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A Temporal Model of Linear Anti-Hebbian Learning

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Abstract. A temporal variant of Foldiak's first model with lateral inhibitory synaptic weights is proposed. The usual symmetric scalar values of the lateral weights are replaced with data driven asymmetric memory based lateral weights, which take the form of Finite Impulse Response (FIR) coefficients. Linear anti-Hebbian learning, as defined by Foldiak (IEEE/INNS International Joint Conference on Neural Networks, 1989) and Matsuoka et al. (Neural Networks, Vol. 8, pp. 411–419, 1995), is employed in the self-organisation of the network weights. The temporal anti-Hebbian learning, when applied to the separation of convolved mixtures of signals, causes the network weights to converge to the truncated FIR filter coefficients of the unmixing transfer function and so recover the original signals. Simulation results are presented for separating two natural speech sources convolved and mixed by *a priori* unknown direct and cross-coupled transfer functions. We compare temporal anti-Hebbian learning with information maximisation learning when applied to the blind separation of convolved sources.

1. Introduction

Foldiak's first proposed network model of anti-Hebbian learning [1] was motivated by Barlow's 'law of repulsion' [2]. The law states that a 'repulsion constant' increases between variables which are correlated. If two cells have pre- and post-synaptic activities y_i and y_j, and the synaptic strength between them is w_{ij} then the law of repulsion is governed by the anti-Hebbian rule $\Delta w_{ij} = -\eta\, y_i y_j : i \neq j$. If the activities of the cells are positively (or negatively) correlated a negative (or positive) weight will build up between them making simultaneous firing more difficult and so eliminating the correlation between them. The model, (a two input/output version is discussed for clarity of presentation) is shown in Figure 1, at convergence the output covariance matrix $\mathbf{C}_{\mathrm{yy}}$ will be diagonal with the off-diagonal terms all being zero, thus indicating the reduction in the output correlation. The governing equation of the neuron dynamics is given as

$$\tau \frac{dy_i}{dt} = -y_i + x_i + \sum_{j=1}^{N} w_{ij} y_i \tag{1}$$

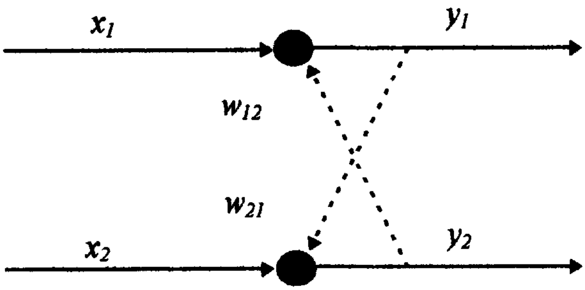

Figure 1. Two neuron version of Foldiak's lateral inhibition network.

Taking the adiabatic approximation (1) can then be written as

$$y_i = x_i + \sum_{j=1}^{N} w_{ij} y_i \tag{2}$$

and in matrix format

$$\mathbf{y} = (\mathbf{I} - \mathbf{W})^{-1} \mathbf{x} \tag{3}$$

2. Blind Separation of Sources

Jutten and Herrault [3, 15] were the first to propose the network architecture of Figure 1 within the context of blind separation of source signals. They developed a nonlinear learning algorithm for weight update (which can be considered as nonlinear anti-Hebbian learning) and exhibited higher order decorrelating properties. This established the initial understanding of the requirements of the blind separation of sources problem, where a priori unknown instantaneous mixtures of source signals could be separated into the original components (up to a permutation and scaling). The asymmetric nonlinear learning introduced higher order statistics into the self organisation and broke up the symmetry of the weight matrix. Recently, Cichocki [4] has derived a nonlinear learning algorithm and the associated nonlinearity from information theoretic principles, this is applied both to the Jutten and Herrault network and a variant which has self connected neurons.

These algorithms deal with the case of instantaneous mixing, which in the domain of signal processing is an artifice. Direct and cross coupling channels can be defined as transfer functions, which in the discrete z domain are given as polynomials of degree M and are modelled as either infinite impulse response (IIR) or FIR filters. Weinstein et al. [5] considered the problem of multi-channel separation, and used the maximisation of a decorrelation criterion in estimating the unmixing transformations. The simplifying assumption of constant gain direct coupling is made by both Weinstein et al. [5] and Yellin and Weinstein [6], which is

