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Model for Real-Time Optimal Flood Control Operation of a Reservoir System

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Abstract. A methodology and model have been developed for the real-time optimal flood operation of river-reservoir systems. This methodology is based upon combining a nonlinear programming model with a flood-routing simulation model within an optimal control framework. The generalized reduced gradient code GRG2 is used to perform the nonlinear optimization and the simulator is the U.S. National Weather Service DWOPER code. Application of the model is illustrated through a case study of Lake Travis on the Lower Colorado River in Texas.

Key words. Flood control, unsteady flow, reservoir operation, optimization.

1. Introduction

1.1. BACKGROUND

Real-time operation of multireservoir systems involves various hydrologic, hydraulic, operational, technical, and institutional considerations. For efficient operation, a monitoring system is essential that provides the reservoir operator with the flows and water levels at various points in the river system including upstream extremities, tributaries and major creeks as well as reservoir levels, and precipitation data for the watersheds whose outputs (runoff from rainfall) are not gauged. A flow routing procedure is needed to predict the impacts of observed and/or predicted inflow hydrographs on the downstream parts of the river system. A reservoir operation policy or a methodology is another component which reflects the flood control objectives of the system, the operational and institutional constraints on flood operations, and other system-related considerations. An integral part of these components is a reservoir operation model that predicts the results of a given operation policy for forecasted flood hydrographs.

Flood forecasting in general, and real-time flood forecasting in particular, have always been an important problem in operational hydrology, especially when the operation of storage reservoirs is involved. The forecasting problem, as in most hydrological problems, can be viewed as a system with inputs and outputs. The system output is related to its causative input through a process, either linear or nonlinear. In the reservoir management problem, the system is the river system that includes a main river and its tributaries, catchments, and natural and manmade structures on the path of the flood waters. The system inputs are inflow hydrographs at the upstream ends of the river system, and runoff from the rainfall (and snowmelt, where applicable) in the intervening catchments. The system outputs are flow rates and/or water levels at control points of the river system. The operations involved are the operations of the reservoir(s) in order to control flood waters. The term 'forecasting' refers to the prediction of the discharges and water surface elevations at various points of a river system as a result of the observed portion of a flood hydrograph.

The real-time reservoir operation problem involves the operation of a reservoir system by making decisions on reservoir releases as information becomes available, with relatively short time intervals which may vary between several minutes and several hours. A new methodology is presented for operating a reservoir system under flooding conditions that incorporates: (a) a simulation model that adequately simulates the hydraulics of the system for a given flood hydrograph and a set of operating decisions, and (b) a systematic way that will improve the trial decisions made previously and generate a set of operating decisions that would cause the least damage to the protected areas.

The model presented in this paper has the following characteristics:

(1) It is deterministic as the inflow hydrographs have to be provided by the user.
(2) It has provisions to incorporate runoff from rainfall, through an option to generate runoff hydrographs resulting from given (deterministic) rainfall hyetographs through a submodule based on a U.S. Soil Conservation Service (SCS) procedure, developed for an earlier real-time flood forecasting model (Unver *et al.*, 1987). In case runoff hydrographs are obtained externally, they can be input to the model.
(3) The releases from reservoirs are realized through the operation of controlled outlet structures (gates) which are hydraulically described by a discharge versus gate setting relationship for various headwater elevations.
(4) Reservoirs which are not controlled by gates, i.e. run-of-the-river type reservoirs, are treated like other flow structures such as bridges, levees, and weirs.
(5) The channel flow as well as the flow through reservoirs and various regulating structures are simulated by state-of-the-art methods, thus the magnitudes and timing of flood flows are accurately estimated.
(6) The data required for computer implementation is basically standard and may be readily available to most potential users as flow routing is accomplished by a modified version of the U.S. National Weather Service DWOPER

(Dynamic Wave OPERation) Model (Fread, 1982), and optimization through a widely used nonlinear optimization code, GRG2 (Lasdon and Waren, 1983).

1.2. PREVIOUS WORK

Development of real-time reservoir operation models has been reported only in the recent literature. Jamieson and Wilkinson (1972) developed a dynamic programming model for flood control with forecasted inflows being the inputs to the model. Windsor (1973) employed a recursive linear programming procedure for the operation of flood control systems, using the Muskingum method for channel routing and the mass balance equation for reservoir computations. The U.S. Army Corps of Engineers developed HEC-5 (1973a) and HEC-5C (1973b) for reservoir operation for flood control, where the releases are selected by applying a fixed set of heuristic rules and priorities that are patterned after typical operation studies. Tennessee Valley Authority (1974) implemented an incremental dynamic programming and successive approximations technique for real-time operations with flood control and hydropower generation being the objectives. Recently, Can and Houck (1984) developed a goal programming model for the hourly operations of a multireservoir system and applied it to the Green River basin in Indiana. The model objective is defined by a hierarchy of goals, with the best policy being a predetermined rule curve. Wasimi and Kitanidis (1983) developed an optimization model for the daily operations of a multireservoir system during floods which combines linear quadratic Gaussian optimization and a state-space mathematical model for flow forecasting. Unver *et al.* (1987) developed a management model for the real-time short-term operations of a multireservoir system on the Colorado River in Texas, which combines a rainfall-runoff routine, the U.S. National Weather Service DWOPER flood routing package, and a graphics package.

Yazicigil (1982) developed the GRBOOM linear optimization model for the daily real-time operations of the Green River basin in Indiana, a system of four multipurpose reservoirs. The primary use of the system is flood control, with recreation and low flow augmentation as secondary purposes. The model inputs are deterministic. The objective of operation is to follow a set of target states, deviations from which are penalized. The channel routing is performed using a linear routing procedure similar to the Muskingum method, called multi-input linear routing. The reservoir calculations are based on mass-balance equations which take into account precipitation input. Other constraints include the minimum and maximum allowable releases, upper limits on flow rates, and nonnegativities. The model variables are the flow rates, storages, and reservoir releases. The objective function is formulated based on a zoning approach. A zone represents a range of deviations from the targets. Associated with each zone is a penalty coefficient, which increases in magnitude as the zone deviates further away from the target, an assumption that makes the objective function convex so the simplex method can be used to solve the problem.

2. Problem Formulation

2.1. PROBLEM STATEMENT

The optimization problem for the operation of multireservoir systems under flooding conditions can be stated as

(1) Objective:

$$\text{Minimize } z = f(\mathbf{h}, \mathbf{Q}). \tag{1}$$

(2) Constraints:

(a) Hydraulic constraints defined by the Saint-Venant equations for one-dimensional gradually varied unsteady flow and other relationships such as upstream, downstream, and internal boundary conditions and initial conditions that describe the flow in the different components of a river-reservoir system,

$$\mathbf{g}\,(\mathbf{h}, \mathbf{Q}, \mathbf{r}) = \mathbf{0}. \tag{2}$$

(b) Bounds on discharges defined by minimum and maximum allowable reservoir releases and flow rates at specified locations,

$$\underline{\mathbf{Q}} \leqslant \mathbf{Q} \leqslant \bar{\mathbf{Q}}\,. \tag{3}$$

(c) Bounds on elevations defined by minimum and maximum allowable water surface elevations at specified locations (including reservoir levels),

$$\underline{\mathbf{h}} \leqslant \mathbf{h} \leqslant \bar{\mathbf{h}}\,. \tag{4}$$

(d) Physical and operational bounds on gate operations,

$$\mathbf{0} \leqslant \underline{\mathbf{r}} \leqslant \mathbf{r} \leqslant \bar{\mathbf{r}} \leqslant \mathbf{1}\,. \tag{5}$$

(e) Other constraints such as operating rules, target storages, storage capacities, etc.

$$\mathbf{W}\,(\mathbf{r}) \leqslant \mathbf{0}\,. \tag{6}$$

The objective z is defined by minimizing the total flood damage or deviations from target levels or water surface elevations in flood areas or spills from reservoirs or maximizing storage in reservoirs. The variables **h** and **Q** are, respectively, the water surface elevation and the discharge at the computational points and **r** is the gate setting, all given in matrix form to consider the time and space dimensions of the problem. Bars above and below a variable denote the upper and lower bounds for that variable, respectively.

2.2. OBJECTIVE FUNCTIONS

The model can be based upon any of a number of objective functions reflecting various approaches to real-time reservoir operation for flood control. The first objective function is based on minimizing total flood damages which are defined as a function of water surface elevations in flood-prone areas. A damage-elevation relationship is provided to the model for each location where flood damage potential exists. The overall damage to be minimized is the summation of the total damages at each location. The mathematical expression for this objective function is:

$$\min z = \sum_i \sum_j c_i h_i^j, \quad i \in I_c, \ j \in T, \tag{7}$$

where z is the objective function value; i is the location index; I_c is the set that contains flood control locations; j is the time index; T is the time horizon; c is the unit flood damage defined as a function of the water surface elevation; h_i^j. The unit flood damage, c, is expressed in terms of the water surface elevation at flood control locations. It must be noted that, unlike the more common approach to damage functions (e.g. Windsor, 1973), the damage is not a function of the maximum water surface elevation for any given location, but rather a function of all elevations that are individually damaging. This approach was chosen to keep all water surface elevations in the nondamaging range individually and when this is not possible to minimize the number of times a damaging elevation occurs. The total damage cost, however, may not have a real meaning in dollar value due to the nature of this formulation.

