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TECHNICAL NOTE

Static and dynamic active earth pressure

B.M. DAS* and V.K. PURI†

**California State University, Sacramento, California, 95819, USA*
†Southern Illinois University, Carbondale, Illinois, 62901, USA

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Summary

The dynamic active earth pressure on retaining structures due to seismic loading is commonly obtained by using the modified Coulomb's approach which is known as the Mononobe–Okabe method. This method has generally been used for cohesionless soils only. A general solution for the determination of total (i.e. static and dynamic) active earth force for a c–ϕ soil as backfill was developed by Prakash and Saran in 1966 based on the simplifying assumption that adhesion between the wall–soil interface is equal to the cohesion of the soil, that the surface of the backfill is horizontal, and that the effect of the vertical acceleration can be neglected. This note presents an improved method for calculating the static and dynamic active force behind a rigid retaining wall based on its geometry, inclination of the backfill, surcharge, strength parameters of the backfill, and the adhesion between the wall face and the soil. The effects of adhesion, inclination of backfill, and vertical components of seismic loading for a typical retaining wall are discussed.

Keywords: Active pressure, dynamic, Mononobe–Okabe method, retaining structure

Introduction

The classical analysis of static active earth forces on retaining structures is generally performed using either Rankine's (1857) or Coulomb's (1773) method. These methods can be found in most textbooks (e.g. Das, 1993; Taylor, 1948). In all seismic areas, the earthquake-induced forces on the retaining walls are computed from an extension of Coulomb's sliding wedge theory in which the transient earthquake forces on the soil backfill are represented by an equivalent static force. This method is usually referred to as the Mononobe–Okabe method (Okabe, 1926; Mononobe, 1929). The Mononobe–Okabe method has been applied to cohesionless soils (Das, 1992). A solution for determination of total (static plus dynamic) active earth pressure with a c–ϕ soil as backfill was developed by Prakash and Saran (1966) and Saran and Prakash (1968). The nature of the problem

*To whom correspondence should be addressed

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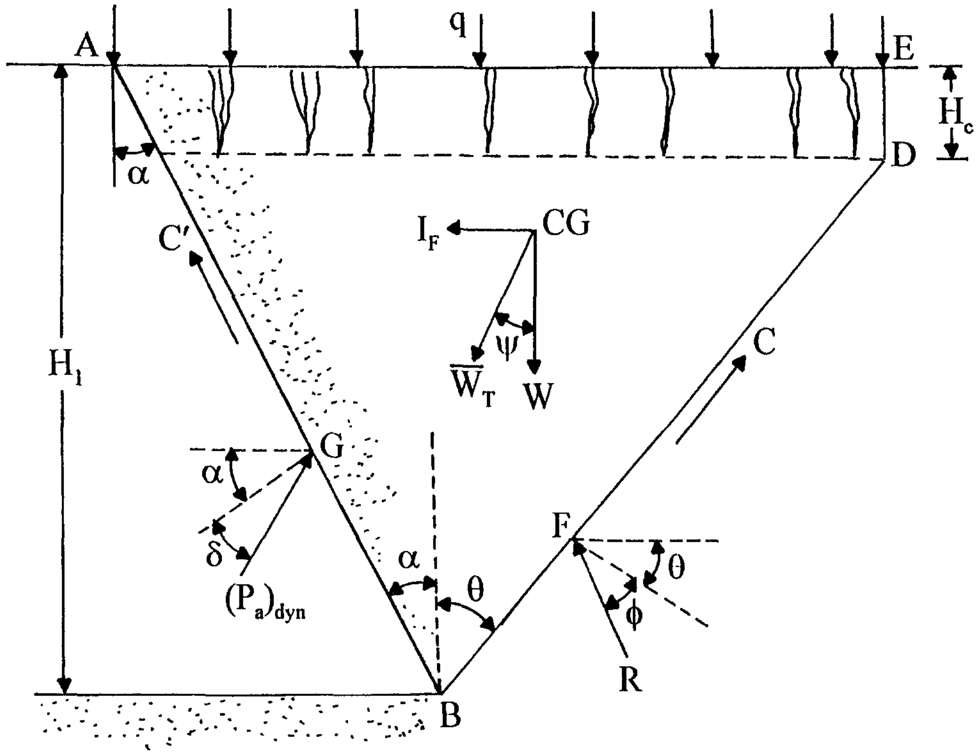

