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MATHEMATICAL EDUCATION VERSUS CRITICAL EDUCATION

ABSTRACT. This paper concentrates upon the relationship between mathematical education (ME) and critical education (CE) connected with the Frankfurt School and Critical Theory. To make the discussion as precise as possible a distinction is made between three alternatives in ME: Structuralism, pragmatism, and process-orientation. These alternatives are related to the key terms of CE in order to show the extent to which ME and CE contradict each other. The conclusion is that there does not exist any integration – nor even any close relationship – between ME and CE.

Finally, this result is discussed in the light of the following two theses:

(A) It is necessary to increase the interaction between ME and CE, if ME is not to degenerate into one of the most important ways of socializing students into the technological society and at the same time destroying the possibilities for developing a critical attitude towards precisely this technological society.

(B) It is important for CE to interact with subjects from the technical sciences, and among these ME, if CE is not to be taken over by the technological development and fade away into an unimportant and uncritical educational theory.

INTRODUCTION

Several times it has been emphasized that mathematical education – to be understood as research into the mathematical educational process – bears a complicated relationship to other subjects of research. In what follows my use of the concept 'mathematical education' should in some cases be identified with the German concept 'Didaktik der Mathematik', in other cases it simply refers to the educational process in mathematics.

The development of mathematical education as a scientific discipline, which was initiated in the late 1960s, has led to the following questions being asked: (1) What is the object(s) of the discipline? (2) What scientific method(s) should be used? (3) What relations does the new field have to other more established scientific disciplines? Anna Zofia Krygowska has given a very precise description of the problems:

> Didactics of Mathematics is developing as a typical 'border' discipline. Every independent discipline is characterised by the specificity of its problems, of its language, and of its research methods. In its first phase of development the border subject has a rather vague status. In particular, its methods of research can be quite inhomogeneous. On the one hand mathematics education develops at the border of mathematics, its philosophy, and its history; on the other hand at the border of pedagogy and psychology. (Krygowska, 1971, p. 118).

In the following I shall discuss a point in relation to the above question (3); I shall look at the connections between mathematical education and

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general educational theories. In my opinion these connections appear to be rather selective. An important branch of general education, interwoven with an important school of epistemology and philosophy, is hardly mentioned and not conceptualized in mathematical education (ME). I am thinking of critical education (CE), critical pedagogy, and ideas of critical theory.[1] And my basic postulates are as follows:

(A) It is necessary to increase the interaction between ME and CE, if ME is not to degenerate into one of the most important ways of socializing students (to be understood as students or pupils) into the technological society and at the same time destroying the possibilities of developing a critical attitude towards precisely this technological society.
(B) It is important for CE to interact with subjects from the technical sciences, and among these ME, if CE is not to be taken over by the technological development and fade away into an unimportant and uncritical educational theory.

In the following I shall concentrate on postulate (A), but postulate (B) should be kept in mind.[2]

1. CRITICAL EDUCATION

CE has several sources of inspiration. There exists a strong association with Karl Marx's understanding of humanism and of society, especially as expounded by the Frankfurt School[3] (or Critical Theory). The original main figures of this school were Theodor W. Adorno, Max Horkheimer, and Herbert Marcuse. They worked at the Institut für Sozialforschung in Frankfurt am Main; the institute was founded in 1923, and in 1923–29 Carl Grünberg was the director of the institute. He was succeeded by Horkheimer, who in his inaugural address (1931) put forward three themes which should characterize the School of Frankfurt:

> The first . . . suggests the necessity of re-specifying 'the great philosophical questions' in an interdisciplinary research programme. The second theme, more implicit but made clearer in later essays, is a call for a rejection of orthodox Marxism and its substitution by a reconstructed understanding of Marx's project. The third emphasizes the necessity for social theory to explicate the set of interconnections (mediations) that make possible the reproduction and transformation of society, economy, culture and consciousness. (Held, 1980, p. 33.)

When the Nazis came to power in Germany, the staff of the institute continued their work outside Germany, especially in the U.S.A. After the war Adorno and Horkheimer returned, but Marcuse remained in the U.S.A. Later Jürgen Habermas became the dominant figure of the school.[4]

Another, but much less important source of inspiration for CE is to be found in the 'Geisteswissenschaftliche Pädagogik'. In a fundamental way this educational theory is inspired by hermeneutics, as worked out by Wilhelm Dilthey.[5] Other important persons in the 'Geisteswissenschaftliche Pädagogik' are Eduard Spranger, Theodor Litt, and Herman Nohl. This pedagogy dominated the educational discussions in Germany between the two World Wars; and again after World War 2.[6] Later Wolfgang Klafki, belonging to this school of education, also shows inspiration from the Frankfurt School.