The second objective function is basically the same as the first one except that flood damages are expressed in terms of discharges instead of water surface elevations, given as:

$$\min z = \sum_i \sum_j c_i' Q_i^j, \quad i \in I_p, \ j \in T, \tag{8}$$

where c' is the unit flood damage as a function of discharge, Q_j. The unit flood damages, c', are expressed in terms of the discharge at the flood control locations. This objective function is provided for cases where it is more convenient to express damages in terms of flow rates for certain locations, or the available data is in this form. It must be noted that this objective function would normally be used for natural channels as the damages in lakes are almost always a function of flood stages.

The third objective function is a combination of the first two for cases where both discharges and water surface elevations are used to define the flood damages given as

$$\min z = \sum_{i \in I_c} \sum_j c_i h_i^j + \sum_{i \in I_p} \sum_i c' Q_i^j, \quad j \in T, \tag{9}$$

where I_c is the set that contains locations where damage is a function of water surface elevation and I_c is the set that contains locations where damage is a function of discharge. The myopic nature of short-term operation is usually handled by constraints that represent the end-of-the-period, or medium-term targets or goals. For example, the possibility of ending up with an empty reservoir is usually prevented by defining a lower limit for the water surface elevation of the headwater location for time step T. An alternative to this is given by the fourth objective function. The objective of operation is defined as the maximization of the total reservoir storages while keeping the water stages and/or flow rates within nondamaging ranges through the constraint set. The fourth objective function is

$$\max z = \sum_i \sum_j Q_i^j, \quad j \in T, \tag{11}$$

where all terms are as defined earlier.

Zoning is another very common approach used in modeling the real-time operation objectives (e.g. Yazicigil, 1982; Can and Houck, 1984; Wasimi and Kitanidis, 1983). In order to use this approach, operation targets (or ideal levels) are defined prior to operation and deviations from these are penalized through a penalty function. Zones are identified for different levels of deviations and a unit penalty (or a penalty coefficient) is assigned to each, almost always in such a way that the resulting function is convex. Although the solution methodology presented in the next section has provisions for violated bounds on discharges and water surface elevations, a penalty-type objective function is presented here, as the sixth objective function, for cases where data are already available or the reservoir operator opts to use a penalty function. The mathematical expression for the sixth objective is:

$$\min z = \sum_i \sum_j c_i h_i^j + \sum_i \sum_j c'_i Q_i^j, \quad i \in l_s, j \in T, \tag{12}$$

where l_s is the set that contains locations for which a target is specified and c and c' are the unit penalties associated with water surface elevation, h, and discharge, Q. It must be noted that water surface elevations in this formulation replace the deviations used in most penalty functions. However, this is justified by the fact that the inclusion of the target into the objective function contributes a constant to the objective value, which does not affect the optimization, within the given range of unit penalties. Different unit penalties for different locations are used to reflect the relative importance of each location.

2.3. CONSTRAINTS

The constraints of the model can be divided into two groups: the hydraulic constraints (Equation (2)) and the operational constraints (Equations (3)–(6)). The hydraulic constraints are equality constraints consisting of the equations that describe the flow in the system. These are (a) the Saint-Venant equations for all computational reaches except internal boundary reaches, (b) relationships to describe the upstream and downstream boundary conditions in addition to the Saint-Venant equations for the extremities, and (c) internal boundary conditions including the continuity equation and a flow relationship.

Internal boundary conditions describe flow that cannot be described by the Saint-Venant equations such as critical flow resulting from flow over a spillway or waterfall. The operational constraints are basically greater-than or less-than type constraints that define the variable bounds, operational targets, structural limitations, capacities, etc. Options for the operator to set or limit the values of certain variables are also classified under this category. The solution methodology used in this study separately solves the hydraulic and operational constraints. The hydraulic constraints are solved implicitly by the simulation model, DWOPER, whereas the operational constraints are solved by the optimization model, GRG2. The DWOPER model performs the unsteady flow computations.

Bound constraints are used to impose operational or optimization-related requirements. Nonnegativity constraints on discharges are not used because discharges are allowed to take on negative values in order to be able to realistically represent the reverse flow phenomena (backwater effects) due to a rising lake or due to large tributary inflows into a lake. Nonnegativity of water surface elevations is always satisfied since the system hydraulics are solved implicitly by the simulation model, DWOPER. The lower limits on elevations and discharges can be used to impose water quality considerations, minimum required reservoir releases, and other policy requirements. The upper bounds on elevations and discharges can be used to set the maximum allowable levels (values beyond are either catastrophic or physically impossible) such as the overtopping elevations for major structures, spillway capacities, etc. When the objective function, Equation (10) or (11) is used, the damaging elevations and/or discharges must be given to the model through the constraints, as neither objective function has any terms to control them.

The third model variable, gate openings, are allowed to vary between zero and one, which corresponds to zero and one hundred percent opening of the available total gate area, respectively. The upper and lower bounds on the model variables are expressed mathematically as

$$\underline{Q}_i^j \leqslant Q_i^j \leqslant \bar{Q}_i^j\,, \quad \forall\, i, j, \tag{13}$$

$$\underline{h}_i^j \leqslant h_i^j \leqslant \bar{h}_i^j\,, \quad \forall\, i, j, \tag{14}$$

$$0 \leqslant \underline{r}_i^j \leqslant r_i^j \leqslant \bar{r}_i^j \leqslant .1\,, \quad \forall\, i, j{:}\; i \in l_r \tag{15}$$

where variables with a bar above them denote upper limits; those with a bar below them denote lower limits; i and j are respectively the time and location index; and l_r is the set containing the reservoir locations. Q, h, and r denote the discharge, water surface elevation, and gate opening, respectively.

The bounds on gate settings are intended primarily to reflect the physical limitations on gate operations as well as to enable the operator to prescribe any portion(s) of the operation for any reservoir(s). Operational constraints other than bounds can be imposed for various purposes. The maximum allowable rates of change of gate openings, for instance, for a given reservoir, can be specified through this formulation, as a time-dependent constraint. This particular formulation may be very useful, especially for cases where sharp changes in gate operations, i.e. sudden openings and closures, are not desirable or physically impossible. It is handled by setting an upper bound to the change in the percentage of gate opening from one time step to the next. This constraint can also be used to model another important aspect of gate operations for very short time intervals, i.e. the gradual settings that have to be followed when opening or closing a gate. For this case, the gate cannot be opened (or closed) by more than a certain percentage during a given time interval. This can be expressed in mathematical terms as follows:

$$-r_{ci} \leqslant r_i^{j+1} - r_i^j \leqslant r_{oi}, \quad i \in I_r \tag{16}$$

where r_c and r_o are the maximum allowable (or possible) percentages by which to open and close the gate. This constraint can be used to model manually operated gates, for example, for all or a portion of the time intervals. The same constraint can be used, for example, to incorporate an operational rule that ties the operations of a reservoir to those of the upstream reservoir such as a multi-site constraint.

3. Solution Approach

3.1. OVERVIEW

The optimization problem stated above is a large mathematical programming problem for most real-world situations. In modeling a river system, computational points are used to discretize the river channels and reservoirs. Each computational point, for each time step of the operation, contributes two flow variables (water surface elevation and discharge) and two hydraulic constraints (the Saint-Venant equations or other flow relations) to the problem. In addition, each reservoir contributes another variable (the setting of the equivalent gate) per time step. The external boundaries each contribute an additional hydraulic relationship. Thus, a typical 24 h operation horizon with 1 h time steps for a river system with 5 reservoirs and 150 computational points would give rise to a problem with more than 7200 flow equations (two times the product of the number of time steps and computational nodes) and over 7200 flow variables. This is beyond the capacity of existing nonlinear programming codes. The logical approach in solving a problem this large would be to reduce its size. Traditionally, the problem size has been reduced by replacing the unsteady flow equations by more simplistic relationships. In this work, a different approach is taken to alleviate the dimensionality problem. The optimum control model presented here leads to an efficient algorithm to solve the optimization problem without sacrificing the hydraulic model accuracy.

The basic idea is to solve the hydraulic constraints (Saint-Venant equations) using an unsteady flow routing model such as the U.S. National Weather Service Dynamic Wave Operational (DWOPER) model. For each iteration of the optimization model, the simulator (DWOPER) solves for the water surface elevations, h, and the flow rates, Q, given the gate operations which are the control variables. This allows the constraints and the objective function of the reservoir optimization problem to be viewed as a function of only the controllable variables. Since there are relatively few controllable variables, the resulting reduced problem is easier to solve. The major remaining difficulty is to compute the first partial derivatives of the objective and constraint functions with respect to the controllable variables. Once the derivatives are determined, several efficient nonlinear optimization routines could be used to solve the reduced optimization problem.

