Letters in Mathematical Physics **26**: 211–218, 1992.

Representations of Twisted Yangians

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(Received: 1 September 1992)

Abstract. We study highest weight representations of certain Yangian-type 'quantum' algebras connected with the series B, C, D of complex classical Lie algebras. In the symplectic case, we obtain a complete parametrization of irreducible finite-dimensional representations in terms of their highest weights. We apply these results to the well-known missing label problem in the reduction $\mathrm{sp}(2n)\downarrow\mathrm{sp}(2n-2)$.

Mathematics Subject Classifications (1991). 17B65, 22E65.

0. Introduction

An important class of quantized universal enveloping algebras called *Yangians* was introduced by V. G. Drinfeld [1]. Let $\mathfrak{a}$ be a simple finite-dimensional complex Lie algebra. Then the Yangian $\mathrm{Y}(\mathfrak{a})$ is a deformation of the universal enveloping algebra $\mathrm{U}(\mathfrak{a}\otimes\mathbb{C}[\lambda])$ in the class of Hopf algebras. One can also define the Yangian $\mathrm{Y}(\mathrm{gl}(N))$ of the reductive complex Lie algebra $\mathrm{gl}(N)$. Now let σ be the involution of $\mathrm{gl}(N)$ corresponding to the orthogonal subalgebra $\mathrm{o}(N)\subset\mathrm{gl}(N)$ or symplectic subalgebra $\mathrm{sp}(N)\subset\mathrm{gl}(N)$, $N=2n$, and consider the twisted polynomial current Lie algebra

$$\{f(\lambda)\in\mathrm{gl}(N)\otimes\mathbb{C}[\lambda]\,|\,f(-\lambda)=f^{\sigma}(\lambda)\}. \tag{0.1}$$

It was shown by G. I. Olshanskiĭ [2] that the enveloping algebra of (0.1) possesses a remarkable deformation. It is called the *twisted Yangian* and denoted by $\mathrm{Y}^{\pm}(N)$, where the upper sign corresponds to the orthogonal case and the lower sign to the symplectic case.

Note that $\mathrm{Y}^{\pm}(N)$ is contained in $\mathrm{Y}(N)$. It has no natural structure of a Hopf algebra, but $\mathrm{Y}^{\pm}(N)$ is a left coideal in $\mathrm{Y}(N)$. Note also that there exists a canonical homomorphism of $\mathrm{Y}^{\pm}(N)$ on $\mathrm{U}(\mathrm{o}(N))$ or $\mathrm{U}(\mathrm{sp}(N))$, the enveloping algebras of $\mathrm{o}(N)$ or $\mathrm{sp}(N)$. Moreover, there is a remarkable connection between the representations of $\mathrm{Y}^{\pm}(N)$ and these enveloping algebras, see [2].

The goal of this Letter is to develop for the twisted Yangians an analog of E. Cartan's theory of highest weights. For the ordinary Yangians, this was done by V. G. Drinfeld. To do this, we use some important ideas of his paper [3], and also that of V. O. Tarasov [4, 5].

For the simplest twisted Yangians $\mathrm{Y}^{\pm}(2)$ we obtain a full description of finite-dimensional irreducible representations including their realization. For $\mathrm{Y}^{-}(N)$, $N=2n$, we find a parametrization of finite-dimensional irreducible representations which is very similar to Drinfeld's classification results. For $Y^+(N)$, $N>2$, our results are somewhat less complete. Finally, we identify the $Y^-(2)$-modules arising in the spaces of $\mathrm{sp}(2n-2)$-highest weight vectors of irreducible finite-dimensional $\mathrm{sp}(2n)$-modules.

1. Preliminaries

In this section, we formulate the main definitions and give the necessary information about the structure of the Yangians and twisted Yangians (see [1, 2]).

The *Yangian* $Y(N) = Y(\mathrm{gl}(N))$ is defined as the complex associative algebra with the unity 1, countably many generators $t_{ij}^{(1)}, t_{ij}^{(2)}, \ldots$, where $i,j \in \{-n, -n+1, \ldots, n\}$, $n = [N/2]$, and in the case of even N, the zero value is excluded. It is convenient to write the defining relations in terms of the generating series

$$t_{ij}(u) = \sum_{k=0}^{\infty} t_{ij}^{(k)} u^{-k}, \qquad t_{ij}^{(0)} = \delta_{ij}.$$

They have the following form

$$(u-v)[t_{ij}(u), t_{kl}(v)] = t_{kj}(u)t_{il}(v) - t_{kj}(v)t_{il}(u).$$

Let $\varphi(u) = 1 + \Sigma_{k=1}^{\infty} \varphi^{(k)} u^{-k}$ be a formal power series. The transformation

$$\varphi : t_{ij}(u) \to t_{ij}(u)\varphi(u) \tag{1.1}$$

is extended to the automorphism of the algebra $Y(N)$. We define the *Yangian* $Y(\mathrm{sl}(N))$ as the subalgebra of $Y(N)$ consisting of the elements which are stable under all the automorphisms (1.1).

