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Optimization of Energy Expenditure during Level Walking

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Abstract. An analytical relationship between the basic variables of walking — step rate, step length, and metabolic energy expenditure — is formulated with the aid of data derived from the only two complete studies in the literature (Atzler and Herbst, 1927; Molen *et al.*, 1972b). The relationship in its hyperbolic form indicates that for any given step length within the normal range of walking speeds (approximately up to 145 m/min) there is a unique step rate which requires minimal energy expenditure per unit distance traversed. Matching the given step length with any other step rate results in greater energy demand. As derived here, the condition of optimality requires that the step rate be directly proportional to the step length with a subject-dependent proportionality constant. Imposing this optimality constraint on the hyperbolic form yields an optimal pattern equation which virtually coincides, up to speeds of about 100 m/min, with an empirical equation of quadratic form, found by a number of investigators to adequately relate energy expenditure to speed under moderate walking conditions.

Key words: Energy Expenditure (Minimization) — Human Walking — Step Length/Step Rate.

A number of authors (Atzler and Herbst, 1927; Margaria, 1938; Müller and Hettinger, 1952; Reitemeyer, 1955; Ralston, 1958; Cotes and Meade, 1960; Corcoran and Brengelmann, 1970; to mention just a few) have discussed the energy expenditure per unit distance during walking. The curve of such energy expenditure, plotted as ordinate against speed as abscissa, is concave upward, and while fairly flat over quite a wide range of speeds (65 to 100 m/min), still exhibits a minimal value (Fig. 1, dashed curve).

Ralston (1958) showed that the mathematical form of such a curve could be derived from an empirical quadratic equation relating energy expenditure to the square of the speed, over a range of speeds of about 25 to 100 m/min:

$$E_w = b + mv^2 \tag{1}$$

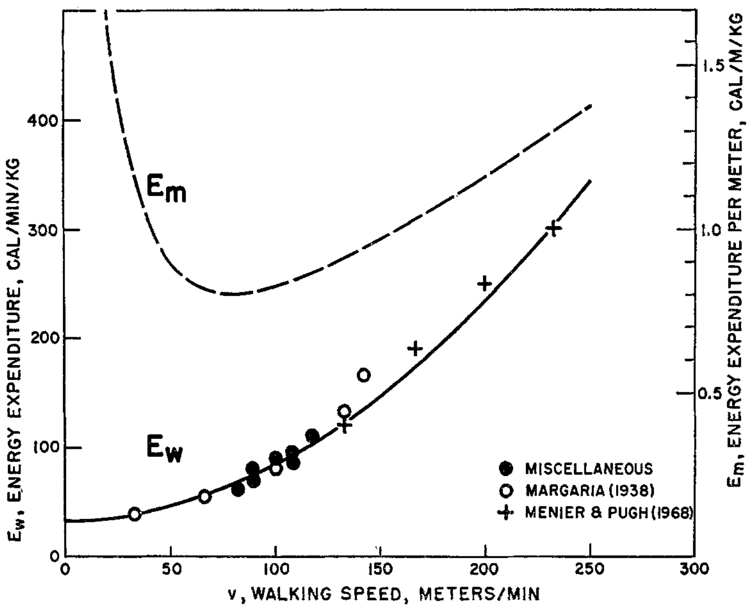


Fig. 1. Energy expenditure per unit distance during walking; solid line relation between E_w and v calculated from Eq. (3); dashed line relation between E_m and v, calculated from Eq. (2)

where E_w is expressed in gram-calories per minute per kilogram body weight and v in meters per minute, and b and m are the vertical intercept and slope, respectively. Dividing by v:

$$\frac{E_w}{v} = E_m = \frac{b}{v} + mv \qquad (2)$$

where E_m is the energy expenditure in gram-calories per meter walked per kilogram body weight.

Differentiating E_m with respect to v and equating to zero, E_m has a minimal value $= 2\sqrt{bm}$ when $v = \sqrt{b/m}$.

Since 1958 a number of authors (Cotes and Meade, 1960; Bobbert, 1960; Molen and Rozendal, 1967; Corcoran and Brengelmann, 1970) have also found that an equation of form (1) adequately represented their data. The present authors have collated all these results, weighting the values of the constants according to the number of subjects, and arrived at a grand average equation, for a range of speeds 25 to 100 m/min:

$$E_w = 32 + 0.0050\, v^2 \qquad (3)$$

based upon 57 normal male and 29 normal female adults.

Fig. 1, solid line, shows the relation between E_w and v, calculated from Eq. (3), for values of v ranging from 25 to 240 m/min.