Fig. 1. Forces acting on a wall retaining c–ϕ soil and subjected to an earthquake-type load

considered by Prakash and Saran (1966) is shown in Fig. 1, in which, ϕ = angle of friction of backfill soil, θ = inclination of the assumed failure wedge with the vertical, I_F = horizontal inertia force given by $\alpha_H W$, α_h = horizontal seismic coefficient, W = weight of assumed wedge ABDE, C = cohesive force along BD, C' = adhesive force along AB, q = surcharge per unit area on the surface of the backfill, ψ = inclination of the resultant of weight W and inertia force I_F with the vertical, and δ = inclination of the dynamic active force with the normal drawn to the back face of the wall. The wall face AB in contact with the soil is vertically inclined at an angle α. The soil retained is horizontal. If the depth of the tension crack is H_c, then

$$H_c = n(H_1 - H_c) = nH \tag{1}$$

in which H_1 is the height of the retaining wall, and

$$H = H_1 - H_c$$

In this analysis, only the horizontal inertia force is considered. From a consideration of equilibrium of forces on the assumed failure wedge and simplification of the resulting mathematical expressions, Prakash and Saran (1966) obtained the following equation for determination of the total active earth force

$$(P_{\mathrm{a}})_{\mathrm{dyn}} = \gamma H^2 (N_{\mathrm{a}\gamma})_{\mathrm{dyn}} + qH(N_{\mathrm{aq}})_{\mathrm{dyn}} - cH(N_{\mathrm{ac}})_{\mathrm{dyn}} \tag{2}$$

In which $(N_{\mathrm{ac}})_{\mathrm{dyn}}$, $(N_{\mathrm{aq}})_{\mathrm{dyn}}$ and $(N_{\mathrm{a}\gamma})_{\mathrm{dyn}}$ are earth pressure coefficients dependent on α, n, ϕ, δ and θ, and are given by Equations 3, 4 and 5, respectively.

$$(N_{\mathrm{ac}})_{\mathrm{dyn}} = \frac{\cos\beta \sec\alpha + \cos\phi \sec\theta}{\sin(\beta + \delta)} \tag{3}$$

$$(N_{\mathrm{aq}})_{\mathrm{dyn}} = \frac{[(n+1)\tan\alpha + \tan\theta][\cos(\theta+\phi) + \alpha_{\mathrm{h}}\sin(\theta+\phi)]}{\sin(\beta+\delta)} \tag{4}$$

$$(N_{\mathrm{a}\gamma})_{\mathrm{dyn}} = \frac{\left[(n+\frac{1}{2})\tan\alpha + \tan\theta + n^2\tan\alpha\right]\left[\cos(\theta+\phi) + \alpha_{\mathrm{h}}\sin(\theta+\phi)\right]}{\sin(\beta+\delta)} \tag{5}$$

In arriving at these equations, the unit adhesion c' between the wall and the backfill was assumed to be equal to the unit cohesion of the soil, c. The values of $(N_{\mathrm{ac}})_{\mathrm{dyn}}$, $(N_{\mathrm{aq}})_{\mathrm{dyn}}$ and $(N_{\mathrm{a}\gamma})_{\mathrm{dyn}}$ were separately maximized to determine the maximum value of $(P_{\mathrm{a}})_{\mathrm{dyn}}$ in Equation 2. The total static earth force may be obtained from the equations by substituting $\alpha_{\mathrm{h}} = 0$. Note that the right-hand side of Equation 3 does not contain α_{h} and, therefore, the values of N_{ac} will be the same for both the static and dynamic cases.

The approach by Prakash and Saran (1966) provides a convenient method for determining the static and dynamic lateral earth force for a typical soil; however, it has the following limitations:

(1) the effect of the vertical component of the dynamic acceleration (α_{v}) or the vertical seismic coefficient (which may be significant under some conditions) has been neglected;

(2) the backfill surface is assumed to be horizontal, which may actually be inclined in many cases;

(3) the unit adhesion between the back face of the retaining wall and the soil is assumed to be equal to the unit cohesion of the soil which may not be so in a practical case.

This paper presents a method for calculation of the dynamic active earth force for a typical c–ϕ soil backfill behind a retaining wall which includes the effects of the following parameters (Fig. 2): (1) the effect of cohesion, c, and adhesion, c'; (2) the inclination of the backfill, i; (3) horizontal and vertical seismic coefficients, α_{h} and α_{v}, respectively; (4) surcharge, q; (5) inclination of the wall face, α; (6) depth of tension cracks, H_{c}.

Coulomb's approach has been modified to account for the effect of these parameters. The value of total dynamic active force (static active force plus the dynamic increment) have been calculated for some typical cases.

