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Paretian Efficiency, Rawlsian Justice and the Nozick Theory of Rights*

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Abstract. In this paper the circumstances under which a Nozickian libertarian claim can produce self-consistent results are studied. These circumstances are shown to consist of technological separability of social alternatives and partial separability of individual preferences. Under these conditions some restricted versions of the Pareto principle and of the Rawlsian maximin can be reformulated consistently with the Nozickian libertarian claim. Also, in absence of external utility, Paretian efficiency, Rawlsian distributive justice and the Nozickian libertarian claim are consistent and can be together satisfied by a choice procedure. Moreover, the Nozick libertarian claim is shown to be satisfied by a perfectly competitive economy. A parallelism between a general equilibrium market system and a Nozickian rights system is pointed out.

1. Introduction

Sen [6] and [7] was the first to bring rights and personal liberty to the attention of economists. He showed a conflict to exist between the criterion of economic efficiency expressed by the Pareto principle and a mild libertarian condition. Gibbard [3] moved the problem forward underlining that a stronger but still very weak libertarian claim can perfectly well be in conflict with itself. The respect of others' personal spheres is a condition which must be guaranteed but which can only with difficulty be guaranteed without restricting the possible manifestation of preferences or the magnitude of the rights assignment. Sen's result and Gibbard's result have been followed by a huge literature; Nozick's contribution to it can be summarized by the following quotation [5, p. 166]:

"Individual rights are co-possible; each person may exercise his rights as he chooses. The exercise of these rights fixes some features of the world. Within the constraints of these fixed features, a choice can be made by a social choice

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mechanism based upon a social ordering, if there are any choices left to make! Rights do not determine a social ordering but instead set the constraints within which a social choice is to be made, by excluding certain alternatives, fixing others, and so on".

My opinion on the general matter of rights and on the dilemma between Paretianism and libertarianism is very close to Sen's [8]: only a case study can tell us which condition is to be maintained and which one should be modified in any particular case. There is no general answer but different answers in different contexts[1]. On the other hand, Nozick's proposal is a clear-cut scheme: the rights must be coherent and personal liberty comes before any other condition. As a result of these assumptions Pareto-inefficient outcomes are perfectly possible. True, the question of outcomes in Nozick's view is not that important. The priority of rights is beyond any discussion and the final outcomes have to be accepted precisely because they descend from the right social rules, not because they are judged as good results. The nonconsequentialist, procedural approach of Nozick is quite different from the usual economists' assessment of a social system in terms of the goodness of results. For an economist, in general, rights are only functional, they are of interest in so far as they produce good results. The importance of some form of nonconsequentialist reasoning for welfare economics could certainly be discussed, but that is not within the scope of this paper. On the other hand, I think it is hard if not impossible, expecially if we are interested in assessing the value of economic environments and institutions, to disregard the final outcomes completely[2].

In this paper I try to translate Nozick's point of view into a consequentialist language. My aim is to see if there are circumstances under which a libertarian claim in the spirit of Nozick can produce good social results. Good results meaning self-consistency of the claim and compatibility with the requirement of Pareto efficiency.

I will show that such circumstances exist and that they consist of a particularly separable social structure, i.e. technological separability of alternatives, partial separability of individual preferences, separable social choice functions, etc. If these conditions hold, it is not only possible to solve the Gibbard paradox and to show that the Nozick libertarian claim is consistent with a restricted version of the Pareto principle, but, what is more, we can reformulate the Rawlsian maximin principle in a way that makes it consistent with Nozick's theory of rights. In other words it is possible to consider Nozick's rights entitlements scheme as a specification of Rawls' first principle of justice (equality in the assignment of basic rights and duties) and to show it is compatible with the second Rawlsian principle, the difference principle or maximin. Moreover the existence of a relation of complementarity between the Nozickian view of liberty and the general equilibrium description of a competitive market system will be elucidated. A pure exchange economy satisfies a generalized version of the Nozickian libertarian claim; on the other hand, the absence of externalities guarantees that the social outcome arising from a Nozickian rights system may be economically (Pareto) efficient. The appropriate economic environment of a consequentialist Nozickian is therefore shown to be the pure neoclassical

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1 See also [13] for a similar point of view

2 See [10] for a discussion of the ethical validity of the market system

world of a perfectly competitive economy without externalities. The Nozickian libertarianism provides a moral justification for the market system whereas the working of the market system supports the Nozickian view by assuring the goodness of consequences. The descriptive limits of the perfectly competitive analysis of the market system and the diffuse extension of market failures pose serious doubts to the general accettability of the Nozickian rights system in terms of consequences.