Also shown in Fig. 1 are average values taken from various authors: Margaria (1938): open circles; Menier and Pugh (1968): + +; Benedict and Murschhauser (1915), Atzler and Herbst (1927), Daniels *et al.* (1953), Mahadeva *et al.* (1953), Durnin and Mikulicic (1956), Booyens and Keatinge (1957): all closed circles.

It should be emphasized that Eqs. (1), (2) and (3) refer to what shall be designated a *free walk:* that is to say, a walk in which the person adopts whatever speed and step rate (or step length) he chooses. As a general rule, the value of E_m in such a walk tends to be a minimum for the chosen speed.

An *optimal-speed walk* will here refer to a free walk of such a speed that the value E_m tends to be a minimum over the widest range of speeds of which the person is capable. Thus, Eq. (3) yields a minimal value of $E_m = 2\sqrt{bm} = 0.8$ cal/m/kg at $v = \sqrt{b/m} = 80$ m/min.

A *speed-controlled walk* will refer to a walk in which the speed is specified, as in walking on a motor-driven treadmill. As a general rule, the value of E_m tends to be a minimum for that particular speed.

Any walk in which speed *and* step rate (or step length) are specified, will be designated a *forced walk.* In such a walk, the value of E_m may be grossly different from that of a free, optimal-speed, or controlled-speed walk.

It is clear that energy equations like Eq. (3), which are functions of speed alone, are incomplete because they do not consider the energy requirements of a forced walk (Lamoreux, 1971; Molen *et al.*, 1972b). Only by use of data derived from both free and forced walks can the mathematical relationship between energy expenditure, step rate, and step length be established.

It is the purpose of this paper to formulate such a mathematical relationship, based upon our own studies and those of other investigators.

Methods

Ten normal male and 10 normal female adults, ranging in age from 20 to 55 and 20 to 49 years, respectively, were studied during treadmill walking at speeds of 24.4, 48.8, 73.2, and 97.6 m/min. Step rates were directly counted, and step lengths calculated from speed = step rate × step length. Average energy values were calculated from Eq. (3).

Results and Discussion

The following symbols will be used: E_w, cal/min/kg; E_m, cal/m/kg; n, steps/min; s, step length, meters; v, speed, m/min $= n \times s$; E_o, value of E_w when $n = s = 0$; n_u, upper limit of n as E_w approaches infinity; s_u, upper limit of s as E_w approaches infinity; v_u, upper limit of $v = n_u s_u$;

Table 1. Average values of v, n, s, s/n, E_w and E_m, for 10 male and 10 female subjects. Standard deviations in brackets

	v m/min	n steps/min	s m	s/n m/steps/min	E_w cal/min/kg	E_m cal/m/kg
Males	24.4	59.5 (4.33)	0.41 (0.025)	0.0069 (0.00065)	35.0	1.43
	48.8	84.4 (6.48)	0.59 (0.041)	0.0070 (0.00072)	43.9	0.90
	73.2	102.2 (5.43)	0.72 (0.056)	0.0070 (0.00066)	58.8	0.80
	97.6	116.3 (3.13)	0.84 (0.020)	0.0072 (0.00026)	79.6	0.82
	Mean			0.0070 (0.0012)		
Females	24.4	60.0 (3.95)	0.41 (0.029)	0.0068 (0.00065)	35.0	1.43
	48.8	86.7 (7.26)	0.57 (0.037)	0.0066 (0.00070)	43.9	0.90
	73.2	109.0 (5.49)	0.67 (0.037)	0.0061 (0.00045)	58.8	0.80
	97.6	126.8 (8.98)	0.77 (0.031)	0.0061 (0.00050)	79.6	0.82
	Mean			0.0064 (0.0012)		

v_{opt}, optimal speed, at which E_m is a minimum over the entire range of speeds possible.

Table 1 summarizes the relations between v, n, s, s/n, E_w, and E_m for the 20 subjects. Standard deviations are shown in brackets.

The difference in the value of s/n at the higher speeds in males and females is highly significant. For example, at $v = 73.2$ m/min, the odds against the difference being due to chance are greater than 1,000:1. Even at $v = 48.8$, the odds are about 4:1. The smaller values of s and s/n in females reflect the shorter average leg length in women as compared with men.

The values shown in Table 1 correspond to a speed-controlled walk, as earlier defined, since the treadmill imposed the speed. However, from Eq. (3), optimal speed in an optimal-speed walk $= \sqrt{b/m} = 80$ m/min, which is fairly close to the speed 73.2 m/min in the table. Therefore the value s/n at the latter speed should be close to s/n in an optimal-speed walk.