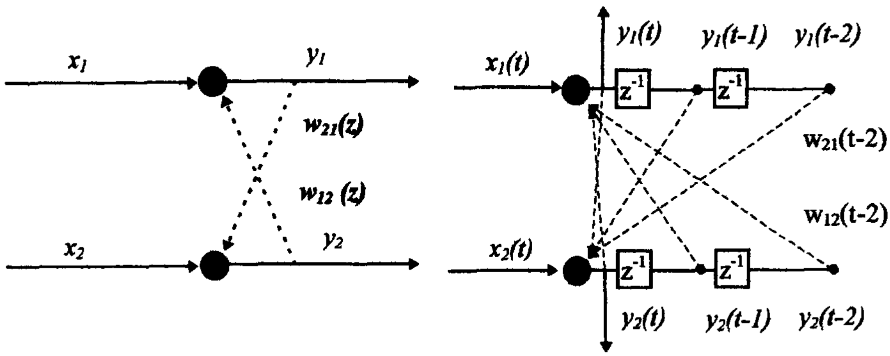

Figure 2. Temporal model of Foldiak's anti-Hebbian network.

unrealistic in acoustic environments. The use of higher order statistics to order four is used in [6] as the criterion for identifying the inverse filters, with decorrelation being the special case of order two.

Nguyen et al. [16] extend the original network and learning proposed by Jutten and Herrault to deal with convolutive mixtures, proposing algorithms based on fourth order and second order statistics. Van Gervan et al. [17] provide a comparative empirical study of the separating performance of algorithms based on second order statistics and those based on fourth order statisitics. Although making the simplifying assumption of constant gain direct coupling, they show that there is no real advantage of using higher order statistics over second order for separating strictly causal convolutive mixtures of sources.

Torkkola [7] has recently developed a variant of the Bell and Sejnowski [8] information maximisation network and learning algorithm to separate mixtures of delayed and convolved sources and has reported encouraging results in acoustic environments. The simulations reported in [7] are more general than those in [5, 6, 16, 17] in that the constant gain direct coupling assumption is dispensed with.

We show in the following sections that for causal convolutive mixtures of sources temporal anti-Hebbian learning yields comparable results to the entropy maximisation algorithm developed in [7] by Torkkola.

3. Temporal Anti-Hebbian Model

We take Foldiak's first model, Figure 1, and apply 'memory based' synaptic lateral connections. Figure 2 shows the temporal model where the weights are written as z-transforms, and where the z^{-1} operator is shown explicitly for two tapped delays.

The network output is now given as a time delayed weighted version of (2), that is

$$y_i(t) = x_i(t) + \sum_{j=1}^{N} \sum_{k=0}^{M} w_{ij}(t-k) y_i(t-k) \tag{4}$$

where N is the number of neurons and M is the total number of delays. We note that this structure is similar to the double feedback adaptive filter proposed in [16, 17]. Written as a z domain matrix we have the compact form of (4)

$$\mathbf{y}(z) = (\mathbf{I} - \mathbf{W}(z))^{-1}\, \mathbf{x}(z) \tag{5}$$

The anti-Hebbian rule given in Section 1, has been shown to yield an output with identity covariance [1]. For second order independance the following should be satisfied $\{y_i(t)y_j(t-k)\} = 0\ \forall i \neq j \wedge k = 1...M$ and so we require an adaptive algorithm which adapts the dependant variable of the network output (i.e the weights) to yield decorrelated outputs in the expectation. By utilising the anti-Hebbian rule given in Section 1 for each temporal output we can then force the output cross-correlation terms to zero. Foldiak proves the inherent stability of the anti-Hebbian rule for the instantaneous case in [18], a similar stability analysis of the fixed points is required for the temporal case to identify potential spurious attractors and will be the topic of further work. However, our simulations, and those reported by Van Gervan et al. [17] have not found this to be a problem. The proposed anti-Hebbian learning for each of the tapped delay weights is therefore

$$\Delta w_{ij}(k) = -\eta\, y_i(t)y_i(t-k) \quad \forall i \neq j \wedge k = 1...M \tag{6}$$

So the cross weights will grow in an inhibitory fashion if there is correlation between the output $y_i(t)$ and each of the tapped outputs of the adjacent neuron $y_i(t-k)$ until the following holds. $E\{y_i(t)y_j(t-k)\} = 0\ \forall i \neq j \wedge k = 1...M$. The weight connecting the current output at node i to the historical output at time $(t-k)$ of node j is denoted as $w_{ij}(k)$. Note that in the non-temporal model $\Delta w_{ij} = -\eta\, y_i y_j = \Delta w_{ji} = -\eta\, y_j y_i$ and so the anti-Hebbian learning will give a symmetric weight matrix. In the temporal case $\Delta w_{ij}(k) \neq \Delta w_{ji}(k) : -\eta\, y_i(t)y_j(t-k)$ need not equal $-\eta\, y_j(t)y_i(t-k)$ and so an asymmetric polynomial weight matrix is generated. As $\langle \Delta w_{ij}(k)\rangle = \langle y_i(t)y_j(t-k)\rangle \to 0\ \forall i \neq j \wedge k$ and using (5) the following polynomial matrix expressions are given.