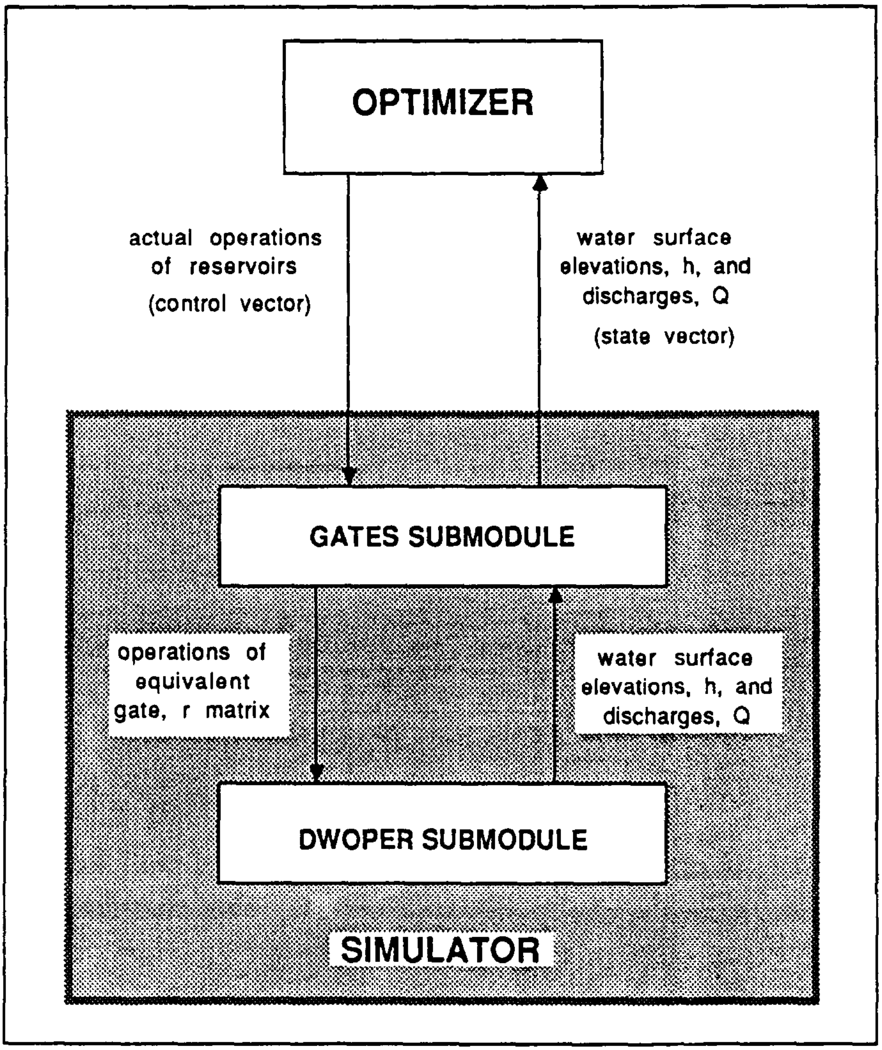

Fig. 1. Optimal control approach to operations problem.

3.2. THE REDUCED PROBLEM

The operations problem (Equations (1)–(6)), referred to as the general operations model (GOM) has certain characteristics that can be used in reducing it to a smaller problem. The GOM has the general structure of a discrete time control with three basic groups of constraints: those concerning the state of the system (hydraulic constraints) and those describing the system controls (bound and operation constraints). The GOM yields to an efficient solution algorithm when the state variables (discharges and water surface elevations) and the control variables (gate settings) are treated separately, in a coordinated manner. The hydraulic constraints (Equation (2)) can be solved sequentially forward in time for the water surface elevations,

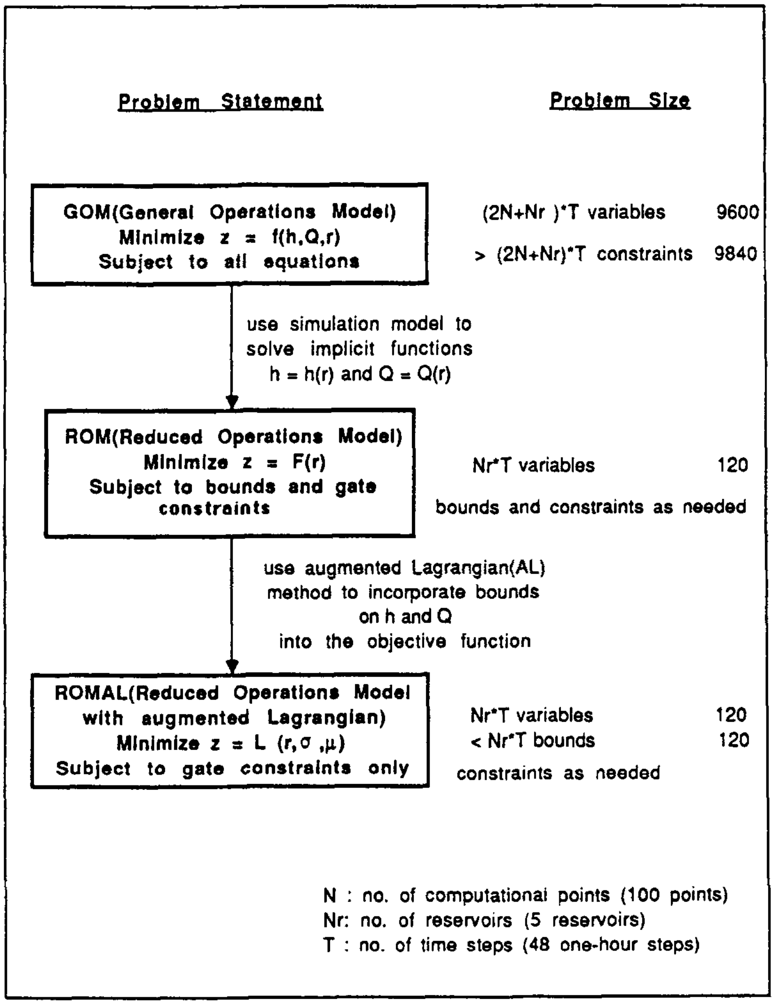

Fig. 2. Transformation of operations problem.

h and the flow rates, Q by using the DWOPER simulation model, once the gate settings, r are specified. The general optimal control approach to the real-time reservoir operation problem is shown in Figure 1. Through this simulator-optimizer formulation, the problem is solved efficiently by incorporating the simulation model into a procedure when a set of gate operations, **r**, (control vector) is chosen, the simulation model is run subject to the selected control vector, to solve the hydraulic constraint set, g, for the elevations and discharges (state vector). Then the objective function is evaluated, the bound constraints are checked for any violations and the procedure is repeated with an updated set of gate operations until a convergence criterion is satisfied and no bound constraints are violated.

It must be noted that the optimization is performed only on the gate settings in this procedure. The new optimization problem, called the reduced operations model (ROM) has $N_r{*}T$ variables compared to the $(2N{*}T{+}N_r{*}T)$ variables of the GOM, where N, T, and N_r are the total number of computational points, time steps, and reservoirs, respectively. The number of constraint equations has also been reduced by the same amount, $(2N{*}T)$, with the elimination of the hydraulic constraints, **g**. The transformation of the operations problem is shown in Figure 2, along with the problem size at each step of the transformation for an example system. The problem size for an example with 100 computational points, 5 reservoirs, and 48 time steps is drastically reduced, from over 9000 variables and constraints to 120 variables and 120 bound constraints because of the simulator-optimizer formulation.

The hydraulic constraint set, **g**, has a special staircase banded structure that can be exploited to construct an efficient overall algorithm. The model presented herein combines the simulation model, DWOPER, and the optimization model, GRG2, within the framework of an optimum control formulation. The transformation of the original problem into the reduced one is similar to the generalized reduced gradient approach, which is also used to solve the reduced (transformed) problem.

The original problem, GOM, can be converted into a reduced problem as suggested by the implicit function theorem (Luenberger, 1973). The implicit function theorem states that if some of the problem variables can be solved in terms of the remaining variables, then a reduced problem can be devised which can be manipulated more easily. The approach is applied to the problem given by Equations (1)–(6) in such a way that the hydraulic constraints (Equation (2)) are handled separately by the simulator and the other constraints by the optimizer. The simulation model computes the values of the state variables, **h** and **Q** for given values of the control variables **r** and the optimization model seeks the optimal values of **r** that will minimize the objective function. The implicit function theorem states that **h**(**r**) and **Q**(**r**) exist if and only if the basic matrix (the Jacobian of the system of equations given by (Equation (2)) is nonsingular. This condition is always satisfied when a solution is possible, as the simulator (DWOPER), uses the same matrix for the finite-difference unsteady flow computations.

Expressing the water surface elevation and discharge as a function of the control variable, **r**,

$$\mathbf{h} = \mathbf{h}(\mathbf{r}) \tag{17}$$

and

$$\mathbf{Q} = \mathbf{Q}(\mathbf{r}) , \tag{18}$$

then, the objective function, now called the reduced objective function is expressed as

Minimize

$$z = F(\mathbf{r}) = f\,[\mathbf{h}(\mathbf{r}),\mathbf{Q}(\mathbf{r})] . \tag{19}$$

The objective function can be evaluated once the state variables, **h** and **Q**, are computed for the given set of control variables, **r**.

The reduced problem, which is called the reduced operations model (ROM), is now expressed by the reduced objective function, Equation (19), subject to Equation (3)–(6). The ROM is much smaller in size than the GOM with the simulator determining the implicit functions **h**(**r**) and **Q**(**r**), by performing the unsteady flow computations thus eliminating the constraint matrix **g** that describes the hydraulics.

