were such a thing as what he called "negative assertion" then the equivalence thesis would hold for it. Finally he argued that it is more *economical* to analyze "negative assertion" in terms of affirmative assertion plus negation (of the content), for then we need only assume content negation plus one kind of judging (or asserting) whereas otherwise we would also need an extra kind of judging. However, in all of Frege's examples he explicitly limits his argument to sentences which have truth-values.[3] But sentences or propositions without truth-values are exactly the cases in which E.T. is most doubtful.

There is in the literature a recent and radical proposal which suggests a different kind of failure of the equivalence thesis. I have in mind views by Meyer and Routley to the effect that in certain cases a sentence and its negation may *both* be true (Routley, Meyer *et al.*, forthcoming). If these authors are right, there are occasions on which both of the following acts are justified:

$$\vdash S$$

and

$$\vdash \sim S.$$

But as I understand them, they are by no means suggesting that in such a situation either or both of S and $\sim S$ should be rejected; that is, they would not regard as justified either of the acts

$$\vdash^* S$$

or

$$\vdash^* \sim S.$$

If this adequately represents their position, they must also deny the equivalence between denying something and asserting its negation.

Reflecting on considerations of this sort leads to the following general view. We expect a scientific or philosophical theory to classify sentences or propositions into three classes: those that should be accepted (according to the theory), those that should be rejected (according to the theory), and those regarding which the theory is silent. If we make certain classical assumptions, a theory which specifies the first class automatically specifies both of the others. For the sentences that it tells us to reject are exactly the negations of the ones that it tells us to accept, and all the remaining ones are in the third class. But if the general equivalence of rejection with assertion of negation breaks down then a theory cannot be identified with the class of sentences that it tells us to accept, as is normally done; the theory must independently specify the class of sentences to be rejected. This is a very different view of theories than that which is normally held.[4]

There are a number of possible "logics" of acceptance and rejection. They all agree in one basic principle: one cannot be justified in both asserting and rejecting the same claim. If this is the only principle to be adopted then even certain extreme philosophical stances may be regarded as having a certain kind of rationality, stances such as:

Complete nihilism: Reject everything: $\vdash^* S$, for each S.

Ultimate eclecticism: Accept everything: $\vdash S$, for each S.

(The latter is a version of Post-inconsistency, often thought to be the worst kind.) Given the equivalence thesis (plus the view that S is always equivalent to $\sim\sim S$) these views are identical. I doubt that they are.

For the remainder of this paper I will ignore these radical alternatives. I will assume only that E.T. is wrong in the sense that sometimes we deny S without thereby asserting $\sim S$, and that this is rational in cases in which we believe that both S and $\sim S$ are without truth-value. I assume that sometimes the use of a negative word in English merely indicates denial or rejection, and that generally whether it is doing this or instead is expressing part of the content cannot be decided apart from context.

In Sections 2 and 3 of this paper I will apply these ideas to certain standard "solutions" to the Liar paradox. These solutions all depend on not interpreting the negation in the Liar sentence as exclusion negation. This will be defended in Section 4.

2. THE LIAR PARADOX

In this section I want to apply the ideas sketched above to indicate a solution to a certain problem concerning the liar paradox. I am *not* proposing a solution to the paradox itself; rather, I am proposing a solution to a *puzzle* that arises out of a solution that others have given. First let us see what that solution is.

Suppose that we have a "liar sentence", i.e., a sentence that "says of itself that it is not true". An example would be a sentence of the form

'$\sim T(L)$', where 'L' names that very sentence; i.e., where the following is true:

$$L = \text{'}\sim T(L)\text{'}.$$

Using this sentence we can then give an argument that appears to be a proof that a contradiction is true. The argument employs the following special principle regarding the truth predicate:

[T] From S one may infer $T'S'$, and vice versa.

Note that principle [T] seems acceptable even if we allow that certain sentences may lack truth-value. For the principle only tells us that if we have already got S then we may conclude $T'S'$ (and vice versa), and that seems safe enough.