$$\mathbf{C}_{\mathbf{yy}}(z) = (\mathbf{I} - \mathbf{W}(z))^{-1}\, \mathbf{C}_{\mathbf{xx}}(z)\, (\mathbf{I} - \mathbf{W}(z))^{-T} = \mathbf{I} \tag{7}$$

$$\Rightarrow\ ((\mathbf{I} - \mathbf{W}(z)) = (\mathbf{C}_{\mathbf{xx}}(z))^{1/2} \tag{8}$$

Which imposes the restriction that the input polynomial covariance is decomposable. The linear anti-Hebbian learning decorrelates the delayed outputs of all neurons $i\ :\ (i \neq j\, \forall\, i)$ with the instantaneous outputs of neuron j.

4. Blind Separation of Convolved Source Signals

The use of higher order statistics (HOS) has become almost ubiquitous in this particular problem domain. This is driven by the need for cross cumulants of all orders to be null if two distributions are factorable and independent. Statistical independence is the criterion on which most blind separation algorithms are based [3, 6–13]. Weinstein et al. [5] and Van Gervan et al. [17] proposed the use of second order independence as the criterion for the identification of the inverse coupling filters. We now reconsider this criterion in view of the temporal model of anti-Hebbian learning developed.

Matsuoka et al. [14] propose an algorithm based on second order statistics for blind separation of symmetric instantaneous mixtures of signals. By exploiting the nonstationarity of certain signals, Matsuoka et al. [14] argue that source separation is possible by minimising a time varying scalar function of the network weights and the output signal variance. The algorithms developed utilise second order statistics only and can be considered as a normalised version of the anti-Hebbian rule. We utilise a temporal form of these algorithms in separating asymmetric convolved signal mixtures and report on the improved performance over the standard temporal anti-Hebbian rule.

Consider two speech sources s_1 and s_2 the signals received at two points displaced in space (x_1 and x_2) from the sources will be given as the matrix multiplication of the transfer function matrix **H(z)** and the source vector vis,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} h_{11}(z) & h_{12}(z) \\ h_{21}(z) & h_{22}(z) \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \tag{9}$$

$$\mathbf{x} = \mathbf{H}\mathbf{s} \tag{10}$$

For clarity we drop the use of (z) as it is now implied in the relevant equations. To recover the original sources then $\mathbf{s} = \mathbf{H}^{-1}\mathbf{x}$ must be satisfied up to an arbitrary filter, using (5) then for $\mathbf{y}$ to be an approximation to $\mathbf{s}$ at convergence we can write

$$\mathbf{y} = \hat{\mathbf{s}} = (\mathbf{I} - \mathbf{W})^{-1}\mathbf{x} \equiv \mathbf{H}^{-1}\mathbf{x} \tag{11}$$

Some algebraic manipulation will yield, in the discrete z domain

$$(1 - w_{12}w_{21})y_1 = (h_{11} + w_{12}h_{21})s_1 + (h_{12} + w_{12}h_{22})s_2 \tag{12}$$

$$(1 - w_{12}w_{21})y_2 = (h_{22} + w_{21}h_{12})s_2 + (h_{21} + w_{21}h_{11})s_1$$

and so

$$w_{12}(z) = -h_{12}(z)h_{22}^{-1}(z) \quad \text{and} \quad w_{21}(z) = -h_{21}(z)h_{11}^{-1}(z) \tag{13}$$

The values of the network output will then be a filtered representation of the original uncorrupted speech source.

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \Delta_H \begin{bmatrix} \frac{s_1}{cofactor(h_{11})} \\ \frac{s_2}{cofactor(h_{22})} \end{bmatrix} \tag{14}$$

Table I. FONMC measures of original, mixed and retrieved signals.

	FONMC Voice 1	FONMC Voice 2	SSFONMC
Original voices	4.46	7.55	76.89
Mixed voices	2.76	5.10	33.63
Retrieved voices	4.30	7.42	73.54

It is straightforward to see that the temporal weights of the network can then be forced to converge to the inverse filters given in (13) using a suitable criterion. It is also clear that although this can be generalised to N sources the complexity and the length of the inverse filters required may grow to untenable lengths.