Molen *et al.* (1972a) studied 309 males and 224 females in free walk along pavement and path. At an average speed of 83.4 m/min, the average value of s/n for males was 0.0072, in good agreement with the values 0.0070 and 0.0072 at speeds of 73.2 and 97.6 m/min in Table 1. For females, they found an average value for $s/n = 0.0060$, at an average speed of $v = 76.1$ m/min, in good agreement with the value 0.0061 at 73.2 m/min in Table 1.

The near-constancy in the value of s/n in free walk has deep physiological significance, as Molen *et al.* (1972a, b) emphasized. Not only does the value of s/n characterize the walk of the male and of the female,

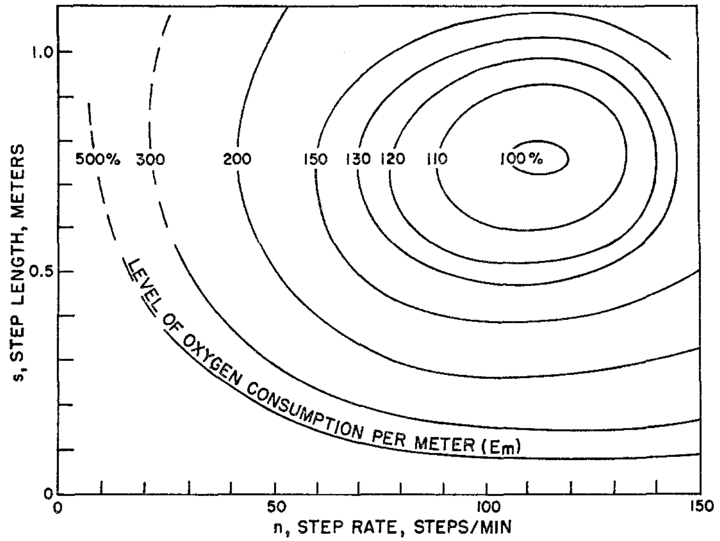


Fig. 2. E_m as a function of n and s (modified, with permission, from Molen *et al.*, 1972b)

but it also plays a fundamental role in determining the optimal speed of walking as related to energy expenditure.

Atzler and Herbst (1927) and Molen *et al.* (1972b) demonstrated in graphical form the relationship between energy expenditure and the total walking pattern of the individual subject. They plotted equal energy (E_m) contour lines on a grid of n and s (Fig. 2).

The regularity of pattern of the energy curves suggests that a mathematical description of the relationship is possible. As a first attempt to study the geometric nature of this relationship, the energy data of Molen *et al.* (1972b) on a single male subject were plotted against n for several constant values of s (Fig. 3). Each curve resembled a hyperbola with non-perpendicular asymptotes, raising the possibility of replacing it with a straight line by plotting $1/E_w$ against n.

It was found, however, that the step rate squared gave a better linear fit (Fig. 4). The family of lines in Fig. 5 has a common horizontal intercept, and each has a vertical intercept that varies approximately as s^2.

In Appendix I it is shown that the relationships displayed in Figs. 4 and 5 can be summarized by expressing E_w as a function of n and s:

$$E_w = \frac{E_o}{\left(1 - \frac{s^2}{s_u^2}\right)\left(1 - \frac{n^2}{n_u^2}\right)} \tag{4}$$