In solving the ROM by a nonlinear programming algorithm, the Jacobian of the matrix **g**(**h**,**Q**,**r**) will be required as well as the gradients of the functions $F(\mathbf{r})$, **h**(**r**), and **Q**(**r**), which are also called the reduced gradients. The Jacobian matrix is defined as

$$\mathbf{J}(\mathbf{h},\mathbf{Q},\mathbf{r}) = [\partial \mathbf{g}/\partial \mathbf{h}, \partial \mathbf{g}/\partial \mathbf{Q}, \partial \mathbf{g}/\partial \mathbf{r}] = [\mathbf{B},\mathbf{C}] \tag{20}$$

or

$$\mathbf{J}(\mathbf{y},\mathbf{r}) = [\partial \mathbf{g}/\partial \mathbf{y}, \partial \mathbf{g}/\partial \mathbf{r}] = [\mathbf{B},\mathbf{C}]\ , \tag{21}$$

where **y** denotes the state variable (**h**,**Q**) and **B** is the basis matrix. The basis matrix of the optimal control problem is the same as the Jacobian matrix used in the Newton–Raphson solution procedure in the simulation model (DWOPER). Thus, the two elements of the Jacobian matrix **J** are available (with the basis **B** explicitly computed, and terms in **C** already available) after a simulation run. The basis matrix is a banded sparse matrix with at most four nonzero elements in each row around the matrix's main diagonal.

The reduced gradients can be calculated by applying the two-step scheme used by Lasdon and Mantell (1978) and also by Wanakule *et al.* (1986). Letting $B_t = \partial g_t/\partial y_t$ denote the basis matrix for time step t, the following scheme is adapted for the ROM:

(i) Solve the system of finite difference equations for the last time step T to find the values of the Lagrange multipliers π_T

$$\pi_T B_T = \partial f/\partial y_T\ , \tag{22}$$

then solve for the π_t backward in time

$$\pi_t B_t = \partial f/\partial y_t - \pi_{t+1}(\partial g_{t+1}/\partial y_t), \quad \text{for } t = T-1, T-2, \ldots, 2, 1\ . \tag{23}$$

(ii) Calculate the value of the reduced gradient

$$\partial F/\partial r_t = \partial f/\partial r_t - \pi_t\,(\partial g_t/\partial r_t), \quad \text{for } t = 1, 2, \ldots, T\ . \tag{24}$$

The Lagrange multipliers, π_t can be used in a sensitivity analysis as they show the effect of a small change in the corresponding term in the objective value.

3.3. SOLUTION OF REDUCED PROBLEM

The reduced problem, ROM, can be solved by a nonlinear programming algorithm.

As the reduced problem still contains bound-type constraints on the state variables **h** and **Q**, the algorithm adopted should have provisions to assure the feasibility of the simulation model solutions for the state variables. An augmented Lagrangian (AL) algorithm that incorporates the bounds on the state variables into the objective function is used for this purpose. An application of this type can be found in Hsin (1981) where the bounds on the state variables are violated until the solution converges. The reduced problem with AL terms is

$$\min L_A(\mathbf{r},\mu,\sigma) = F(\mathbf{r}) + 0.5\ \Sigma_i\, \sigma_i \min\,[0, (b_i - \mu_i/\sigma_i)]^2 + 0.5\ \Sigma_i \mu_i^2/\sigma_i\,, \tag{25}$$

where i denotes the constraint set which is formed of the bounds on the state variables, i.e. the water surface elevations and discharges, and σ_i and μ_i are, respectively, the penalty weight and the Lagrange multiplier associated with the ith bound. The term b_i is the violation term defined as

$$b_i = \min\,[(y_i - \underline{y}_i), (\bar{y}_i - y_i)]\,. \tag{26}$$

The constraints of the new problem are the bounds on the control variables and the operating constraints.

A reduced gradient approach is adopted to solve the reduced problem with AL terms. This new problem, which will be referred to as the reduced operations model with augmented Lagrangian (ROMAL) can be expressed as

$$\text{Minimize } L_A(\mathbf{r},\sigma,\mu) \tag{27}$$

subject to Equations (5) and (6).

The solution to this is a two-step procedure with an inner and an outer problem that must be solved. The objective function of this inner-outer problem combination is

$$\min_{\sigma,\mu} \Big[\min_{\mathbf{r}\in S} L_A(\mathbf{r},\sigma,\mu) \Big]\,, \tag{28}$$

where **r** is selected from S, the set of feasible gate settings defined by Equation (5). The inner problem involves the optimization of the augmented Lagrangian objective by using GRG2 to determine optimal values of **r** while keeping μ and σ fixed. Then the outer problem is iterated by updating the values of μ and σ for the next solution run of the inner problem. The overall optimization is attained when μ and σ need no further updating, within a given tolerance interval. The updating formula used for μ is

$$\mu_i^{(k+1)} = \begin{cases} \mu_i^{(k)} - \sigma_i b_i, & \text{if } c_i < \mu_i/\sigma_i\,, \\ 0, & \text{otherwise,} \end{cases} \tag{29}$$

where k is the number of the current iteration. The value of σ is normally adjusted once during early iterations and then kept constant (Powell, 1978).

In applying the generalized reduced gradient approach to the ROMAL formulation the gradient of the new objective function is evaluated as

$$\nabla L_A(\mathbf{r},\mu,\sigma) = \partial L_A/\partial r_i - \pi\ (\partial g/\partial r_i), \quad \text{for all } i = 1 \text{ to } 2N\,. \tag{30}$$

The solution of the inner problem, i.e. finding the optimal $\mathbf{r}$ for fixed μ and σ is accomplished by GRG2 (Lasdon and Waren, 1983), which is based on the generalized reduced gradient technique. The basic steps of the optimal control algorithm are shown in Figure 3.

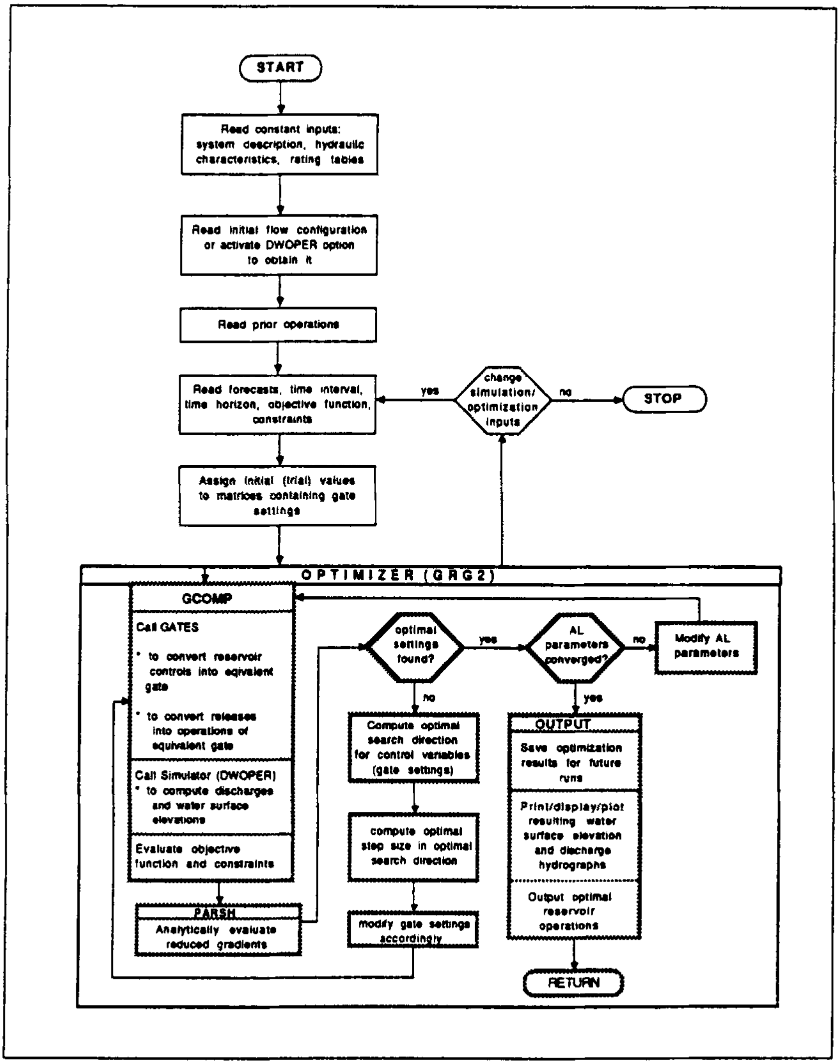

Fig. 3. Block diagram of optimal control algorithm.