The assumptions of technological and preferences separability together with the concept of separable social choice function (SSCF) are defined in Sect. 2. An SSCF is a choice procedure which gives an answer only if the set of alternatives from which to choose has a particular structure: i.e. if it can be seen as the product space of separate individual feature spaces. In Sect. 3 two versions of the Nozick libertarian claim are presented, and it is shown how the Gibbard paradox can be solved using them: there is an SSCF satisfying both the strong and the weak version of the Nozick libertarian claim. Section 4 is devoted to the analysis of a restriction of the Pareto principle which is consistent with the Nozick libertarian claim. Such a restriction aims at limiting the working of the Pareto principle to that part of a social alternative which is of clear public interest. In Sect. 5 the Rawlsian maximin principle is taken into consideration and it is demonstrated that a restricted version of it is consistent with the Nozick libertarian claim. It is argued that this result shows how the Nozick libertarian claim can be interpreted as Rawls' first principle of justice, the principle of maximum amount of liberty for all. If this is done, the apparent conflict between the two theories of justice is solved in a compromise which embeds a strong form of Nozickian libertarianism in a Rawlsian framework. Section 6 is devoted to the analysis of the effect of excluding external utility: it is demonstrated that in such a circumstance the Nozick libertarian claim and the public Pareto principle imply the Pareto principle. The section ends showing how a very simple exchange economy satisfies a slightly generalized version of the Nozick libertarian claim, and how in that environment the condition of no external utility assures the achievement of the Pareto efficiency. In Sect. 7 the conclusions are drawn.

2. Alternatives, Preferences, Rights and Choice Functions

I will adopt the approach of formalizing social alternatives in terms of issues introduced by Gibbard [3]. Each alternative x is represented by a set of distinct features. My formalization of a rights system is different from the one usual in the literature[3]: a rights system D is just an assignment of sets of features to individuals. For simplicity I assume each individual has only one set of features and there is only one set of public features, but all the results of this paper can be easily projected onto more general situations.

[3] See [11] for an example

Definition 1. 1) Society I is composed of n individuals; $I=:\{1,\ldots,n\}$. 2) The set of alternatives X is the cartesian product $\prod_{i=0}^{n} X_i$ where any X_i is a compact space. 3) Given X the rights assignment D is defined as $D=:\{(X_1,1),\ldots,(X_n,n)\}$; if $(X_j,j)\in D$ we say that the rights system assigns the set of features X_j to the individual j. 4) If Definition 1.3 is not to be empty it is necessary that there exists a j such that $X_j\neq\emptyset$; a rights system is said to be full if, for any $j\in I$, $X_j\neq\emptyset$.

For any $i\in I$, X_i is to be interpreted as the set of features regarding individual i and therefore assigned to him by the rights system D. X_0 is the set of public features. Definition 1.2 is a fundamental assumption of technological separability: every social alternative can be decomposed into separate features. The assumption of compact feature spaces is a generalization of the usual social choice setting of a finite space of alternatives. In order to describe a situation under which the control individuals have over their personal spheres gives rise to effective exercise of their rights, some restrictions on the possible manifestation of preferences have to be made. It will be shown that partially separable individual preferences are sufficient for the exercise of rights being co-possible.

An individual's preference relation is partially separable if it can be represented by a utility function which is separable with respect to his and others' personal feature spaces and to the public feature one. In what follows I assume that all the individuals have partially separable utility functions. This is obviously a restriction on the usual domain of preferences in social choice theory, i.e. weak orderings.

