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Adaptive Control of Single Rigid Robotic Manipulators Interacting with Dynamic Environment – An Overview

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Abstract. This paper presents the current state of the art in the adaptive control of single rigid robotic manipulators in the constrained motion tasks. A complete mathematical model of a single rigid robotic manipulator in contact with dynamic environment is presented. The basic approaches in deriving the environment model are given. The significance of the dynamic environment in the scope of the stability problem of the whole system robot-dynamic environment is emphasized. A classification of the adaptive contact control concepts in manipulation robotics is presented. The main characteristics of the most important adaptive strategies in constrained manipulation are given. The advantages and the drawbacks of the presented methods are emphasized. The paper covers results published a few years ago, as well as some recent trends in this field. One important result in the stability analysis of robotic manipulators in the constrained motion tasks is reported. Finally, some concluding remarks are given and possible future investigation trends in adaptive control of robotic manipulators are indicated.

Key words: contact tasks, adaptive control laws, adaptive stabilization, robot environment.

1. Introduction

The nature of relation between a robot and its environment can be categorized in two classes [1]. The first one concerns the non-contact, e.g. unconstrained, motion in a free workspace, without any relevant environmental influence exerted on the robot. A limited number of most frequently performed simple robotic tasks in practice, such as pick-and-place, spray painting, gluing or welding, belong to this group. In contrast to these tasks, many complex advanced robotic applications such as grinding, cutting, drilling, insertion, fastening, joining, contour following, debarring, scribing, drawing, sweeping, assembly, require the manipulator to be mechanically coupled to other objects. These tasks are refereed to as essential contact tasks, because they include phases where the robot end-effector must come into a contact with objects in its environment, produce certain forces upon them, and move along their surfaces. The terms constrained or compliant motion are usually referred to contact tasks.

Contact tasks are characterized by dynamic interaction between the robot and its environment, which often cannot be predicted accurately. In many cases, the magnitude of mechanical work exchanged between the robot and its environment during contact may vary drastically. Therefore, for a successful completion of contact tasks either the interaction forces have to be monitored and controlled, or control concepts ensuring compliant interaction with the environment must be applied. Compliance can be considered as a measure of the ability of a manipulator to react on interaction forces. The term refers to a variety of different control methods in which the end-effector motion is modified by the contact forces [1].

The type of contact tasks may vary for specific requirements, but in all cases, the robot has to perform three kinds of motion: so-called gross motion, related to robot movement in free space, compliant or fine motion, related to robot movement constrained by an environment, and interface or approach motion, representing all transitions between gross and compliant motion.

The state of the art in the non-adaptive control of robotic manipulators in constrained motion tasks is presented in [1, 2]. There are also several reviews of adaptive control of unconstrained (free) motion of robotic manipulators [3–5]. In this paper we will consider the current state-of-the-art in the adaptive constrained motion control of single rigid robotic manipulators.

The adaptive control algorithms should be employed only when simple classical non-adaptive controllers cannot achieve desirable performances. For example, when the predicted variation of the robot or environment parameters would exceed the capability of the robust control to 'overcome' it, the adaptive control, that is control with variable gains, should be introduced. Adaptive control is often necessary in the constrained motion tasks because: (i) in the contact tasks the robot works in the ambient conditions which may not be completely known in advance; (ii) in defining the environment model it is difficult to take into account all the existing effects, that is, in deriving the environment model the imprecise modeling is always unavoidable; (iii) the parameters of the robot or environment dynamics usually are not known in advance, they cannot be predicted accurately, and need to be estimated on-line; (iv) the robot or environment parameters may vary in a wide range, and it is not possible to solve the contact control task with the non-adaptive robust control with fixed gains.

The adaptive control algorithm allows the manipulator to identify its own dynamic model, or the environment model, and to adjust to changes in the manipulator or environment dynamics, in order to successfully complete the control task. Generally adaptive control is composed of two parts: (i) an identification part, which identifies parameters of the plant itself, or controller parameters, or environment parameters; (ii) a control law part, which implements a control law which is the function of the parameters identified. Hence, adaptive control techniques include automated identification scheme and control design scheme. Both schemes may be determined in different ways.

The key adaptive techniques are automatic tuning, gain scheduling, and continuous adaptation. In any case, the main problem in the synthesis of the adaptive controllers is to prove that the resulting system is stable. The desirable goals in the design of adaptive robot controllers are: (i) insensitivity to robot and environment parameter uncertainties; (ii) decoupled joint response; (iii) low demand for on-line computations.

The plan of this paper is as follows. In Section 2.1 the robot dynamics model and the model of actuators driving the robot joints are presented. In Section 2.2 two basic approaches in deriving the environment model are given. In Section 3 a classification of the adaptive contact control concepts in manipulation robotics is presented. The most important adaptive strategies in constrained manipulation and one important result in the stability analysis of robotic manipulators in the constrained motion tasks are presented in Section 4. Finally, in Section 5 some concluding remarks are given and possible future investigation trends in adaptive control of robotic manipulators are indicated.

2. Dynamic Model of Robot in Contact Tasks

2.1. The Robot Dynamics Model

The mathematical model of the robot in the constrained motion tasks consists of the model of the robot mechanism, model of the actuators driving its joints, and the environment model (contact force model) [6, 7].

Dynamic model of a robot mechanism having n degrees of freedom and interacting with its environment can be written as:

$$H(q,\xi)\ddot{q} + C(q,\dot{q},\xi)\dot{q} + g(q,\xi) = \tau - J^T(q,\xi)F, \tag{1}$$

where q is the n-dimensional vector of the robot joint angles; $H(q,\xi)$ is the $(n \times n)$ positive definite matrix of the moments of inertia of the manipulation mechanism of the robot and its actuators system; $C(q,\dot{q},\xi)\dot{q}$ is the n-dimensional nonlinear vector function representing centrifugal and Coriolis moments; τ is the n-dimensional vector of driving forces in joint space; $J(q,\xi)$ is the $(m \times n)$ full rank Jacobian matrix connecting the velocities of the robot end-effector with the velocities of the robot joint angles; i.e., $\dot{p} = (\partial f/\partial q)\dot{q} = J(q,\xi)\dot{q}$, where p is the $(m \times 1)$ vector of external robot coordinates which determines the end-effector position and orientation, f is the function which defines the relationship between joint (internal) coordinates q and external coordinates p: $p = f(q)$; $\xi = \xi(t)$ is the l-dimensional vector function of the time dependent robot parameters; $F = F(t)$ is the m-dimensional vector function of the generalized forces and moments acting on the end-effector from the environment. The dimension of the vector F can be adopted in the Cartesian coordinates to be 3 or 6. Three cases may arise: (i) $m = n$, in that case it is possible to determine the joint velocities $\dot{q}$ corresponding to the given external velocities $\dot{p}$; (ii) $m > n$, in that case it is not possible to determine the joint velocities $\dot{q}$ corresponding to the given external velocities $\dot{p}$ (except in some special cases); and (iii) $m < n$, when some additional criteria should be introduced in order to determine the unique $\dot{q}$ corresponding to the given $\dot{p}$.

The above dynamic model can be transformed into the equivalent model which is very suitable for analysis and synthesis of a robot controller in the contact tasks. This model describes the end-effector motion in the Cartesian (operational) space, Khatib [8], that is the space where manipulation tasks are naturally specified. The dimension of this space m is less or equal to the dimension of the joint space n. The dynamic model of the robot mechanism in the Cartesian space may be written as:

$$\Lambda(p)\ddot{p} + \Omega(p,\dot{p})\dot{p} + \Theta(p) = u - F, \tag{2}$$

where p is the $(m \times 1)$ vector of external robot coordinates which determines the end-effector position and orientation, $\Lambda, \Omega\dot{p}, \Theta, u$ are counterparts of $H, C\dot{q}, g, \tau$, respectively. If $m = n$, the relationships between the corresponding matrices and vectors from the joint space and the Cartesian space are given by:

$$\begin{aligned}
&\Lambda(p,\xi) = J^{-T}(q,\xi)H(q,\xi)J^{-1}(q,\xi),\\
&\Omega(p,\dot{p})\dot{p} = J^{-T}(q,\xi)C(q,\dot{q},\xi)\dot{q} - \Lambda(p,\xi)\dot{J}(q,\xi)\dot{q},\\
&\Theta(q) = J^{-T}(q,\xi)g(q,\xi),\\
&u = J^{-T}(q,\xi)\tau.
\end{aligned} \tag{3}$$

In the case of electric DC motors it is sufficiently accurate to adopt their dynamic models in the form:

$$\dot{x}^i = A^i x^i + b^i N(u^i) + f^i u^i, \quad i = 1,2,\ldots,n, \tag{4}$$

where x^i is the $(n_i \times 1)$ state vector of the ith actuator model; A^i is the $(n_i \times n_i)$ actuator matrix; b^i and f^i are the $(n_i \times 1)$ input distribution and load distribution vectors, respectively; u^i is the scalar input to the ith actuator; $N(u_i)$ is the nonlinearity of the amplitude saturation type; n_i is the order of the actuator state model. The actuator model is usually of third or second order, where $x^i = (q^i, \dot{q}^i, i_R^i)^T$ or $x^i = (q^i, \dot{q}^i)^T$ respectively, and i_R^i is the ith rotor current.