The argument now goes as follows:

(1)	$L = \text{'}\sim T(L)\text{'}$	Given
(2a)	Assume $T(L)$	Assumption (for R.A.A.)
(2b)	$T' \sim T(L)'$	(1) and (2a) by Subst. of Iden.
(2c)	$\sim T(L)$	(2b) by [T]
(3)	$\sim T(L)$	R.A.A. from (2a)–(2c)
(4)	$T' \sim T(L)'$	(3) by [T]
(5)	$T(L)$	(1) and (4) by Subst. of Iden.
(6)	$T(L) \,\&\sim T(L)$	(3) and (5) by conjunction

The classical attack on the argument is due to Tarski: we simply cannot have a predicate which obeys principle [T] and also applies to the very language in which it occurs itself. But this has seemed unduly restrictive to many people, and they have sought other ways out. One such way has now achieved a widespread acceptance; I will call it the "Standard Solution". According to this solution, the liar sentence lacks a truth-value, and so step (3) in the argument is fallacious. For step (3) presumes the validity of R.A.A. in a context in which sentences may lack truth-value, and this is not generally truth-preserving. The argument in (2a)–(2c) only shows that '$T(L)$' cannot be true if line (1) is true; it does *not* show that '$T(L)$' *has* a truth-value, and so the assertion of '$\sim T(L)$' in line (3) has not been fully justified.

There are actually two different reasons why one might believe that the liar sentence lacks truth-value. One reason is simply that it allows one to escape the above argument, whereas other escapes may seem less plausible. Another quite different reason that some people have (e.g., Martin and Woodruff, 1975; Kripke, 1975), is that they have a theory of truth which yields this result as a by-product. I will not be concerned here with motivations for the solution, but rather with the question of whether the solution can be consistently maintained.

The Standard Solution tells us:

SS: The liar sentence lacks truth-value.

But this statement is as disconcerting as it is plausible. For it seems to entail:

A. The liar sentence is not true.

The trouble is, the liar sentence *is* L, and so A in turn seems to commit one to:

B. L is not true.

But B is just an Englishized version of the liar sentence itself, and in fact it leads to paradox just as easily as does the original liar.[5] The Standard Solution, then, seems committed in turn to holding that:

C. Sentence B also lacks truth-value.

But now we have a situation in which we are told something (in B), and also told that that very thing is not true. But if I am assured that B is not true, then it would be irrational of me to believe either it or anything (e.g., SS) which entails it. It appears then that accepting the Standard Solution requires that we reject it as well. It is this problem – the apparent self-falsifying nature of the Standard Solution – that I want to address in this section.[6]

Given what I have said in Section 1, my suggestion here should surprise no one. The Standard Solution may be correct, provided that it is accurately formulated so as to reflect the understanding that we have of the liar sentence. This understanding should not be expressed in terms of an assertion of the negation of the claim that the liar sentence is true; is should be merely a rejection of that claim. Claim A should not be analyzed as

A′ $\vdash \sim$ (the liar sentence is true),

but rather as

A″ $\vdash^{*}$ The liar sentence is true.

Likewise, B should not read

B′ $\vdash \sim$ (L is true),

but rather

B″ $\vdash^{*}$ L is true.

I am not saying that the claims should be given the latter formulations just because those formulations keep one out of the trouble discussed above. They do keep one out of trouble, granted, but they should be formulated in that way because that is the correct way to articulate the insights that are being expressed. Having discovered that a sentence or proposition does not have truth-value, we want to reject it, *not* to assert a related sentence (its negation) which we also wish to reject.

3. REFORMULATIONS

It is not uncommon to think that you have solved a paradox only to have it come back to haunt you in revised form. In this section I will go over two revisions of the liar paradox; neither will prove to be problematic.

