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TECHNICAL NOTE

Bearing capacity of a strip foundation on a sand layer overlying a smooth rigid stratum

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The effect of a smooth rigid stratum, located beneath a dense sand layer, on the bearing capacity and settlement of surface and shallow strip footings is investigated using an advanced experimental model. A theoretical analysis is presented for the bearing capacity of surface footings. The results indicate that the bearing capacity reaches a minimum value at a specific sand-layer thickness. Any increase in the layer thickness above this value causes an increase in the bearing capacity up to that corresponding to a continuous media.

Keywords: Bearing capacity, bearing capacity factors, sand, layered soil, rigid stratum, laboratory testing

Introduction

Soil in nature may exist in a stratified manner. A possible condition is the case of a rigid stratum underlying a sand layer. Published data deal mainly with the case where the interface between the sand layer and the rigid stratum is rough (Meyerhof, 1974; Pfeifle and Das, 1979). The results indicate that the bearing capacity increases with the decrease in the layer thickness. To the authors' knowledge there is a lack in the literature of any experimental work or theoretical solution when the interface is smooth. Mandel and Salençon (1972) however, published a solution for the effect of overburden pressure (shallow footings) on the bearing capacity. They showed that the bearing-capacity factor N_{qs} (the subscript $_s$ is used here to denote a smooth interface) was dependent on the angle of internal friction (ϕ) and the ratio of the layer thickness (H) to the footing width (B) as shown in Fig. 1. The figure illustrates that for a given value of ϕ there is a limiting thickness after which N_{qs} remains constant.

In this paper, results of tests on surface and shallow footings overlying a smooth rigid stratum, together with a theoretical approach for the bearing capacity of surface footings are presented.

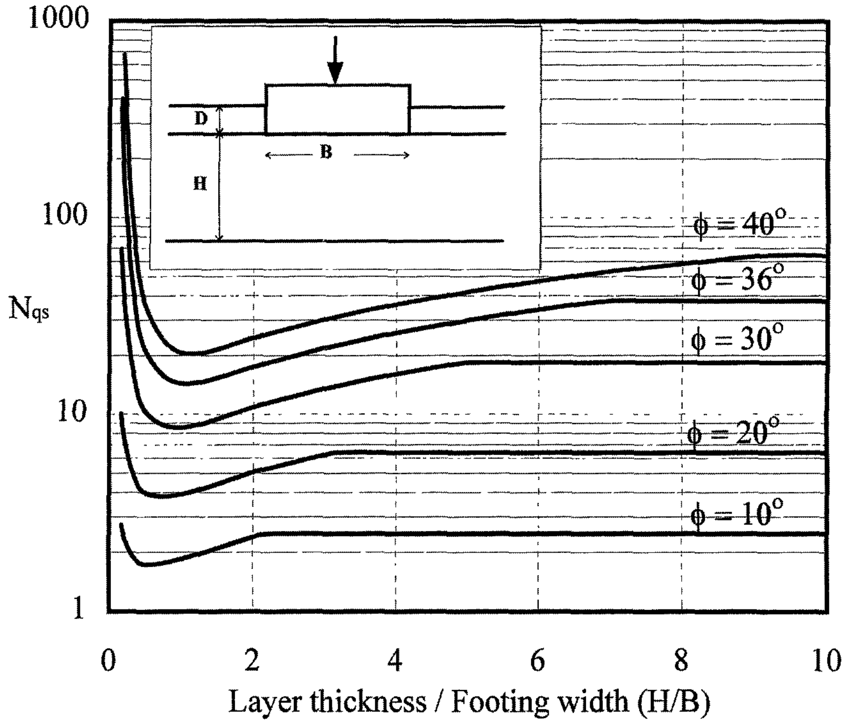

Fig. 1. Theoretical variation of N_{qs} with H/B (after Mandel and Salençon, 1972)

Experimental system

Tests were conducted in a rigid steel framed tank 0.90 m wide × 1.2 m long. The sides of the tank were 18 mm glass plates which were supported against lateral deflection. A frictionless interface was achieved by placing silicon grease/latex lubrication in panels over the inside glass surface and at the tank base (stratum surface). The tank base was a rigidly supported steel plate 5 mm thick.