2.2. The Environment Model

The environment model describes the complex relation between the reaction force F and the end-effector position p, velocity $\dot{p}$ and acceleration $\ddot{p}$, e.g. $F = F(p,\dot{p},\ddot{p})$. There are two basic approaches in deriving the environment model:

First approach: when the contact between the robot and the environment does not imply energy transfer or dissipation, i.e. when the environment imposes purely kinematic constraints on the end-effector motion. (Yoshikawa [9], McClamroch and Wang [10], Mills and Goldenberg [11]). In this case the dynamics of the environment is neglected and only a static balance of forces and torques occurs at the contact. This environment model is only valid for frictionless contact surfaces, and since most real contact surfaces do have friction, application of this model need to be further investigated. According to this approach, the environment is described by a set of m rigid mutually independent hypersurfaces:

$$\Phi(p) = 0, \qquad \Phi(p) = [\phi_1(p), \ldots, \phi_m(p)]^T, \quad m \leqslant n. \tag{5}$$

The normal component of the interaction force F can be written as:

$$F_n = D^T(p) \cdot \lambda, \qquad D(p) = \partial\Phi(p)/\partial p, \tag{6}$$

where:

- λ is the m-dimensional vector of generalized Lagrange multipliers associated with the constraints which represent normal contact force components;
- F_n represents the normal contact force in Cartesian space;
- $D(p)$ is the $(m \times n)$ matrix.

So, in this approach the normal component of the contact force between the robot end-effector and the constraint surface can be expressed in the Cartesian space in terms of the constraint multiplier vector λ (it is assumed that the friction force $F_t = 0$).

Second approach: when the contact between the robot and the environment does imply energy transfer or dissipation, i.e. when the robot end-effector is coupled with the dynamic environment (De Luca and Manes [12], Yao and Tomizuka [13], [14]). In this case dynamic interactions between the robot and its environment are taken into account together with the purely kinematic constraints imposed on the end-effector. The robot exerts active forces at the tip, i.e. forces which are not compensated by a constraint reaction and produce work on the environment. These additional active contact forces are responsible for the energy transfer between the robot and the environment and have to be introduced when the dynamics of the environment is included.

In general case of treating the contact tasks, the complete environment dynamic model must be taken into account because the dynamic environment does not have to be passive in the consideration of the complete stability problem of the robot in contact. The environment dynamics has a significant influence on the robot stability and insufficiently accurately modeled environment dynamics can significantly influence the contact task performances. Beside that, without knowing a sufficiently accurate environment model it is not possible to determine the nominal contact force $F^0(t)$. So the assumptions as environment passivity and its stability which have been widely used in the papers up to now, do not guarantee robot stability when the robot comes into a contact with dynamic environment. This fact significantly narrows the class of the real contact tasks in which the methods which use the assumption that the dynamics of the environment can be neglected, may be practically implemented.

The overall dynamics of the robot-environment system is derived in a unique framework in [12]. The general dynamic model of the environment with which the robot is interacting may be described in the form of a vector nonlinear differential equation, [12]:

$$M(q)\ddot{q} + L(q, \dot{q}) = S^T(q)F, \tag{7}$$

where $M(q)$ is the nonsingular, positive definite and continuous $(n \times n)$ inertia matrix; $L(q, \dot{q})$ is the continuous nonlinear n-dimensional vector function; $S^T(q)$ is the continuous $(n \times m)$ matrix with a rank equal to m, i.e., $\text{rank}(S) = m$. The parametric description of the environment dynamics has been given in [12].

However, in some practical cases, it is reasonable to assume that some of the dynamic effects of the environment may be neglected. In these cases, taking into account only the dominant effects, it is sufficiently accurate to adopt a simplified linearized environment model [15–17]:

$$F = -g_E(s)(p - p_E), \tag{8}$$

where s is the Laplace operator, p_E is the $(m \times 1)$ vector of coordinates of the point of impact between the end-effector (tool) and the environment, $g_E(s)$ is the $(n \times m)$ matrix which establishes a linear mapping between $(p - p_E)$ and F. The environment model may take one of the following forms which are special cases of the general model (8):

(i) Stiffness model $F = -K_E(p - p_E)$, (9)

(ii) Damping model $F = -D_E(\dot{p} - \dot{p}_E)$, (10)

(iii) General impedance model

$$F = -M_E(\ddot{p} - \ddot{p}_E) - D_E(\dot{p} - \dot{p}_E) - K_E(p - p_E). \tag{11}$$

In the above equations K_E is the $(m \times m)$ semi-definite environment stiffness matrix, D_E is the $(m \times m)$ semi-definite environment damping matrix, and M_E is the $(m \times m)$ positive definite inertia matrix.

Supposing that the contact surface is not frictionless, i.e. that the friction force $F_t \neq 0$, Yao and Tomizuka [13, 14], modeled the interaction force F between the robot and its environment in the following way:

$$F = F_n + F_t = D^T(p)\lambda + a_t f_t(\mu, \dot{p}, \lambda) = [D^T(p) + L^T(\mu, p, \dot{p})]\lambda, \tag{12}$$

where (i) $F_t = a_t f_t(\mu, \dot{p}, \lambda)$ is the vector of the friction force; its directions are specified by a_t, the unit tangent directions of the surfaces opposite to the end-effector velocity $\dot{p} \in R^m$; the magnitude is linearly proportional to the normal contact force F_n, i.e. λ, and friction coefficient $\mu \in R^m$, with sign determined by the end-effector velocity $\dot{p}$; (ii) L is the $(m \times n)$ matrix which elements are linear with respect to the friction coefficient $\mu \in R^m$. The other terms have the same meaning as in (6).

3. Classification of Adaptive Control Methods

In general, adaptive control methods can be classified into direct, indirect and composite methods of adaptive control.

In the indirect adaptive control the adaptation algorithm is driven by the prediction error associated with the identification algorithm (Li and Slotine [18], Schwartz, Warshaw and Janabi [19], Middleton and Goodwin [20]). The prediction errors are defined as the difference between the actual torque and the predicted torque applied to the manipulator. The drawbacks of the indirect method of adaptive control are: (i) the actual tracking errors are not taken into consideration; (ii) the manipulator trajectory must be persistently excited.

In the direct adaptive control the adaptation algorithm is driven by the trajectory tracking errors. (Slotine and Li [21], Craig, Hsu and Sastry [22], Colbaugh, Glass and Seraji [23]). The drawbacks of the direct method of adaptive control are: (i) the manipulator trajectory must be persistently excited in order to ensure convergence of the parameter estimates to the true parameter values; (ii) the error dynamics is a function of the estimated parameters, i.e. it cannot be arbitrarily specified; (iii) computational complexity.

Composite adaptive control, Slotine and Li [24], bases the adaptation algorithm both on the trajectory tracking error and prediction error, and therefore represents a combination of a direct and indirect approach.

The first attempt to contribute in the field of adaptive control of robotic manipulators was made in 1979 by Dubowski and DesForges [25]. Since then, research in adaptive robot control has been very active, for both single and multiple robot manipulators. The literature on the adaptive control of rigid robot manipulators may be grouped in the following way:

(i) literature on the adaptive control of single rigid robot manipulators in unconstrained motion tasks (Ortega and Spong [5], Middleton and Goodwin [20], Slotine and Li [21], Craig, Hsu and Sastry [22], Colbaugh, Glass and Seraji [23], Koivo and Guo [26], Vukobratovic and Kircanski [27, 28], Vukobratovic, Stokic and Kircanski [29], Sadegh and Horowirz [30], Reed and Ioannou [31], Carelli, Kelly and Ortega [32]), etc.;

(ii) literature on the adaptive control of single rigid robot manipulators in constrained motion tasks (Yao and Tomizuka [13, 14], Fukuda, Kitamura and Tanie [33], Jean and Fu [34], Lu and Meng [35], Kelly, Carelli, Amestegui and Ortega [36], Slotine and Li [37], Arimoto, Liu and Naniwa [38], Walker [39], Vukobratovic and Ekalo [40, 41], Pourboghrat [42], Mo and Bayoumi [43], Lozano and Brogliato [44], Liu, Arimoto and Kitagaki [45], Zhen and Goldenberg [46], Hu et al. [47]), etc.;

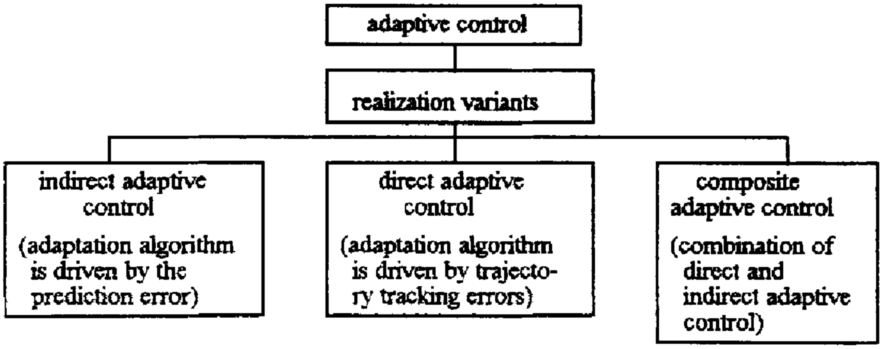

Figure 1. General classification of adaptive control methods.