Since the solution sketched above replaced negation by denial in a certain crucial context, one might think to try to reformulate the paradox in terms of denial. Suppose that instead of considering a problem case of the form

$$\vdash \sim T(L)$$

we consider an act of the form

$$\models^* T(L),$$

where, as usual, L = '$\sim T(L)$'. Then such an act does not commit one to an act of the first form, for denying a sentence or proposition does not commit one to asserting its negation. If one is committed to substitutivity of identity within denials, then acknowledging that L = '$\sim T(L)$' will commit one to the correctness of an act of the form

$$\models^* T' \sim T(L)',$$

and this in turn may even commit one to

$$\models^* \sim T(L)$$

by dint of an analogue of the Tarskian principle [T]. Thus one may be committed to denying both $T(L)$ and $\sim T(L)$. The analogy with the original paradox is apparent, but these denials are not paradoxical; indeed they are just what one expects from one who does not accept the liar sentence as being either true nor false.

(When I talk of being committed to a certain kind of act I do not mean that one is committed to performing the act, but only something like this: that one is committed to the act being correct if performed. Though even this is not quite correct; see below.)

If one cannot get in trouble by saying "I deny that this very sentence is true", perhaps one can get in trouble by saying "I deny that this very sentence is denied". Let me explore this road. In addition to asserting and denying sentences we can and do *say that* people assert and deny certain sentences. We also can and do say that *we* assert and deny various sentences. So let me assume that among the predicates of our language we have one that means 'is true', one that means 'I assert', and one that means 'I deny' or 'I reject'. I will write these predicates as 'T', 'A', and 'R'. I assume that whether or not the speaker in question actually asserts or rejects a given sentence is an unproblematic factual question, so if 's' is a sentence of the language then '$A(s)$' and '$R(s)$' will each definitely be either true or false. Note that I have in mind only explicit assertion and denial; I do not count someone to have asserted s if they merely say something *else* which entails s. I assume that both '$A(s)$' and '$R(s)$' may be true, if the speaker is inconsistent, and both may be false if he keeps his mouth shut or only asserts or denies sentences other than s.

Although the point I am making is quite general, I should say something specific about the semantics. So let me assume a semantic theory of the sort given in Kripke (1975), and let me assume that the allowable models are always minimal fixed points.[7]

I will define a *testimony* to be any pair of sets of sentences. Intuitively, the first set represents a set of sentences that some possible speaker explicitly asserts, and the second set represents the set of sentences that that speaker explicitly rejects. Then a *coherent testimony* will be a testimony, $\langle S_1, S_2 \rangle$, for which there is a model for the language in which (1) the extension of 'A' $= S_1$ and the extension of 'R' $= S_2$, and (2) every sentence in S_1 is true and no sentence in S_2 is true. Coherent testimonies are easy to produce. A speaker who never says anything thereby producing a coherent testimony in which both S_1 and S_2 are the empty set. A speaker who never uses the predicates 'T', 'A', or 'R' may produce a coherent testimony simply by insuring that the set of sentences asserted together with the negations of the sentences rejected constitute a set that is consistent in the usual logical sense. A speaker who uses the predicate 'T' must be more careful; he has the option, e.g., of rejecting the liar sentence, but he may not assert either it or its negation. Of present interest is the use of 'A' and 'R', for they may be used to form analogues of the liar. Suppose, for example, that 'c' names the sentence '$\sim A(c)$'; i.e., 'c' names a sentence which says "I do not assert this very sentence". This sentence cannot appear in the assertion component of any coherent testimony. But that is not because the sentence cannot be true. Whether it is true or not is a straightforward empirical question: it is true if the speaker does not assert it and false otherwise. So it is a sentence which can be true, but cannot be truly asserted. And not asserting it makes it true. This is analogous to a paradoxical situation, but it is not actually paradoxical. We have a paradox only if we have some argument that leads us by acceptable means from acceptable premises to unacceptable conclusions. The sentence:

I do not assert this very sentence

is no more paradoxical than the sentence:

Nobody is alive.