Definition 2. An utility function $U_i: X\to R_+$ is said to be partially separable if there exist continuous functions $v_i^0, v_i^i, v_i^{-i}(v_i^0: X_0\to R_+, v_i^i: X_i\to R_+, v_i^{-i}: X_{-i}\to R_+$, where $X_{-i}=\prod_{j\neq 0,i} X_j)$, and there exists a continuous function $V_i: R_+^3\to R_+$ such that $U_i(x)$ $=V_i(v_i^0(x_0), v_i^i(x_i), v_i^{-i}(x_{-i}))$. The utility function is said to be additively separable if it is such that, for any $x\in X$, $U_i(x)=V_i(v_i^0(x_0)+v_i^i(x_i)+v_i^{-i}(x_{-i}))$.

Let $\mathscr{V}$ be the set of partially separable preferences and $\mathscr{V}^n$ the cartesian product of $\mathscr{V}$ n times. Let $\mathscr{Z}$ be the set of all non void closed subsets of X; define $\mathscr{W}$ as the following subset of $\mathscr{Z}$; $\mathscr{W}=:\left\{S\subset X/S=\prod_{i=0}^{n} S_i \text{ where } S_i \text{ is any non void closed subset of } X_i\right\}$. Note that any S belonging to $\mathscr{Z}$ or $\mathscr{W}$ is also compact because it is a closed subset of the compact space X.

Definition 3. A separable social choice function (SSCF) is a map $f: \mathscr{V}^n\times\mathscr{W}\to\mathscr{W}$ such that for any $S\in\mathscr{W}$ and for any $A\in\mathscr{V}^n$, $f(A,S)$ is a non void closed subset of S.

An SSCF is a rule which assigns a choice set to every subset S of X which can be written as the cartesian product of closed sets S_i. The ideas underlying the definition of an SSCF are those of separability and decentralization: an SSCF f can be seen as the cartesian product of $n+1$ choice functions f_i each of them mapping into $\mathscr{Z}_i$, the set of non void closed subsets of X_i; $f_i: \mathscr{V}^n\times\mathscr{W}\to\mathscr{Z}_i$, $f(A,S)=\prod_{i=0}^{n} f_i(A,S)$ such that, for any $S\in\mathscr{W}$ and for any $A\in\mathscr{V}^n$, $f_i(A,S)$ is a non void closed subset of S_i.

3. The Nozick Libertarian Principle

Having specified the space of social alternatives and the assignment of rights we now need a rule for implementing them. I will give two rules which aim to follow Nozick's idea of the unrestricted exercise of rights. My intention is to describe a decentralized system which gives every individual control over his personal sphere. Even though personal control over decisions regarding rights is not always possible, as Sen [9] has pointed out, it seems desirable to have such a control whenever possible. It is clear, anyway, that in Nozick's view individual control is an essential feature of personal liberty. Personal control together with the assumption of separable preferences will be shown to assure to every individual the possibility of real unilateral exercise of their rights: formal control can be effectively implemented. Actually Nozick is really concerned only with giving control to individuals. On the contrary, implicit consequentialist assumption in what follows will be that assigning rights is not enough if they cannot be exercised in the way individuals would like to, i.e. if desired results cannot be achieved.

Condition NL_1 (Strong Nozick libertarian claim). Given a full D and given any x, $y \in X$, if $A \in \mathscr{V}^n$ is such that there exists $j \in I$ with $v_j^j(x_j) > v_j^j(y_j)$ then $y \notin f(A, S)$ when $x \in S$.

Condition NL_2 (Weak Nozick libertarian claim). Given a full D and given any x, $y \in X$, if for any $i = 0, 1, \ldots, n, i \neq j \in I$, $x_i = y_i$, and if $A \in \mathscr{V}^n$ is such that $v_j^j(x_j) > v_j^j(y_j)$ then $y \notin f(A, S)$ when $x \in S$.

In formal social choice theory the problem of a possible conflict between the rights of different individuals was firstly posed by Gibbard [3] who proposed a self-consistent libertarian claim according to which only individuals who express 'unconditional preferences' can implement their rights. Unconditional preferences are those preferences which are separable with respect to every individual's own feature space. Apart from the more general context of compact rather than finite feature spaces, condition NL_2 is equivalent to the Gibbard self-consistent libertarian claim. Gibbard assumes that the individual preferences domain comprises all the weak orderings but that only individuals showing unconditional preferences can implement their rights. I assume the domain of preferences comprises only separable preferences but I do not constrain the implementations of rights. As I said, the need for restricting the domain of individual preferences arises from the intention to describe the circumstances under which no social mechanism is necessary for deciding who can effectively exercise his rights. Under my decentralized scheme personal control is not purely formal but can be exercised unilaterally.