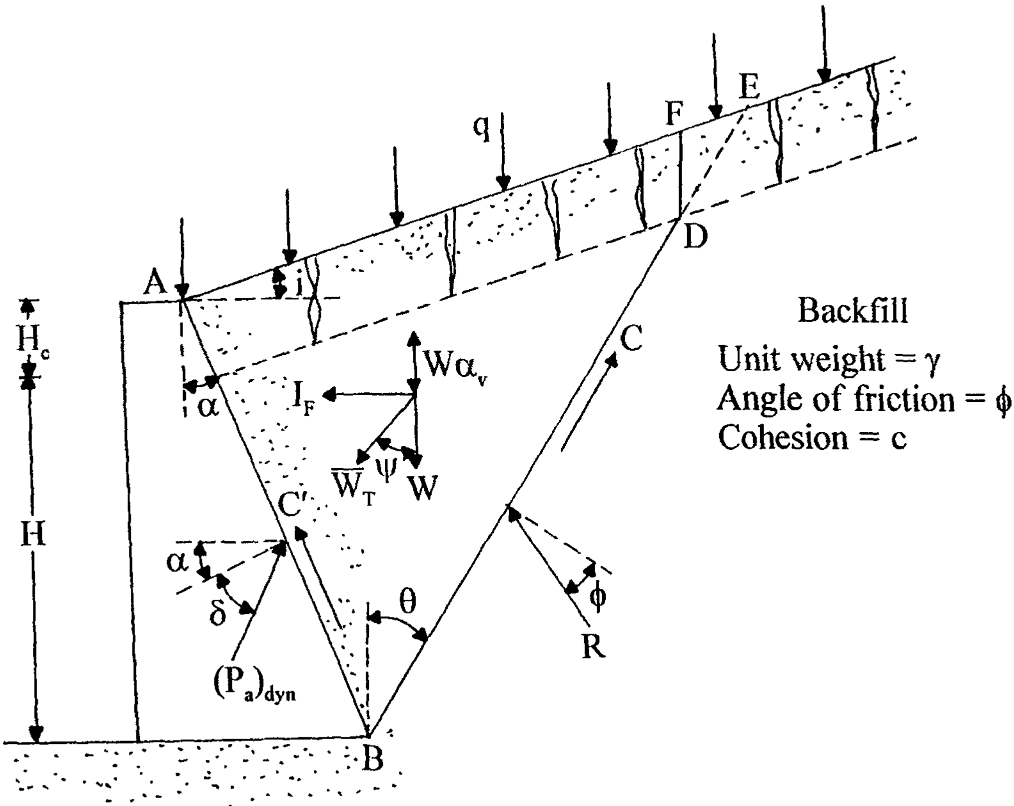

Fig. 2. Typical assumed failure wedge with forces acting on it

Mathematical model

A schematic diagram of the dynamic active earth force problem considered in this paper is shown in Fig. 2. ABDF is an assumed failure wedge. Considering the unit length of the wall, $(P_a)_{dyn}$ = total active force, R = soil reaction, I_F = horizontal inertia force, W = weight of assumed failure wedge, $\overline{W}_T$ = resultant of weight W and I_F, $W\alpha_v$ = inertia force due to the vertical component of earthquake acceleration, C = cohesive force, C' = adhesive force, q = surcharge, α = inclination of wall face with the vertical ($\alpha \geq 0$ only is considered), and i = inclination of the backfill ($0 \leq i \leq \phi$ only is considered).

The relationship for the forces on the wedge under consideration is as follows

$$W = \frac{1}{2}\gamma H^2 \left\{ \tan\,\alpha + \tan\,\theta + \frac{n}{\cos\,i\,\cos(\theta+i)}[(2+n)\tan\,\alpha\cos\,\theta + 2\,\sin\,\theta] + \frac{\sin^2(\alpha+\theta)\sin i}{\cos^2\alpha\cos\theta\cos(\theta+i)} \right\} \quad (6)$$

in which

$$n = \frac{H_c}{H} \tag{7}$$

$$H_c = \text{depth of tensile cracks} = \frac{2c}{\gamma}\sqrt{K_a} \tag{8}$$

in which K_a = Rankine's active earth pressure coefficient and γ = unit weight of soil

$$H_1 = H + H_c \tag{9}$$

in which H_1 = total height of the retaining wall.

$$\text{Surcharge, } Q = \frac{qH}{\cos(\theta + i)}\left[\frac{\sin(\alpha + \theta)}{\cos\alpha} + n \tan\alpha \cos\theta\right] \tag{10}$$

$$\text{Cohesive force, } C = c\frac{H}{\cos\alpha}\frac{\cos(\alpha - i)}{\cos(\theta + i)} \tag{11}$$

$$\text{Adhesive force, } C' = e\ c\ \frac{H}{\cos\alpha} \tag{12}$$

in which

$$e = \frac{c'}{c}$$

where c' = unit adhesion between the back face of the wall and the backfill and c = unit soil cohesion.

The reaction, R, cancels from the final expression and its magnitude need not be calculated. However the direction of R is known.