This latter sentence can only be asserted if it is not true. This may sound bizarre, but the reason is clear and there is no paradox. By similar reasoning one can establish that:

I deny this very sentence

cannot be coherently denied (i.e., cannot be a member of the second component of any coherent testimony), though it can be untrue. Further,

I do not deny this very sentence

may always be coherently denied. (Denying it is thus on a par with asserting "I assert this very sentence".)

The existence of sentences such as c has an interesting effect on the "logic" of assertion. Suppose that R has the form of an ordinary rule of inference in logic. Let us say that R is *coherence-preserving* if coherent testimony is always closed under applications of R, i.e., if whenever $\langle S_1, S_2\rangle$ is a coherent testimony, and S follows from some subset of S_1 by an application of rule R, then $\langle S_1 \cup \{S\}, S_2\rangle$ is also a coherent testimony. Then *most* of our ordinary rules of inference are *not* coherence-preserving. Let 'c' name '$\sim A(c)$', let 'd' also name '$\sim A(c)$', and let E be the sentence 'Snow is white'. Then '$\sim A(c)$ & E' may easily be part of a coherent testimony (just imagine a situation in which the speaker asserts that conjunction without ever asserting either conjunct). But, as noted above, '$\sim A(c)$' cannot be part of any coherent testimony. So simplification is not coherence-preserving.

Likewise, '$\sim A(d)$' and '$c = d$' can both be parts of the same coherent testimony, even though '$\sim A(c)$' cannot be, so substitutivity of identity is also not coherence-preserving. In fact, substitutivity of identity behaves here in application to sentences containing 'A' in much the same way that Skyrms says that it behaves in application to sentences containing 'T' (Skyrms, 1982).[8]

4. EXCLUSION NEGATION

It is common in the literature on the paradoxes to distinguish two kinds of negation: choice negation and exclusion negation. They may be characterized as follows.

(D) A unary connective $\sim$ is a *choice negation* if, for any sentence S, $\sim S$ is true when S is false, false when S is true, and undefined otherwise.

(E) A unary connective $\neg$ is an *exclusion negation* if, for any sentence S, $\neg S$ is true when S is false or undefined, and false when S is true.

The negation utilized in the discussion above of the liar paradox was choice negation, and that was no accident. For the standard solutions of the paradox, with the exception of Skyrms', do not work if the negation in question is exclusion negation. Recall the argument given above that apparently led to a contradiction. The fallacy was supposed to be at step (3), where we asserted '$\sim T(L)$' simply because '$T(L)$' had been found to lead to contradiction. This was unjustified because '$T(L)$' might not be false; it might merely be without truth-value, and so we should not be able to *assert* '$\sim T(L)$'. That is, we should not be able to assert this if '$\sim$' is choice negation. But, having shown (by R.A.A.) that '$T(L)$' is either false or without truth-value, we ought to be able to assert its *exclusion* negation. For the exclusion negation of a sentence must be forthrightly true in such a situation. In fact, if the negation in question is taken to be exclusion negation, it is very difficult to find anything wrong with the argument at all.

The commonest way of dealing with exclusion negation is to ignore it. Occasionally it will be mentioned that exclusion negation is "of course" not present in the language. The reason, which might or might not be made explicit, is that having exclusion negation in the language tends to mess things up. It tends to bring back the paradoxes. Of course, most people who discuss the semantic paradoxes are not engaging in solving the paradoxes, but are rather engaged in seeing how much semantics can be done without running into them. This is a laudable goal, but it is little help to someone who is curious about the paradoxes themselves.

It is commonly said, regarding the paradoxes, that one must "buy consistency at the price of expressive completeness". That is, in order to avoid paradox one must give up the ability to express certain notions in one's language. The absence of exclusion negation is often thought to be a case of this kind. Only by excluding exclusion negation can we exclude inconsistency.

This seems a rather timid approach to the bogyman of exclusion negation. I would like to suggest a bolder approach. When we "exclude exclusion negation" from our language we are not in fact excluding anything at all. For there is no such thing as exclusion negation in any formal language which accurately reflects our own native speech. This cannot be proved, of course, but I think that the view is plausible. In the remainder of this section I will articulate it more fully.