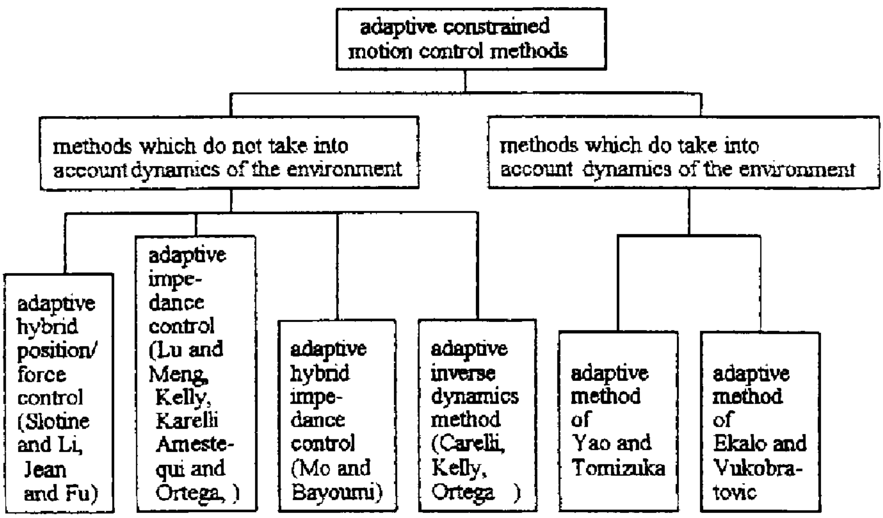

Figure 2. Classification of adaptive constrained motion control methods.

(iii) literature on the adaptive control of multiple rigid robot manipulators in unconstrained motion tasks (Koivo [48], Pittelkay [49], Seraji [50], Mo and Bayoumi [51], Walker, Kim and Dionise [52], Bolandi, Carroll and Chen [53], Zribi and Ahmad [54], Uchiyama and Yamashita [55], Hsu [56], Jean and Fu [57], Damm [58]), etc.;

(iv) literature on the adaptive control of multiple rigid robot manipulators in constrained motion tasks (Yao et al. [59], Hu and Goldenberg [60], Su and Stepanenko [61], Yao and Tomizuka [62]), etc. Since the literature on the adaptive control of rigid robot manipulators is very extensive, it should be noted that in this paper we consider only the current state-of-the-art in the adaptive control of *single* rigid robot manipulators in *constrained* motion tasks. According to the reported contributions in the field of adaptive constrained motion control methods, they can be classified into:

(i) methods which do not take into account the dynamic environments: adaptive hybrid position/force control, adaptive impedance control, adaptive inverse dynamics method (also called adaptive computed torque method), adaptive hybrid impedance control.

(ii) methods which do take into account the dynamic environments: adaptive method of Yao and Tomizuka and adaptive method of Ekalo and Vukobratovic.

The classifications reported in this section are summarized in Figure 1 and Figure 2.

4. The Most Important Adaptive Concepts in Constrained Manipulation

4.1. Adaptive Impedance Control

The adaptive impedance control problem can be formulated in the following way. Consider the robot manipulator described by (2). The dynamic vector parameter ξ of the manipulator and the payload is constant, but unknown. The Jacobian matrix $J(q)$ is assumed to be non-singular and, since it does not depend on the dynamic parameters, is assumed to be known. If p_d denotes a desired bounded motion trajectory, $e(t)$ denotes a motion error and $F(t)$ is interaction force measured at the end-effector, the impedance control problem can be stated as to design a controller which computes torque τ so that the following control aim is satisfied:

$$e(t) = p_d(t) - p(t) \longrightarrow -[s^2 M_E + sD_E + K_E]^{-1} F(t) \quad \text{as} \quad t \to \infty,$$

where $s = \mathrm{d}/\mathrm{d}t$, M_E, D_E and K_E are $(m \times m)$ diagonal positive definite matrices which define a target impedance (see (11)). So, if the impedance error μ is defined as:

$$\mu = e + [s^2 M_E + sD_E + K_E]^{-1} F(t), \tag{13}$$

then the control aim is to provide

$$\mu(t) \to 0 \quad \text{as} \quad t \to \infty.$$

The most important adaptive impedance controllers are proposed by Lu and Meng [35], and Kelly et al. [36]. Here we will present the adaptive impedance controller given in [36]. It requires measurement of only position, velocity and force. The controller consists of two parts: a control law part and adaptive part (parameter estimator). The control law part is given by

$$\tau = J^T[\widehat{\Lambda}\ddot{p}_r + \widehat{\Omega}\dot{p}_r + \widehat{\Theta} - K_D\beta + F], \tag{14}$$

$$\beta = -(\dot{\mu} + \Psi\mu), \tag{15}$$

where $\widehat{\Lambda}$, $\widehat{\Omega}$, and $\widehat{\Theta}$ are the estimates of $\Lambda(p), \Omega(p, \dot{p})$, and $\Theta(p)$, respectively; Ψ is the $(m \times m)$ matrix whose eigenvalues are strictly in the right-half complex plane; K_D is the $(m \times m)$ positive definite matrix, possibly time-varying; μ is the impedance error (13), $\dot{p}_r$ and $\ddot{p}_r$ can be determined by

$$\begin{aligned}\dot{p}_r &= \dot{p}_d + \Psi e + (sI_m + \Psi)(s^2 M_E + sD_E + K_E)^{-1} F,\\ \ddot{p}_r &= \ddot{p}_d + \Psi \dot{e} + s(sI_m + \Psi)(s^2 M_E + sD_E + K_E)^{-1} F,\end{aligned}$$

where I_m is the $(m \times m)$ identity matrix.

The authors used the following well-known fundamental property of the robot dynamics, [63]:

PROPERTY 1. All the constant parameters (link masses, moments of inertia, etc.) appear as coefficients of known functions of the generalized coordinates. If each coefficient is defined as a separate parameter, a linear relationship results so it is possible to write the dynamic model of the robot mechanism in the Cartesian space (2) as:

$$\Lambda(p,\xi)\ddot{p} + \Omega(p,\dot{p},\xi)\dot{p} + \Theta(p,\xi) = Y(p,\dot{p},\ddot{p})\xi = u - F, \tag{16}$$

where $Y(p,\dot{p},\ddot{p})$ is the $(m \times l)$ matrix of known functions, ξ is the l-dimensional vector of parameters, and the other terms have the same meaning as in (2).

In view of Property 1, the control law (14) can be written as:

$$\tau = J^T[Y\hat{\xi} - K_D\beta + F], \tag{17}$$

that is,

$$Y\hat{\xi} = \hat{\Lambda}\ddot{p}_r + \hat{\Omega}\dot{p}_r + \hat{\Theta},$$

where Y is the $(m \times l)$ matrix whose elements depend on $p, \dot{p}, p_d, \dot{p}_d, \ddot{p}_d, F$ and β.

To update the parameter vector $\hat{\xi}$ an integral adaptive law is considered:

$$\dot{\hat{\xi}} = -\Gamma Y^T \beta, \tag{18}$$

where $\Gamma = \Gamma^T$ is the $(l \times l)$ positive definite adaptation gain matrix.

The authors [36], have proved that if the control law (17) with the adaptive law (18) is applied in closed loop with the manipulator (2), then the control aim $\mu(t) \to 0$ as $t \to \infty$ is fulfilled.

The drawbacks of this method are: (i) the effects of uncertainties of the robot or environment dynamics, as well as the uncontrollable external perturbations are not studied; (ii) the environment model is adopted in a simplified linearized form (12), that is, the complete environment dynamic model is not taken into account; (iii) there is no experimental analysis on laboratory equipment.

4.2. Adaptive Hybrid Control

The most important adaptive hybrid controllers are proposed by Jean and Fu [34], Slotine and Li [37] and Lozano and Brogliato [44]. Here we will present the controller given in [37].

The problem in the adaptive hybrid control considered in [37] can be stated as that of designing a hybrid control law and an adaptation law for a robot with unknown dynamic parameters so that the tool can accurately follow the desired trajectory p_d in the unconstrained directions and impose the desired force F_d in the constrained directions. The proposed controller only adapts to the unknown parameters of the tool or the robot, and it is not adaptive to the environment parameters. The estimation of the environment parameters may be also included in the proposed law if the deformation measurements or the derivatives of the contact force are available.

First, a compliance frame R_c is set up to describe the compliant motion task and the vectors p_d and F_d representing the desired position trajectory and desired force are expressed in the compliance frame. The dynamic model of the robot is transformed into a representation in terms of velocity and acceleration vectors in the compliance frame. The tool velocity vector $\dot{p}_c$ is related to the generalized velocity $\dot{p}$ by a transformation matrix R

$$\dot{p} = R\dot{p}_c. \tag{19}$$

Therefore, the accelerations are related by

$$\ddot{p} = R\ddot{p}_c + \dot{R}\dot{p}_c. \tag{20}$$

Using (2), (19) and (20), the dynamic model of the robot mechanism in the compliance frame may be written as:

$$\Lambda_c(p)\ddot{p}_c + \Omega_c(p,\dot{p})\dot{p}_c + \Theta_c(p) = u_c - F_c, \tag{21}$$

where $F_c = R^T F$ is the contact force in the compliance frame, $\Lambda_c = R^T\Lambda R$ is the inertia matrix in the compliance frame, $\Omega_c = R^T\Omega R + R^T\Lambda\dot{R}$ is the compliance frame matrix which corresponds to matrix $\Omega(2)$, $\Theta_c = R^T\Theta$ is the gravity force in the compliance frame, and $u_c = R^T u$ is the driving force expressed in the compliance frame.

