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TECHNICAL NOTE

Plane failure analysis of rock slopes

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Summary

Hoek and Bray (1981) gave an analytical approach for plane failure analysis for rock slopes that is limited to those slopes in which the upper slope surface is horizontal and the tension crack is vertical. An analysis is presented here which can take these factors into account. It is found that varying the angle of the upper slope from 0° to 30° causes a significant reduction in the factor of safety. Varying the tension crack from vertical to 70° only has an effect when the upper slope angle is less than 20°.

Keywords: Rock slope stability; plane failure; upper slope; tension crack; factor of safety.

Introduction

Plane failures in rock slopes occur when a geological discontinuity strikes parallel or nearly parallel to the slope face and dips at an angle greater than the angle of internal friction. Hoek and Bray (1981) gave an analytical solution for the plane failure mode in rock slopes. In this analysis, they assumed that the upper slope surface is horizontal and the tension crack is vertical. However, this method does not account for those rock slopes in which the upper slope surface and tension crack are inclined. In the present paper an attempt has been made to modify their approach.

Geometry of the slope

The geometry of the slope considered in the present analysis is defined in Fig. 1. The various symbols used in this figure are: α_f, slope face angle; α_s, upper slope surface angle; α_p, dip of potential failure plane; α_t, angle of tension crack; h, height of slope; Z_L, height of tension crack; W, weight of the sliding block; U, uplift water force acting on the block; V, water force in the tension crack, acting on the rear face of the block.

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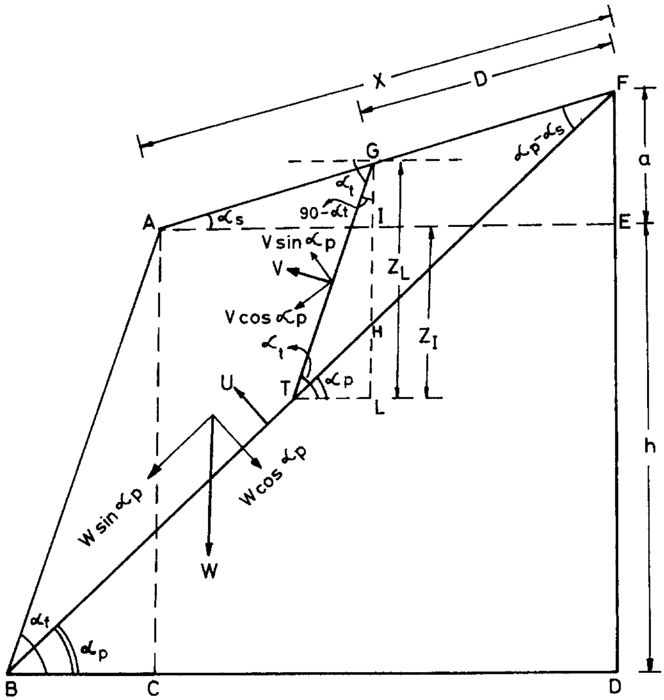

Fig. 1. Geometry of the slope considered in the analysis

Plane failure analysis for inclined upper slope surface and tension crack

In the present analysis, the general conditions as assumed by Hoek and Bray (1981) remain the same for plane failure except that the upper slope surface and tension crack are inclined. For the present analysis the following general conditions must be satisfied:

(1) The failure plane must strike parallel or nearly parallel (approximately ± 20°) to the slope face;

(2) The dip of the failure plane must be smaller than the dip of the slope face ($\alpha_p < \alpha_f$);

(3) The angle of internal friction (ϕ) of the failure plane must be smaller than the dip of the failure plane ($\phi < \alpha_p$);

(4) The upper slope surface and the tension crack must be inclined;

(5) The dip of the upper slope must be smaller than the dip of the failure plane ($\alpha_s < \alpha_p$);

(6) The tension crack must be present on the upper slope surface.

The following assumptions are made in this analysis:

(1) The tension crack is filled with water to a vertical depth of Z_w. The water from the tension crack seeps along the failure surface and escapes out on the slope face where the failure surface daylights.

(2) It is presumed that there is no resistance to sliding at the lateral boundaries of the slide.

Area of the sliding block

The area of the sliding surface, A, is represented by the length of the surface visible in a cross section drawn through the slope. Hence, from Fig. 1:

$$\text{Area}, A = (h - Z_I) * \operatorname{cosec} \alpha_p \tag{1}$$

$$b_C = \text{AI} = h[\sqrt{}(\cot \alpha_f * \cot \alpha_p) - \cot \alpha_f] \tag{2}$$

$$Z_C = \text{IH} = h[1 - \sqrt{}(\cot \alpha_f \ * \ \tan \alpha_p)] \tag{3}$$

These equations are equivalent to those given by Hoek and Bray (1981).