Why might we think that there must be such a thing as exclusion negation? One reason might be that we think it can be *defined*. But that is not obvious. There are two kinds of definitions available here. Definition (E) given above is one kind; it tells you how a connective must behave in order to be an exclusion negation. But (E) does not entail that there are any such connectives. To get a definition that does that we need one of the form "Exclusion negation is *the* connective which . . . ". But definitions of this kind are clearly creative; they *assume* that there is such a thing, and that assumption needs to be justified. If it is not, perhaps nothing has been defined.

Perhaps I have been looking in the wrong place. It might be agreed that there is no such connective, but insisted that there is such a sense or meaning. That is, there is a sense or meaning appropriate to a connective that *would* work as characterized in (E), and the problem is that no language that is rich in expressive resources can consistently contain a connective that expresses this sense. The trouble with a view of this sort, however, is that one wants to know why there should be such a sense. If the answer is that the sense is given by definition, then one wants to see the definition – for definitions of senses can be creative too. Or perhaps another way to put it is that a creative definition does not guarantee that the term it defines expresses a sense, or that there is any such sense to express.

Now it might be replied that in the case of exclusion negation there clearly is an appropriate *truth-function* which can be defined, and which can be define noncreatively. Consider:

$f =_{df}$ the function which maps T to F and maps both F and N to T.[9]

I do not deny that such a function exists. However, the existence of exclusion negation requires more than the existence of the truth function; it requires also that the truth-function in question can be assigned as the denotation of a unary connective that consistently forms falsehoods from truths and truths from sentences that are either false or neuter. This is not guaranteed by the definition of the truth-function, and the existence of Liar-type sentences shows that it cannot be done without giving up some very plausible principles.

I could sum up the point of my remarks as follows. The titles "choice negation" and "exclusion negation" do not pick out kinds of negation; instead they pick out theories about the logical behavior of negation. Theories of the latter kind are not true.

I think that there is actually a more basic reason why people believe in exclusion negation. We are sometimes greeted by a claim that we do not accept, but one that is in some sense defective. We want to reject it, but we do not want to take responsibility for the defect. The classic case is "I have (or have not) stopped beating my wife". Perhaps for many philosophers "The purpose of life is to serve mankind" is also an example. Now sometimes what we do in such a case is to reply with an emphatic 'not', perhaps together with an explanation. "I have *not* stopped beating my wife; I couldn't stop because I never started", or, "The purpose of life is *not* to serve mankind; it doesn't make sense to ascribe purpose to life". It is then easy to think of the emphasized 'not' as a specially strong kind of negation, as exclusion negation.

I think, of course, that such a view confuses the rejection of a sentence with an assertion of its negation. We think of the negative aspect of the act as being located in its content, and then naturally take the act to be an assertion of something negative. If we think in these terms then we are practically forced to believe in exclusion negation, and to look askance at purported solutions of the paradoxes that ignore it. But there is another option: the emphasized 'not' does not stand for part of the content of an assertion; it rather signals rejection of the remaining content.

Some philosophers might grant that this is a correct account of ordinary usage, but criticize it as being inadequate. They would suggest that if you think that a sentence is defective then you should not use it or some variant of it yourself; rather you should ascend to the formal mode and make an assertion whose subject matter is the sentence itself. It is exactly this move, however, that is unhelpful in dealing with the liar paradox. For the liar sentence is already in the formal mode, and ascending doesn't get you anywhere. We do not need to ascend in order to reject; we need only exploit the usage that is already there in ordinary language.

NOTES

[1] One may or may not also want to add the claim that denying $\sim S$ is always the same as asserting S.

[2] It makes little difference in this paper whether we construct the objects of speech-acts as sentences or as propositions, as long as we assume that sentences without truth-value may still express propositions.