$$\text{Horizontal inertia force, } I_F = (W + Q)\alpha_h \tag{13}$$

$$\text{Vertical inertia force, } I_{FV} = (W + Q)(1 \pm \alpha_v) \tag{14}$$

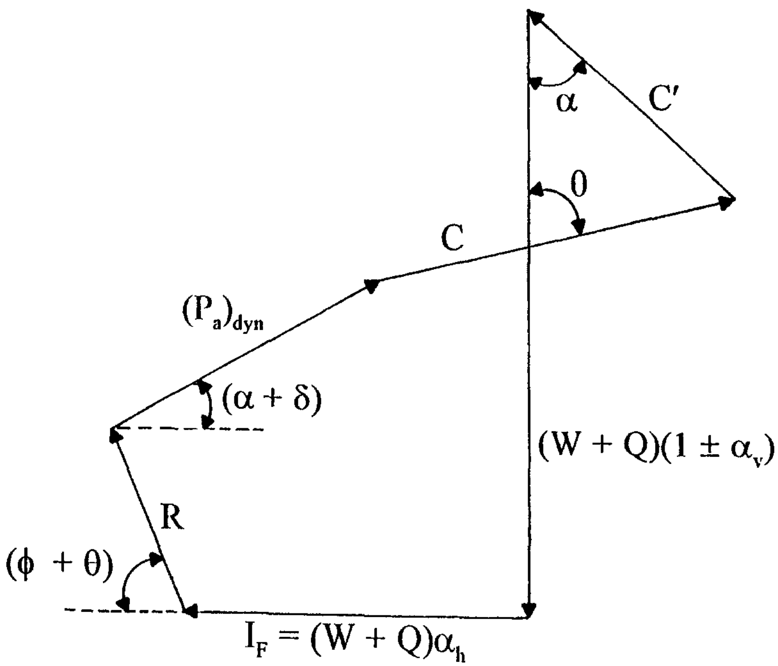

Fig. 3. Force polygon

The polygon of forces acting on the wedge may then be drawn as shown in Fig. 3. Applying the conditions for static equilibrium, namely $\Sigma F_{\text{horizontal}} = 0$ and $\Sigma F_{\text{vertical}} = 0$, one obtains

$$(P_a)_{dyn}\cos(\alpha+\delta) - R\cos(\phi+\theta) - (W+Q)\alpha_h$$
$$+C\sin\theta - C'\sin\alpha = 0 \tag{15}$$

$$(P_a)_{dyn}\sin(\alpha+\delta) + R\sin(\phi+\theta) + C\sin\theta + C'\cos\alpha$$
$$-(W+Q)(1\pm\alpha_v) = 0 \tag{16}$$

Multiplying Equation 15 by $\sin(\phi+\theta)$ and Equation 16 by $\cos(\phi+\theta)$ and simplifying, a relationship for $(P_a)_{dyn}$ can be obtained, or

$$(P_a)_{dyn} = \frac{1}{2}\gamma H^2 (N_{a\gamma})_{dyn} + qH(N_{aq})_{dyn} - cH(N_{ac})_{dyn} \tag{17}$$

in which

$$(N_{ac})_{dyn} = \left[\frac{\cos(\alpha - i)\cos\phi}{\cos\alpha\cos(\theta + i)} + \frac{e\ \cos(\alpha + \phi + \theta)}{\cos\alpha}\right] \times \left[\frac{1}{\sin(\alpha + \delta + \phi + \theta)}\right] \tag{18}$$

$$(N_{aq})_{dyn} = \left[\frac{\sin(\alpha + \theta)}{\cos\alpha} + n\tan\alpha\cos\theta\right] \times \left[\frac{\alpha_h \sin(\phi + \theta) + (1 \pm \alpha_v)\cos(\phi + \theta)}{\sin(\alpha + \delta + \phi + \theta)\cos(\theta + i)}\right] \tag{19}$$

and

$$(N_{a\gamma})_{dyn} = \frac{1}{\sin(\alpha + \delta + \phi + \theta)}\left\{\tan\alpha + \tan\theta + \frac{n}{\cos i\cos(\theta + i)} \times [(2 + n)\tan\alpha\cos\theta + 2\sin\theta] + \frac{\sin^2(\alpha + \theta)\sin i}{\cos^2\alpha\cos\theta\cos(\theta + i)}\right\} \times [\alpha_h \sin(\phi + \theta) + (1 \pm \alpha_v)\cos(\phi + \theta)] \tag{20}$$

It may be noted that the right-hand side of Equation 18 does not contain α_h and α_v and therefore the value of N_{ac} will be the same for static and dynamic cases. It must however be recognized that the value of N_{ac} does depend upon the orientation of the failure wedge (defined by angle θ) and should be expected to vary if the failure wedge in the static and dynamic cases is different. What is implied here is the fact that the orientation of the failure wedge giving maximum values of $(P_a)_{dyn}$ and $(P_a)_{stat}$ may be different and, because of this, the values of N_{ac} may vary somewhat.

The static active earth pressure $(P_a)_{stat}$ may be obtained as follows

$$(P_a)_{stat} = \frac{1}{2}\gamma\ H^2(N_{a\gamma})_{stat} + qH(N_{aq})_{stat} - cH(N_{ac})_{stat} \tag{21}$$

in which

$$(N_{ac})_{stat} = (N_{ac})_{dyn} \tag{22}$$

Relationships for $(N_{aq})_{stat}$ and $(N_{a\gamma})_{dyn}$ may be obtained from Equations 19 and 20, respectively, by substituting $\alpha_h = 0$ and $\alpha_v = 0$. Thus

$$(N_{\mathrm{aq}})_{\mathrm{stat}} = \left[\frac{\sin(\alpha+\theta)}{\cos\alpha} + n\tan\alpha\cos\theta\right] \times \left[\frac{\cos(\phi+\theta)}{\sin(\alpha+\delta+\phi+\theta)\cos(\theta+i)}\right] \tag{23}$$

$$(N_{\mathrm{a}\gamma})_{\mathrm{stat}} = \frac{1}{\sin(\alpha+\delta+\phi+\theta)}\left\{\tan\alpha + \tan\theta + \frac{n}{\cos i\cos(\theta+i)} \times [(2+n)\tan\alpha\cos\theta + 2\sin\theta] + \frac{\sin^2(\alpha+\theta)\sin i}{\cos^2\alpha\cos\theta\cos(\theta+i)}\right\} \times [\cos(\phi+\theta)] \tag{24}$$