The model footing was 0.12 m wide fabricated from rigid steel plates bolted together. Glass paper (BS grade S2, grit No. 40) was glued onto the footing base. The footing settlement was measured using displacement transducers having a minimum resolution of 0.04 mm. A unit of 13 narrow load cells, which could measure the normal load was housed near the footing centre.

Another unit of four cells with relatively wide active faces, which could measure both normal and shear loads, was housed adjacent to the narrow load cells. A loading frame was used to apply a constant rate of footing penetration of 5 mm/h.

The sand used is Loch Aline sand classified as a uniform sand of medium size. The maximum and minimum porosities are 45% and 33% respectively and the specific gravity is 2.64. The model tests were performed at a relative density, D_r, of 88.8% (unit weight 16.93 kN/m^3) corresponding to a triaxial friction angle, ϕ, of 37°. Sand beds of uniform density were deposited using a mechanical spreader. Full details of the experimental system and test methods can be found in Al-Omari (1984).

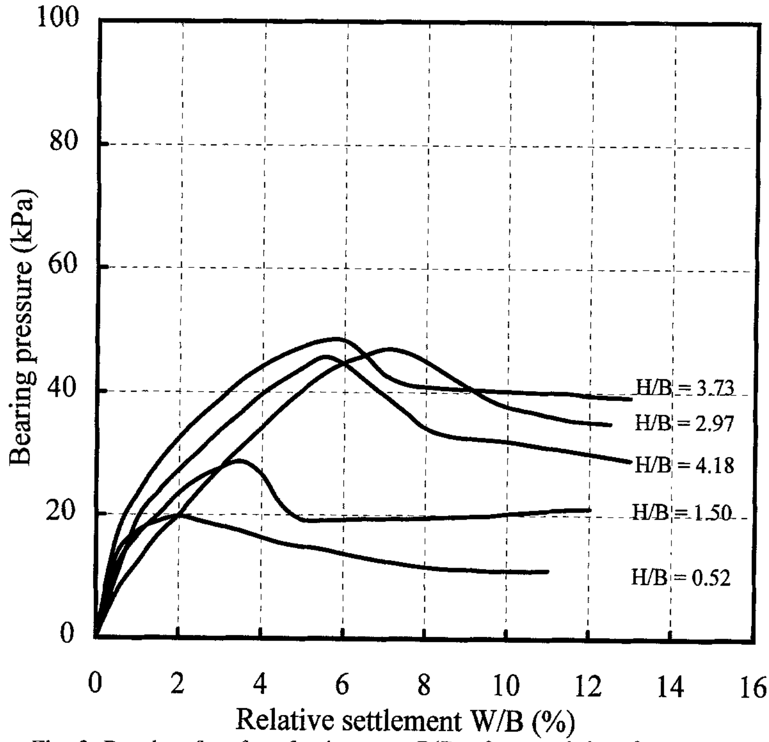

Fig. 2. Results of surface footing tests $D/B = 0$, smooth interface

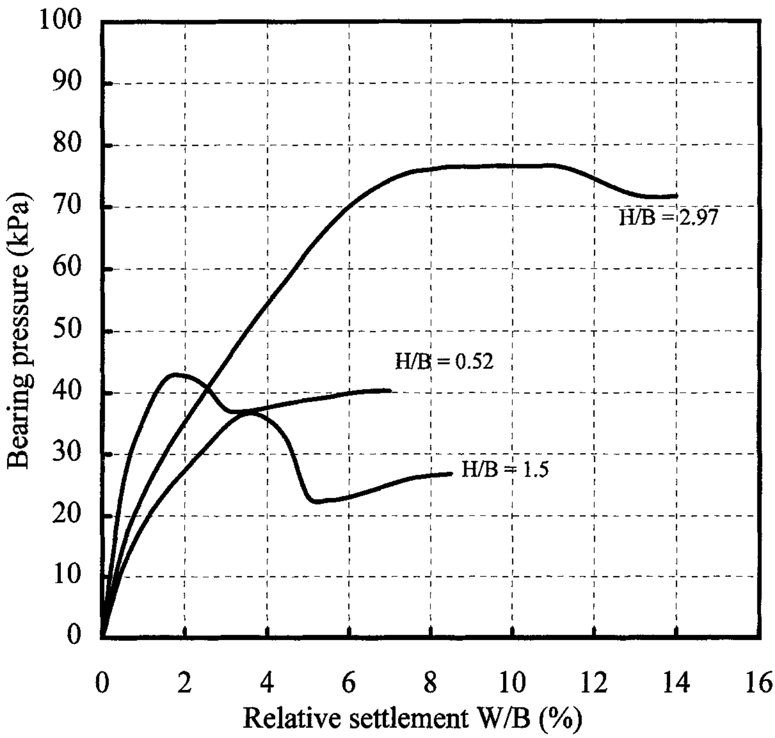

Fig. 3. Results of shallow footing tests $D/B = 0.5$, smooth interface

Bearing capacity

The bearing stress (q) versus relative settlement relationships for surface and shallow footing tests and for different values of *H/B* are shown in Fig. 2 and Fig. 3 respectively. Distinct peaks in the stress were obtained in every test and this was taken as the bearing capacity (q_u).

Experimental bearing capacity factors

Variation of the bearing capacity with the layer thickness is shown in Fig. 4. For a surface footing ($D/B = 0$) the bearing capacity increases with increasing H/B and remains constant after a certain limiting value of H/B which is approximately 3.0. After this limiting value, the classical failure mechanism for a continuous media becomes valid. It is reasonable to assume that the bearing capacity will have a high value as H/B approaches zero. This implies that there is a minimum value of bearing capacity which occurs at a certain small value of H/B. This can clearly be observed in the case of shallow footings ($D/B = 0.5$) illustrated in Fig. 4 where the minimum bearing capacity occurred at $H/B \approx 1.3$. For surface footings, the value of H/B at which the minimum value of bearing capacity occurs is expected to be around 0.50.