Using a hybrid control approach, the velocity vector $\dot{p}_c$ can be partitioned according to the division of constrained and unconstrained directions

$$\dot{p}_c = \begin{pmatrix} \dot{p}_p \\ \dot{p}_f \end{pmatrix}. \tag{22}$$

If the robot arm with the tool and the contact surface are assumed to be rigid, then the motion of the robot end-effector in the constrained directions is negligible compared with motion in the unconstrained directions, and therefore $\dot{p}_c$ can be written as:

$$\dot{p}_c = \begin{pmatrix} \dot{p}_p \\ 0 \end{pmatrix}. \tag{23}$$

The dynamic equations for the unconstrained and constrained directions can be partitioned as:

$$\Lambda_{c1}(p)\ddot{p}_p + \Omega_{c1}(p,\dot{p})\dot{p}_p + \Theta_{c1}(p) = u_{c1}, \tag{24}$$

$$\Lambda_{c2}(p)\ddot{p}_p + \Omega_{c2}(p,\dot{p})\dot{p}_p + \Theta_{c2}(p) = u_{c2} - f_c, \tag{25}$$

where f_c is the contact force in the force controlled directions, $u_c = \{u_{c1} \;\; u_{c2}\}^T$ is the driving force expressed in the compliance frame.

In the position controlled directions the following control and adaptation laws are considered

$$u_{c1} = \widehat{\Lambda}_{c1}(p)\ddot{p}_{pr} + \widehat{\Omega}_{c1}(p,\dot{p})\dot{p}_{pr} + \widehat{\Theta}_{c1}(p) - K_p\Psi, \tag{26}$$

$$\dot{\widehat{\xi}} = -\Gamma_p^{-1}Y_p^T\Psi, \tag{27}$$

where ξ is the l-dimensional vector containing the suitably selected set of unknown manipulator parameters, $\widehat{\xi}$ is its estimate, $\widetilde{\xi} = \widehat{\xi} - \xi$ is the parameter estimate error, Γ_p is $(l \times l)$ positive definite matrix, K_p is $(m \times m)$ positive definite matrix which may be chosen to be time-varying, $\psi = \dot{p}_p - \dot{p}_{pr}$ is $(m \times 1)$ vector which is a measure of the tracking accuracy, $\dot{p}_{pr} = p_d - Z_p\widetilde{p}_p$, where $\dot{p}_d$ is the desired motion velocity, Z_p is the $(m \times m)$ positive definite matrix, $\widetilde{p}_p$ is the position tracking error, Y_p is the $(m \times l)$ matrix which has parallel meaning as Y in Section 4.1, i.e.

$$\begin{aligned} &\widetilde{\Lambda}_{c1}(p)\ddot{p}_p + \widetilde{\Omega}_{c1}(p,\dot{p})\dot{p}_p + \widetilde{\Theta}_{c1}(p) = Y_p\widetilde{\xi}, \\ &\widetilde{\Lambda}_{c1} = \widehat{\Lambda}_{c1} - \Lambda_{c1}, \quad \widetilde{\Omega}_{c1} = \widehat{\Omega}_{c1} - \Omega_{c1}, \quad \widetilde{\Theta}_{c1} = \widehat{\Theta}_{c1} - \Theta_{c1}, \end{aligned} \tag{28}$$

where '$\widehat{\cdot}$' denotes estimates of the corresponding matrices.

In the force controlled directions the following control and adaptation laws are considered

$$u_{c2} = u_{c21} + u_{c22}, \tag{29}$$

$$u_{c21} = \widehat{\Lambda}_{c2}\ddot{p}_{pr} + \widehat{\Omega}_{c2}\dot{p}_{pr} + \widehat{\Theta}_{c2}, \tag{30}$$

$$u_{c22} = f_d - k_f(f - f_D) - k_v\dot{p}_f, \tag{31}$$

where '$\widehat{\cdot}$' denotes estimates of the corresponding matrices, f_d and f are the desired and measured contact forces in the constrained directions, k_f is the force feedback gain and k_v is the positive definite matrix which can be chosen to be an appropriate constant matrix or the following time-varying matrix

$$k_v = k_0\widehat{\Lambda}_{c22},$$

where k_0 is a positive constant, and $\widehat{\Lambda}_{c22}$ is computed using the estimated parameters from (27).

Finally, using (26), (29)–(31), it is possible to determine $u_c = \{u_{c1} \ \ u_{c2}\}^T$, and to compute the driving forces in the joint space

$$\tau = J^T R^{-T} u_c.$$

This algorithm requires measurements of the joint position, joint velocity and contact force. The drawbacks of the proposed method are: (i) it is based on the conventional hybrid control which has significant theoretical shortcomings [64], [41]; (ii) it is not adaptive to the environment parameters; (iii) it requires knowledge of the link lengths.

4.3. Adaptive Inverse Dynamics Method

The adaptive inverse dynamics method proposed by Carelli et al. [32], addressed the problem of designing an adaptive force controller which achieves a pure tracking force objective under the following assumptions: (i) the dynamic parameters of the robot are unknown, (ii) object stiffness is unknown, (iii) only position, velocity and interaction force measurements are required. This method includes a non-linear robot model, identifies both the robot and environment parameters, and does not use measurement of the acceleration nor the derivative of the interaction force.

The adaptive controller consists of a control law and a parameter update law. The control law is given by

$$F = F_e + \varphi(p, \dot{p}, F_e, F_d, \dot{F}_d, \ddot{F}_d)\widehat{\xi} + Y\dot{\widehat{\xi}}, \tag{32}$$

where F is the real force trajectory, F_d is the desired force trajectory, F_d, $\dot{F}_d$, and $\ddot{F}_d$ are bounded functions to be specified in advance, F_e is the force measured by the force sensor, $\varphi \in R^{m\times l}$ is a matrix function, $\xi \in R^l$ is the parameter vector, $\widehat{\xi}, \dot{\widehat{\xi}}$ are given by the update law, '$\widehat{\cdot}$' denotes the estimated value, Y is the $(m \times l)$ matrix defined as

$$Y = L^{-1}(s)\varphi; \quad s = \mathrm{d}/\mathrm{d}t,$$

where $L^{-1}(s)$ is a stable realizable filter, $L(s) = s + \rho, \rho$ is a positive scalar which is chosen together with α_D, α_P, so that $(k + \rho)/(k^2 + \alpha_D k + \alpha_P)$ is a strictly positive real transfer function.

The first two terms in (32) represent the adaptive inverse dynamics part of the control law and the third term is similar to the one used in the adaptive control of linear plants with relative degree two [65]. The estimated parameter vector $\widehat{\xi}$ is updated with the standard gradient type parameter update law

$$\dot{\widehat{\xi}} = -\Gamma Y^T \widetilde{F}, \tag{33}$$

where Γ is the $(l \times l)$ positive definite adaptation gain matrix, $\widetilde{F} = F_e - F_d$.

The main stability result is given by the following theorem proved in [32].

THEOREM. *Consider the control law* (32) *with the adaptive law* (33) *in a closed-loop with the combined robot-environment system* (2), (9). *Denote the parameter error by* $\tilde{\xi} = \hat{\xi} - \xi$. *Assuming that:* (a) *the contact point* p_E *is constant, and* (b) Λ^{-1} *is a constant matrix, then the control objective is ensured, that is*

(i) $\tilde{\xi}, \tilde{F}$ *are bounded, and* (ii) $\tilde{F}(t) \to 0$ *as* $t \to \infty$.

The drawbacks of the proposed method are: (i) the environment model is adopted in a simplified linearized form (9), that is, the complete environment dynamic model is not taken into account; (ii) the assumptions that the contact point p_E and the inertia matrix Λ are constant often cannot be justified. For example, the assumption that the inertia matrix is constant is justifiable when $\dot{p}$ is small, that is, when we deal with small motions of a heavy mechanism in contact with its environment; (iii) the effects of uncontrollable external perturbations are not included in the control law synthesis.

4.4. Adaptive Method of Yao and Tomizuka

Yao and Tomizuka [13, 14], have recently proposed a new method for adaptive control of single rigid robot manipulators in constrained motion tasks. Starting from the dynamic model (1), and the environment model (5), (12), the authors introduced a following transformed constrained dynamic model which is the basic equation for their adaptive controller design:

$$M(r_p)v + C_h(r_p, \dot{r}_p)\dot{r} + g(r_p) + B_m(\mu, r_p, \dot{r}_p)\lambda = T_r + \overline{G}_f\lambda, \tag{34}$$

where r is a set of curvilinear coordinates

$$\begin{aligned} r &= [r_f^T, r_p^T]^T, \quad r_f = [\phi_1(p), \ldots, \phi_m(p)]^T, \\ r_p &= [\psi_1(p), \ldots, \psi_{n-m}(p)]^T. \end{aligned} \tag{35}$$

The constraints are simply described by $r_f = 0$, that is, the robot motion is uniquely determined by the coordinates r_p; $\phi_i(p), i = 1, 2, \ldots, m$, according to (5), describe the environment by a set of m rigid hypersurfaces; $\psi_i(p), i = 1, 2, \ldots, n-m$, are $(n-m)$ twice continuously mutually independent curvilinear coordinates defining which $(n-m)$ position robot coordinates need to be controlled in the constrained motion; the choice of $\psi_i(p)$ is flexible and can be found in [66].