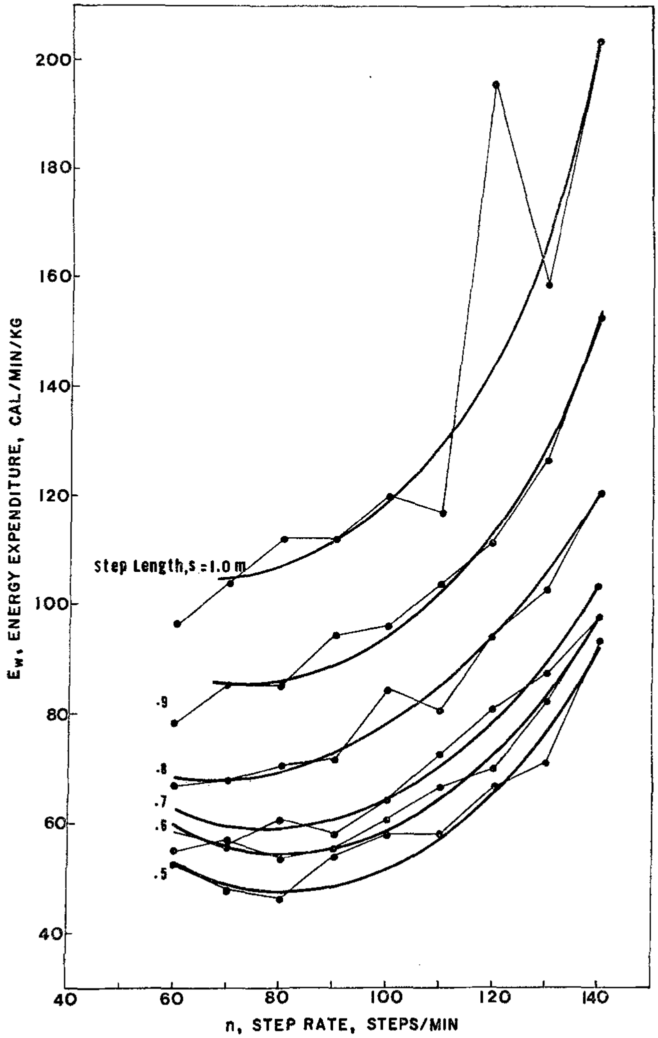


Fig. 3. E_w as a function of n for several constant values of s. The spacing between curves varies approximately as s^2 (data from Molen *et al.*, 1972b)

E_o is the rate of energy expenditure, cal/min/kg, when $n = s = 0$. This is the reciprocal of the vertical intercept of the dashed line in Fig. 5, and has the value 35.0. s_u is the square root of the horizontal intercept in Fig. 5. The common intercept of Fig. 5 is n_u^2.

It is of interest that E_w may be expressed in the form of the dimensionless ratio E_w/E_o.

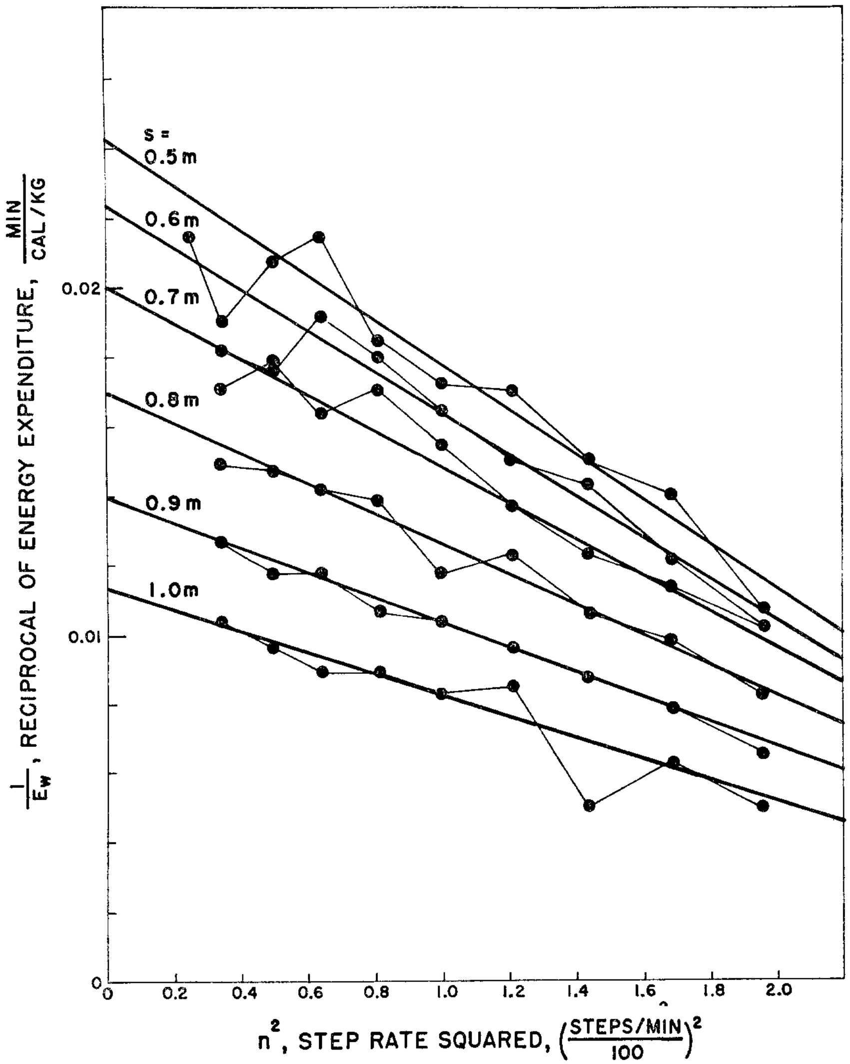


Fig. 4. Test of the hyperbolic relationship between E_w and n^2 (same data as in Fig. 3)

Fig. 6a and b shows the characteristics of gait in dimensionless and dimensional form. See Appendix II for details of construction.