A set of conditions imposed on a procedure of choice is said to be weakly consistent if there exists an SSCF which satisfies them. With an abuse of language a set of conditions will be said to weakly imply another set of conditions if, whenever a certain SSCF satisfies the first set, it also satisfies the second one. It is easy to see that NL_1 weakly implies NL_2; that means that NL_1 is a stronger condition to be imposed on a social choice procedure. I show that an SSCF satisfying NL_1 exists.

Proposition 1. *NL_1 is weakly self-consistent.*

Proof. Define the following SSCF: given any $A \in \mathscr{V}^n$ and any $S \in \mathscr{W}$, $f(A,S)=Q$ where $Q=:\{q \in S/\text{for any } i \in I \;\; q_i \text{ is a solution to } \max_{|x_i| \in S_i} v_i^i(x_i)\}$. Clearly the q_i's exist for every i because the functions v_i^i are continuous and the sets S_i are compact. Q is a closed subset of S because it is the product of closed sets: the sets of maximum points are close because they are the inverse images of the maximum values. It is plain to see that the defined SSCF satisfies condition NL_1: all the q_i's are maximum points in their respective S_i's. Q.E.D.

Assuming condition NL_1 to hold, it is possible to construct a collective choice rule as a product of individual choice rules: every individual chooses from among his set of features. The social rule, i.e. the SSCF, is simply the product of the individual rules.

4. Paretian Principles

Define the well known Pareto principle.

Condition P. (Pareto principle). Given any $x, y \in X$, if $A \in \mathscr{V}^n$ is such that for any $i \in I$ $U_i(x) > U_i(y)$, then $y \notin f(A,S)$ when $x \in S$.

Sen [5], [6] was the first to notice that the Pareto condition conflicts with rights in social choice. That is particularly disappointing because Pareto optimality or Pareto efficiency, whichever it is called, is a central condition of welfare economics. Under the conditions of this paper, which are different from Sen's, the Paretian libertarian paradox still holds: it is trivial to show that there is no SSCF satisfying both NL_1 (or NL_2) and the weak Pareto principle.

Many attempts have been made to solve the Paretian libertarian dilemma modifying the conditions under which it holds[4]. The purpose of this section is to show that there is a restriction of the Pareto condition which is weakly consistent with the Nozick libertarian claim. What is more important, this Pareto-like condition has a clear and significant interpretation. Sen [8] proposed a modification of the Pareto principle in order to resolve the paradox. The basis of his approach is that the Pareto principle is not always morally justified in terms of the motivations underlying individual preferences. His resolution arises from voluntary individual restrictions of preferences: individuals want their preference for a certain alternative over another not to count in the procedure of social decision. The approach I suggest is different from Sen's: whether individuals want to do so or not, only a subset of their preferences counts for the social choice of the public feature.

Condition PP (Weak public Pareto principle). Given any $x, y \in X$, if for any $i \in I$ $x_i = y_i$ and if $A \in \mathscr{V}^n$ is such that for any $i \in I$ $v_i^0(x_0) > v_i^0(y_0)$, then $y \notin f(A,S)$ when $x \in S$.

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[4] See [1], [2], [3], [8], [11] for some examples, and [13] for a review of the literature

The idea behind PP is that in determining the public feature of the chosen alternative only individual preferences between public features should count, and no consideration should be given to an individual's preferences within his own feature space or within others'. Think of the 0-th feature space as if it were the rights assignment to the group I composed of all members of society; PP can be viewed as an extension to group I of the libertarian conditions NL_1 and NL_2[5]. A public feature must be rejected if there is available a second public feature which all the individuals prefer to the first. This can be not true for a general alternative x because protection of individual rights must be given first and y can be excluded from the social choice on that ground even if all the individuals prefer it to the chosen alternative x. Condition PP is Nozickian in spirit, since it captures Nozick's view that the domain of social choice can only be found in the public feature of social alternatives. To make clear that PP is a weakening of the Pareto principle I will show that, in fact, condition P weakly implies PP.