Some of the main persons in the (first) more theoretical development of CE are Herwig Blankertz, Wolfgang Lempert, Klaus Mollenhauer, and also Wolfgang Klafki. Basically they try to develop pedagogy as a praxiological research discipline as a reaction to the empirical-positivistic tradition in pedagogy; e.g., Blankertz tries to relate pedagogy to Habermas' theory of interests. Similar problems are discussed by Klafki in "Erziehungswissenschaft als kritischkonstuktive Theorie: Hermeneutik – Empirie – Ideologiekritik".

Also Oskar Negt reveals a close relation to critical theory in his theoretical basis, but he gives CE a more independent and original foundation, as he takes into consideration a broad spectrum of political, economical, and psychological aspects. Negt does not limit the development of CE primarily to philosophy, which has been a characteristic tendency in the first phase of CE. In Scandinavia Negt's version of CE has been discussed in great detail and developed for instance by Knud Illeris.[7]

2. MAIN TERMS IN CRITICAL EDUCATION

It is, of course, impossible to sum up the ideas of CE in a few statements. However, I shall make an attempt, although it may create misunderstandings.

For CE the relation between teacher and students plays an important role. Several types of relationship are possible, but CE emphasizes that an important principle is that the partners are equal. Paulo Freire has discussed the teacher–students relationship in connection with what he calls 'emancipatory pedagogy':

> Through dialogue, the teacher-of-the-students and the students-of-the-teacher cease to exist and a new term emerges: teacher–student with students–teachers. The teacher is no longer merely the-one-who-teaches, but one who is himself taught in dialogue with students, who in their turn while being taught also teach. They become jointly responsible for a process in which all grow. (Freire, 1972a, p. 53.)

The ideas concerning the dialogue and the student–teacher relationship are developed from the general point of view that education must belong to a process of democratization. If a democratic attitude is to be developed through education, education as a social relationship should not contain fundamentally undemocratic features. It is not acceptable that the teacher (alone) has the decisive and prescribing role. Instead the educational process must be understood as a dialogue.

Summarily, we can specify the first key term in CE as: Involvement of the students in the control of the educational process. Or, in other words, in CE the students (and the teachers) are attributed a *critical competence.*

This competence is attributed mainly to the students for two reasons. First, for *factual* reasons, that the students, although their experience is faulty, fragmentary etc., also have a general experience which in a dialogue with the teacher enables them to identify relevant subjects for the educational process; relevant both in relation to the immediate interests of the students but also in relation to the general perspective of the educational process. Secondly, for *principle* reasons, that if an education should develop a critical competence, such a competence cannot be imposed upon the students, but must be developed from an already existing capability.

An educational process involves persons (students, teachers) but of course also a subject matter (the curriculum). The next basic key term of CE is: Critical consideration of contents and other subject matter aspects. Formulated otherwise: in CE both the students and the teacher must establish a *critical distance* to the content of the education. In German this key term is "Fachkritik", translated into English "curriculum critique" perhaps could be used.

In a curriculum critique we try to put the seemingly objective and value-free principles for the structuring of the curriculum into a new perspective, as we try to reveal such principles as value-loaded. Questions in a curriculum critique concern the following:

(1) The applicability of the subject matter:
Who use it? Where is it used? Which types of qualifications are developed in ME?
(2) The interests behind the subject:
Which knowledge-constituting interests are connected with the subject?
(3) The assumptions behind the subject:
Which questions and which problems have generated the concepts and the results in mathematics? Which contexts have promoted and controlled the development?

(4) The functions of the subject:
Which possible social functions could the subject have? This question does not primarily concern the possible applications, but the implicit function of a ME with regard to attitudes towards technological questions, the student's attitudes towards his own capacities etc.
(5) The limitations of the subject:
In which areas and in relation to which questions is the subject without any importance?