Eq. (4) is simple in form, but by bringing together many of the gait features described in previous studies by various investigators, it provides a schedule of the metabolic requirements for free, speed-controlled, and forced walking.

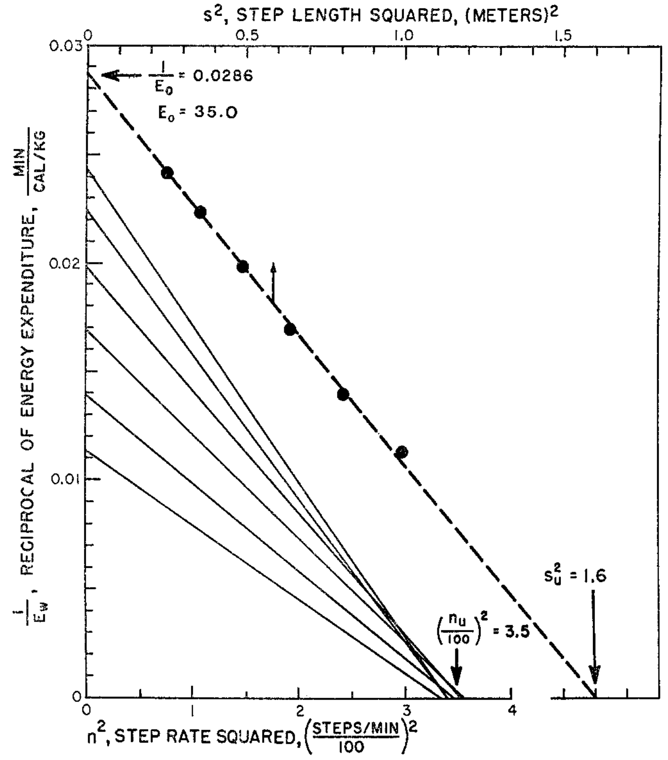


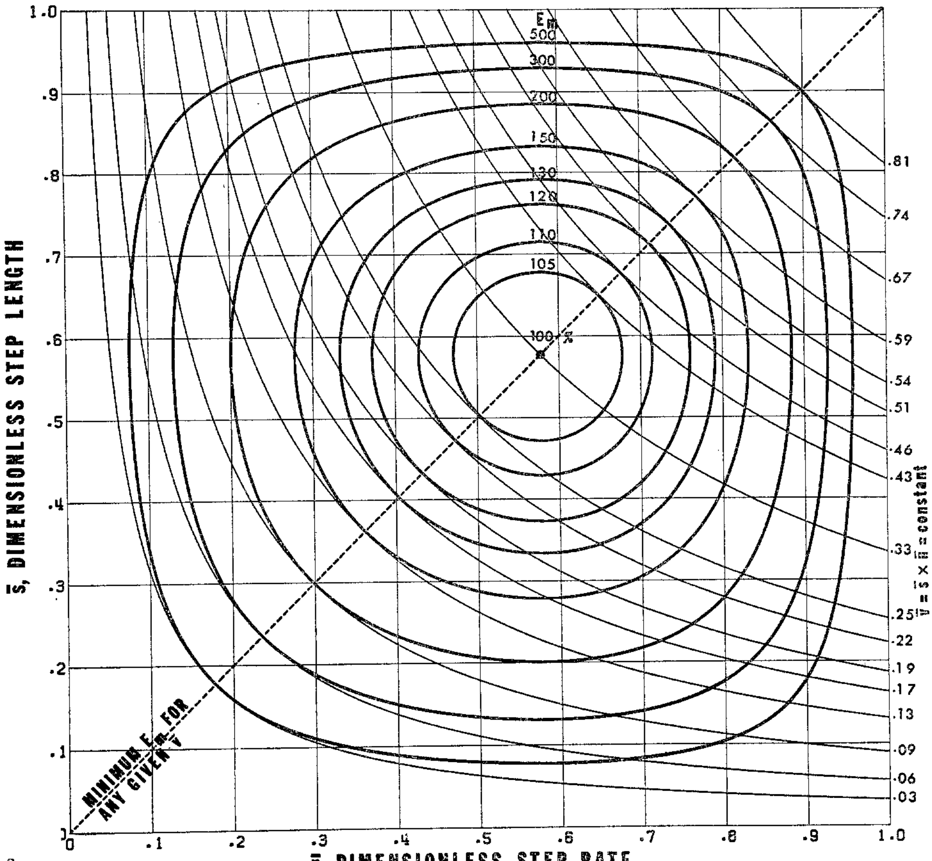
Fig. 5. Determination of the hyperbolic parameters. The family of fitted lines of Fig. 4 is redrawn in the lower left-hand corner to obtain the intercept ($n^2{}_u$), which appears to be common to all lines. The dashed line shows the relation between $1/E_w$ and s^2. This line yields the parameters E_o and s_u (same data as in Fig. 3)