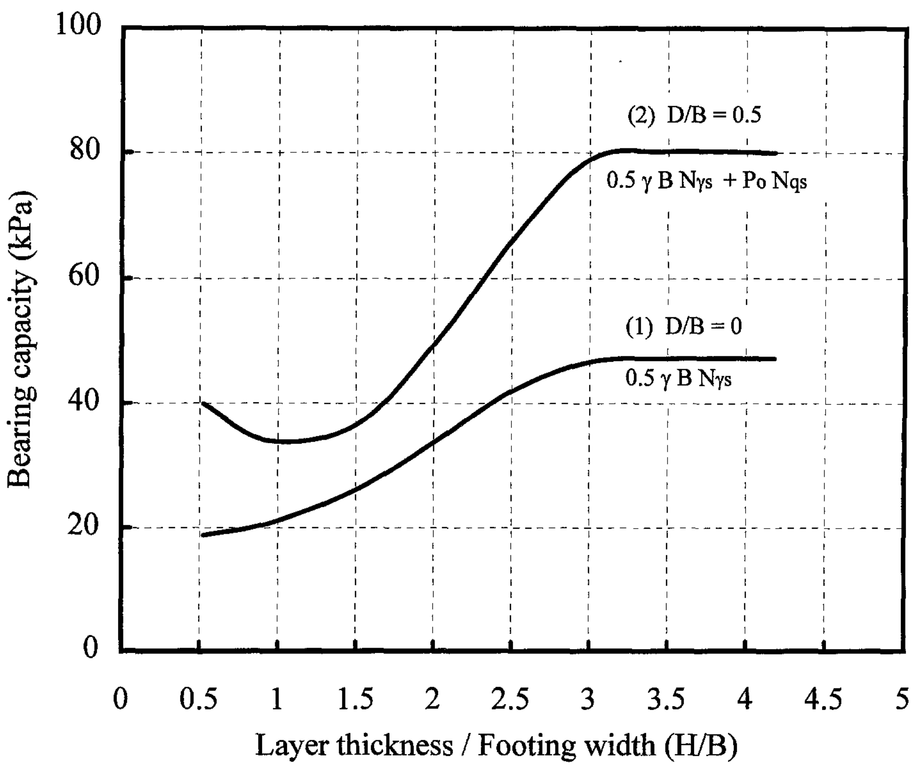

Fig. 4. Experimental results for smooth interface: dependence of bearing capacity on relative layer thickness

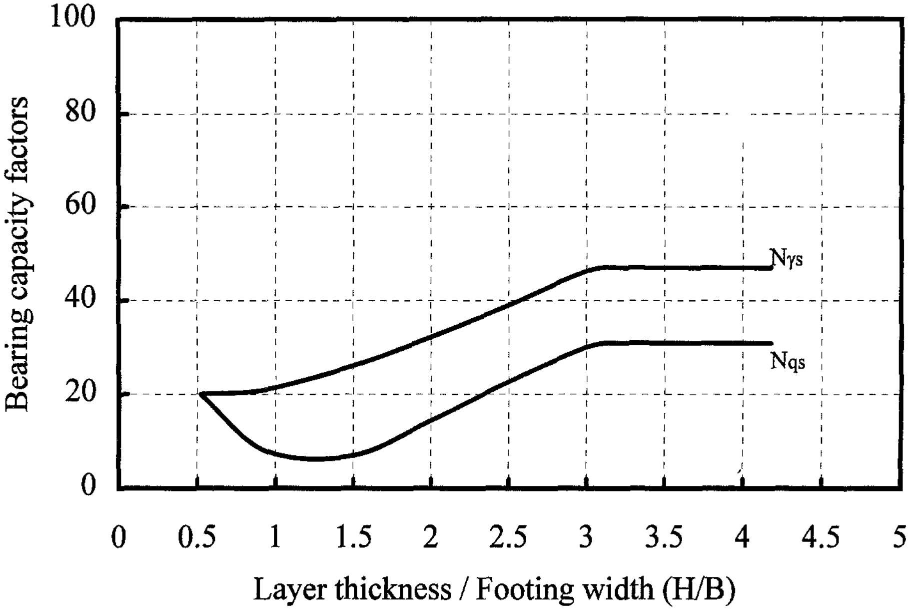

Fig. 5. Experimental results for smooth interface: variation of bearing capacity factors with relative layer thickness

Assuming the bearing capacity equation can still be stated in the form of the classical case for a continuous media, then

$$q_u = 0.5\gamma B N_{\gamma s} + P_o N_{qs} \tag{1}$$

where q_u = bearing capacity, γ = unit weight, B = foundation breadth, $P_o = \gamma D$ = surcharge and $N_{\gamma s}$, N_{qs} are dimensionless bearing capacity factors which depend on ϕ and H/B.

The experimental values of $N_{\gamma s}$ can be determined from curve 1, Fig. 4, thus allowing N_{qs} to be derived from curve 2. The results of these factors are plotted in Fig. 5. It is seen that both factors decrease as H/B decreases from a limiting value and seem to be near a minimum when H/B is small.

The figure shows that the limiting depth ratio (H_o/B) for the factor N_{qs} is similar to that for the factor $N_{\gamma s}$ which is 3.0 at $\phi = 37°$. It should be noted that H_o/B increases with increasing ϕ (Mandel and Salençon, 1972).