We can use the temporal linear anti-Hebbian learning given in (6) to stochastically maximise the second order independence criterion. It is clear that statistical independence is a 'stronger' criterion, indeed an adaption rule based on fourth order cross moments may be considered

$$\Delta w_{ij}(k) = -\eta\, y_i(t) y_i(t-k) y_j(t-k-1) y_j(t-k-2) \quad \forall i \neq j \tag{15}$$

However, by using a temporal form of the Matsuoka anti-Hebbian rules [14] statistical independance of the network output can also be achieved without resorting to higher order statistics.

$$\Delta w_{ij}(k) = -\frac{\eta\, y_i(t) y_j(t-k)}{\Phi_i(t)} \quad \forall i \neq j$$

$$\Phi_i(t+1) = \alpha \Phi_i(t) + (1-\alpha) y_i^2(t) \tag{16}$$

5. Simulation Results

Five seconds of male and female speech was sampled at 8Khz and mixed using the transfer function matrix used by Torkkola [7] in his simulations.

We use the Sum of Squares of Fourth Order Normalised Marginal Cumulants (SSFONMC) (Fourth Order Normalised Marginal Cumulant, FONMC) as a measure of the separation yielded [9]. The impulse response of the ideal inverting filters is graphed along with those of the converged linear temporal network, Figure 4. A memory length of 100 sample lags was used.

We can see from Table 1 the drop in the absolute value of the FONMC of each source caused by the onset of central limit effects due to the convolutive mixing. The restored signals FONMC is within 4% of the original values. Scatter plots of the original, mixed and restored signals amplitude are given in Figure 3. The characteristic orthogonal cross shape shows clearly the independence of the original two signals; the plot of the mixed signals shows clearly the correlation now existing

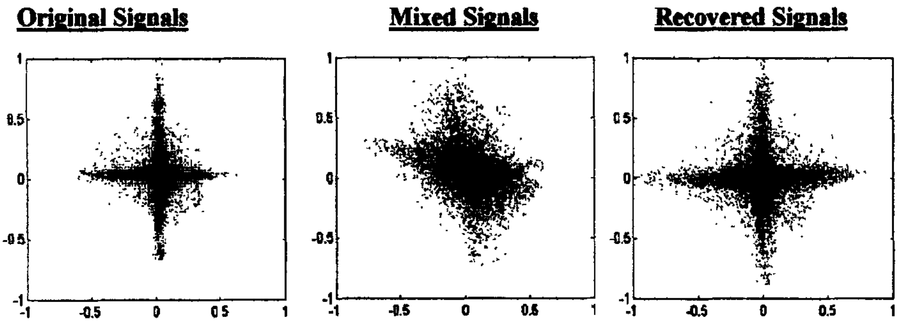

Figure 3. Signal amplitude scatter plots.

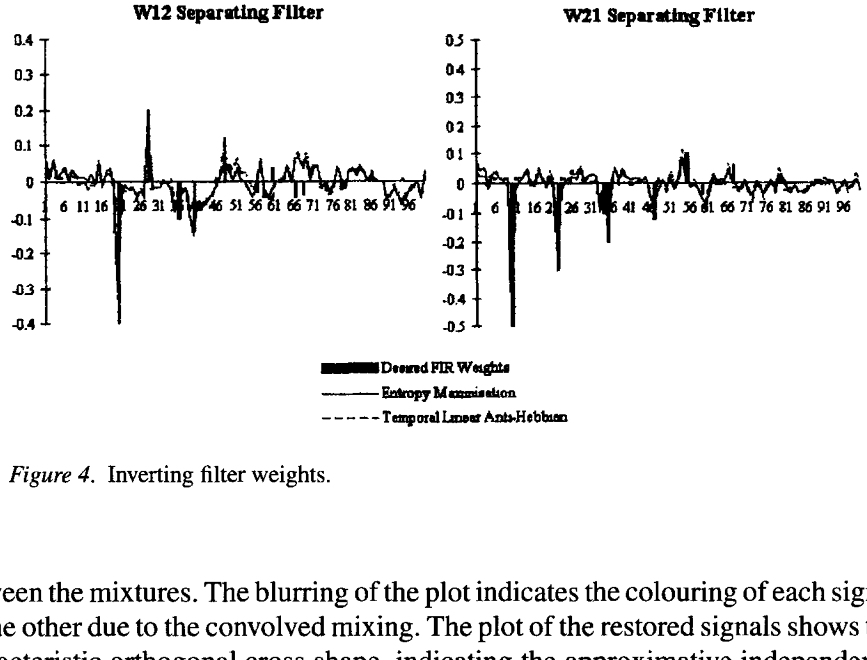

Figure 4. Inverting filter weights.

between the mixtures. The blurring of the plot indicates the colouring of each signal by the other due to the convolved mixing. The plot of the restored signals shows the characteristic orthogonal cross shape, indicating the approximative independence of the output signals.