The values of $(P_{\mathrm{a}})_{\mathrm{dyn}}$ and $(P_{\mathrm{a}})_{\mathrm{stat}}$ obtained from Equations 17 and 21 respectively are for a given assumed failure wedge. In order to obtain the maximum values of the total dynamic earth force, $(P_{\mathrm{a}})_{\mathrm{dyn}}$, the earth pressure coefficients $(N_{\mathrm{ac}})_{\mathrm{dyn}}$, $(N_{\mathrm{a}\gamma})_{\mathrm{dyn}}$ and $(N_{\mathrm{aq}})_{\mathrm{dyn}}$ were optimized. A computer code was developed for this purpose. It must be mentioned here that these earth force coefficients were individually optimized and then $(P_{\mathrm{a}})_{\mathrm{dyn}}$ was obtained by superimposing their effects, that is using Equation 17. The same procedure was followed for the maximum value of static force, $(P_{\mathrm{a}})_{\mathrm{stat}}$. From known values of $(P_{\mathrm{a}})_{\mathrm{dyn}}$ and $(P_{\mathrm{a}})_{\mathrm{stat}}$, the dynamic increment $(\Delta P_{\mathrm{a}})_{\mathrm{dyn}}$ can be obtained as

$$(\Delta P_{\mathrm{a}})_{\mathrm{dyn}} = (P_{\mathrm{a}})_{\mathrm{dyn}} - (P_{\mathrm{a}})_{\mathrm{stat}} \tag{25}$$

Effects of various parameters on dynamic earth pressure

Using the procedure developed in the preceding section, calculations can be made for specific cases to show the effect of parameters such as $e = c'/c$, i and α_{v} on the dynamic active earth force on retaining walls. These factors have not been addressed in the published studies presently available for a typical c–ϕ type soil.

Effect of e

Figures 4–6 show the plots of $(\Delta P_{\mathrm{a}})_{\mathrm{dyn-e}}/(\Delta P_{\mathrm{a}})_{\mathrm{dyn-e=0}}$ for a retaining wall with $H = 10$ m, $\alpha = 10°$, $i = 0$ and $q = 0$. The constant properties of the backfill are $\phi = 30°$, $\delta = 2\phi/3$, and $\gamma = 18$ kN/m^3. The cohesion of the backfill was varied as 10 kN/m^2 (Fig. 4), 20 kN/m^2 (Fig. 5) and 30 kN/m^2 (Fig. 6). It was also assumed that $\alpha_{\mathrm{h}} = 0.2$ and $\alpha_{\mathrm{v}} = 0$. It can be

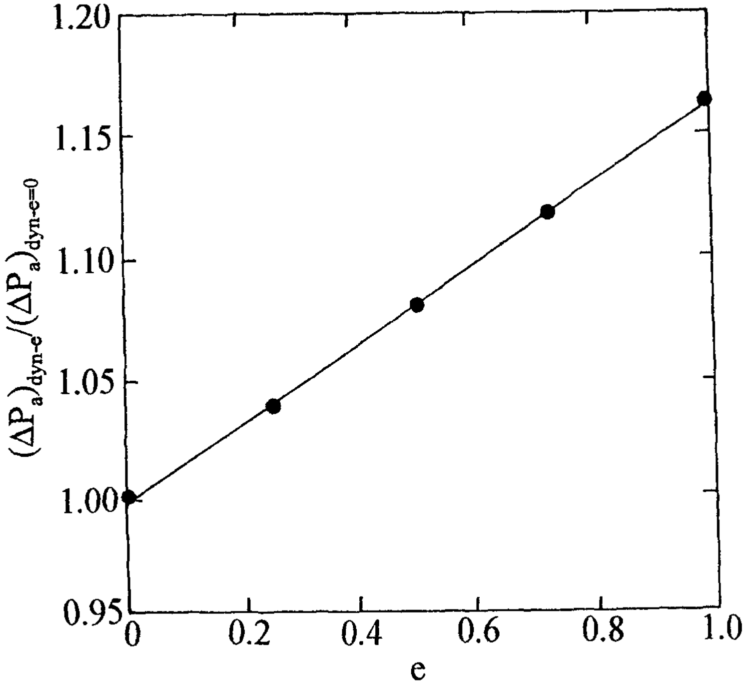

Fig. 4. Normalized dynamic load increment $(\Delta P_a)_{dyn-e}/(\Delta P_a)_{dyn-e=0}$ versus ratio of adhesion to cohesion e for c = 10 kN/m^2. H = 10 m, α = 10°, i = 0, q = 0, ϕ = 30°, $\delta = 2\phi/3$, γ = 18 kN/m^3, α_h = 0.2 and α_v = 0