The *twisted Yangian* $Y^{\pm}(N)$ is defined as the subalgebra of $Y(N)$ generated by all the coefficients $s_{ij}^{(1)}, s_{ij}^{(2)}, \ldots$, $-n \leqslant i, j \leqslant n$, of the series $s_{ij}(u)$, where

$$s_{ij}(u) = \sum_{\alpha} \theta_{\alpha j} t_{i\alpha}(u) t_{-j,-\alpha}(-u),$$

$\theta_{ij} \equiv 1$ for $Y^+(N)$, $\theta_{ij} = \mathrm{sgn}\, i \,\mathrm{sgn}\, j$ for $Y^-(N)$, and α runs over the set $\{-n, -n+1, \ldots, n\}$. The quadratic and linear relations which are defining relations for the generators of the algebra $Y^{\pm}(N)$ are the following [2]:

$$\begin{aligned}
&(u^2 - v^2)[s_{ij}(u), s_{kl}(v)] \\
&\quad = (u+v)(s_{kj}(u)s_{il}(v) - s_{kj}(v)s_{il}(u)) - \\
&\quad - (u-v)(\theta_{k,-j} s_{i,-k}(u) s_{-j,l}(v) - \theta_{i,-l} s_{k,-i}(v) s_{-l,j}(u)) + \\
&\quad + \theta_{i,-j}(s_{k,-i}(u) s_{-j,l}(v) - s_{k,-i}(v) s_{-j,l}(u)),
\end{aligned} \tag{1.2}$$

$$\theta_{ij} s_{-j,-i}(-u) = s_{ij}(u) \pm \frac{s_{ij}(u) - s_{ij}(-u)}{2u}, \tag{1.3}$$

where the upper sign corresponds to $Y^+(N)$ and the lower sign to $Y^-(N)$.

The *special twisted Yangian* $SY^{\pm}(N)$ is defined by the equality $SY^{\pm}(N) = Y(sl(N)) \cap Y^{\pm}(N)$. In other words, $SY^{\pm}(N)$ is the subalgebra of $Y^{\pm}(N)$ consisting of the elements which are stable under all the automorphisms

$$\psi : s_{ij}(u) \to s_{ij}(u)\psi(u), \tag{1.4}$$

where $\psi(u) = 1 + \Sigma_{k=1}^{\infty} \psi^{(2k)}u^{-2k}$. The following decomposition holds

$$Y^{\pm}(N) = Z^{\pm} \otimes SY^{\pm}(N), \tag{1.5}$$

where $Z^{\pm}$ is the center of $Y^{\pm}(N)$ (see [2]). Thus the representation theory of $Y^{\pm}(N)$ and that of $SY^{\pm}(N)$ are essentially similar.

The comultiplication map $\Delta: Y(N) \to Y(N) \otimes Y(N)$ is defined by $\Delta(t_{ij}(u)) = \Sigma_{\alpha} t_{i\alpha}(u) \otimes t_{\alpha j}(u)$. The algebra $Y^{\pm}(N)$ is a left coideal of $Y(N)$ [2] and

$$\Delta(s_{ij}(u)) = \sum_{\alpha,\beta} \theta_{\beta j} t_{i\alpha}(u) t_{-j,-\beta}(-u) \otimes s_{\alpha\beta}(u). \tag{1.6}$$

Due to this remarkable fact, we can define the tensor product $L \otimes V$ of a $Y(N)$-module L and a $Y^{\pm}(N)$-module V; the result is a $Y^{\pm}(N)$-module. This construction appears in (3.2).

2. Highest Weight Representations

DEFINITION 2.1 (cf. [3]). A representation L of the algebra $Y(N)$ (resp. $Y^{\pm}(N)$) is said to be *highest weight* if there is a vector $\Omega \in L$, such that L is generated by Ω, $t_{ij}(u)\Omega = 0$ (resp. $s_{ij}(u)\Omega = 0$) for $i > j$ and $t_{ii}(u)\Omega = a_i(u)\Omega$ (resp. $s_{ii}(u)\Omega = a_i(u)\Omega$) for a certain family of formal series $a(u) = (a_{-n}(u), \ldots, a_n(u))$ called the highest weight of L.