Proposition 2. *P weakly implies PP.*

Proof. Given $A \in \mathscr{V}^n$, suppose P is satisfied but PP is not, i.e. there is a non Pareto dominated y member of the choice set $f(A, S)$ and an $x_0 \in S_0$ such that for any $i \in I$ $v_i^0(x_0) > v_i^0(y_0)$; this is clearly impossible because in that case (x_0, y_{-0}) belongs to S and it Pareto dominates y, against the hypothesis $\left(\text{remember } S = \prod_{i=0}^{n} S_i \text{ and } y_{-0} = (y_1, \ldots, y_n)\right)$. Q.E.D.

The Nozick libertarian claim and the weak public Pareto principle are not inconsistent.

Proposition 3. *1) NL_1 and PP are weakly consistent.*

Proof. Define the following SSCF: given any $A \in \mathscr{V}^n$ and any $S \in \mathscr{W}$, $f(A, S) = Q$ where $Q = :\left\{q \in S / \text{for any } i \in I,\ q_i \text{ is a solution to } \max_{x_i \in S_i} v_i^i(x_i),\ q_0 \text{ is a solution to } \max_{x_0 \in S_0} \sum_{i=1}^{n} v_i^0(x_0)\right\}$. As in the proof of Proposition 1 the q_i's clearly exist for every i but q_0 also exists because the sum of continuous function is continuous and S_0 is compact. Q is closed because it is the product of closed sets. It is elementary to show that the defined SSCF satisfies both NL_1 and PP[6]. Q.E.D.

Suppose every individual is left free to choose the preferred issue in his feature space. When every individual has made this choice any single characteristic is fixed except the public one. Proposition 3 says that a mild version of the Pareto principle

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[5] See [4] for an interpretation of the Paretian libertarian conflict as a particular case of the conflict amongst individuals' own liberty spheres

[6] Note that the proof could have been omitted, because Proposition 3 descends as a corollary from Propositions 7 and 5. This also means that the assumption of cardinal unit comparability, implicit in the definition of Q in the proof, is not really necessary. Ordinal comparability of levels is sufficient

can hold true in the choice of the public characteristic in order to exclude the public issues Pareto dominated by other public issues. In evaluating that sort of Pareto domination only the utility accruing to individuals from the public feature is considered. A point which has to be brought out clearly is that, although this Paretian claim is certainly weak, it is possibly as far as one can go in general if the individual rights are all taken as unrelaxable. I doubt that this extreme position, which always allows rights to prevail upon Pareto efficiency considerations, is sustainable in every circumstances. This is better understood if we consider that it does not either allow the voluntary bargaining of rights which is for example considered by Gibbard. NL_1 and NL_2 do not permit individuals to decide whether to waive or exercise, their rights. Rights must be automatically exercised, they cannot be marketed.

5. Rawlsian Maximin Principles

Usually it is argued that Nozick and Rawls have two contrasting views about social organization; this is certainly true in general. What I want to point out here is that my consequentialist formulation of Nozick theory of rights can be consistent with a restricted form of Rawls' maximin principle. As far as I know economists have paid attention mainly to Rawls' second principle of justice, the so called maximin principle, without giving much importance to the first one, the principle of maximum amount of liberty for all. I suggest that account should be taken also of Rawls' first principle of justice and I propose to do so by considering the Nozick libertarian claim as expressing the principle of maximum liberty for all. If that is accepted we get a partial reconciliation between two theories up to now considered completely alternative[7]. Let us now write Rawls' maximin as it has been considered by the social choice literature[8].

Condition M. (Rawls maximin). Given any $x, y \in X$, if $A \in \mathscr{V}^n$ is such that there is $j \in I$ with $U_j(y) < U_i(x)$ for any $i \in I$, then $y \notin f(A, S)$ when $x \notin S$.

It is well known that Condition M is self-consistent. It is also well known and obvious from the definitions that Condition M weakly implies Condition P. It is therefore true that NL_1 and NL_2 which are not weakly consistent with P cannot be weakly consistent with M: the standard Rawls maximin conflicts with the Nozick libertarian claim.