The last key term of CE concerns conditions outside the educational process. It could be formulated as problem-orientation of the teaching-learning process. The essential thing is that the educational process is related to problems existing outside the educational universe. Moreover several criteria can be used to select these problems. The two fundamental criteria are the following. The subjective one: The problem must be conceived as relevant from the perspective of the students, it must be possible to encircle and define the problem in close connection to the experience and conceptual framework of the students. And the objective one: The problem must have a close relation to objective existing social problems.

This last key term about problem-orientation gives perspective to the previous terms concerning critical competence and critical distance. The problem-orientation implies that the dimension of *critical engagement* must be involved in education.

3. ALTERNATIVES IN MATHEMATICAL EDUCATION

Mathematical education includes several very different, even inconsistent, ideas about mathematics and education. To make the discussion as precise as possible I shall distinguish between three alternatives in ME: (1) structuralism, (2) pragmatism, and (3) process-orientation.[7a] Furthermore I shall relate the above alternatives to the key terms of CE in order to show the extent to which ME and CE contradict each other.

Structuralism is characterized by the following assumptions: The essence of mathematics can be determined by crystallizing fundamental concepts through logical analysis of existing mathematical theories; these fundamental concepts can be conveyed to the learner by means of suitable concretizations in accordance with the epistemological potentials of the child. The structural point of view is then characterized by an idea about mathematics (to be associated with the name of Nicolas Bourbaki), an idea about educational communication and transformation (Jerome S. Bruner), and an idea about epistemology (Jean Piaget).

The assumptions of structuralism now exemplify an almost complete negation of CE, a fact which does not seem to have affected the proponents of structuralism in any way. I shall illustrate this by a remark of Jean Dieudonné:

> . . . if the people responsible for drawing up school curricula may be persudaded to consult professional mathematicians in order to understand the relevance of their decisions to science as it is practiced in the university and beyond, we may yet witness one day some sensible teaching of mathematics from kindergarten to graduate school. (Dieudonné, 1973, p. 19.)

Here we have an explicit negation of the idea about critical competence, and implicitly the ideas about critical distance and critical engagement are also negated.

Structuralism bears a close relationship to the tradition in pedagogy: "Teach the disciplines!" This could be interpreted both as a principle governing the selection of school subjects as well as a principle governing the presentation of the subject matter. Basic to this tradition is the idea that the students' knowledge has to be built up in accordance with structures and contents identified independently of the students.

In accordance with the *pragmatic* trend in ME the essence of mathematics is to be found in its applications, and thus in a sense outside mathematics. In the process of education it is therefore highly important to illustrate the various ways in which mathematics can be useful. This pragmatic trend can be understood in a broad sense, and many arguments have been put forward to support an application-oriented ME.[8] This broad pragmatic trend can be interpreted as a reaction against the structuralism of the 1960s.

However, the pragmatic trend can also be interpreted in a much more specific way. In the article "Mathematics, Applicable versus Pure-and-Applied" Christopher Ormell emphasizes that the work of the Sixth form Mathematics Project at Reading is based on a specific "Working Philosophy" of mathematics which he relates to the philosophy of Charles Sanders Peirce who has noted: "Mathematics is the study of what is true of hypothetical states of things." (Peirce, 1960, p. 193.) Ormell stresses the relevance of this working philosophy about mathematics as follows:

> What has altered the situation today is the emergence of the Peircean point of view; which provides for the first time a definite, coherent account of the role of mathematics in science and affairs. It has become clear that the unique contribution which mathematics makes to society operates not via its relevance to the actual, but via its relevance to the not yet and would be actual: via the light it sheds on the hidden and semi-hidden implications of candidate innovations, hypotheses, proposals, programmes and projects of all kinds. The 'relevance' of mathematics is not to a crass utilitarianism, but to the possibilities of new patterns of explanation in science and new patterns of organization in social and technical affairs." (Ormell, 1972a, pp. 129–130.)

The basic principles of Ormell's interpretation of mathematics are as follows: First: Mathematics is the science of hypothetical situations. Secondly: The central thing in ME is as simply as possible to illustrate how mathematics can be used to handle and to investigate hypothetical situations.