In the case of $Y^{\pm}(N)$, due to (1.3),

$$a_i(u) = a_{-i}(-u) \pm \frac{a_{-i}(u) - a_{-i}(-u)}{2u}.$$

Hence, if $N = 2n$ (resp. $N = 2n + 1$), we can take $a(u) = (a_{-n}(u), \ldots, a_{-1}(u))$ (resp. $a(u) = (a_{-n}(u), \ldots, a_0(u))$ with $a_0(-u) = a_0(u)$) as a family of independent parameters of the highest weight.

It is not difficult to prove that every irreducible finite-dimensional representation of $Y(N)$ (resp. $Y^{\pm}(N)$) is highest weight and for any family of series $a(u) = (a_i(u))$ there exists the only (up to isomorphism) irreducible representation $L(a(u))$ (resp. $V(a(u))$) with highest weight $a(u)$ (cf. [3]). We shall use the following statement below [3].

THEOREM 2.2. *The representation $L(b_1(u), \ldots, b_N(u))$ of $Y(N)$ is finite-dimensional if and only if there exist polynomials $P_1, \ldots, P_{N-1} \in \mathbb{C}[u]$, such that*

$$\frac{b_{i+1}(u)}{b_i(u)} = \frac{P_i(u+1)}{P_i(u)} \quad \textit{for } i = 1, \ldots, N-1.$$

We may assume that any P_i is monic, i.e. its leading coefficient is equal to 1.

The representations V_1 and V_2 of $Y^{\pm}(N)$ are called *similar* if there exists an automorphism (1.4), such that its composition with V_1 is isomorphic to V_2. Due to (1.5), there is a one-to-one correspondence between finite-dimensional irreducible representations of $SY^{\pm}(N)$ and similarity classes of finite-dimensional irreducible representations of $Y^{\pm}(N)$.

3. Finite-Dimensional Irreducible Representations of the Simplest Twisted Yangians

Consider the irreducible highest weight representation $L(1+\alpha u^{-1}, 1+\beta u^{-1})$ of $Y(2)$. This is simply a lifting to $Y(2)$ of the irreducible representation of $U(gl(2))$ with highest weight (α, β). It follows that $L(1+\alpha u^{-1}, 1+\beta u^{-1})$ is finite-dimensional if and only if $\beta - \alpha \in \mathbb{Z}_+$ (then the dimension is equal to $\beta - \alpha + 1$). We shall also need a one-dimensional representation $V((1-\alpha u^{-1})(1+1/2u^{-1})^{-1})$ of $Y^+(2)$.

THEOREM 3.1. *Let $V(a(u))$ be an irreducible highest weight representation of* $Y^{\pm}(2)$.

(i) *$V(a(u))$ is finite-dimensional if and only if there exists an even formal series $\psi(u) = 1 + \Sigma_{r=1}^{\infty} \psi^{(2r)} u^{-2r}$, such that*

$$a(u)\psi(u) = \begin{cases} \prod_{i=1}^{2k+1} (1-\alpha_i u^{-1})(1+1/2u^{-1})^{-1}, & \text{for } Y^+(2), \\ \prod_{i=1}^{2k} (1-\alpha_i u^{-1}), & \text{for } Y^-(2). \end{cases} \tag{3.1}$$

where $k \geqslant 0$ and $\alpha_{2m-1} + \alpha_{2m} \in \mathbb{N}$ for all $m = 1, \ldots, k$.

(ii) *Suppose $a(u)$ has the form* (3.1). *Then we have the following decomposition*:

$$V(a(u)) = \bigotimes_{m=1}^{k} L(1-\alpha_{2m-1}u^{-1}, 1+\alpha_{2m}u^{-1}) \otimes \otimes V((1-\alpha_{2k+1}u^{-1})(1+1/2u^{-1})^{-1}) \tag{3.2}$$

in the case of $Y^+(2)$, *and*

$$V(a(u)) = \bigotimes_{m=1}^{k} L(1-\alpha_{2m-1}u^{-1}, 1+\alpha_{2m}u^{-1}) \tag{3.3}$$

in the case of $Y^-(2)$.