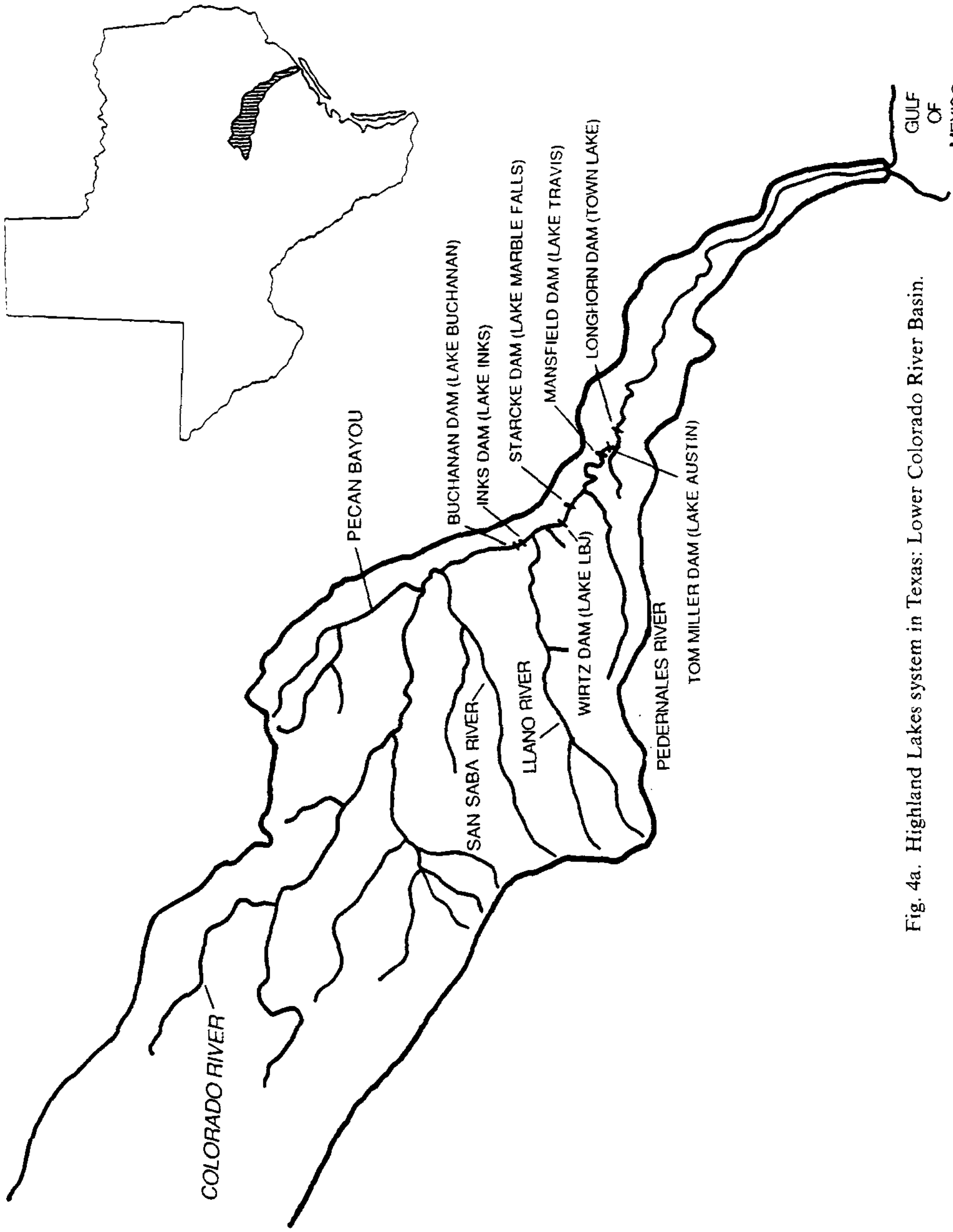

Fig. 4a. Highland Lakes system in Texas: Lower Colorado River Basin.

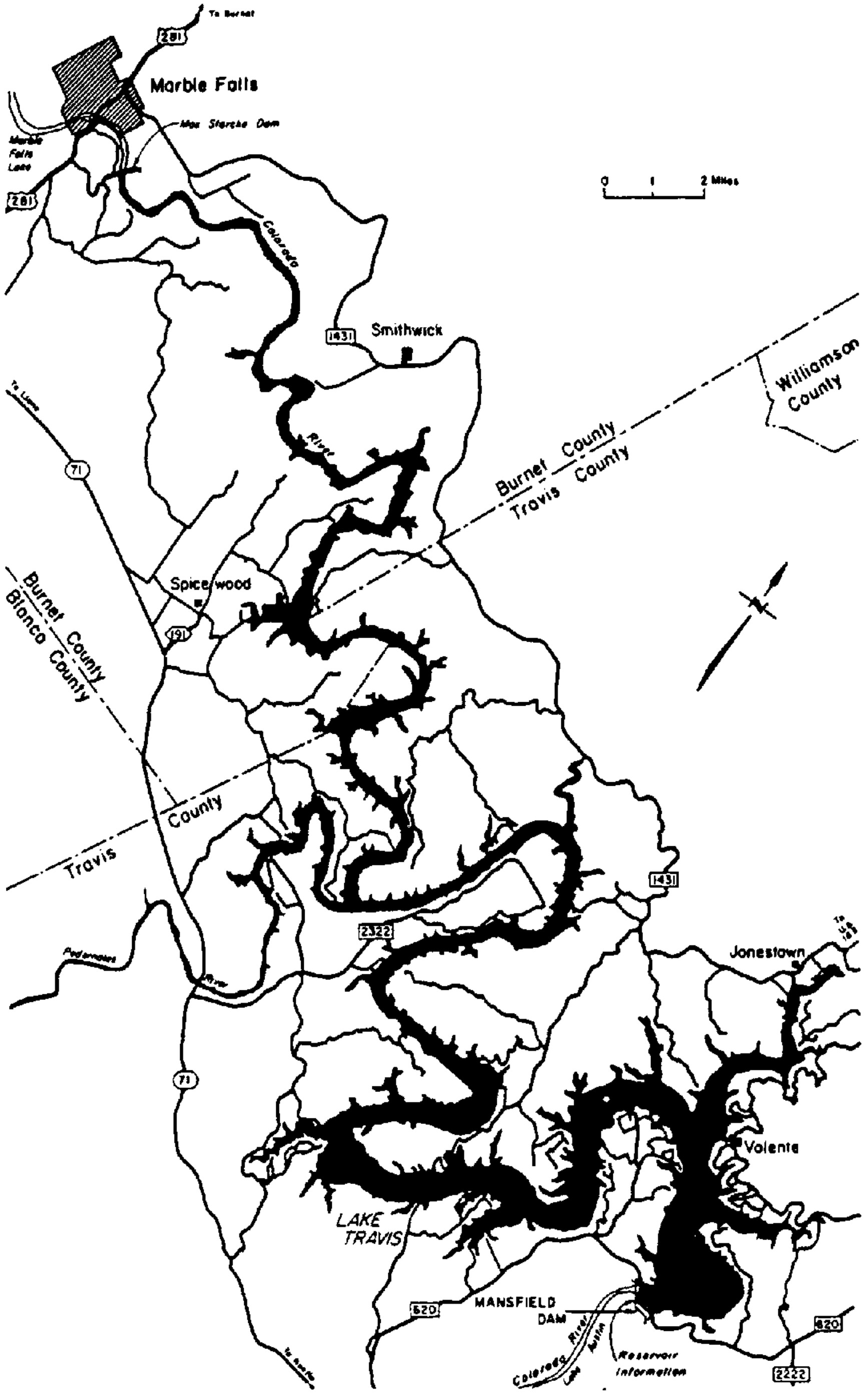

Fig. 4b. Highland Lakes system in Texas: Lake Travis (Texas Water Development Board, 1971).

4. Model Application

4.1. CASE STUDY-OPERATION OF LAKE TRAVIS

Lake Travis is one of the seven reservoirs of the Highland Lake system located on the Colorado River in central Texas near Austin (Figure 4) with a total contributing drainage area of 27 352 miles^2. A major tributary is the Pedernales River, which has a watershed area of approximately 1280 miles^2. Lake Travis is about 64 miles long with a designated flood control capacity of 3 223 000 acre-feet. Mansfield Dam originally was built primarily for flood control and hydroelectric production. Of the seven reservoirs in the lake chain, Lake Travis is the only reservoir with designated flood control stage. Development in the flood plain of the Highland Lakes has caused severe problems in operation of the reservoir under flooding conditions.

Lake Travis is operated by the Lower Colorado River Authority for purposes of flood control, water supply, hydropower generation, and low flow augmentation. Historically, flood operations for the lake were prescribed by a schedule set by the U.S. Army Corps of Engineers, based on the forecasted lake elevations. Unver *et al.* (1987) modeled the lake using DWOPER in the framework of a real-time simulation model. The original DWOPER model consisted of 82 cross-sections; however as an effort to reduce the problem size for the optimization model, 24 cross-sections were selected uniformly from among the previous 82 cross-sections. The storm event that was used for the original model calibration was resimulated. The resulting discharges and water surface elevations were the same as using 82 cross-sections within practical limits. Prior to the application of this model there was no computational procedure used by LCRA for flood forecasting.

The available data and existing operational and legal restrictions on the operation of the lake dictate that the optimization objective consider both lake elevations and releases. Extensive development along the shores of the lake and the existence of a downstream urban area (Austin) are two major concerns in the flood operation of Lake Travis. The critical elevations in the existing flood operations schedule were used to set up an elevation versus penalty weight table for the lake area. Similarly, a release versus penalty weight relationship was established to model the damages resulting from excessive releases from the lake. The objective function,

Table I. Components of two objective functions for Lake Travis

Lake Level[a] (feet)	660	670	680	685	687	689	691	695	700
Penalty weight (feet^{-1})	30	20	10	0	10	15	20	30	40
Release (1000 cfs)	0	50	60	70	80	85	90	100	200
Penalty weight (1000 cfs)	0	0	0.3	0.3	0.4	0.5	0.6	0.9	1.0

[a] Lake elevation at 18 miles upstream of Mansfield Dam.