Also, from Fig. 1

$$\text{AG} = b_C / \cos \alpha_s \tag{4}$$

$$\text{IG} = b_C / \cot \alpha_s \tag{5}$$

Substituting the value of b_C from Equation (2) into Equation (5):

$$\text{IG} = h[\sqrt{}(\cot \alpha_f \ * \ \cot \alpha_p) - \cot \alpha_f] / \cot \alpha_s \tag{6}$$

If we denote Z = GH = IG + IH, from Equations 3 and 6:

$$Z = h[(1 - \cot \alpha_f / \cot \alpha_s) + \sqrt{}\cot \alpha_f / \sqrt{}\cot \alpha_p \ * \ (\cot \alpha_p / \cot \alpha_s - 1)] \tag{7}$$

Again, from Fig. 1

$$Z_L = \text{GL} = Z \sin \alpha_t / (\sin \alpha_t - \tan \alpha_p \ * \ \cos \alpha_p) \tag{8}$$

$$Z_I = \text{IL} = \text{GL} - \text{IG}$$

Thus, from Equations (6) and (8):

$$Z_t = \text{IL} = \frac{Z \sin \alpha \text{I}}{\sin \alpha_t - \tan \alpha_p \ * \ \cos \alpha_t} - b_c \ * \ \tan \alpha_s \tag{9}$$

By substituting the value of Z_I from Equation (9) into Equation 1, the area of the sliding surface can be calculated.

Weight of the sliding block

The weight of the sliding block is calculated from

$$\text{W} = \frac{1}{2} \gamma [(h + a)X - DZ_L] \tag{10}$$

where γ is the unit weight of the rock, X and D are the slope distances AF and GF respectively and a is the height EF as shown in Fig. 1.

$$X = \frac{h \cot \alpha_f}{\cos \alpha_s} * \frac{\tan \alpha_p - \tan \alpha_f}{\tan \alpha_s - \tan \alpha_p} \tag{11}$$

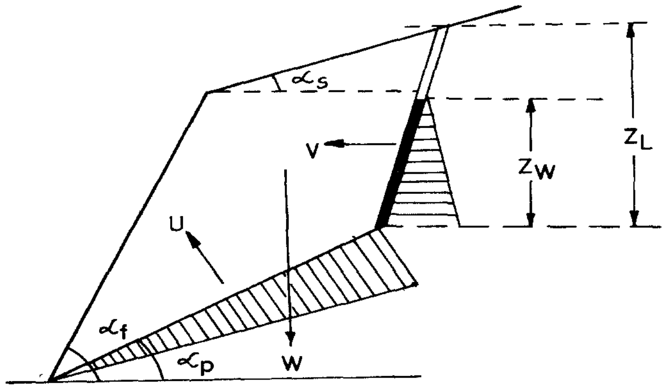

Fig. 2. Water pressure distribution along the tension crack and the base of the sliding block

$$D = \frac{Z}{\tan \alpha_p * \cos \alpha_s - \sin \alpha_s} \tag{12}$$

and

$$a = h\frac{\tan \alpha_S}{\tan \alpha_f} * \frac{\tan \alpha_p - \tan \alpha_f}{\tan \alpha_S - \tan \alpha_p} \tag{13}$$

Horizontal water force

The horizontal force, V, due to water pressure in the tension crack, acting on the rear face of the block (as shown in Fig. 2) is derived from

$$V = \frac{1}{2}\gamma_w Z_w^2 \sin^2 \alpha_t \tag{14}$$

where, γ_w is the unit weight of water and Z_w is the height of the water column in the tension crack.

Uplift water force

The water seeps through the tension crack into the sliding surface and results in an uplift force, U, as shown in Fig. 2. This uplift force is calculated from

$$U = \frac{1}{2} \gamma_w Z_w \sin \alpha_t (h - Z_I) * \operatorname{cosec} \alpha_p \tag{15}$$

Thus, substituting the value of Z_I from Equation 9, the uplift water force acting on the sliding block can be computed.

Factor of safety

The factor of safety of the slope considered in this analysis, can be derived from the equation where c is the cohesive component of strength of the failure plane.

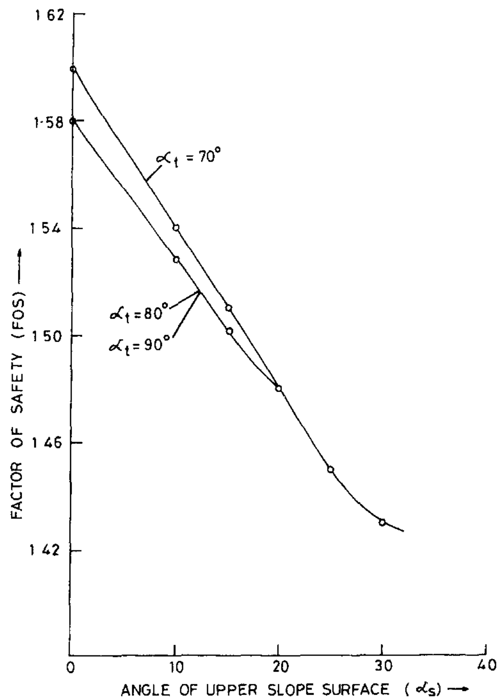

Fig. 3. Effect of upper slope angle on factor of safety

$$F = \frac{cA + (W \cos \alpha_p - U - V \sin \alpha_p) \tan \phi}{W \sin \alpha_p + V \cos \alpha_p} \tag{16}$$