$$v = \begin{bmatrix} K_f\lambda \\ \ddot{r}_p \end{bmatrix}, \qquad M(r_p) = \begin{bmatrix} (I_m + G_f)K_f^{-1} & H_{12}(r) \\ H_{21}(r) & H_{22}(r) \end{bmatrix},$$

$$C_h(r_p, \dot{r}_p) = \begin{bmatrix} 0 & C_{12} \\ C_{21} & C_{22} \end{bmatrix},$$

$$\overline{G}_f = \begin{bmatrix} G_f \\ 0 \end{bmatrix}, \qquad B_m(\mu, r_p, \dot{r}_p) = B(\mu, r, \dot{r}) + B'_m(r_p),$$

$$B'_m(r_p) = -\begin{bmatrix} 0 \\ H_{21}(r)K_f \end{bmatrix},$$

$\lambda \in R^m$ is a vector of Lagrange multipliers (12), $\mu \in R^m$ is a friction coefficient (12), I_m is the $(m \times m)$ identity matrix, m is the number of the rigid hypersurfaces (5) which describe the environment, $K_f = \mathrm{diag}\{k_{f1}, k_{f2}, \ldots, k_{fm}\}$ and $G_f = \mathrm{diag}\{g_{f1}, g_{f2}, \ldots, g_{fm}\}$ are constant diagonal matrices with $k_{fi} > 0, g_{fi} > 0, i = 1, 2, \ldots, m$. Other terms have the following meaning:

$$H(r) = J_q^{-T}(q)H(q)J_q^{-1}(q) = \begin{bmatrix} H_{11}(r) & H_{12}(r) \\ H_{21}(r) & H_{22}(r) \end{bmatrix},$$

$$C(r, \dot{r}) = J_q^{-T}C(q, \dot{q})J_q^{-1} - J_q^{-T}H(q)J_q^{-1}\dot{J}_qJ_q^{-1} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix},$$

$$g(r) = J_q^{-T}g(q) = [g_1^T(r), g_2^T(r)]^T, \qquad T_r = J_q^{-T}(q) \cdot \tau = [T_{r1}^T, T_{r2}^T]^T,$$

$$B(\mu, r, \dot{r}) = J_p^{-T}L^T = [B_1^T, B_2^T]^T,$$

$$J_p = \frac{\partial r(p)}{\partial p}, \qquad J_p = [D^T(p) \;\; J_{ps}^T], \qquad J_{ps} = \frac{\partial \psi(p)}{\partial p} \in R^{(n-m)\times n},$$

$$J_q = \frac{\partial r(p(q))}{\partial q}, \qquad J_q = J_p(p(q))J(q), \qquad J_q, J_p \in R^{n\times n},$$

$J(q)$ is the $(n \times m)$ Jacobian matrix, L^T and $D^T(p)$ are defined by (12).

Similar to the Property 1 from Section 4.1, it is possible to state the following:

PROPERTY 2. The matrices $M(r_p), C_h(r_p, \dot{r}_p), g(r_p), B'_m(r_p)$ are linear in terms of a suitably selected set of robot parameters ξ i.e.

$$M(r_p)z_v + C_h(r_p, \dot{r}_p)z_r + g(r_p) + B'_m(r_p)\lambda = Y_\xi(r_p, \dot{r}_p, z_r, z_v, \lambda)\xi, \quad (36)$$

where z_r, z_v are any reference values. $B(\mu, r, \dot{r})$ is linear in terms of the friction coefficient μ:

$$B(\mu, r, \dot{r})\lambda = Y_\mu(r_p, \dot{r}_p, \lambda)\mu, \quad Y_\mu \in R^{n\times m}. \quad (37)$$

Equation (34) is the base for synthesis of the following control law:

$$\begin{aligned} T_r &= \widehat{M}(r_p, \widehat{\xi})z_v + \widehat{C}_h(r_p, \dot{r}_p, \widehat{\xi})z_r + \widehat{g}(r_p, \widehat{\xi}) + \\ &\quad + (\widehat{B}_m - \overline{G}_f)\lambda - K_s s - \overline{K}_p e_p \\ &= Y_\xi(r_p, \dot{r}_p, z_r, z_v, \lambda)\widehat{\xi} + Y_\mu(r_p, \dot{r}_p, \lambda)\widehat{\mu} - \overline{G}_f\lambda - \overline{K}_p e_p - K_s s \quad (38) \end{aligned}$$

and the adaptation law:

$$\dot{\hat{\xi}}_E = -\Gamma_\xi Y_{\xi E}^T(r_p, \dot{r}_p, z_r, z_v, \lambda)s, \tag{39}$$

$$\dot{\hat{\mu}} = -\Gamma_\mu Y_\mu^T(r_p, \dot{r}_p, \lambda)s, \tag{40}$$

where $\overline{K}_p = [0 \quad K_p]^T$, $K_p, \Gamma_\xi, \Gamma_\mu$ are constant strictly positive definite (s.p.d.) matrices, $K_s(t)$ is a uniformly s.p.d. matrix, s is a measure of motion and force tracking defined as

$$s = \begin{bmatrix} s_f \\ s_p \end{bmatrix} = \begin{bmatrix} K_f \int_0^t e_f(\nu)\,\mathrm{d}\nu \\ \dot{e}_p + De_p \end{bmatrix},$$

where D is chosen as $D = K_p^{-1}R$ and R is any $(n-m)\times(n-m)$ s.p.d. matrix; $e_p = r_p(t) - r_{pd}(t)$ is the motion tracking error; $r_{pd}(t) = \Psi(p(q_d(t))) \in R^{n-m}$ is the desired robot motion trajectory (see (35)); $e_f = \lambda(t) - \lambda_d(t)$ is the force tracking error, $\lambda_d(t) \in R^m$ is the desired force trajectory; z_r and z_v are the corresponding reference velocity and acceleration defined as:

$$z_r = \begin{bmatrix} z_{rf} \\ z_{rp} \end{bmatrix} = \begin{bmatrix} -s_f \\ \dot{r}_{pd} - De_p \end{bmatrix}, \qquad z_v = \begin{bmatrix} z_{vf} \\ z_{vp} \end{bmatrix} = \begin{bmatrix} K_f\lambda_d \\ \ddot{r}_{pd} - D\dot{e}_p \end{bmatrix},$$

$\xi = [\xi_E^T, \xi_R^T]^T$ is the constant parameter set, ξ_E is the unknown parameter set needed to be estimated on-line, ξ_R is the known parameter set. Correspondingly partition Y_ξ as

$$Y_\xi = [Y_{\xi E}, Y_{\xi R}].$$

'$\hat{\cdot}$' denotes the estimate of the corresponding matrix by substituting the estimated $\hat{\xi} = [\hat{\xi}_E^T, \xi_R^T]^T$ and $\hat{\mu}$ for the actual ξ and μ respectively.

Finally, the authors have proved [13], that for the constrained robot manipulator described by Equation (34), and if the control law (38) with update law (39) and (40) is used, the motion tracking error e_p and the force tracking error e_f asymptotically converge to zero, i.e. the robot follows the desired motion and force trajectories.

The method of Yao and Tomizuka has the following advantages: (i) it is based on a new transformed constrained dynamic model which is suitable for controller design and valid for friction surface; (ii) the unknown parameters are updated by both motion and force tracking errors; (iii) the control law can guarantee asymptotical motion and force tracking without persistent excitation condition; (iv) the proposed control is designed in the presence of parametric uncertainties both in the robot and surface friction coefficient; (v) the suggested control structure possesses robustness to the bounded measurement noise in the force and velocity sensors as well as bounded disturbances. The presented control algorithm have to be further investigated in two directions: (i) computer simulations which illustrate the stability and performance of the proposed adaptive controller have to be carried out; (ii) experimental analysis on laboratory equipment should be performed.

4.5. Adaptive Method of Ekalo and Vukobratovic

The adaptive method of Ekalo and Vukobratovic [40, 41] is presented in four sections: in 4.5.1 the task of controlling a robot in contact with its environment is formulated; in 4.5.2 a general scheme of robot adaptive control in contact tasks is given; in 4.5.3 a theorem which proves the stability of the proposed control laws is reported; and in 4.5.4 concluding discussion is given.

4.5.1. *Contact Task Setting*

In order to represent the robot control laws in a convenient way, Ekalo and Vukobratovic [40, 41] have written Equation (1) in the form

$$\tau = U(q, \dot{q}, \ddot{q}, F, \xi), \tag{41}$$

where U is the following n-dimensional vector function

$$U(q, \dot{q}, \ddot{q}, F, \xi) = H(q, \xi)\ddot{q} + C(q, \dot{q}, \xi)\dot{q} + g(q, \xi) + J^T(q, \xi)F.$$

Similarly, the real robot dynamics may be described by the vector differential equation

$$\ddot{q} = \Phi(q, \dot{q}, \tau, F, \xi) + r(t), \tag{42}$$

where

$$\Phi(q, \dot{q}, \tau, F, \xi) = H^{-1}(q, \xi)[-C(q, \dot{q}, \xi)\dot{q} - g(q, \xi) + \tau - J^T(q, \xi)F]$$

is the n-dimensional vector function, $r(t)$ represents the inadequacy of the robot dynamics description by the model (1), and/or the uncontrollable external perturbations. Ekalo and Vukobratovic adopted the general dynamic model of the environment (7), written in the form solved with respect to F

$$F = f(q, \dot{q}, \ddot{q}) + \rho(t), \tag{43}$$

where

$$f(q, \dot{q}, \ddot{q}) = (S^T(q))^{-1}[M(q)\ddot{q} + L(q, \dot{q})],$$

$f(q, \dot{q}, \ddot{q})$ and $\rho(t)$ are the n-dimensional vector functions (it is assumed that $m = n$, (see (1)); $\rho(t)$ represents the inadequacy of description of the environment dynamics by the model (7) and/or the uncontrollable external perturbations of the environment.