Theoretical considerations

A theoretical formula for $N_{\gamma s}$ is derived using the limit equilibrium analysis and a failure mechanism based on the experimental observations of displacements using the stereo photogrammetric technique (Al-Omari and Al-Taweel, 1987). It is assumed that the footing penetration which took place before failure compacts the central zone bounded by the footing base and the stratum. This zone would be densified to allow for the penetration until it reaches a state when further densification is not possible. At this state, failure would be characterized by the uniform bulging of the central zone pushing the surrounding soil and creating passive wedges at both sides. A smooth footing base is assumed, so that the stress normal to it is the major principal stress. The assumed failure mechanism is shown in Fig. 6a.

The above description is similar to the uniform lateral displacement observed in triaxial compression test samples with free ends, with the sample height representing the layer thickness (H). The value of the lateral pressure σ_3 around the central zone depends on the gravity force which in turn increases with increasing the layer thickness (H). Considering the stresses at the middle of the layer, Fig. 6a:

Lateral pressure, $\sigma_3 = 0.5\gamma\ HK_p$ where $K_p = \dfrac{1+\sin\phi}{1-\sin\phi}$

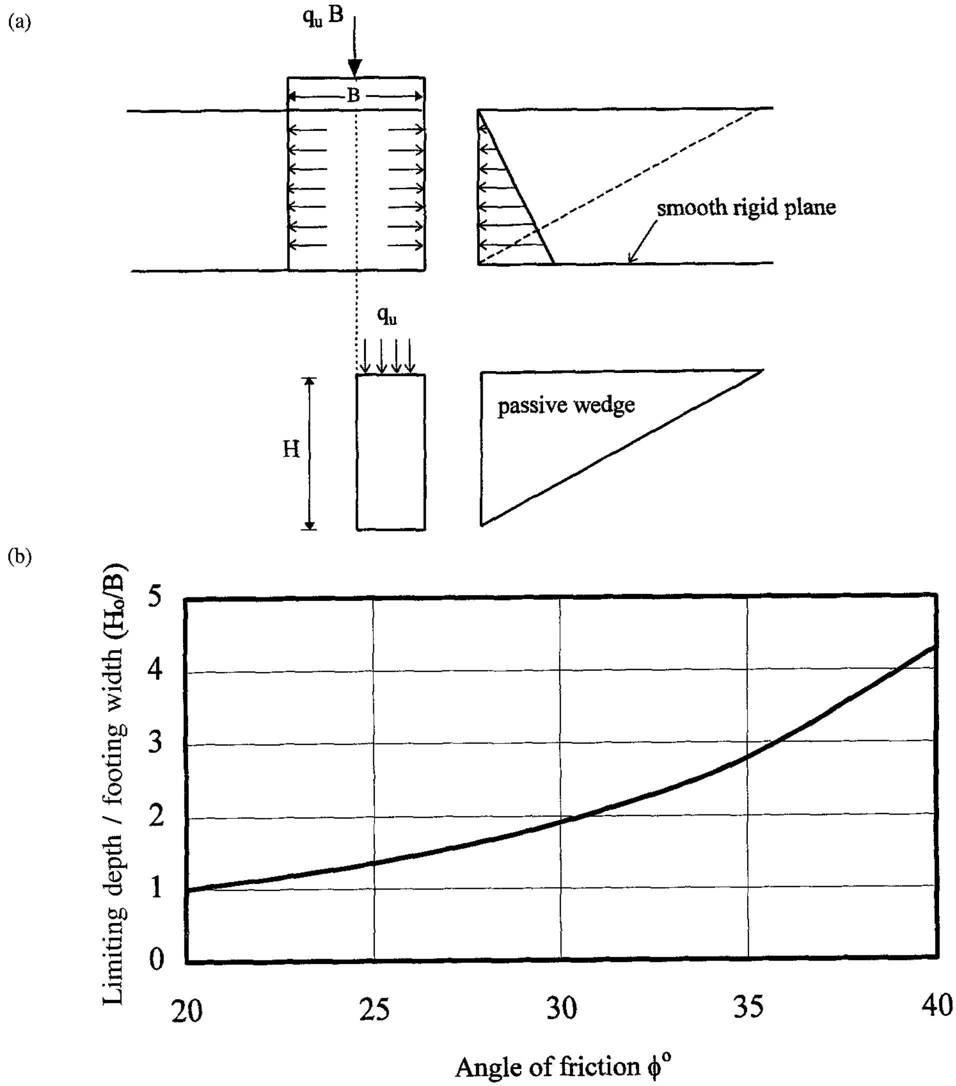

Fig. 6. (a) Assumed failure mechanism. (b) Theoretical limiting depth for $N_{\gamma s}$. (c) Proposed theoretical variation of $N_{\gamma s}$ with H/B for smooth interface

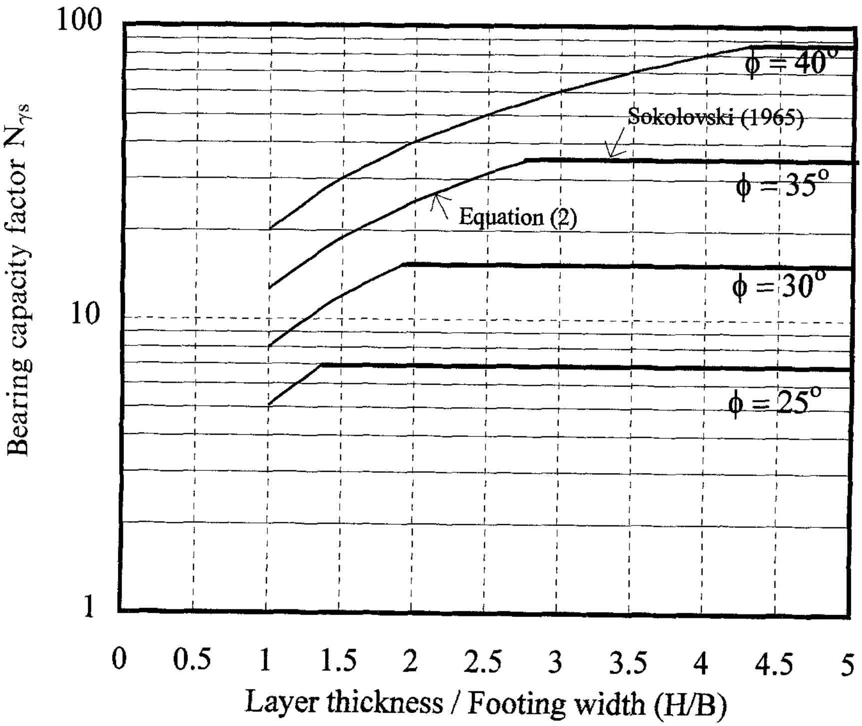

Fig. 6c.

Vertical pressure, $\sigma_1 = q_u + 0.5\gamma\ H$

Also, $K_p = \dfrac{\sigma_1}{\sigma_3} = \dfrac{q_u + 0.5\gamma\ H}{0.5\gamma\ HK_p}$

Thus, $q_u = 0.5\gamma\ H\left(K_p^2 - 1\right)$

Since $q_u = 0.5\gamma\ BN_{\gamma s}$

$\therefore\ N_{\gamma s} = \dfrac{H}{B}\left(K_p^2 - 1\right)$

$$\text{or } N_{\gamma s} = \frac{H}{B}\left[\left(\frac{1+\sin\phi}{1-\sin\phi}\right)^2 - 1\right] \qquad (2)$$

The displacement field was found to depart from the above description when H/B became smaller than 1.0. Therefore, Equation (2) is not valid when $H/B < 1.0$.