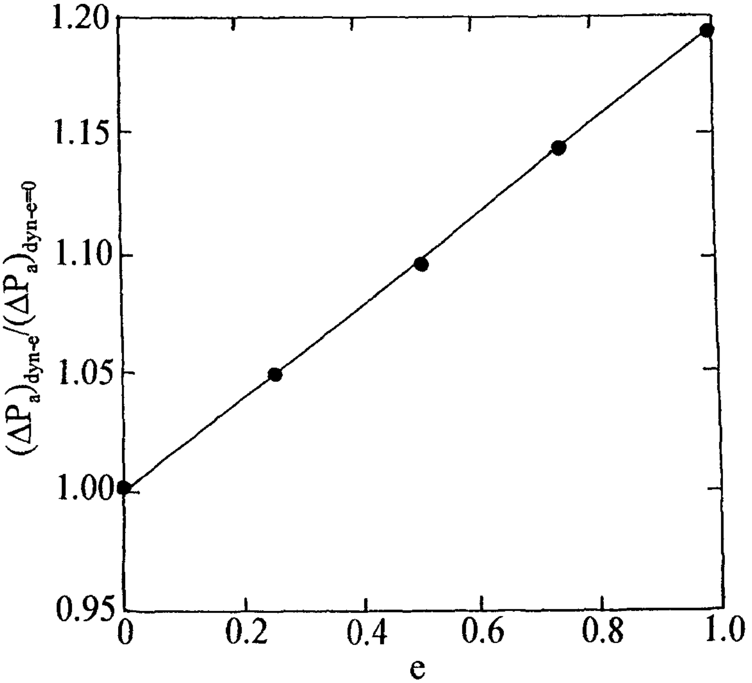

Fig. 5. Normalized dynamic load increment $(\Delta P_a)_{dyn-e}/(\Delta P_a)_{dyn-e=0}$ versus ratio of adhesion to cohesion e for c = 20 kN/m^2. Other parameters as Fig. 4

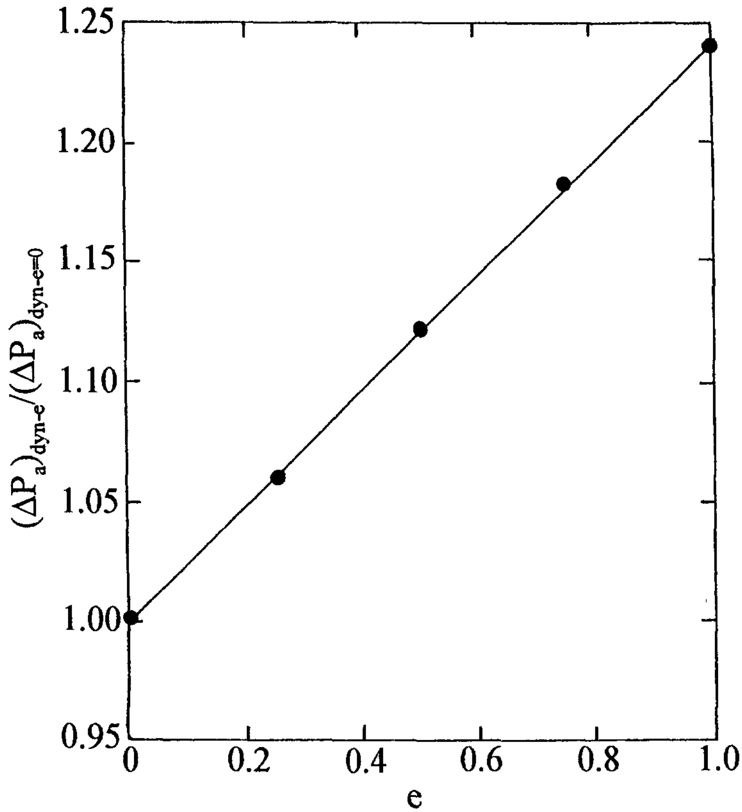

Fig. 6. Normalized dynamic load increment $(\Delta P_a)_{dyn-e}/(\Delta P_a)_{dyn-e=0}$ versus ratio of adhesion to cohesion e or c = 30 kN/m^2. Other parameters as Fig. 4

seen from these plots that, for any given value of c, the magnitude of $(\Delta P_a)_{dyn-e}/(\Delta P_a)_{dyn-e=0}$ increases with an increase in e. Hence it is obvious that an assumption of $e = 1$ leads to somewhat conservative values of dynamic earth force increment.

Effect of the inclination of backfill, i

The effects of the inclination on the dynamic active force are shown in Figs 7 and 8. In obtaining these plots, the following constant parameters were assumed: $H = 10$ m, $\alpha = 10°$, $\phi = 30°$, $\delta = 2\phi/3$, $\gamma = 18$ kN/m^3, $\alpha_h = 0.2$ and $\alpha_v = 0$. In Fig. 7 the magnitudes of $q = 50$ kN/m^2, $c = 0$ and the angle i was varied from zero to 15°. In a similar manner, in Fig. 8 the magnitudes of $q = 0$, $c = 20$ kN/m^2 and i were varied from zero to 15°. These plots show that the value of $(\Delta P_a)_{dyn-i}/(\Delta P_a)_{dyn-i=0}$ increases with the increase in the magnitude of i. This is primarily due to the fact that, for a given retaining wall, an increase in the positive value of i increases the weight of the failure wedge and it generates higher dynamic force increments.