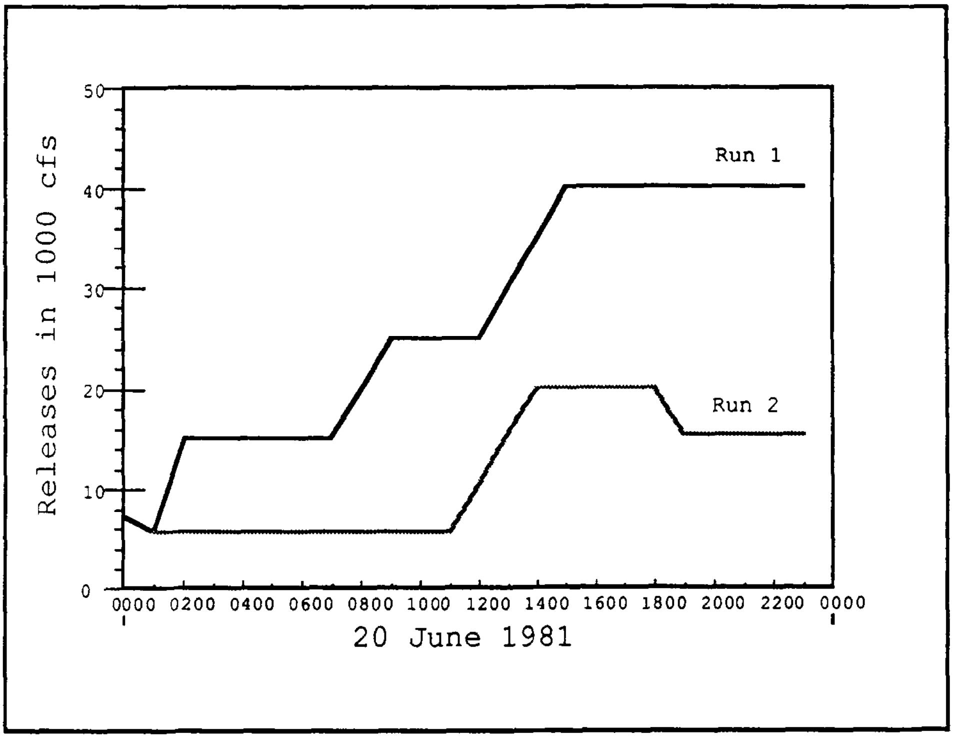

Fig. 5. Initial feasible policy for Lake Travis runs.

given by Equation (12) was adopted for Lake Travis.

The components of the objective function are given in Table I. The penalty weights and the corresponding discharges and water surface elevations were selected so that some nonzero objective value would result even at the optimum. No lower limit was specified to the releases from the lake, whereas lake levels below 680 ft were penalized as well as those over 680. The penalty weights were specified so a higher deviation from the target values would have a higher unit penalty associated with it. The flood control section where excessive water levels are penalized was chosen to be about 18 miles upstream of the Mansfield Dam to eliminate the effects of drawdown resulting from gate operations. The objective function, although based on the existing operations policy, does not function the same way as the standard policy, which takes action only when the set critical value is forecasted, thus eliminating the option of pre-emptying the reservoir to account for otherwise potentially damaging flow situations. This near-sighted nature of the standard operations is not followed by the optimization model, which seeks the set of operations that will result in the overall minimization of the set objective. In addition, the optimization model, through DWOPER, makes use of flood forecasts on a real-time basis.

The optimization model was run to determine the optimal hourly operations of Lake Travis for the June 1981 flood event. Two different starting operation

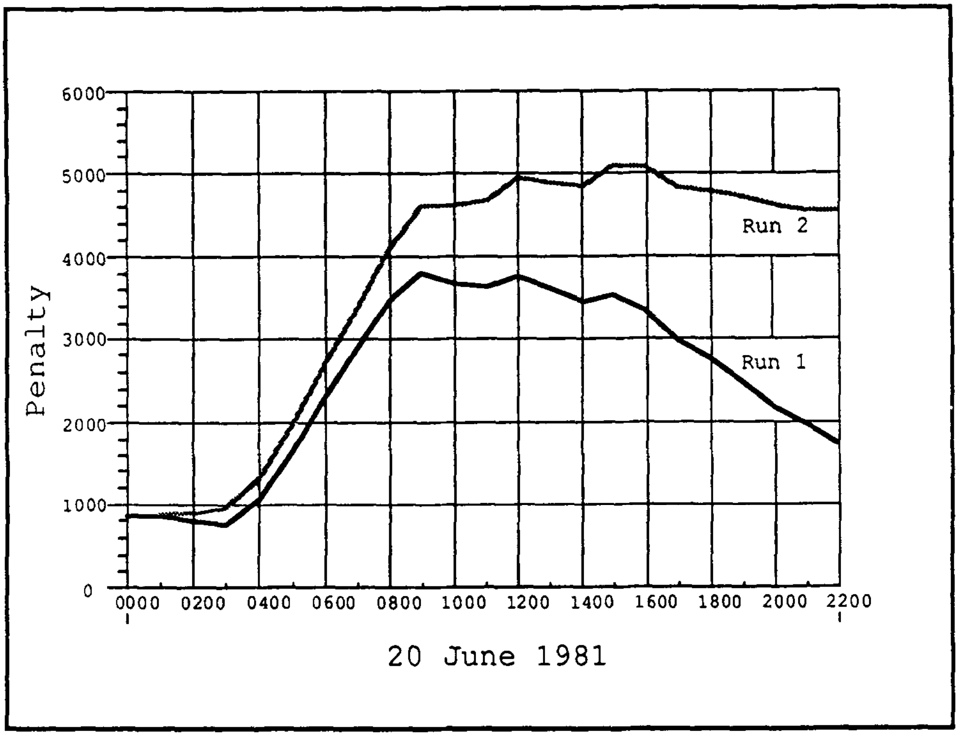

Fig. 6. Initial value of objective function for runs 1 and 2.

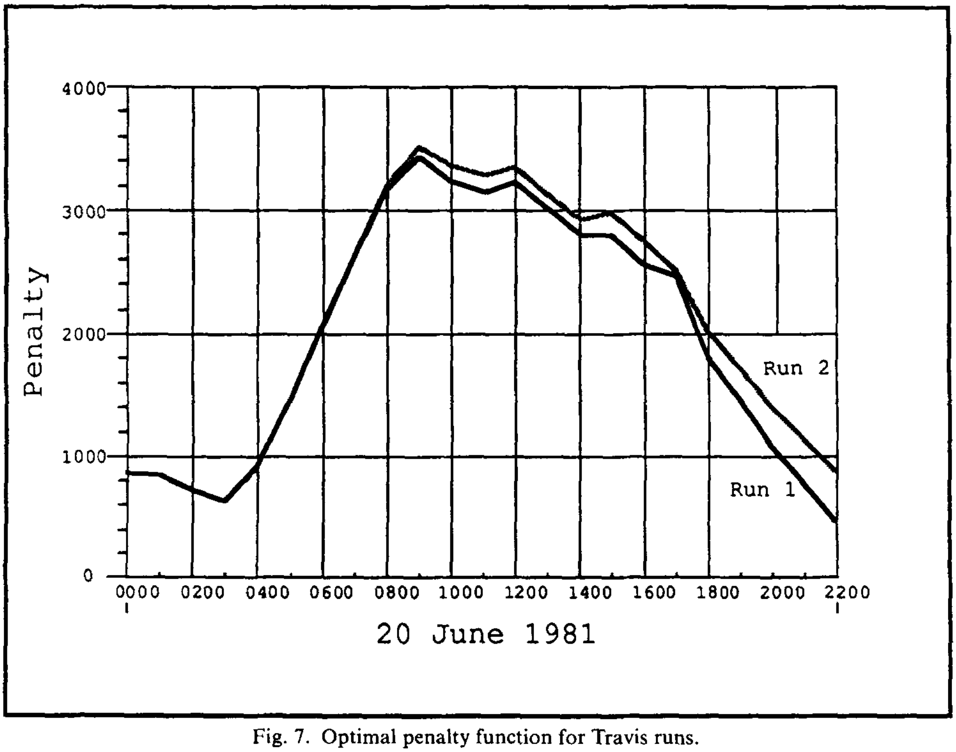

Fig. 7. Optimal penalty function for Travis runs.

policies were selected to test for local optima and study the convergence characteristics of the optimization model. The operation horizon was selected to be 24 h. The two sets of initial gate operations are given in Figure 5.

The first set of initial gate operations (Run 1) was selected at random, whereas the second set (Run 2) is similar to the actual operations for the given period. Figure 6 shows the resulting objective function values corresponding to the initial operations for the two trial runs on a step by step basis, as a function of time period. The total initial objective function value is defined as the area under each of the two curves. The initial operations for run 1 were apparently closer to the optimum than those of Run 2. Solution of the model required 10 iterations of the optimizer (GRG2) to each an optimum for Run 1, whereas 19 iterations were necessary for Run 2. The objective functions at the end of the optimization are shown in Figure 7, as a function of time step.