The impulse response coefficients given by (13) are plotted in Figure 4 along with the converged network weight values using the temporal linear anti-Hebbian learning of (6) and the Entropy Maximisation algorithm developed by Torkkola [7].

The mean square value of the estimation errors caused by the use of linear anti-Hebbian and entropy maximisation algorithms were almost identical, (MSE Entropy Max = 2.28%, MSE Linear Anti-Hebb = 2.91%). The audible results were good with some echo still remaining in the output signals. Applying the learning of (16) gave slightly improved results over those given by the use of (6), though the learning parameters required carefull selection.

Figure 5. Original, convolved / mixed and retrieved voices.

We then took the same original speech sources and used a polynomial mixing matrix which introduced convolved delays up to 7.8 mS to give our mixed and convolved signals.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 - 0.45z^{-10} + 0.3z^{-15} - 0.2z^{-35} - 0.1z^{-60} & 0.5z^{-30} - 0.3z^{-53} - 0.1z^{-63} \\ 0.5z^{-15} + 0.25z^{-26} - 0.15z^{-32} - 0.1z^{-50} & 1 - 0.2z^{-25} - 0.2z^{-34} + 0.15z^{58} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$

The mixing was strong enough that both voices competed at almost the same volume. We now used the network learning of (16) and a memory length of 150 time lags, the network weights converged to very good approximations (2% MSE)

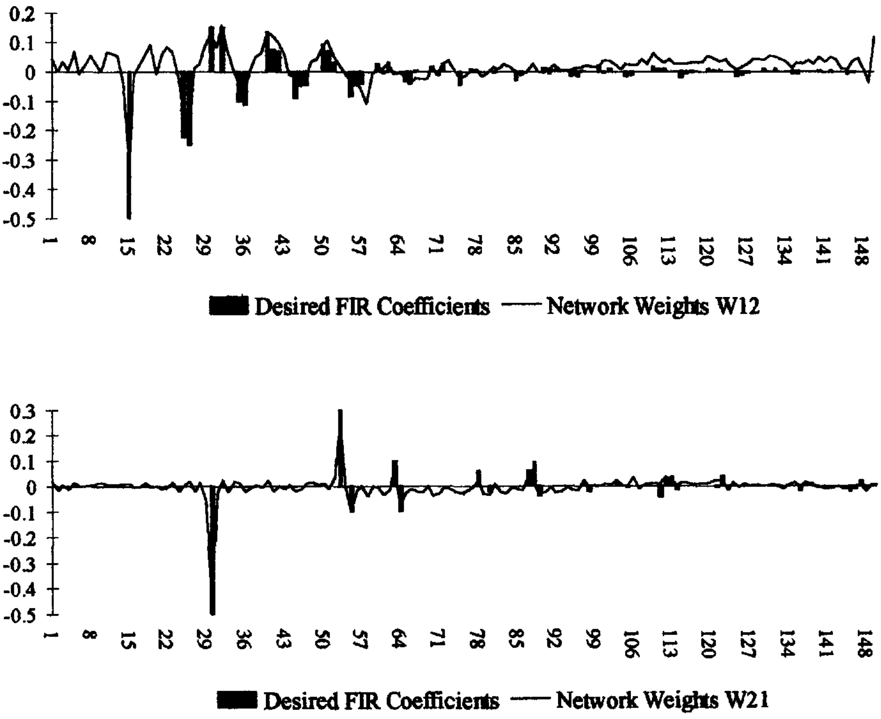

Figure 6. Converged network weights and required FIR coefficients.

of the required FIR coefficients Figure 6, audibly the unmixing was near perfect as is indicated by Figure 5.

6. Conclusions

We have proposed a temporal model of Foldiak's first linear anti-Hebbian decorrelating network. This network is similar to the filter model and network proposed in [16, 17] and is indicative of the convergence of research from the neural network and signal processing communities. By employing the linear anti-Hebbian learning (6, 16) we have shown that second order based learning is capable of identifying the inverse filter coefficients required to separate two convolved mixtures of naturally occurring speech. This is a more general case than that considered in [5, 6, 16, 17]. We have empirically compared linear anti-Hebbian learning with the maximum entropy algorithm recently developed by Torkkola and have found the performance to be remarkably similar. This finding, of course, requires to be more assiduously tested, however initial results indicate that temporal linear anti-Hebbian learning may be sufficient for adaptive blind separation of two convolved speech sources. The simulations reported have shown promising results, and so this temporal second order model will be the subject of further work and analysis in this area.

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