Minimal Energy Criterion

It has long been observed that people, when walking freely, use step lengths that are approximately proportional to step rates (Dean, 1965; Molen and Rozendal, 1967; Lamoreux, 1971). This simple relationship can be deduced from Eq. (4) by finding the combination of s and n that minimizes E_m. The condition of minimal E_m is derived in Appendix II, and is shown as:

$$s = (s_u/n_u)n \quad (5)$$

This condition appears in Fig. 6a as a 45° line through the origin, depicting all tangency points between the E_m contours and the speed hyperbolas. Each point on the line indicates a combination of n and s requiring the least amount of energy per meter walked.



a

Fig. 6. (a) Relation between E_m and any combination of $\bar{s}$ and $\bar{n}$, in dimensionless form. The closed ellipsoidal curves represent an assemblage of $\bar{s}$ and $\bar{n}$ pairs requiring equal values of E_m. The hyperbolas are curves of constant speed, chosen to illustrate the points of tangency to the E_m contours. The overall minimal E_m contour degenerates to a point labeled 100%. Other E_m contours are computed by setting $E_m = 5\%$, 10%, etc., higher than minimal E_m. (b) Dimensional form of (a). By introducing the individual's parameters as found by experiment, the general plot (a), can be transformed into this specific dimensional one (same data as in Fig. 3)

For the subject of Molen *et al.* (1972b), s_u was 1.265 m and n_u was 187 steps/min. The derived slope s_u/n_u of Eq. (5) is 0.0068, which is in good agreement with Molen's typical experimental value 0.0066.

Energy Demand of the Optimal Walking Pattern

If the subject assumes a combination of s and n as given by Eq. (5), he can no longer vary his step length independently of his step frequency.

b

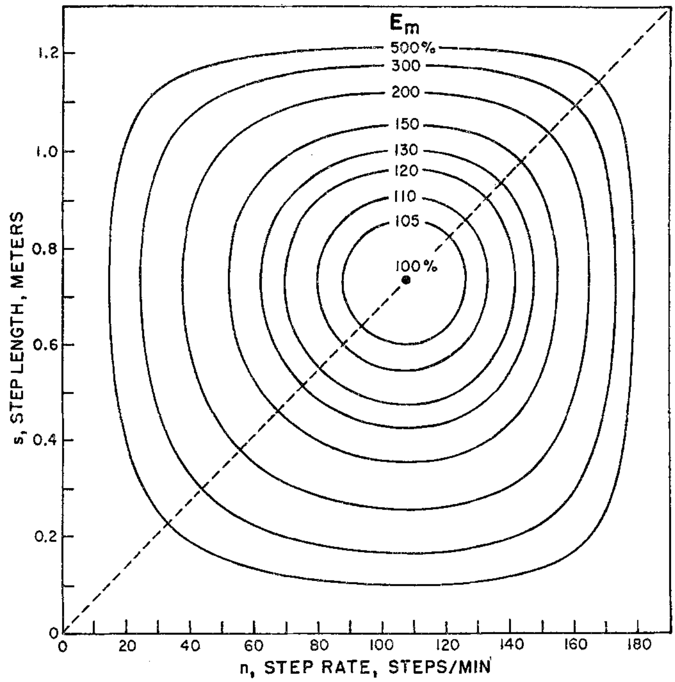


Fig. 6b (for legend see page 301)

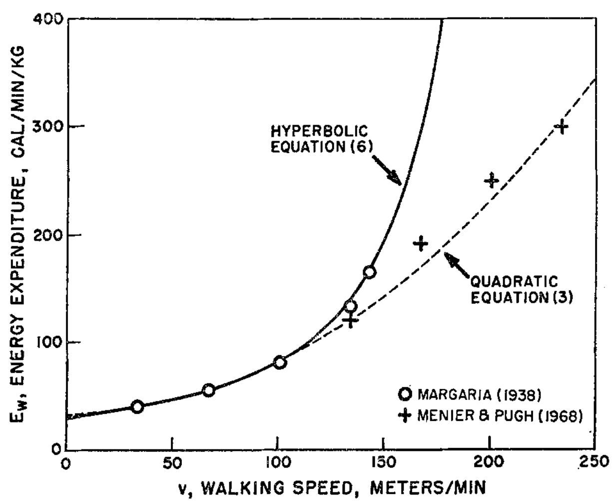


Fig. 7. Relation between E_w and v, calculated from the hyperbolic Eq. (6), using $E_o = 28.3$ and $v_u = 242$, solid line, and the quadratic Eq. (3), dashed line