Effect of inclined upper slope surface and tension crack on factor of safety

To examine the effect of the inclined upper slope surface and tension crack on the factor of safety, a hypothetical example was considered. The data for this example is as follows:

Slope face angle (α_f) = 50°
Upper slope surface angle (α_S) = 0° to 30°
Dip of potential failure plane (α_p) = 35°
Angle of tension crack (α_t) = 90° to 70°
Height of slope (h) = 60 m
Cohesion of rock mass (c) = 120 kN m^{-2}
Angle of internal friction (ϕ) = 45°
Unit weight of the rock (γ) = 26 kN m^{-3}
Unit weight of water (γ_w) = 10 kN m^{-3}
Height of water column in the tension crack (Z_w) = 14 m

Table 1. Stability analysis of hypothetical slope at varying upper slope angles

Case No.	Upper slope angle (α_S)	Area (A) M^2	Weight (W) kN	Horizontal water force (V) kN	Uplift water force (U) kN	Factor of safety (FOS)
For an angle of tension crack (α_t) at 70°						
1	0°	71.845	2267.68	86.472	472.59	1.60
2	10°	70.243	3317.43	86.472	462.05	1.54
3	15°	69.382	4433.85	86.472	456.38	1.51
4	20°	68.487	6715.23	86.472	450.50	1.48
5	25°	67.565	12998.24	86.472	444.43	1.45
6	30°	66.505	71425.55	86.472	335.97	1.43
For an angle of tension crack (α_t) at 80°						
1	0°	76.72	2340.37	95.044	528.83	1.58
2	10°	76.09	3456.77	95.044	524.53	1.53
3	15°	75.73	4636.49	95.044	522.00	1.50
4	20°	75.38	7032.68	95.044	519.67	1.48
5	25°	74.99	13465.16	95.044	516.90	1.45
6	30°	74.59	46627.40	95.044	514.14	1.43
For vertical tension crack (α_t) = 90°						
1	0°	80.191	2392.03	98.000	561.267	1.58
2	10°	80.191	3558.34	98.000	561.267	1.53
3	15°	80.191	4785.03	98.000	561.267	1.50
4	20°	80.191	7254.02	98.000	561.267	1.48
5	25°	80.191	13932.64	98.000	561.267	1.45
6	30°	80.191	47526.01	98.000	561.267	1.43

The results are presented in Table 1. A plot of factor of safety against the angle of the upper slope (α_S) is given in Fig. 3. This indicates that the factor of safety decreases considerably as the upper slope inclination increases. For the case where the upper slope is horizontal, the calculated factor of safety would be 1.6. If the upper slope inclination is increased to 30°, the factor of safety drops to 1.43. This means that if the effect of the upper slope is not included, the calculated factor of safety will be higher, giving an unconservative design.

The impact of the tension crack inclination (α_t) on factor of safety is shown in Fig. 4. The effect is appreciable for upper slope angles (α_S) up to 20° but has no effect for steeper values. For the range of upper slope angles where it does have an effect (0–20°) the factor of safety actually increases as the angle of the tension crack (α_t) decreases.

Conclusion

The plane failure analysis given by Hoek and Bray (1981) has been extended to incorporate an inclined upper slope and a non-vertical tension crack. It is shown that the

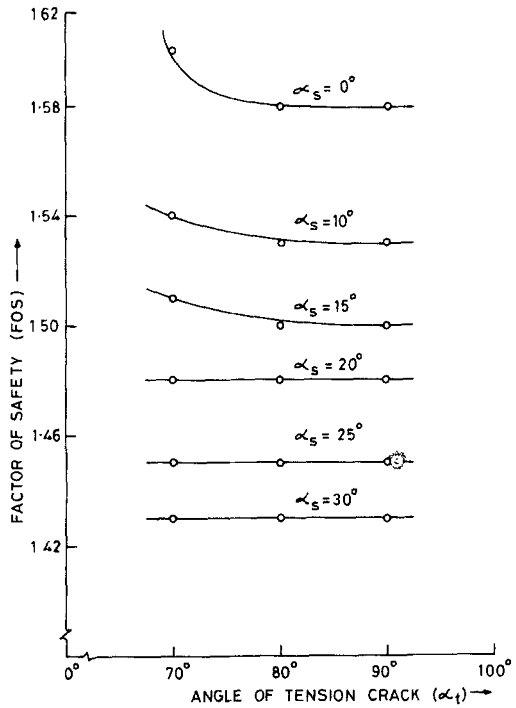

Fig. 4. Effect of tension crack inclination on factor of safety

factor of safety of the slope is significantly reduced if the inclination of the upper slope is included. For a slope which has a factor of safety of 1.6, if the top of the slope is horizontal, the effect of increasing the upper slope angle to 30° is to reduce the factor of safety to 1.43. The inclination of the tension crack has some effect if the upper slope angle is less than 20° but for steeper values it has no effect on the factor of safety.

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