By eliminating the force F from the real robot dynamics (42), in accordance with the real environment dynamics (43), the following equation of the dynamic interaction of the robot with the environment is obtained

$$\ddot{q} = \varphi(q, \dot{q}, \tau, \xi) + D_1(q, \xi)r(t) + D_2(q, \xi)\rho(t), \tag{44}$$

where

$$\varphi(q, \dot{q}, \tau, \xi) = \overline{H}^{-1}(q, \xi)(-C(q, \dot{q}, \xi)\dot{q} - g(q, \xi) + \tau - N(q, \xi)L(q, \dot{q})),$$

$$D_1(q, \xi) = \overline{H}^{-1}(q, \xi)H(q, \xi),$$

$$D_2(q, \xi) = \overline{H}^{-1}(q, \xi)J^T(q, \xi),$$

$$\overline{H}(q, \xi) = H(q, \xi) - J^T(q, \xi)(S^T(q))^{-1}M(q),$$

$$N(q) = J^T(q, \xi)(S^T(q))^{-1}.$$

The following assumptions are used:

ASSUMPTION 1. The constraints of the robot motion $q(t), \dot{q}(t), \ddot{q}(t)$, the control action $\tau(t)$, the interaction force $F(t)$, and the variation of the robotic parameters $\xi(t)$ are set up in the form of the relations:

$$q(t) \in V_q \subset R^n, \qquad \dot{q}(t) \in V_{\dot{q}} \subset R^n, \qquad \ddot{q}(t) \in V_{\ddot{q}} \subset R^n, \tag{45}$$

$$\tau(t) \in \overline{V}_\tau \subset R^n, \quad \forall t \geqslant t_0, \tag{46}$$

$$F(t) \in V_F \subset R^m, \quad \forall t \geqslant t_0, \tag{47}$$

$$\xi(t) \in \overline{V}_\xi \subset R^l, \quad \forall t \geqslant t_0, \tag{48}$$

where $V_q, V_{\dot{q}}, V_{\ddot{q}}, V_\tau, V_F, V_\xi$ are the given open, constrained and simply connected sets in the corresponding spaces, $\overline{V}_\tau$ is the closure of the set V_τ in R^n, $\overline{V}_\xi$ is the closure of the set V_ξ in R^l.

ASSUMPTION 2. The inadequacy levels of the robot and environment model and/or of the external perturbations are constrained by

$$\|r(t)\| \leqslant C_r, \quad \forall t \geqslant t_0, \tag{49}$$

$$\|\rho(t)\| \leqslant C_\rho, \quad \forall t \geqslant t_0. \tag{50}$$

ASSUMPTION 3. The vector function U (41) on the set of arguments values $V_q \times V_{\dot{q}} \times V_{\ddot{q}} \times V_F \times \overline{V}_\xi$, and the vector function f (43) on the set of arguments values $V_q \times V_{\dot{q}} \times V_{\ddot{q}}$, satisfy the Lipschitz conditions with respect to each variable with the Lipschitz constants $L_q^u, L_{\dot{q}}^u, L_{\ddot{q}}^u, L_F^u, L_\xi^u$ and $L_q^f, L_{\dot{q}}^f, L_{\ddot{q}}^f$, for the functions U and f respectively. Similarly, the vector function Φ (42) on the set of arguments values $V_q \times V_{\dot{q}} \times \overline{V}_T \times V_F \times \overline{V}_\xi$, and the vector function φ (44) on the set of arguments values $V_q \times V_{\dot{q}} \times \overline{V}_\tau \times \overline{V}_\xi$, satisfy the Lipschitz conditions with respect to each variable with the Lipschitz constants $L_q^\Phi, L_{\dot{q}}^\Phi, L_\tau^\Phi, L_F^\Phi, L_\xi^\Phi$ and $L_q^\varphi, L_{\dot{q}}^\varphi, L_\tau^\varphi, L_\xi^\varphi$ for the functions Φ and φ respectively.

ASSUMPTION 4. The unknown robot parameters $\xi(t)$ belong to the known closed convex set $\overline{V}_\xi$ of the constraining variation of the robotic parameters (see (48)). C_ξ is defined as

$$C_\xi = \sup_{\xi \in \bar{V}_\xi} \|\xi\|. \tag{51}$$

ASSUMPTION 5. The robot dynamic model (41) has the Property 1 from the Section 4.1:

$$\tau(t) = U(q(t), \dot{q}(t), \ddot{q}(t), F(t), \xi(t)) = G(q, \dot{q}, \ddot{q}, F)\xi(t), \tag{52}$$

G denotes a known continuous $(n \times l)$ matrix function.

Under these conditions, Ekalo and Vukobratovic formulated the task of controlling a robot in contact with its environment in a following way. For the complete equations of the robot dynamics (42) and the environment (43), it is necessary to synthesize the admissible (i.e. satisfying the constraint (46)) control action $\tau(t), t \geqslant 0$, which, in solving the contact task, will ensure that: (i) the real robot motion $q(t), \dot{q}(t), \ddot{q}(t)$ satisfies the constraints (45); (ii) the real force of interaction of the robot and the environment $F(t)$ satisfies the constraint (47); (iii) starting from a time instant $t_p \geqslant t_0$, the following goal conditions are satisfied:

$$\|\mu(t)\| < \delta, \tag{53}$$

$$\|x(t)\| < \varepsilon, \tag{54}$$

where

$$x(t) = (\eta(t), \dot{\eta}(t))^T, \quad \eta(t) = q(t) - q_p(t), \quad \mu(t) = F(t) - F_p(t); \tag{55}$$

ε and δ are given numbers which define the stabilization accuracies, $\|\cdot\|$ denotes the Euclidean norm, $q_p(t)$ is the programmed motion and $F_p(t)$ is the programmed interaction force.

4.5.2. *General Scheme of Robot Adaptive Control in Contact Tasks*

The scheme of the adaptive control system function is based on a recurrent procedure, in which, starting from the time instant t_0, a sequence of increasing time instants $t_0, t_1, t_2, \ldots, t_k, \ldots$, is produced. On each time interval $T_k = [t_k, t_{k+1}]$ the following adaptive control law is proposed:

$$\tau_k(t) = U(\widehat{q}, \widehat{\dot{q}}, \ddot{q}_p + \Gamma_1(\widehat{\dot{q}} - \dot{q}_p) + \Gamma_2(\widehat{q} - q_p), \widehat{F}, \xi_k), \tag{56}$$

or, using (43):

$$\begin{aligned} \tau_k(t) &= U(\widehat{q}, \widehat{\dot{q}}, \ddot{q}_p + \Gamma_1(\widehat{\dot{q}} - \dot{q}_p) + \Gamma_2(\widehat{q} - \dot{q}_p), \\ &\qquad f(\widehat{q}, \widehat{\dot{q}}, \ddot{q}_p + G_1(\widehat{\dot{q}} - \dot{q}_p) + G_2(\widehat{q} - q_p)), \xi_k), \end{aligned} \tag{57}$$

where Γ_1 and Γ_2 are constant $(n \times n)$ matrices, such that the eigenvalues of the $(2n \times 2n)$ matrix

$$\Gamma = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ \Gamma_2 & \Gamma_1 \end{bmatrix} \tag{58}$$

have negative real parts; $I_{n\times n}$ is the $(n\times n)$ identity matrix, $\widehat{q}, \widehat{\dot{q}}, \widehat{F}$ are the sensors indications of the position, velocity and force respectively; ξ_k is a constant estimate on the time interval T_k; an arbitrary initial estimate vector ξ_0 is chosen from the set $\overline{V}_\xi$, and successive estimates $\xi_1, \xi_2, \xi_3, \ldots, \xi_k, \ldots$, are obtained using a special recurrent algorithm. In the text to follow this algorithm is explained.

The auxiliary function $\overline{\tau}_k(t)$ described by the relation:

$$\overline{\tau}_k(t) = U(\widehat{q}, \widehat{\dot{q}}, \widehat{\ddot{q}}, \widehat{F}, \xi_k) \tag{59}$$

is introduced. For $t \geqslant t_k$, the system of inequalities in the form

$$\|\tau_k(t) - \overline{\tau}_k(t)\| < h, \tag{60}$$

where $h > 0$ is the control scheme parameter, is considered, and the first time instant $t'_k \geqslant t_k$ at which the inequality (60) is violated is determined. So $t'_k \geqslant t_k$ is the first time instant when

$$\|\tau_k(t'_k) - \overline{\tau}(t'_k)\| \geqslant h. \tag{61}$$

Then

$$\xi_{k+1} = A(\xi_k, t'_k) \tag{62}$$

is an algorithm for correcting the estimates of the parameters, i.e. an algorithm for determining the estimate ξ_{k+1} of the current values of the unknown parameters $\xi(t)$ at the time instant t'_k, such that $\xi_{k+1} \in \overline{V}_\xi$. For example, the following algorithm for the correction of the parameters estimates may be considered:

$$\xi_{k+1} = P_{\overline{V}_\xi}\left[\xi_k + \frac{G^T(t'_k)(\tau_k(t'_k) - \overline{\tau}_k(t'_k))}{\|G(t'_k)\|^2}\right], \tag{63}$$

where

$$G(t'_k) = G(\hat{q}(t'_k), \hat{\dot{q}}(t'_k), \hat{\ddot{q}}(t'_k), \hat{F}(t'_k)) \tag{64}$$

for the case of applying the control law (56), and

$$G(t'_k) = G\Big(\hat{q}(t'_k), \hat{\dot{q}}(t'_k), \hat{\ddot{q}}(t'_k), f(\hat{q}(t'_k), \hat{\dot{q}}(t'_k), \hat{\ddot{q}}(t'_k))\Big) \tag{65}$$

for the control law (57). Here $P_{\overline{V}_\xi}$ is the orthogonal projection operator onto the set $\overline{V}_\xi$, G is $(n \times l)$ matrix function defined by (52).