In particular, any irreducible finite-dimensional $Y^-(2)$*-module is obtained as the restriction of a* $Y(2)$*-module.*

Outline of the proof. We use the arguments similar to [4, 5] (see also [6]). First we prove that if $\dim V(a(u)) < \infty$, then there exists an even formal series $\psi(u)$, such that $a(u)\psi(u)(1+1/2u^{-1})$ (in the case of $Y^+(2)$) or $a(u)\psi(u)$ (in the case of $Y^-(2)$) is a polynomial, which has no divisors of the form $1-\gamma u^{-2}$. It means that (3.1) is true for certain numbers α_i. Renumerating the α_i's, we may assume that for every $m = 1, \ldots, k$ the following condition holds: if the set $\{\alpha_i + \alpha_j \mid 2m-1 \leqslant i < j\} \cap \mathbb{N}$ is not empty, then its minimal element is $\alpha_{2m-1} + \alpha_{2m}$. Further, we verify that in this case, (3.2) and (3.3) hold. It is easy to see that these tensor products are finite-dimensional if and only if $\alpha_{2m-1}+\alpha_{2m}\in\mathbb{N}$ for $m=1,\dots,k$ which completes the proof. □

Now we introduce the following

DEFINITION 3.2. The monic polynomials $P_1, P_2\in\mathbb{C}[u]$ are called equivalent if there exist polynomials $q_1, q_2\in\mathbb{C}[u]$ such that

$$\frac{P_1(u+1)}{P_1(u)}q_1(u^2)=\frac{P_2(u+1)}{P_2(u)}q_2(u^2).$$

The couples (P_1,γ_1) and (P_2,γ_2), where $\gamma_1,\gamma_2\in\mathbb{C}$, are called equivalent if there exist polynomials $q_1, q_2\in\mathbb{C}[u]$ such that

$$\frac{P_1(u+1)(u-\gamma_1)}{P_1(u)}q_1(u^2)=\frac{P_2(u+1)(u-\gamma_2)}{P_2(u)}q_2(u^2).$$

It follows from Definition 3.2 that equivalent polynomials have the same degree and every equivalence class $\langle P\rangle$ contains a finite number of polynomials.

COROLLARY 3.3. *The finite-dimensional irreducible representations of* $\mathrm{SY}^-(2)$ *(resp.* $\mathrm{SY}^+(2)$*) are parametrized by the equivalence classes of polynomials* $\langle P\rangle$ *(resp. the equivalence classes of couples* $\langle(P,\gamma)\rangle$*).*

Proof. Using the notation of Theorem 3.1, put

$$b(u)=\prod_{m=1}^{k}(1-\alpha_{2m-1}u^{-1});\qquad c(u)=\prod_{m=1}^{k}(1+\alpha_{2m}u^{-1}).$$

The equality

$$\frac{c(u)}{b(u)}=\frac{P(u+1)}{P(u)}$$

defines the only monic polynomial $P\in\mathbb{C}[u]$. We correlate the equivalence class $\langle P\rangle$ (resp. $\langle P,-\alpha_{2k+1}\rangle$) with the representation $V(a(u))$. It is easy to verify that this is a well-defined one-to-one correspondence. □

4. Representations of $\mathrm{Y}^{\pm}(N)$, $N>2$

THEOREM 4.1. *If the representation* $V(a(u))$ *of* $\mathrm{Y}^{\pm}(N)$ *is finite-dimensional, then there exist monic polynomials* $P_2,\dots,P_n\in\mathbb{C}[u]$ *such that*

$$\frac{a_{-i+1}(u)}{a_{-i}(u)}=\frac{P_i(u+1)}{P_i(u)}\quad \textit{for } i=2,\dots,n.$$

Proof. Let I be the left ideal of $\mathrm{Y}^{\pm}(N)$ generated by the elements $s_{i,-j}^{(1)}, s_{i,-j}^{(2)},\dots, 1\leqslant i,j\leqslant n$, and let

$$\mathrm{Norm}\, I=\{y\in\mathrm{Y}^{\pm}(N)\mid Iy\subset I\}$$

be the normalizer of I. The subspace

$$V(a(u))^I = \{v \in V(a(u)) \mid Iv = 0\}$$

has a natural structure of Norm I/I-module. It contains the highest vector Ω of $V(a(u))$. The relations (1.2) show that the map $\xi: t_{ij}^{(k)} \to s_{ij}^{(k)}$ mod I defines a homomorphism of algebras $\xi: \mathrm{Y}(n) \to$ Norm I/I. Here we identify $\mathrm{Y}(n)$ with the subalgebra of $\mathrm{Y}(2n)$ generated by $t_{ij}^{(1)}, t_{ij}^{(2)}, \ldots, -n \leqslant i, j \leqslant -1$. By making use of ξ, we can regard $V(a(u))^I$ as a $\mathrm{Y}(n)$-module. The cyclic span of Ω is a finite-dimensional representation of $\mathrm{Y}(n)$ with highest weight $(a_{-n}(u), \ldots, a_{-1}(u))$. Applying Theorem 2.2, we complete the proof. □