When v is chosen as the independent variable, substitution of Eq. (5) into Eq. (4) yields a relationship predicting the optimal energy expenditure for *any particular speed:*

$$E_w = \frac{E_o}{(1 - v/v_u)^2} \tag{6}$$

Solving Eqs. (3) and (6) simultaneously, using values of E_w and v from Table 1, results in $E_o = 28.3$ cal/min/kg and $v_u = 242$ m/min. This value of v_u lies between the value $v_u = 237$ for the single subject of Molen *et al.* (1972b) and $v_u = 249$ as found by Atzler and Herbst (1927) for their two subjects. These values of v_u correspond closely to speeds attained in competition walking.

The solid curve in Fig. 7 shows the calculated values of E_w for various values of v, using Eq. (6) with $E_o = 28.3$ and $v_u = 242$. It is evident that this curve is virtually coincident with the dashed (quadratic) curve repeated from Fig. 1, for speeds up to about 100 m/min (approx. 0.4 v_u), but that considerable divergence occurs at higher speeds.

Walking speeds up to about 145 m/min may be regarded as "natural" speeds, *i.e.*, not requiring the special pelvic rolling characteristic of competition walking. The hyperbolic (solid) curve of Fig. 7 may therefore be regarded as describing the relation between E_w and v in natural walking.

Rather unexpectedly, the quadratic curve of Figs. 1 and 7 not only adequately describes the relation between E_w and v up to speeds of about 100 m/min, but also predicts, within about 10% or less, the values of E_w for speeds attained in competition walking.

The differentiation of Eq. (6), after dividing by v, yields:

$$v_{\text{opt}} = v_u/3 \tag{7}$$

where v_{opt} is the overall optimal velocity which minimizes the energy expenditure per meter, E_m. From $v_u = 242$ m/min, $v_{\text{opt}} = 242/3 = 80.7$ m/min. Eq. (3) yields $v_{\text{opt}} = \sqrt{b/m} = \sqrt{32/0.005} = 80$ m/min.

The effectiveness of the hyperbolic formulation [Eq. (6)] for natural walking can be exhibited by applying it to cases where the total walking pattern is determined for only one speed. Cotes and Meade (1960) provide such data on their subject 2. A least-squares fit of their data yields $E_o = 24$ cal/min/kg, $v_u = 202$ m/min, and $n_u = 168$ steps/min. The value of s_u therefore is $v_u/n_u = 1.202$ m, and $s/n = s_u/n_u = 1.202/168 = 0.00715$. The average experimental value for the same subject was 0.00735.

Appendix I

Derivation of the Total Pattern Eq. (4)

By inspection of Fig. 4, the relationship of the reciprocal of the metabolic energy expenditure to both step length, s, and step rate, n, can be deduced. Each curve is a line where both the intercept and the slope are a function of s:

$$1/E_w = a(s) + b(s)n^2 \tag{1a}$$

Since the family of lines shown has a common horizontal intercept, n_u^2, the slopes are proportional to the vertical intercepts. Each slope $b(s)$ is the negative of the intercept $a(s)$ divided by n_u^2:

$$b(s) = -a(s)/n_u^2. \tag{2a}$$

Substituting this equation into Eq. (1a) results in:

$$1/E_w = a(s)\,(1 - n^2/n_u^2.) \tag{3a}$$

The intercept $a(s)$ varies approximately as s^2 (Fig. 5):

$$a(s) = \frac{1}{E_o}(1 - s^2/s_u^2) \tag{4a}$$

Combining Eq. (4a) with Eq. (3a) yields:

$$1/E_w = \frac{1}{E_o}(1 - s^2/s^2_u)\,(1 - n^2/n_u^2) \tag{5a}$$

which is the inverted form of Eq. (4).

Eq. (5a) shows that there are limiting values to both s and n. As these variables approach their limiting level, the energy expenditure becomes increasingly large, a condition which is not physiologically possible. Consequently these limiting values are never attained during normal human locomotion and should only be considered as a mathematical convenience.

Appendix II

Optimization of the Total Pattern

The expression for energy expenditure for the total walking pattern when written on per distance basis is:

$$E_m = \frac{E_o/v_u}{\bar{s}\bar{n}\,(1 - \bar{s}^2)\,(1 - \bar{n}^2)} \tag{6a}$$

where $\bar{s} = s/s_u$, the ratio of s to the limiting step length, s_u.