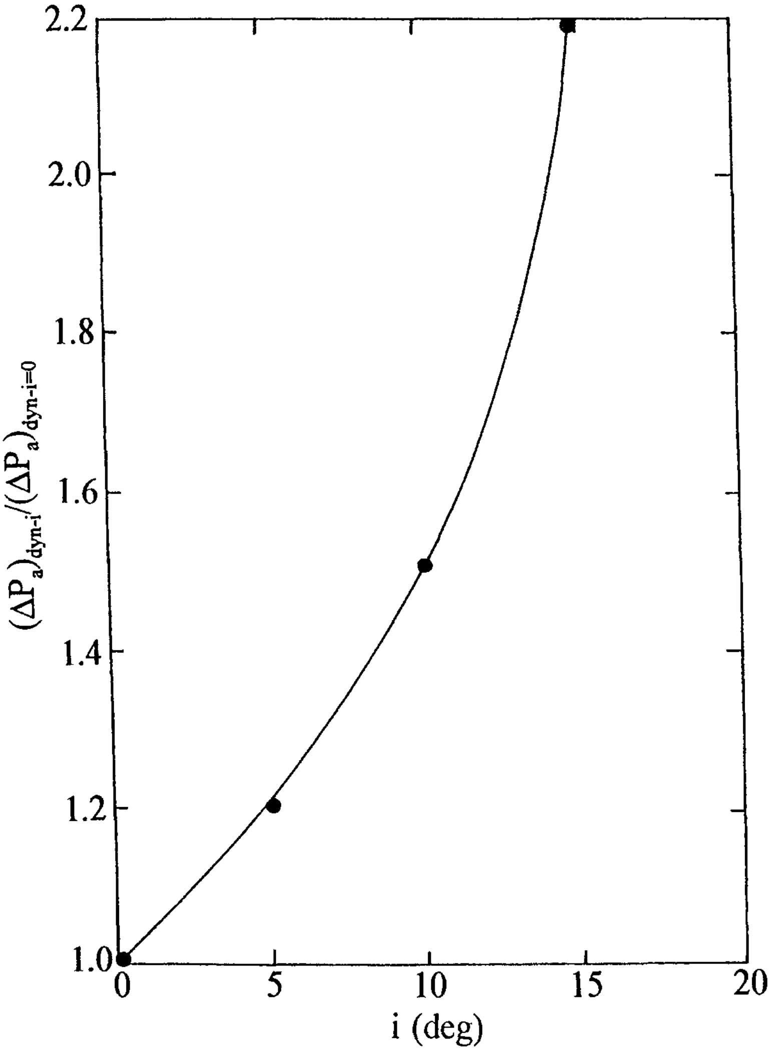

Fig. 7. Normalized dynamic load increment $(\Delta P_a)_{dyn-i}/(\Delta P_a)_{dyn-i=0}$ versus backfill inclination i for q = 50 kN/m^2 and $c = 0$. H = 10° m, α = 10°, ϕ = 30°, $\delta = 2\phi/3$, γ = 18 kN/m^3, α_h = 0.2 and α_v = 0

Effect of vertical seismic coefficient, α_v

Figure 9 shows plots of $(\Delta P_a)_{dyn-\alpha_v}/(\Delta P_a)_{dyn-\alpha_v=0}$ against α_v/α_h. In developing these plots, it was assumed that H = 10 m, $c = 0$, α = 10°, $q = 0$, ϕ = 30°, γ = 18 kN/m^3, δ = $2\phi/3$ and $i = 0$. From these plots it can be seen that the dynamic force increment depends on the magnitude of α_v/α_h for $\alpha_h < 0.5$. When α_h is small, the dynamic force increment increases with the increase in α_v; however, for $\alpha_h \geq 0.5$ the magnitude of α_v has an insignificant effect.

The fact that the vertical seismic coefficient α_v can have a significant effect (when α_h is small) is in agreement with the conclusion reached by Richards and Elms (1979).