The time instant t_{k+1} which follows the time sequence $t_0, t_1, t_2, \ldots, t_k$, is determined by

$$t_{k+1} = t'_k + \theta, \tag{66}$$

where θ is the time necessary for calculation of a new parameters estimate ξ_{k+1} according to the algorithm (62). The value θ plays the role of a parameter in the control scheme characterizing the workspeed of the adaptive algorithm.

By determining the time instant t_{k+1} and the estimate ξ_{k+1} the recurrence of the adaptive control scheme is completed. The algorithm for correction of the parameters estimates (62), together with the procedure of determining the time instants t'_k violating the inequality (60) represents itself the essence of the adaptation algorithm used in forming the adaptive control laws (56) and (57).

4.5.3. *Stability of the Adaptive Control Laws Proposed by Ekalo and Vukobratovic*

The authors [40, 41], have proved the stability of the adaptive control laws (56), (57) by establishing one main theorem which proof, due to the lack of the space, here will be omitted. The following definitions are used [40, 41]:

DEFINITION 1. Under the α-narrowing of open set A in R^n is understood its non-empty subset A^α, such that

$$A = \bigcup_{x \in A^\alpha} B_\alpha(x), \tag{67}$$

where $B_\alpha(x)$ is an open sphere in R^n with the center at the point x and the radius $\alpha > 0$.

DEFINITION 2. Variation of the real function $g(t)$ on the segment $[a, b]$ is described as

$$\underset{[a,b]}{\mathrm{Var}}(g) = \mathrm{Sup} \sum_{k=1}^{N} |g(t_k) - g(t_{k-1})|, \tag{68}$$

where the supremum is taken over all divisions $a = t_0 < t_0 < t_1 < \cdots < t_N = b$ of the segment $[a, b]$. If there exists a constant M, such that $\mathrm{Var}_{[a,b]}(g) \leqslant M$, the function g is called the function of bounded variation on the segment $[a, b]$.

DEFINITION 3. The function $g\colon [t_0, \infty] \to R$ is called the function of uniformly bounded variation of the order $p > 0$ if on an arbitrary segment $[a, b] \subset [t_0, \infty]$ of the length p, the function g is the function of bounded variation with the one and the same constant $M(p)$.

The following notations are used:

$$C_G^{(1)} = \text{Sup}\|G(q, \dot{q}, \ddot{q}, F)\|, \tag{69}$$

$$C_G^{(2)} = \text{Sup}\left\|G(q, \dot{q}, \ddot{q}, f(q, \dot{q}, \ddot{q}))\right\|, \tag{70}$$

where G is defined by (52) and the two suprema are taken over all points $q \in \overline{V}_q, \dot{q} \in \overline{V}_{\dot{q}}, \ddot{q} \in \overline{V}_{\ddot{q}}, F \in \overline{V}_F$.

Taking into account the a priori given set V_F, the authors established the following:

THEOREM. *Let us suppose that the following conditions are satisfied* [40, 41]:

CONDITION 1. The components $\xi_j(t), j = 1, \ldots, l$ of the robot parameters function $\xi(t)$ are the functions of uniformly bounded variations of the order $p > 0$ with the constants $M_j(p)$ (see Definition 2 and Definition 3).

CONDITION 2. The class of programmed motion $\{q_p(t)\}$ is determined with the aid of the inclusions (see Definition 1):

$$\begin{aligned} &q_p(t) \in V_q^{C_\Gamma\|x(t_0)\|+a+\delta_q+\delta_0}, \qquad \dot{q}_p(t) \in V_{\dot{q}}^{C_\Gamma\|x(t_0)\|+a+\delta_{\dot{q}}+\delta_0}, \\ &\ddot{q}_p(t) \in V_{\ddot{q}}^{C_\Gamma\|x(t_0)\|+a+\delta_{\ddot{q}}+\delta_0}, \quad \forall t \geqslant t_0, \end{aligned} \tag{71}$$

where $C_\Gamma = \sqrt{2n}\|T\|\|T^{-1}\|$, T is the non-singular transformation matrix converting the $(2n \times 2n)$ matrix T (58) into its diagonal form

$$T^{-1}\Gamma T = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{2n}),$$

$\lambda_i, i = 1, 2 \ldots, 2n$ are the eigenvalues of the matrix Γ, n is the number of the robot degrees of freedom, $\|x(t_0)\|$ is the initial perturbation (see (55));

$$a = \overline{\delta} + L_\tau C_\Gamma \lambda^{-1} h + CK \left(\sum_{j=1}^{l} M_j(p)\right) \theta,$$

where $\overline{\delta} = C_\Gamma(\delta_{\ddot{q}} + \|\Gamma_1\|\delta_{\dot{q}} + \|\Gamma_2\|\delta_q)\lambda^{-1}$, $\lambda = -\max_i \lambda_i$, δ_q and $\delta_{\dot{q}}$ determine the sensors errors

$$\|\Delta_q(t)\| \leqslant \delta_q, \qquad \|\Delta_{\dot{q}}(t)\| \leqslant \delta_{\dot{q}}, \quad \forall t \geqslant t_0,$$

where

$$\Delta_q(t) = \widehat{q}(t) - q(t), \qquad \Delta_{\dot{q}}(t) = \widehat{\dot{q}}(t) - \dot{q}(t), \quad \forall t \geqslant t_0,$$

are the error functions of the sensors, $\widehat{q}$ and $\widehat{\dot{q}}$ are the sensors indications of the position and velocity, respectively, Γ_1 and Γ_2 are defined by (58); $L_\tau = L_\tau^\Phi$ for the control law (56), and $L_\tau = L_\tau^\varphi$ for the control law (57), where L_τ^Φ and L_τ^φ are the Lipschitz constants (see Assumption 3 from Section 4.5.1).

$$C = C_\Gamma(2C_{\ddot{q}} + 2\delta_{\ddot{q}} + d_1 C_\tau + d_2 C_\rho + \|\Gamma_1\|\delta_{\dot{q}} + \|\Gamma_2\|\delta_q),$$

where

$$C_{\ddot{q}} = \mathrm{Sup}\|\varphi(q, \dot{q}, \tau, \xi)\|, \text{ over all } (q, \dot{q}, \tau, \xi) \in \overline{V}_q \times \overline{V}_{\dot{q}} \times \overline{V}_\tau \times \overline{V}_\xi,$$

$\delta_{\ddot{q}}$ determines the sensor error

$$\|\Delta_{\ddot{q}}(t)\| \leqslant \delta_{\ddot{q}}, \quad \forall t \geqslant t_0,$$

where $\Delta_{\ddot{q}}(t) = \widehat{\ddot{q}} - \ddot{q}$ denotes the deviation of the estimated value of the second derivative of the robot motion $\widehat{\ddot{q}}$ from its real value $\ddot{q}$. The estimate $\widehat{\ddot{q}}$ can be obtained by sensors (accelerators) or by numerical differentiation of the estimate $\widehat{\dot{q}}$.

$$d_1 = \mathrm{Sup}\|D_1(q, \xi)\|, \qquad d_2 = \mathrm{Sup}\|D_2(q, \xi)\|, \quad \text{at } (q, \xi) \in V_q \times \overline{V}_\xi.$$

$D_1(q, \xi)$ and $D_2(q, \xi)$ are defined by (44), C_τ is defined by (49), C_ρ is defined by (50).

The quantity $K\left(\sum_{j=1}^{l} M_j(p)\right)$ is defined by

$$K\left(\sum_{j=1}^{l} M_j(p)\right) = \frac{(\mathrm{diam}\,\overline{V}_\xi)^2 + 4C_\xi\left(\sum_{j=1}^{l} M_j(p)\right)}{hh_0 C_G^{-2}},$$

where $\mathrm{diam}(\overline{V}_\xi)$ is the diameter of the set $\overline{V}_\xi$; other terms have the following meaning:

$$C_G = \begin{cases} C_G^{(1)}, & \text{for control law (56)} \\ C_G^{(2)}, & \text{for control law (57)} \end{cases},$$
$$h_0 = \begin{cases} h - 2h_1, & \text{for control law (56)} \\ h - 2h_2, & \text{for control law (57)} \end{cases}, \tag{72}$$

where $C_G^{(1)}$ and $C_G^{(2)}$ are defined by (69) and (70), respectively; $h > 0$ (60), is the parameter of the adaptation scheme, chosen such that the quantity h_0 is positive; h_1 and h_2 are defined by

$$\begin{aligned} h_1 &= L_q^u\delta_q + L_{\dot{q}}^u\delta_{\dot{q}} + L_{\ddot{q}}^u(\delta_{\ddot{q}} + C_\tau) + L_F^u\delta_F < h, \\ h_2 &= L_q^u\delta_q + L_{\dot{q}}^u\delta_{\dot{q}} + L_{\ddot{q}}^u(\delta_{\ddot{q}} + C_\tau) + L_F^u(L_q^f\delta_q + L_{\dot{q}}^f\delta_{\dot{q}} + L_{\ddot{q}}^f\delta_{\ddot{q}} + C_\rho) < h. \end{aligned}$$

In the above equations $L_q^u, L_{\dot{q}}^u, L_{\ddot{q}}^u, L_F^u, L_\xi^u$ and $L_q^f, L_{\dot{q}}^f, L_{\ddot{q}}^f$, are the Lipschitz constants for the functions U (41), and f (43), respectively, (see Assumption 3 from Section 4.5.1); δ_0 is a suitably small fixed number, and the algorithm scheme parameter θ is defined by (66).