Equation (2) indicates that the value of $N_{\gamma s}$ increases with the increase in H/B. In order to determine the limiting values of H/B after which $N_{\gamma s}$ remains constant, Sokolovski's (1965) analysis for a semi-infinite layer is used by substituting in Equation (2) Sokolovski's values of N_γ (for continuous deep layer) for different values of ϕ and calculating the corresponding values of H_o/B. The results are shown in Fig. 6b. Variation of the predicted $N_{\gamma s}$ with H/B for different values of ϕ is presented in Fig. 6c.

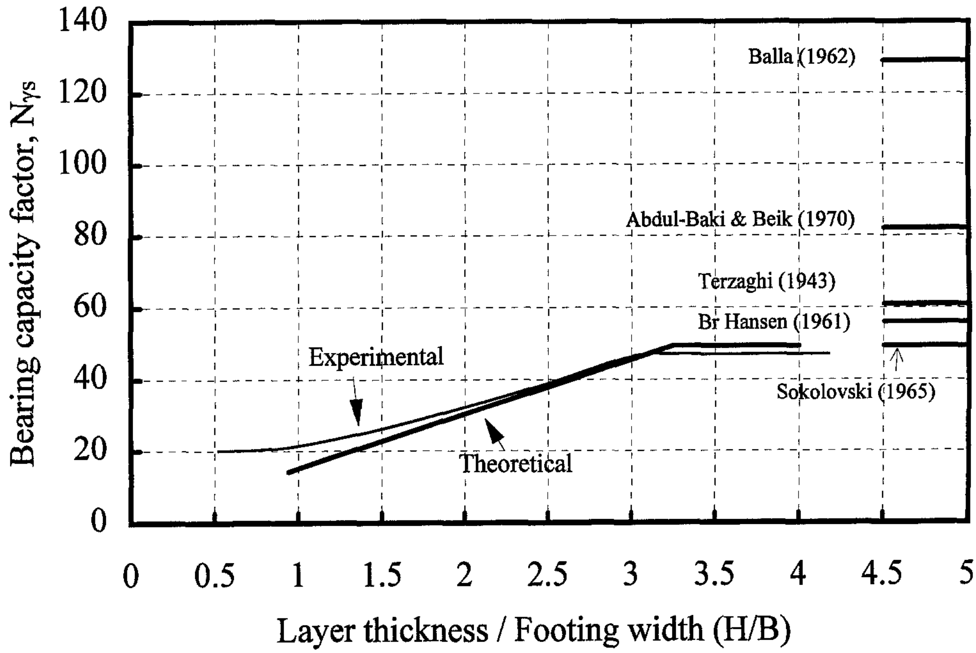

Fig. 7. Comparison between experimental and theoretical values of $N_{\gamma s}$ (ϕ = 37°)

Discussion

The experimental values of $N_{\gamma s}$ are compared with the predicted values (Equation 2) in Fig. 7. The comparison shows that when the triaxial ϕ is used, the theoretical results agree closely with the experimental results. Although this is a plane-strain problem, the use of the triaxial ϕ value seems reasonable since the phenomenon of progressive rupture reduces the mobilized angle of friction along the slip lines below that corresponding to the plane-strain conditions. It is to be noted that the use of Equation (2) when *H*/*B* is less than 1.0 gives safe but uneconomical values.

For deep layers (*H*/*B* > 3.0) the experimental results agree with the theory of Sokolovski (1965) while other theories (e.g. Terzaghi (1943), Brinch Hansen (1961), and Balla (1962)) give overestimated values even if the triaxial ϕ is used (Fig. 7).

The roughness of the footing is expected not to cause significant alteration to the results. The measured mobilized angle of footing base friction was small with an average of 11.66°. Ko and Davidson (1973) found that the effect of footing roughness on their bearing capacity results is within 10%. Abdul Baki and Beik (1970) stated that for a rough foundation, failure would occur before full friction is mobilized, and the value of the angle of footing base friction that yields minimum N_γ (failure) is somewhere between 8° and 10°; in other words there is no significant difference between a smooth and a rough foundation.