THEOREM 4.2 (cf. [3]). *The finite-dimensional irreducible representations of* $\mathrm{SY}^-(2n)$ *are parametrized by the families* $\{\langle P_1\rangle, P_2, \ldots, P_n\}$, *where* $P_i \in \mathbb{C}[u]$ *is a monic polynomial for* $i = 1, \ldots, n$.

Proof. Take a finite-dimensional representation $V(a(u))$ of $\mathrm{Y}^-(2n)$ and consider the polynomials $P_2, \ldots, P_n$ defined in Theorem 4.1. Note that they only depend on the restriction of $V(a(u))$ to $\mathrm{SY}^-(2n)$. Further, we see from (1.2) and (1.3) that the subalgebra of $\mathrm{Y}^-(2n)$ generated by $s_{ij}^{(1)}, s_{ij}^{(2)}, \ldots,$ with $i, j \in \{-1, 1\}$ is isomorphic to $\mathrm{Y}^-(2)$. The cyclic $\mathrm{Y}^-(2)$-span of the highest vector Ω is the finite-dimensional representation of $\mathrm{Y}^-(2)$ with highest weight $a_{-1}(u)$. Using Corollary 3.3, we associate to it the equivalence class $\langle P_1\rangle$.

Conversely, given a family $\{\langle P_1\rangle, P_2, \ldots, P_n\}$, we consider the representation $V(a(u))$, where $a_{-1}(u)$ is defined by Corollary 3.3 and $a_{-2}(u), \ldots, a_{-n}(u)$ are defined from Theorem 4.1. Applying a suitable automorphism (1.4) and taking a similar representation, we may assume that $a_{-1}(u), \ldots, a_{-n}(u)$ are polynomials. Write

$$a_{-r}(u) = \prod_{i=1}^{2k} (1 - \alpha_i^{(r)} u^{-1}).$$

By Theorems 3.1 and 4.1, we may assume that

$$\alpha_{2m-1}^{(1)} + \alpha_{2m}^{(1)} \in \mathbb{Z}_+ \quad \text{for } m = 1, \ldots, k$$

and

$$\alpha_i^{(r+1)} - \alpha_i^{(r)} \in \mathbb{Z}_+ \quad \text{for } i = 1, \ldots, 2k;\ r = 1, \ldots, n-1.$$

For $r = 1, \ldots, n$ set

$$b_{-r}(u) = \prod_{i=1}^{k} (1 - \alpha_{2i-1}^{(r)} u^{-1}) \quad \text{and} \quad b_r(u) = \prod_{i=1}^{k} (1 + \alpha_{2i}^{(r)} u^{-1}).$$

Then the $\mathrm{Y}(2n)$-module $L(b_{-n}(u), \ldots, b_n(u))$ is finite-dimensional by Theorem 3.2. The cyclic $\mathrm{Y}^-(2n)$-span of its highest vector is the finite-dimensional representation with the highest weight $a(u)$. Hence, $\dim V(a(u)) < \infty$ and the Theorem is proved.