[3] What Frege actually assumes is that in all the cases under discussion the Thought expressed does not "belong to fiction". In his other writings (e.g., in "On Sense and Reference") he takes sentences from fiction as paradigms of sentences which lack truth-value.

[4] For example, it entails that an axiomatic presentation of the ordinary sort does not succeed in picking out a unique theory – unless the equivalence thesis is presupposed.

[5] The argument goes as follows.

(1)	L = '$\sim T(L)$'	Given
(2)	L is not true	New given
(3)	$\sim T(L)$	Synonymous with (2)
(4)	$T'\sim T(L)'$	From (3) by [T]
(5)	$T(L)$	(1) and (4) by subst. of iden.
(6)	$T(L)$ & $\sim T(L)$	(3) and (5) by conjunction

[6] This puzzle was noted by Keith Donnellan in Donnellan (1970). Since then various people have proposed various ways out. Kripke (in Kripke, 1975) suggests that the concept of truth used in SS is a different one than that which appears in the liar sentence. One wonders then if this different notion could be used to construct a revitalized version of the paradox. The answer is clearly "yes", and Kripke is led to remark "The ghost of the Tarski hierarchy is still with us" (*ibid.* p. 714). Skryms suggests that we can express our insight that the liar is not true by saying

A_{Sk} '$\sim T(L)$' is not true.

We can then avoid concluding that

B_{Sk} L is not true,

by dening the substitutivity of identity. This is technically beyond reproach, but it has not resulted so far in a semantics which helps us see which of these should be true, and why. Martin (in Martin, 1979) would accept SS (or something like it) but deny that SS entails A, on the grounds that A (unlike SS) contains a category mistake.

[7] A fixed point is a model in which the extension of 'true' is the set of true sentences of the language, and the anti-extension of 'true' is the set of false sentences of the language. The minimal fixed point is the fixed point which does not make a sentence true or false unless all fixed points do also.

[8] None of the facts described in the last two paragraphs depend in any way on rejecting the Equivalence Thesis. They are meant to illustrate some paradoxical-sounding fact which, upon examination, turn out not to be paradoxical at all. Our rules of logic tell us that certain things must be true if certain other things are; they do not tell us that these things would remain true if they were articulated.

[9] Of course, if sentences which are neither true nor false have no truth-values at all, then this definition fails, for it assumes that such sentences have a "special" value, "neuter". If we insist that the only truth-values are truth and falsehood then it is easy to see that there is no such truth-function as exclusion negation. For in the case of a sentence of the form $\sim S$, where S lacks truth-value, the function would have no argument, and therefore could not yield a value for the sentence as a whole to denote.

REFERENCES

Arnauld, A., *La Logique, ou L'Art de Penser*, 1662.

Austin, J., *How to Do Things With Words*, Oxford University Press, London, 1962.

Donnellan, K., 'Categories, Negation, and the Liar Paradox', in R. L. Martin (ed.) *The Paradox of the Liar*, Yale University Press, New Haven, 1970.

Frege, G., 'Negation', in M. Black and P. T. Geach (eds.), *Translations from the Philosophical Writings of Gottlob Frege*, Blackwell, Oxford, 1960.

Hare, R. M., *The Language of Morals*, Oxford University Press, Oxford, 1952.

Kripke, S., 'Outline of a Theory of Truth', *Journal of Philosophy* (1975) 690–716.

Martin, R. L., 'Reply to Hugly and Sayward', *Analysis* 39 (1979) 169–174.

Martin, R. L. and Woodruff, P., 'On Representing 'True-in-*L*' in *L*', *Philosophia* 5 (1975) 213–217.

Routley, R., Meyer, R. K. *et al.*, *Relevant Logics and Their Rivals*, Research School of Social Sciences, Australian National University, Canberra (forthcoming).

Skyrms, B., 'International Aspects of Semantical Self-Reference', in R. L. Martin (ed.) (forthcoming).

Department of Philosophy
University of California at Irvine,
Irvine,
CA 92717, U.S.A.