What is possible now is to look at an amendment of Condition M which makes it consistent with NL_1 and/or NL_2. First I move in a private-oriented direction exposing a version of the Pareto principle and a version of the Rawls maximin which look only to the individual's personal spheres; they clearly make sense only with separable preferences.

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[7] It is worth emphasizing that I will follow the popular use among economists of interpretating Rawls' two principles of justice in terms of utility levels rather than in terms of access to primary social goods

[8] There is also a most refined version of Rawls' second principle of justice which is called leximin; see [12, p. 157]. I could have used it obtaining quite similar results to the ones I have obtained here. If I have not done so it is only for the sake of computational simplicity

Condition PRP (Private Pareto principle). Given any $x, y \in X$, if $A \in \mathscr{V}^n$ is such that for any $i \in I$ $v_i^i(x_i) > v_i^i(y_i)$, then $y \notin f(A, S)$ when $x \in S$.

Condition PRM (Private maximin). Given any $x, y \in X$, if $A \in \mathscr{V}^n$ is such that there is $j \in I$ with $v_j^j(y_j) < v_i^i(x_i)$ for any $i \in I$, then $y \notin f(A, S)$ when $x \in S$.

Condition PRP says that if every individual agrees in preferring his personal feature in x to the one in y then y cannot be chosen when x is available. Condition PRM says that the social choice upon x and y must reflect the preferences on his personal feature space of the individual who gets the lowest utility from it. That the two conditions mentioned are not too strong is made clear by the following result.

Proposition 4. *NL_1 weakly implies PRM which weakly implies PRP.*

Proof. For simplicity write the three conditions in the following symbolic way. NL_1: A implies D. PRM: B implies D. PRP: C implies D. Where it is clear from the definitions that D is the same for all the conditions. From the definitions it is also clear that C implies B which implies A; the proposition follows. Q.E.D.

Some, very mild, sort of Paretianism and Rawlsian justice is implied by the Nozick libertarian claim: this is what Proposition 4 says.

The next step is to explore more public-oriented versions of the Rawlsian second principle of justice; in the following conditions the right to choose amon the public features is given to the individual worst-off in terms of utility accruing from the public characteristic.

Condition PM_1 (Strong public maximin). Given any $x, y \in X$, if $A \in \mathscr{V}^n$ is such that there is $j \in I$ with $v_j^0(y_0) < v_i^0(x_0)$ for any $i \in I$, then $y \notin f(A, S)$ when $x \in S$.

Condition PM_2 (Weak public maximin). Given any $x, y \in X$, if for any $i \in I$ $x_i = y_i$ and if $A \in \mathscr{V}^n$ is such that there is $j \in I$ with $v_j^0(y_0) < v_i^0(x_0)$ for any $i \in I$, then $y \notin f(A, S)$ when $x \in S$.

It is plain to see that PM_1 implies PM_2.

The public nature of the two conditions is underlined by the following proposition.

Proposition 5. *PM_2 weakly implies PP.*

Proof. Trivial. It is sufficient to look at the definitions. Q.E.D.

As M weakly implies P so PM_2 implies PP; the implication of the Pareto principle by the Rawls maximin holds true also for the public versions of them. Unfortunately we cannot prove for PM_1, PM_2 and M a result similar to Proposition 2, i.e. PM_1 and PM_2 are not weakly implied by M. Furthermore we see that the public and the standard maximin are not consistent.

Proposition 6. *M and PM_2 are not weakly consistent.*

Proof. The following example is sufficient. Let $I = \{1, 2\}$, $S_0 = \{x_0, y_0\}$, $S_1 = \{x_1\}$, $S_2 = \{x_2\}$. Take the following $A \in \mathscr{V}^n$, such that every individual has an addi-

tively separable utility function: U_1: $v_1^0(x_0)=2$, $v_1^0(y_0)=5$, $v_1^1(x_1)=6$, $v_1^2(x_2)=0$. U_2: $v_2^0(x_0)=4$, $v_2^0(y_0)=3$, $v_2^1(x_1)=0$, $v_2^2(x_2)=2$. For any SSCF satisfying PM_2 $a=(x_0, x_1, x_2)$ is excluded from $f(A,S)$ because $v_1^0(x_0)<v_i^0(y_0)$ $i=1,2$. For any SSCF satisfying M, $b=(y_0, x_1, x_2)$ is excluded from $f(A,S)$ because $U_2(b)<U_i(a)$ $i=1,2$. But $S=\{a,b\}$ and therefore $f(A,S)=\emptyset$. Q.E.D.