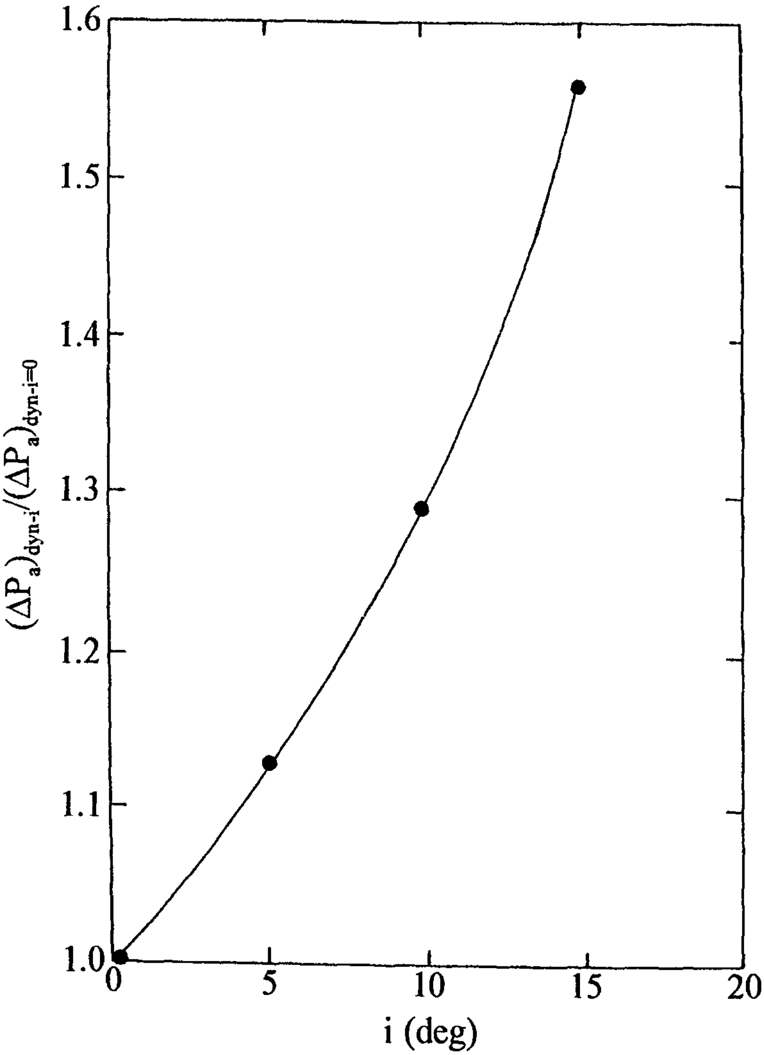

Fig. 8. Normalized dynamic load increment $(\Delta P_a)_{dyn-e}/(\Delta P_a)_{dyn-e=0}$ versus backfill inclination i for $q = 0$ and $c = 20$ kN/m^2. Other parameters as Fig. 7

Conclusions

A procedure has been developed to estimate the magnitude of the static and dynamic active force on a retaining wall with a c–ϕ soil as backfill. This is an improvement over the existing methods available in the literature. Based on the present analysis, the following general conclusions can be drawn:

(1) The assumption that cohesion is equal to adhesion leads to a conservative assumption of the dynamic active force;

(2) For a given retaining wall, the increase in the slope of the backfill leads to an

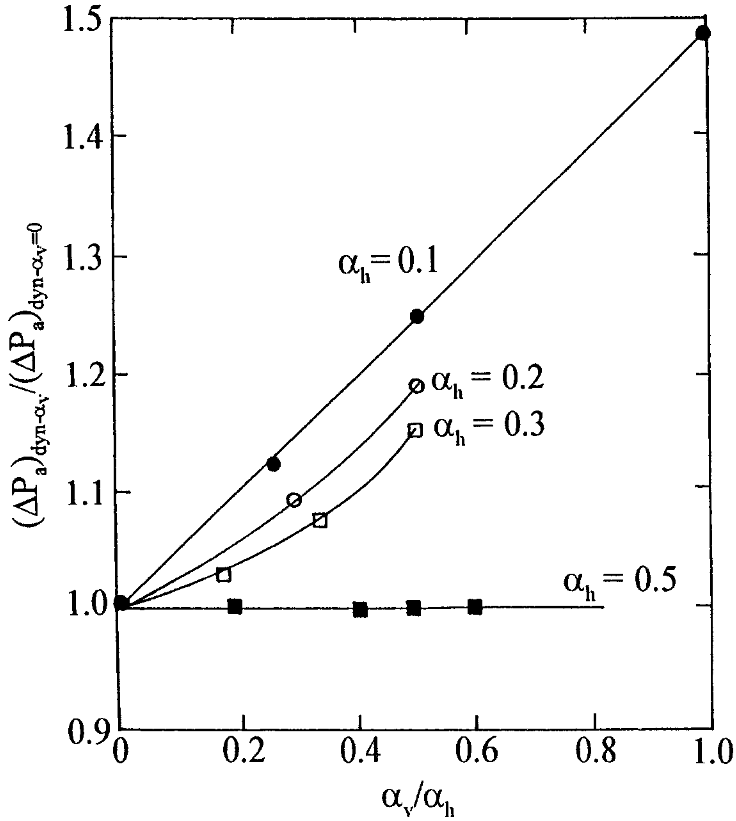

Fig. 9. Normalized dynamic load increment $(\Delta P_a)_{dyn-\alpha_v}/(\Delta P_a)_{dyn-\alpha_v=0}$ versus vertical to horizontal seismic coefficients α_v/α_h. $H = 10$ m, $\alpha = 10°$, $\imath = 0$, $q = 0$, $\phi = 30°$, $\delta = 2\phi/3$, $\gamma = 18$ kN/m^3 and $c = 0$

increase in the dynamic active force;

(3) In some cases (when the horizontal seismic coefficient is small) the vertical seismic coefficient α_v can have a significant effect on the dynamic active force.

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