As improvement in the hourly objective value for Run 1 is shown in Figures 8a and 8b. As Figures 7 and 8 indicate, a steady and consistent decrease in the objective function value was observed for Run 1, although the improvement during the second five iterations was somewhat slower (approximately half the rate during the first five iterations). The optimization results are guaranteed to be at least at

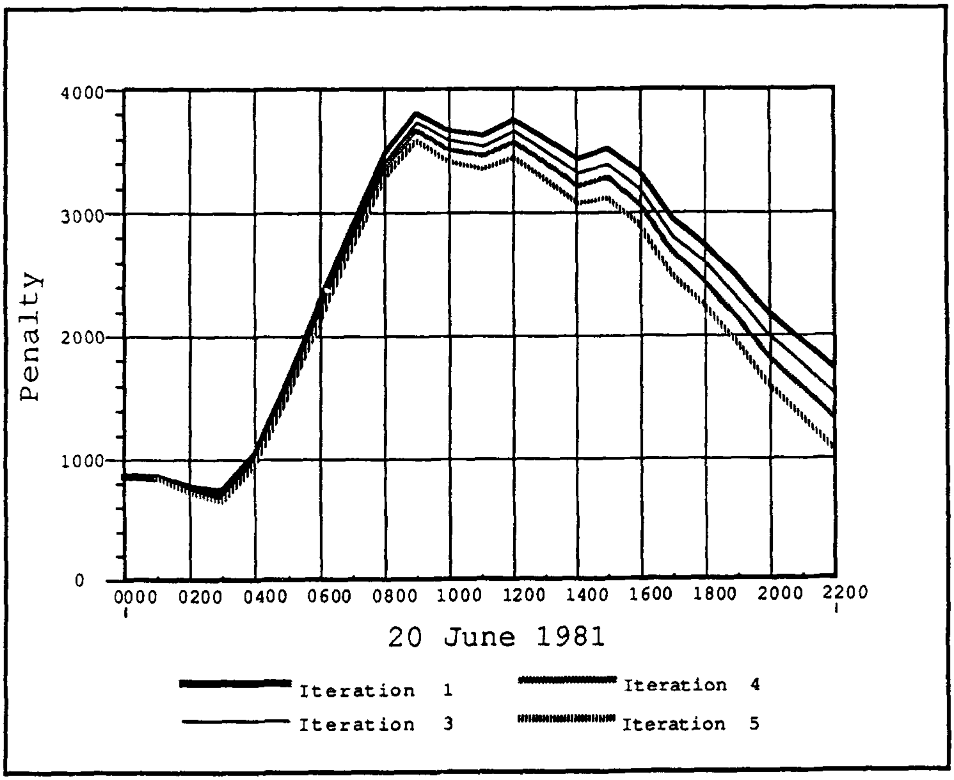

Fig. 8a. Penalty function for Travis Run 1, iterations 1, 3, 4, 5.

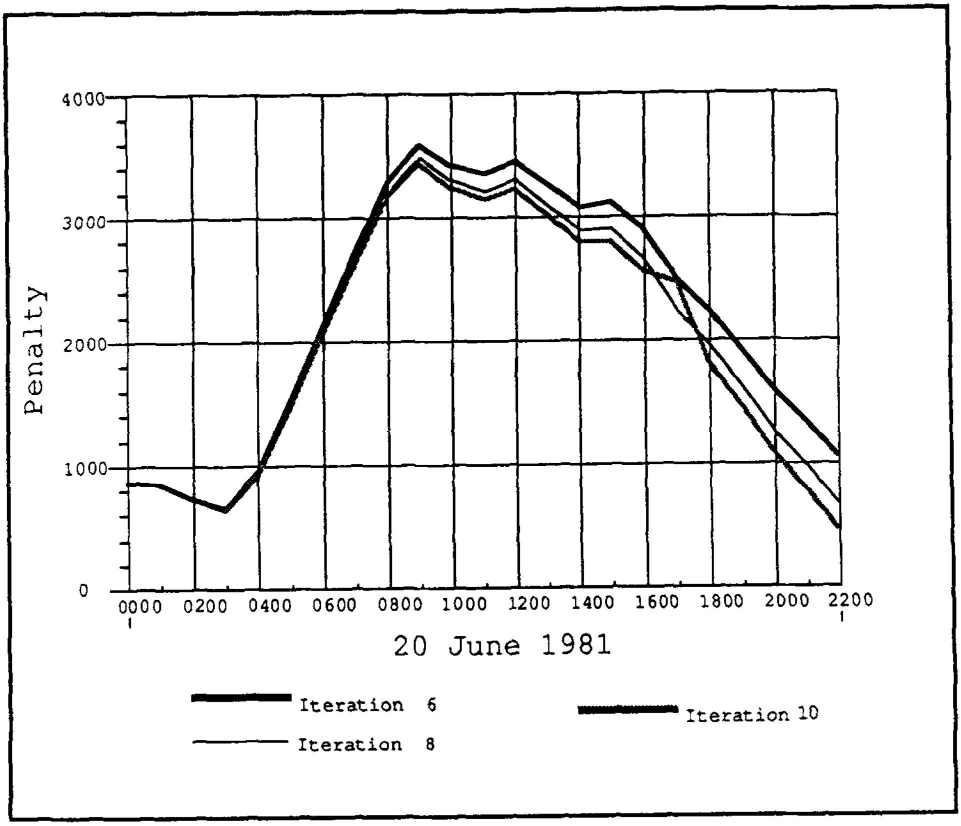

Fig. 8b. Penalty function for Travis Run 1, iterations 6, 8, 10.

a local optimum since the Kuhn–Tucker conditions were satisfied at the final point.

Figures 9a and 9b show the improvement in the hourly objective function value for Run 2. The improvement in the total objective function value for Run 1 and Run 2 is given as a function of the iteration number in Figure 10. (The total objective function value for an iteration is defined as the area under the hourly objective function for that iteration.) As the figure indicates, very little improvement was obtained during the last seven iteration cycles due to the very conservative value assigned as the stopping criteria. However, the model produced significantly different hourly objectives, as shown in Figure 8b, during the last five iterations, in an attempt to improve the total value of the objective function by an amount that is insignificant for actual operations. A realistic convergence criteria would terminate the optimization at the end of the 12th or 13th iteration cycle. The optimal operations found in Run 2 are also guaranteed to be at least a local optimum since the Kuhn–Tucker conditions were satisfied at the final point. The releases associated with the initial and optimal gate operations for the two runs are shown in Figure 11.

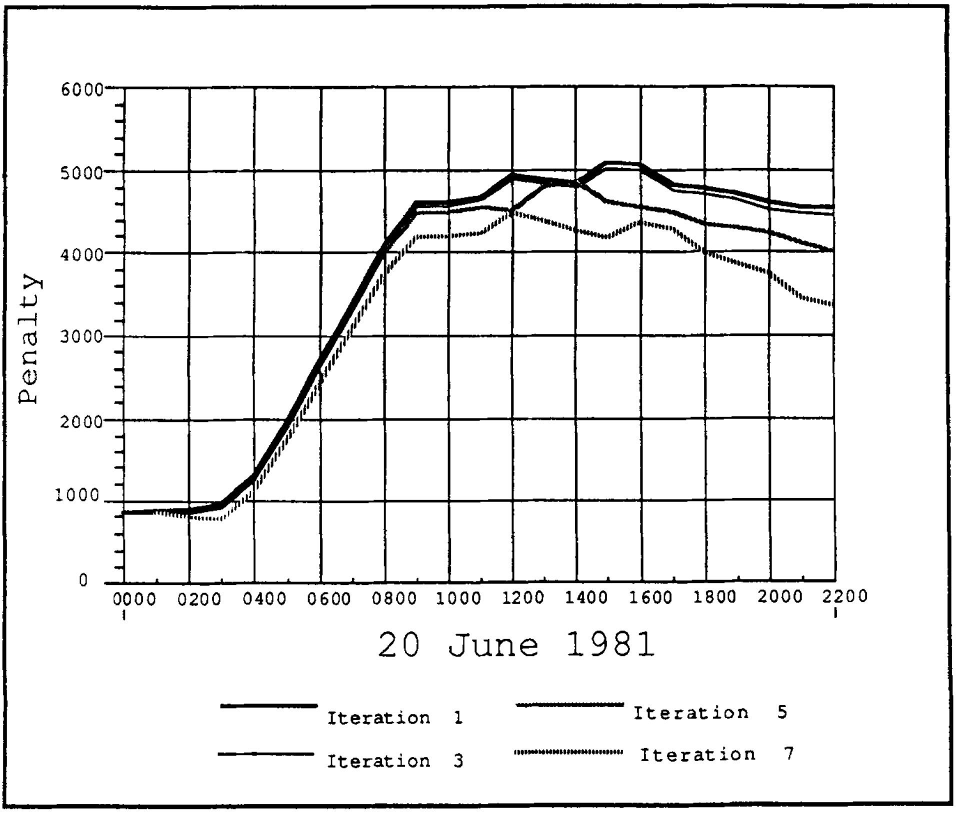

Fig. 9a. Penalty function for Travis Run 2, iterations 1, 3, 5, 7.