Obviously PM_1 as well is not weakly consistent with M.

Let us now move to the most important result of this section. The existence will be proved of choice procedures which satisfy both the Nozick libertarian claim and the public Rawlsian maximin.

Proposition 7. *1) NL_1 and PM_1 (and therefore PM_2) are weakly consistent.*

Proof. Define the following SSCF: given any $A\in\mathscr{V}^n$ and any $S\in\mathscr{W}$, $f(A,S)=Q$, where $Q=:\left\{q\in S/\text{for any } i\in I\ q_i \text{ is a solution to } \max_{x_i\in S_i} v_i^i(x_i),\ q_0 \text{ is a solution to } \max_{x_0\in S_0}\min_{i\in I} v_i^0(x_0)\right\}$. The q_i's exist for every $i\in I$ because the v_i^i's are continuous functions and the S_i's are compact sets. For the same reason also q_0 exists: I and S_0 are compact sets, v_i^0 and $\min_{i\in I} v_i^0$ are continuous functions. Q is closed because it is the product of closed sets; the set of the maximum points in S_i and of the maximin points in S_0 are closed, being the inverse images of closed sets: the maximum and maximin value respectively. It is easy to see that the defined SSCF satisfies both conditions NL_1 and PM_1. Q.E.D.

According to the last proposition Rawls' theory of justice can be compatible with Nozick's theory of rights. Furthermore, if we are prepared to accept the Nozick libertarian claim as a way of expressing the Rawlsian principle of maximum liberty for all, then the two theories become complementary. To be precise, the protection of personal rights, a theme of strong Nozickian flavour, is incorporated in the Rawlsian conception of justice.

6. No External Utility

In this section I will scrutinize the effect of imposing an additional restriction on the domain of individual preferences. Having assumed partial separability of preferences it is always possible to divide the total utility of an individual in a state x into three parts. The utility coming from his personal feature, the utility coming from the public feature and the one coming from other individuals' features. In a sense this latter component of an individual utility represents a sort of external effect of others' situations on a person. According to the different circumstances this interest in others' position can represent forms of altruism, meddlesomeness, love, nosiness, jealousy, envy and so on. These forms of concern for others certainly cannot be neglected when we aim at a complete and faithful description of a social situation. On the other hand it is also true that greater part of economic theory has developed assuming the absence of any external effect among individuals. It is

therefore interesting to look at what happens when we introduce the same restrictive assumptions in the theoretical social choice framework of this paper. Let us hence suppose that no individual has any interest in others' affairs.

Condition NEU (No external utility). The domain of any SSCF is restricted to the individual utility functions which satisfy the following assumption: given any $x \in X$, for any $i \in I$, $v_i^{-i}(x_{-i}) = 0$.

Proposition 8. *NEU, NL_2 and PP weakly imply P.*

Proof. If NEU holds and $U_i(x) > U_i(y)$, then either $v_i^i(x_i) > v_i^i(y_i)$ or $v_i^0(x_0) > v_i^0(y_0)$ or both hold true. Suppose for any $i \in I$ $U_i(x) > U_i(y)$ and NEU, NL_2, PP hold true; two cases are possible: i) for any $i \in I$ $v_i^0(x_0) > v_i^0(y_0)$, and then by PP y is excluded from $f(A, S)$, $S \in \mathcal{W}$, by the alternative (x_0, y_{-0}). ii) There is at least one j such that $v_j^j(x_j) > v_j^j(y_j)$, and then by NL_2 y is excluded from $f(A, S)$, $S \in \mathcal{W}$, by the alternative (x_i, y_{-i}). Note that the smallest $S \in \mathcal{W}$ containing x and y is $S = \prod_{i=0}^{n} S_i$ where $S_i = \{x_i, y_i\}$. Q.E.D.