The experimental values of N_{qs} are compared with the values extracted from the theory of Mandel and Salençon (1972) for ϕ = 37° in Fig. 8. The experimental results indicate that the limiting depth at which N_{qs} reaches a constant value occurs at *H*/*B* = 3.0 which does not agree with the predicted value of 7.35. However, both the theory and the

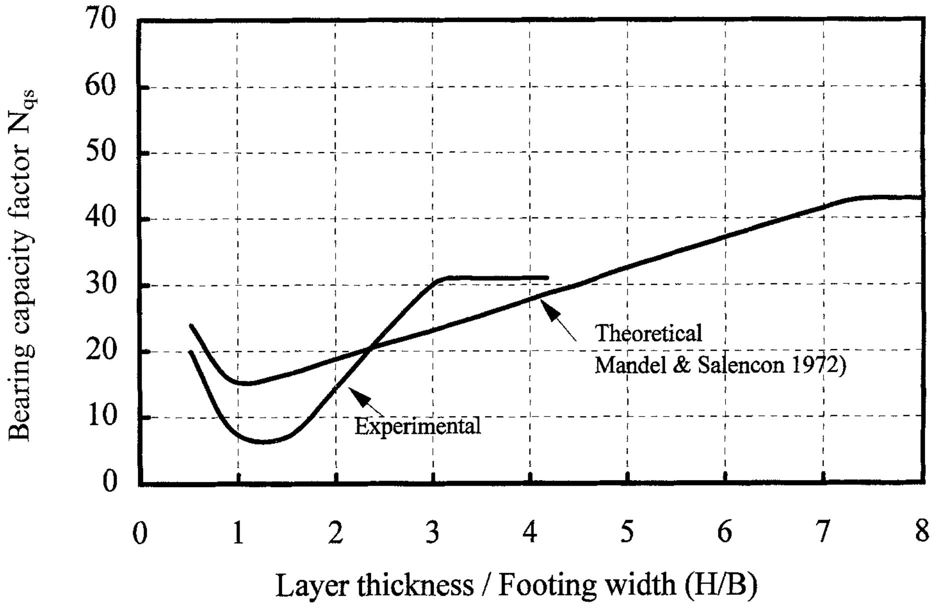

Fig. 8. Comparison between experimental and theoretical values of N_{qs}

experiments suggest that the minimum value of N_{qs} occurs at $H/B \approx 1$ although the theory gives higher values of N_{qs} at this region. Figure 8 also indicates that the experimental value of N_q for deep layers is smaller than those predicted by Sokolovski (1965) and Terzaghi (1943).

Conclusions

The bearing capacity of foundations founded on a finite layer of sand underlain by a smooth rigid stratum varies with the layer thickness. The results indicate that the bearing capacity decreases to a minimum value with the decrease in layer thickness. The factors $N_{\gamma s}$ and N_{qs} decrease to minimum values when H/B is about 0.50 and 1.3 respectively. This behaviour contrasts with the case of a rough interface where the bearing capacity increases with the decrease in the layer thickness. The limiting depth at which $N_{\gamma s}$ and N_{qs} become equal to those for a semi-infinite mass is three times the foundation breadth for the sand used at $\phi = 37°$. The theoretical approach presented in this paper is in close agreement with the experimental results if the triaxial ϕ is used. The predicted values of N_{qs} by Mandel and Salençon (1972) are not in close agreement with the experimental results. However, they can be used in conjunction with the presented values of $N_{\gamma s}$ to achieve an approximate estimation of the bearing capacity of shallow foundations.

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Notation

H	= thickness of the sand layer
B	= foundation width
N_q and N_γ	= bearing capacity factors for a semi-infinite layer
N_{qs} and $N_{\gamma s}$	= bearing capacity factors for a finite layer
H_o/B	= limiting depth
D_r	= relative density
ϕ	= angle of soil internal friction
M	= model width
D	= depth of surcharge
q	= bearing stress, pressure applied on the footing
q_u	= bearing capacity
γ	= unit weight of sand