5. Application to the Missing Label Problem

We regard the Lie algebra sp(2n) as the linear span of matrices

$$a_{ij} = e_{ij} - \varepsilon_i\varepsilon_j e_{j'i'}, \tag{5.1}$$

where e_{ij} are matrix units, $1 \leqslant i, j \leqslant 2n$, $\varepsilon_i = 1$ for $1 \leqslant i \leqslant n$, $\varepsilon_i = -1$ for $n+1 \leqslant i \leqslant 2n$ and $i' = 2n + 1 - i$. Let $L(\lambda)$ be the finite-dimensional irreducible representation of sp(2n) with the highest weight $\lambda = (\lambda_1, \ldots, \lambda_n)$, where $\lambda_1 \geqslant \cdots \geqslant \lambda_n$ are nonnegative integers, i.e., $L(\lambda)$ is generated by a vector v_0 such that $a_{ii}v_0 = \lambda_i v_0$, $a_{ij}v_0 = 0$ for $i < j$. The subalgebra sp(2n − 2) is the span of the elements (5.1) with $i, j \neq n, n'$. Let $L(\lambda)^+_\mu$ be the subspace of $L(\lambda)$ consisting of sp(2n − 2)-highest vectors of weight $\mu = (\mu_1, \ldots, \mu_{n-1})$. It was shown in [2, Theorem 4.5] that there exists a natural homomorphism from $Y^-(2)$ to the centralizer $U(\mathrm{sp}(2n))^{\mathrm{sp}(2n-2)}$. Thus $L(\lambda)^+_\mu$ has a natural structure of a $Y^-(2)$-module. One can show that this representation is irreducible. The generators of $Y^-(2)$ act by the rule

$$s_{ij}(u) = \left(1 - \frac{A}{u + n + \frac{1}{2}}\right)^{-1}_{\bar{\imath}\bar{\jmath}} f(u),$$

where

$$A = (a_{i'j'})_{i,j=1}^{2n}, \qquad \bar{\imath} = \begin{cases} n, & \text{for } i = 1, \\ n', & \text{for } i = -1, \end{cases}$$

$$f(u) = \prod_{i=1}^{n} \frac{(u + \lambda_i + \rho_i + \frac{1}{2})(u - \lambda_i - \rho_i + \frac{1}{2})}{(u + \rho_i + \frac{1}{2})(u - \rho_i + \frac{1}{2})}, \quad \rho_i = n + 1 - i.$$

THEOREM 5.1. *$Y^-(2)$-module $L(\lambda)^+_\mu$ is isomorphic to $V(a(u))$, where*

$$a(u) = \prod_{i=1}^{n} \frac{(u + \sigma_i + \rho_i - \frac{1}{2})(u - \tau_i - \rho_i + \frac{1}{2})}{(u + \rho_i - \frac{1}{2})(u - \rho_i + \frac{1}{2})},$$

where

$$\sigma_i = \max\{\mu_i, \lambda_{i+1}\}, \qquad \tau_i = \min\{\mu_{i-1}, \lambda_i\} \quad (\mu_n = \lambda_{n+1} = 0, \mu_0 = +\infty).$$

In particular, as a $SY^-(2)$-module it is isomorphic to the restriction of

$$\bigotimes_{i=1}^{n} L(1 + (\sigma_i + \rho_i - \tfrac{1}{2})u^{-1}, 1 + (\tau_i + \rho_i - \tfrac{1}{2})u^{-1}).$$

Outline of the proof. We use the transvector algebra method (see [7, 8]). Let p be the extremal projector for the algebra sp(2n − 2) [7]. Set

$$x_i = pa_{in'}, \qquad x_{-i} = pa_{n'i}, \qquad y_i = pa_{in}, \qquad y_{-i} = pa_{ni}.$$

The elements $x_{\pm i}, y_{\pm i}$ $(i = 1, \ldots, n - 1)$ and $a_{nn'}, a_{n'n}$ generate an algebra Z with quadratic defining relations [8]. One can verify that

$$\Omega = x_{-1}^{\lambda_2 - \tau_2} \cdots x_{-(n-1)}^{\lambda_n - \tau_n} y_{-1}^{\lambda_1 - \sigma_1} \cdots y_{-(n-1)}^{\lambda_{n-1} - \sigma_{n-1}} v_0$$

is the highest vector of $Y^{-}(2)$-module $L(\lambda)_{\mu}^{+}$. In order to find the weight of Ω, we use the relations in Z and the commutation relations between $x_{\pm i}, y_{\pm}$, and the elements $(A^{k})_{\bar{i},\bar{j}}$.

6. Remarks

Due to Theorem 4.1, the classification problem for finite-dimensional irreducible representations of $Y^{+}(N)$ is reduced to the cases $Y^{+}(3)$ and $Y^{+}(4)$. To solve this problem, we need more information about the structure of the representation of $Y(N)$, $N \geqslant 3$. In this connection, we would like to formulate the following question: is it true that every irreducible representation of $Y(N)$ with polynomial highest weight is isomorphic to a tensor product of representations of the form $L(b(u))$, where $b_i(u) = 1 + \beta_i u^{-1}$ for $i = 1, \ldots, N$? It was proved in [5] for $N = 2$.

Acknowledgement

I am very grateful to G. I. Olshanskiĭ for numerous stimulating discussions and help in preparing this Letter.

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