$\bar{n} = n/n_u$, the ratio of n to the limiting step rate, n_u.

v_u = upper speed limit = $s_u n_u$.

All the parameters E_o, s_u, and n_u are calculated from experimental results. The equation for a specific energy expenditure level is given by setting E_m = constant. The equation of a tangent to this specific contour line is found by differentiating E_m. This derivative is:

$$dE_m = 0 = \frac{\partial E_m}{\partial s}ds + \frac{\partial E_m}{\partial n}dn \tag{7a}$$

Rearranging to solve for the tangent, ds/dn gives

$$\frac{ds}{dn} = -\frac{\partial E_m/\partial n}{\partial E_m/\partial s} \tag{8a}$$

The equation of a tangent to constant velocity hyperbola is expressed as:

$$\frac{ds}{dn} = -\frac{s}{n} \tag{9a}$$

Differentiating Eq. (6a):

$$\frac{\partial E_m}{\partial n} = -\frac{E_m\,(1 - 3\,\bar{n}^2)}{n\,(1 - \bar{n}^2)} \tag{10a}$$

and

$$\frac{\partial E_m}{\partial s} = -\frac{E_m (1 - 3\,\bar{s}^2)}{s\,(1 - \bar{s}^2)} \tag{11a}$$

The condition of minimal E_m can be found by equating both right sides of Eqs. (8a) and (9a) and substituting from Eqs. (10a) and (11a) for the partial derivatives:

$$\frac{s}{n} = \frac{(E_m/n)\,(1 - 3\,\bar{n}^2)\,(1 - \bar{s}^2)}{(E_m/s)\,(1 - 3\,\bar{s}^2)\,(1 - \bar{n}^2)} \tag{12a}$$

Canceling and simplifying results in the following simple condition of optimality:

$$\bar{s} = \bar{n} \tag{13a}$$

Eq. (13a) is rewritten in a dimensional form as follows:

$$s = (s_u/n_u)n \qquad \text{(14a) and (5)}$$

Geometrically, Eq. (14a) is a line representing the locus of tangency points between the E_m contour curves and the speed hyperbolas on the s and n plane. Hence, it depicts all of the combinations of s and n which require a minimal metabolic energy per distance walked.

Only one independent variable remains in the total pattern expression of Eq. (6a), if the $\bar{s}$ and $\bar{n}$ combinations are restricted to those described by Eq. (13a). Choosing v to be the independent variable, the energy expenditure associated with the free walking pattern at any speed can be found by combining Eqs. (13a) and (6a):

$$E_{m,\,\text{opt}} = \frac{E_o/v_u}{\bar{v}\,(1 - \bar{v})^2} \tag{15a}$$

There is a unique velocity $\bar{v}_{\text{opt}}$ for which $E_{m,\,\text{opt}}$ is a minimum. This speed is formed by differentiating Eq. (15a) and setting the derivative, $dE_m/d\bar{v} = 0$:

$$\bar{v}_{\text{opt}} = 1/3 \tag{16a}$$

or

$$v_{\text{opt}} = v_u/3 \qquad \text{(17a) and (7)}$$

Since, from Eq. (13a), $\bar{s} = \bar{n} = \sqrt{\bar{v}}$, it follows that the optimal step length and step frequency are:

$$s_{\text{opt}} = s_u/\sqrt{3} \tag{18a}$$

$$n_{\text{opt}} = n_u/\sqrt{3} \tag{19a}$$

The energy contours in Fig. 6 are found by setting E_m equal to a constant which is selected on the basis of the minimal energy level corresponding to v_{opt}:

$$E_{m,\,\text{min}} = \frac{E_o}{v_u\,(1/3)\,(1 - 1/3)^2} = \frac{27}{4}\,(E_o/v_u) \tag{20a}$$

When this least energy value is substituted in Eq. (15a), the contour curve degenerates to a point which is labeled 100% in Fig. 6a. Other contours are computed by setting E_m at 5, 10, 20% etc. higher than $E_{m,\,\text{min}}$ of Eq. (20a) In the normalized form, Fig. 6a, in addition to E_m, a normalized step rate must also be selected to solve for two acceptable roots of the polynomial:

$$\bar{s}\,(1 - \bar{s}^2) - C = 0$$

where

$$C = E_o/(E_m v_u \bar{n}\,(1 - \bar{n}^2))$$

A trial and error method can be used to find the roots, but the use of a digital computer is more appropriate.

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