4.2. COMPUTATIONAL ASPECTS

The model development and optimization runs were made on the Control Data Corporation (CDC) Dual Cyber computer system at the University of Texas at Austin. The execution times for the two problems were 565 and 712 sec, respectively. However, increasing the convergence criteria of the optimization model will significantly speed up the decision process. Similarly, larger tolerance intervals for the iterative simulation computations will result in less overall computation time. Although the execution times for the Lake Travis runs may seem excessive, since the results of each iteration are displayed on the screen during the model run, it may be sufficient, for practical purposes, to stop at a suboptimal point when sufficient improvement over the initial policy is obtained or when the rate of improvement is small. It is also recommended the initial policy be obtained by running the real-time flood forecasting model first. This will guarantee the feasibility of the initial solution given to the optimization model, assuming a successful simulation run can be obtained. It is also anticipated that the user will want to make a final run with the flood management model by modifying the optimal operations to reflect the actual operation of the dam (fractional values are typical

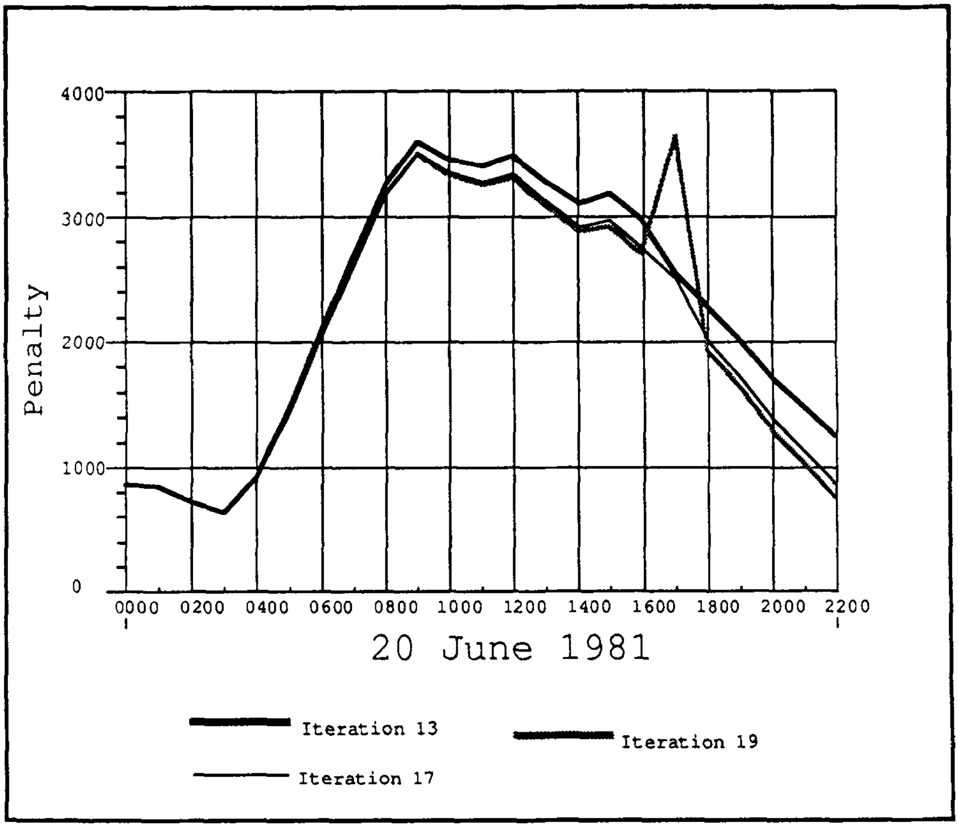

Fig. 9b. Penalty function for Travis Run 2, iterations 13, 17, 19.

for optimization results whereas it may be desirable to operate gates by certain increments).

5. Summary and Conclusions

Multi-reservoir operation can be characterized by the integrated operation of multiple facilities on river systems for multiple objectives, flood control being one of the major purposes. Many reservoirs were built several years ago and operation policies were established. However, many of these reservoirs cannot be operated in the manner that they were initially intended to be operated. One of the major reasons is the uncontrolled urbanization into the floodplains of the river and reservoirs. Other reasons are due to inadequate spillways for passing floods, legal constraints, etc. Many of the reservoir systems are characterized by conditions which result in significant backwater conditions due to tributary flows, hurricane surge flows, tidal conditions, flow constrictions in the rivers, etc. These conditions cannot be described by the use of hydrologic routing methods or conceptual models, and as a result must be described by more accurate hydraulic routing models such as DWOPER.

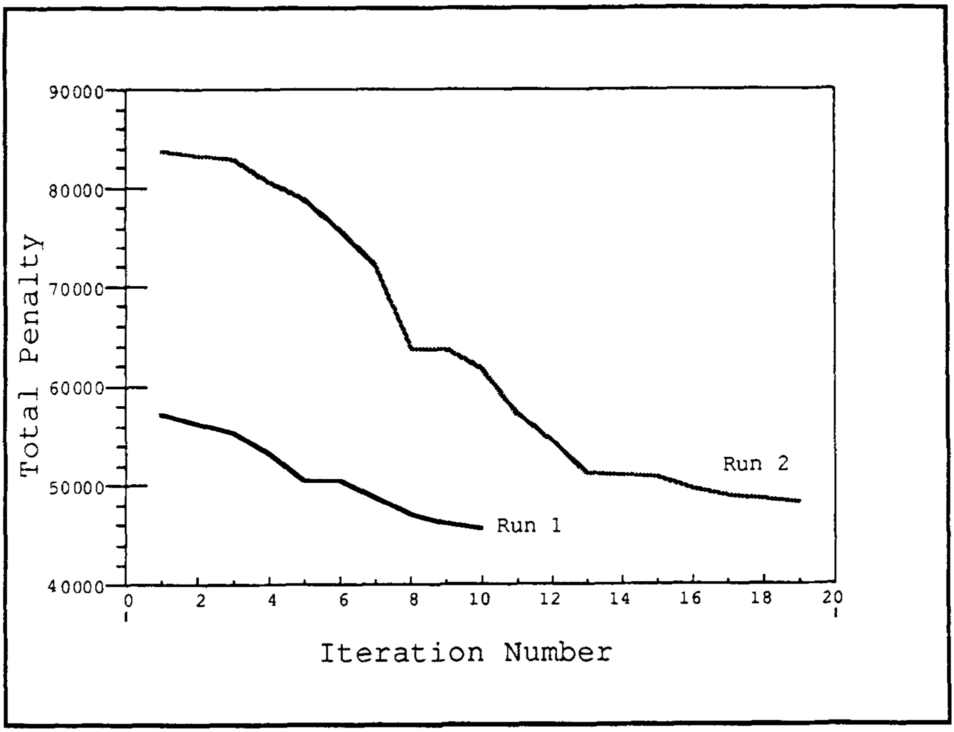

Fig. 10. Total penalty function for Lake Travis runs.

One example of the type of river-reservoir system described above is the Lower Colorado River-Highland Lake System in Texas. Development of the floodplains of the Highland Lake System has caused severe problems in operation of the reservoirs under flooding conditions as in many other systems around the U.S. Because of the severe limitations placed on many of these reservoir systems during flooding conditions, real-time data collection and transmission systems have been implemented. However, even with such real-time data collection systems in place, there is still a lack of available methodologies and software to use in conjunction with the real-time data to make the best possible flooding estimates and to optimally operate these systems in order to minimize flood damages.

The modeling effort reported herein is essentially a step forward to enhance the previous type of modeling effort by Unver *et al.* (1987). Using the concepts of optimal control theory, it is now possible to link nonlinear optimization models with unsteady flow routing models such as DWOPER to solve the large-scale nonlinear programming problems associated with reservoir operation under flooding conditions. The next step forward will be to build upon and expand this model to include expert systems capability, reliability analysis, and updating and self-correction schemes. The benefits of real-time flood data and transmission systems and real-time operation models such as the one presented herein, go far beyond

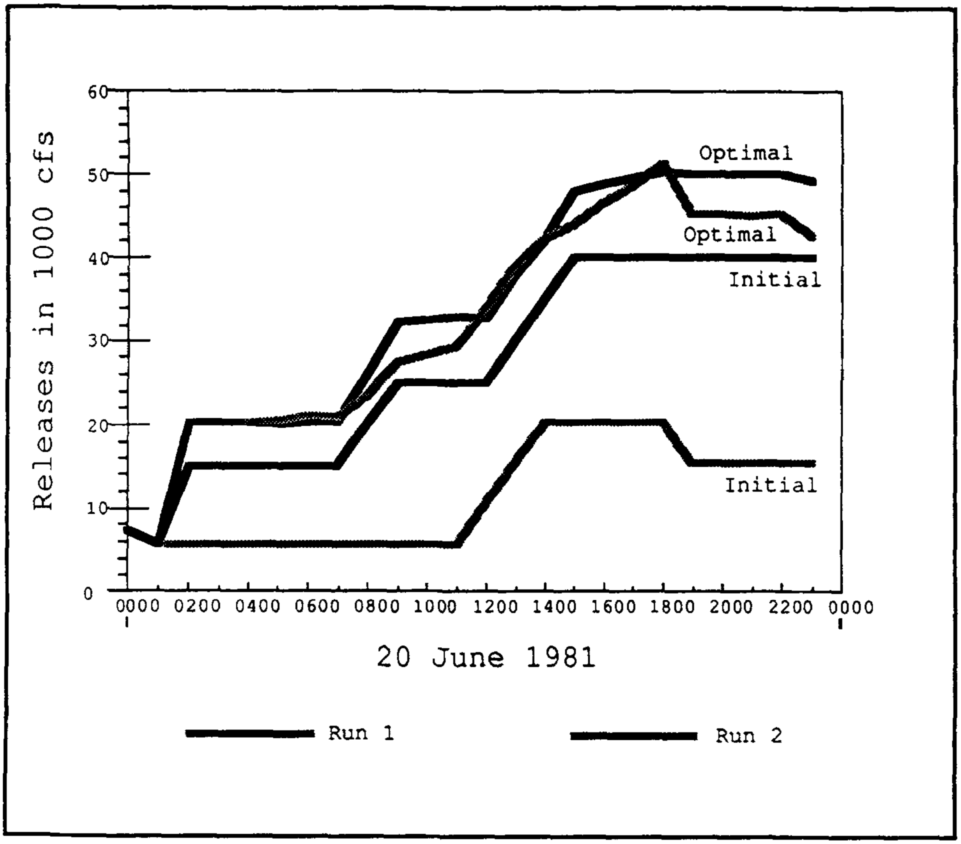

Fig. 11. Initial and optimal releases for Lake Travis runs.

the potential economic losses averted and greatly reduce the social disruption, deaths and injuries caused by floods.

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