CONDITION 3. The class of the programmed interaction force $\{F_p(t)\}$ is determined with the aid of the inclusion

$$F_p(t) \in V_F^{L(C_\Gamma\|x(t_0)\|+a+\delta_0)+C_\rho+\delta_F}, \quad \forall t \geqslant t_0,$$

where

$$L = L_q^f + L_{\dot{q}}^f + L_{\ddot{q}}^f,$$

$L_q^f, L_{\dot{q}}^f, L_{\ddot{q}}^f$ are the Lipschitz constants (see Assumption 3 from Section 4.5.1); δ_F denotes the sensor error

$$\|\Delta_F(t)\| \leqslant \delta_F,$$

where $\Delta_F(t) = \widehat{F}(t) - F(t)$ is the error function of the force sensor, $\widehat{F}(t)$ is the force sensor indication.

CONDITION 4. The levels of the initial perturbation $\|x(t_0)\|$, of the external perturbations C_r and C_ρ, of the sensors errors $\delta_q, \delta_{\dot{q}}, \delta_{\ddot{q}}$ and δ_F, and of the adaptation algorithm scheme parameters h and θ are such that these classes are non-empty, $a < \|x(t_0)\|$ and the quantity h_0 defined by (72) is positive.

CONDITION 5. The stabilization accuracies δ (53) and ε (54) are given numbers which satisfy the inequalities

$$a < \varepsilon, \qquad La + C_\rho < \delta. \tag{73}$$

CONDITION 6. The order p satisfies the inequality

$$p \geqslant \max\{p_1, p_2, p_3\},$$

where

$$p_1 = \frac{1}{\lambda} \ln \frac{C_\Gamma\|x(t_0)\|}{\|x(t_0)\| - a}, \quad p_2 = \frac{1}{\lambda} \ln \frac{C_\Gamma\|x(t_0)\|}{\varepsilon - a}, \quad p_3 = \frac{1}{\lambda} \ln \frac{LC_\Gamma\|x(t_0)\|}{\delta - La - C_\rho}.$$

If the conditions (1)–(6) are satisfied, then for all $t \geqslant t_0$, the following statements will hold [40, 41]:

Statement 1. For the transient processes determined by the adaptive control laws (56) and (57), the estimates

$$\|x(t)\| < C_\Gamma\|x(t_0)\| + a, \qquad \|\mu(t)\| < L(C_\Gamma\|x(t_0)\| + a) + C_\rho$$

are fulfilled.

Statement 2. The real robot motion $q(t), \dot{q}(t)$ will satisfy the constraints (45).

Statement 3. The control $\tau(t)$ will be admissible, that is, the constraint (46) will be fulfilled.

Statement 4. The real force of interaction of the robot with the environment $F(t)$ will satisfy the constraint (47).

Statement 5. The goal condition (54) will be fulfilled not later than the time instant:

$$t_{p_2} = \begin{cases} t_0 + p_2, & \text{if } C_\Gamma \|x(t_0)\| + a \geqslant \varepsilon \\ t_0, & \text{in opposite case} \end{cases}.$$

Statement 6. The goal condition (53) will be fulfilled not later than the time instant:

$$t_{p_3} = \begin{cases} t_0 + p_3, & \text{if } LC_G \|x(t_0)\| + La + C_r \geqslant \delta \\ t_0, & \text{in opposite case} \end{cases}.$$

4.5.4. *Concluding Discussion*

Ekalo and Vukobratovic [40, 41], proposed a general approach to the synthesis of adaptive control laws solving the task of simultaneous stabilization of motion and interaction force of the robot with the environment for the case when the robot parameters may vary with time in an unknown way. This approach is based on the unified approach to control laws synthesis for robotic manipulators in contact with dynamic environment proposed by Vukobratovic and Ekalo in [67–69]. Their approach, differing from the impedance control (Hogan [17]), and the hybrid control (Raibert and Craig [70]), is based on the general dynamics equation of the environment (7), and is solving simultaneously both the stabilization tasks of position and interaction force with the environment. The unified approach proposed by Vukobratovic and Ekalo is aimed at direct realization of a desired robot motion and desired interaction force, which, as a pair of time-functions, satisfy the nonlinear differential equation of the environment dynamics model (7). In the adaptive case, this goal is achieved by designing a special adaptive control scheme (Section 4.5.2) and using the finite-convergence adaptation algorithms, [71].

Ekalo and Vukobratovic have established the dependencies of the classes of stabilized motion and forces, and their stabilization accuracies, on the levels of initial and external perturbations of the robot and environment dynamics, as well as of the sensors errors, workspeed of the adaptation algorithm, and of other parameters of the adaptive control scheme. The degree of parameter drift and the degree of inadequacy of the robot model are determined by the class of functions of uniformly bounded variation. Ekalo and Vukobratovic assumed that the function of variation of robotic parameters is not apriori known, and only the constrained, closed set $\overline{V}_\xi \in R^l$ of possible values of the robot parameters has been given. The adaptive scheme is based on recurrent procedure, in which a sequence of increasing time instants $t_0, t_1, t_2, \ldots, t_k, \ldots$, is produced. On each time interval $T_k = [t_k, t_{k+1}]$, either the adaptive control law (56) or (57) is utilized, in which the robot parameter estimate $\hat{\xi}(t)$ is replaced by the constant estimate $\xi(t)$. At this, an arbitrary vector from the set $\overline{V}_\xi$ is chosen as the initial estimate ξ_0, and successive estimates $\xi_1, \xi_2, \ldots, \xi_k, \ldots$, are obtained using a special recurrent algorithm (62). So, Ekalo and Vukobratovic proposed the algorithm for correction of the parameter estimates (62) and a system of inequalities (60) on the basis of which the procedure of adaptation algorithm is actually functioning. The algorithm for parameter correction (62) and the system of inequalities (60) represent the essence of the adaptation algorithm used in forming the adaptive control laws (56), or (57).

This approach ensures: (i) solution of the contact tasks for robots with both stationary and nonstationary dynamics; this method is practically the first with this characteristic; (ii) direct realization of a desired robot motion and desired interaction force which, as a pair of time functions, satisfy the general nonlinear second-order differential equation of the environment dynamics model (7); (iii) adaptation to any unknown deviation from the class of functions of uniformly bounded variation; (iv) stabilization of the proposed control laws to the initial and external perturbations as well as the measuring sensors errors; (v) adaptation to the essentially inadequate description of the robot dynamics by its mathematical model.

5. Conclusion

During the past several years, adaptive constrained robot control has emerged as one of the most attractive and fruitful research areas in robotics. The adaptive control of constrained motion of robots is a challenging research area whose successful solution will considerably affect further application of robots in industry and increase their efficiency and productivity.

The adaptive control is often necessary in the constrained motion tasks because the modeling of the robot and environment is often imprecise and the parameter uncertainties in the robot and environment model always exist. Therefore, in order to successfully solve the contact control task, the adaptive control methods which allow the manipulator to identify its own dynamic model, or the environment model, and to adjust to changes in the manipulator or environment dynamics, are introduced.

In this paper we have attempted to present an overview of the state of the art in adaptive contact control concepts in manipulation robotics on the base of the work reported in the open literature. First, a complete mathematical model of a rigid robot manipulator in contact with dynamic environment is presented together with several different environment models. In the general case of treating the contact tasks, the complete real environment dynamic model which has been neglected in the papers up to now, must be taken into account because the dynamic environment has a significant influence on the robot stability and the contact task performances. However, uncertainties, particularly in the dynamic model of the environment, represent one of the main problems in a synthesis of control laws. Thus, for this type of control tasks the practical stability appears to be more appropriate than the asymptotic (exponential) stability since it enables to consider effects of uncertainties in the control laws which cannot guarantee asymptotic stability at all, but can fulfill practical stability conditions [72].

A classification of the adaptive contact control concepts in manipulation robotics is presented. Certainly there are other schemes which may not fit well into this classification. The main characteristics of the several most important adaptive strategies in constrained manipulation are presented. The advantages and the drawbacks of the presented methods are emphasized. It may be concluded that the method of Yao and Tomizuka (Section 4.4) and especially the method of Ekalo and Vukobratovic (Section 4.5) has several significant advantages over the other methods. First of all, the method of Yao and Tomizuka and the method of Ekalo and Vukobratovic take into account the dynamics environment: the first one is based on a new transformed constrained dynamic model which is valid for friction surface, and the second one assumes that the environment dynamics model is described by the general nonlinear second-order differential equation (7).

The method of Ekalo and Vukobratovic is the first one which gives solution of the contact tasks of robots with both stationary and nonstationary dynamics; it is also the first one which ensures direct realization of a desired robot motion and desired interaction force which satisfy the general nonlinear second-order differential equation of the environment dynamics model; the proposed control laws are adaptive to any unknown deviation from the class of functions of uniformly bounded variation and stable to the initial and external perturbations as well as the measuring sensors errors. The theorem which proves the stability of the adaptive control laws proposed by Ekalo and Vukobratovic is reported in this paper. More appropriate stability investigation of the adaptive control has to be a very challenging topic in robotic contact tasks.

All the results have the following general drawback: they have not been experimentally evaluated and implemented in real robot control. So the next step in the adaptive contact control synthesis is direct practical application of the proposed strategies and verification of their effectiveness.

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