The weak Nozick libertarian claim and the weak public Pareto principle, in absence of external utility, are sufficient to weakly imply the Pareto principle. Because, as I have proved in Proposition 5, PP is weakly implied by the weak public maximin, we can say that, in absence of external utility, the Nozick libertarian claim and the public Rawlsian maximin imply the efficiency of the social choice rule. In the framework of absence of external effects, the usual one for economic theory, Paretian-efficiency, Rawlsian distributive justice and Nozickian liberty are consistent and can be together satisfied by a choice procedure. According to the Nozick libertarian claim the individual utility arising from external interest must be in most cases completely disregarded by the social choice mechanism. In practice utility deriving from other individuals' characteristics matters only when the interested individual is indifferent as regards the choice between a pair of features concerning himself. It is clear therefore that when some individual has strong preferences between features regarding others it can be the case that there is a conflict between the Nozick libertarian claim and the Pareto principle. This conflict is clearly no longer possible when no individual has any external utility.

Let us now show how the working of a very simple exchange economy can be depicted following the lines of this paper. First of all it is necessary to generalize the hypothesis of technological separability. Up to now the individual feature spaces have been taken as given and fixed. Henceforth let us assume that they depend on some vector of parameters. To be precise let us suppose that, for any $i \in I$, X_i is the budget set of individual i, which is assumed to be compact and non empty for any vector of prices $p > 0$ on which it depends. The Nozick libertarian claim simply says that every individual is free to choose the preferred consumption bundle in his budget set. Disregard for simplicity the space of public features X_0, which could be taken to represent quantities of public goods and private contributions to the public budget. Suppose a social choice function of this sort: if the individuals' choices on their feature spaces x_i satisfy the specified feasibility and compatibility conditions

(e.g. equality of total demand and supply of any single good) $f(A, S) = \prod_{i=1}^{n} x_i$. If they do not, change the prices (vector of parameters) according to some rule and let individuals choose again in their feature spaces. If everything is well specified we can hope to get, at the end of this process, a stable outcome which can be seen as our economic equilibrium. Clearly the defined social choice function mimics the price system in a competitive economy. What should be clear from the example is that a perfectly competitive system satisfies the conditions necessary for the Nozick libertarian claim's holding true. In fact such an economy satisfies a generalized version of condition NL_1. Moreover, if there are not externalities, i.e. if the hypothesis of no external utility holds true, then the equilibrium of that exchange economy is also Pareto-efficient, the last condition being implied by the Nozick libertarian claim when X_0 is empty as we have supposed. It is interesting to point out the parallelism between the general equilibrium market system and the Nozick rights system. Though for Nozick rights are essential, whereas for general equilibrium theory they are instrumental, general equilibrium theory uses rights (property rights in "endowments", exchange rights in "contracts") which are somewhat Nozickian in spirit. In some sense a Nozickian theory of rights can be used for an ethical justification of the market system wehereas general equilibrium theory can provide a theory of rights with support in terms of goodness of consequences. We could question the degree of libertarianism of a system which makes the feature space of every individual dependent on a vector of parameters (prices). It is certainly possible that for some value of the parameters some individual space becomes very small. What can we say if that happens? How can we face the related problem of equality of rights? Whereas these and other important questions should be answered by a complete theory of rights and justice, they go beyond the scope of this paper.

7. Conclusions

In this paper a consequentialist analysis of a Nozickian libertarian claim has been carried out. The central point was to show that this claim can be compatible with both some restricted form of the Pareto principle and the spirit of Rawls' distributive justice. The proposal advanced was to consider the Nozick libertarian claim as equivalent to a specification of Rawls' first principle of justice. To make clear the consistency of that purpose I have demonstrated that specification to be coherent with a reformulation of the difference principle (maximin) in terms of the public feature space only. In developing the model I have given room to some possible ways out of Gibbard's and Sen's paradoxes. Both the solutions need a particularly separable social structure. To cope with Sen's dilemma I have underlined the necessity of restraining the Pareto principle when individuals have external utility, i.e. utility coming from interest in others' affairs. A final discussion of a simple exchange economy aimed at relating the social choice framework with standard economic theorizing: the Nozick libertarian claim has been shown to be satisfied by a perfectly competitive economy. In absence of a public feature space and of external utility, that claim appeared sufficient to assure the Pareto-efficiency of an economic equilibrium.

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