Ormell seems to argue in favour of the first proposition on two levels – not clearly separated; and it is not evident whether Ormell finds that the two levels ought to be separated. At one time he says that mathematics could be interpreted as if it *in fact* is a science of possible situations. In "Mathematics, Applicable versus Pure-and-Applied" Ormell develops this point of view by, among other things, looking at the history of mathematics. He finds that even the development of Greek geometry fits into the pattern of the Peircean view. In "Mathematics, Science of Possibility" Ormell gives further reasons for the descriptive part of the argument. At other times he says that mathematics *ought* to be interpreted as a science of possible situations, if we are to develop an adequate interpretation of mathematics. This is the normative part of the argument in favour of the Peircean view.

Secondly it was stated that the crucial point of ME is to explain how mathematics can become a science of hypothetical situations; and the textbooks "Mathematics Applicable" illustrate how this can be done in elementary ways.[9]

If we now turn to the key terms of CE it is obvious that "Mathematics Applicable" does not consider the passage about critical competence. Although the adoption of the Peircean approach implies that mathematics must be seen primarily as a model building activity, and not as a body of results, in "Mathematics Applicable" it is difficult to find a platform for an authentic teacher–student dialogue. The textbooks contain examples of mathematical modelling, and the process of education must – at least according to the exposition – move from one thoroughly prepared example to another thoroughly prepared example. What the students have to do is to calculate, to find the solutions, and to solve already well defined problems.

With regard to the key term: critical consideration of contents and other subject matter aspects, the situation is somewhat different. An important element in the curriculum critique concerns the question: Which (type of) problems and which (type of) questions have generated the technical terms? To some extent this aspect could be compatible with the pragmatic trend. But the element of "propaganda for the usefulness of mathematics" which is a rather characteristic feature of "Mathematics Applicable" is quite foreign to curriculum critique.

The pragmatic trend is problem-oriented. However, in CE it is essential that the problems have to do with fundamental social situations and conflicts, and it is important that the students simultaneously can recognize the problems as "their own problems", cf. the objective and the subjective criteria of problem identification in CE. Problems must not belong to "play-realities" without any significance except as an illustration of: mathematics as a science of hypothetical situations. In general, therefore there exists a large gap between CE and the pragmatic trend in ME.

When looking at the general theory of education we can easily relate the pragmatism in ME to the pragmatic educational philosophy, as for instance worked out by John Dewey. It is, however, not possible to equate the two types of pragmatism. Dewey emphasizes the experimental activity of the students to a much higher degree than the textbook system "Mathematics Applicable" does. Furthermore it is difficult to use any ready-made educational programme in the framework of the pedagogy of Dewey. However, in a metaperspective we find the CE criticism of the Dewey-pragmatism to be parallel to the above sketched criticism of the Ormell-pragmatism.

The third trend in ME which should be mentioned is *process-orientation*. According to this viewpoint the essence of mathematics is neither connected to particular concepts nor to the applicability (the usefulness) of mathematics as such, but to the processes of thought which have led to the mathematical insight. And it is stressed that the main concern of mathematical education is to give students opportunities for making re-inventions of their own.[10]

One of the most important proponents of this view is Hans Freudenthal. In the preface of "Mathematics as an Educational Task" Freudenthal writes: "My educational interpretation of mathematics betrays the influence of L. E. J. Brouwer's view on mathematics (though not on education)". (Freudenthal, 1973, p. ix.) If Brouwer's intuitionistic interpretation of mathematics should be summarized in few words it could be: Mathematics is a human construction. The constructivism implies that the mathematical concepts and truths are to be constructed, not to be discovered.[11] Freudenthal uses the formulation: "Mathematics is a human activity." Here he stresses: First, that the essential thing is the activity, the process of thought, which led up to the mathematical concepts. Secondly, that this activity is a general human activity – not an exclusive one, only for specific talented persons.

From this point of view the fundamental thing in education is to make the students able to create mathematics. Freudenthal writes: "Science at its summit has always been creative invention, and today it is even so at levels lower than that of the masters. The learning process has to include phases of directed invention, that is, of invention not in the objective but in the subjective sense, seen from the perspective of the student. It is believed that knowledge and ability acquired by re-invention are better understood and more easily preserved than if acquired in a less active way." (Freudenthal, 1973, p. 118.) In many ways the work of IOWO (Instuut voor de Ontwikkeling van het Wiskunde Onderwijs) has illustrated possible practical implications of such general phrases.[12]

This process-orientation has something in common with CE. What I especially have in mind has to do with critical competence. In the work of IOWO a crucial point is the refusal to use all sorts of pre-structured and ready-made material as developed in connection with structuralism. Instead IOWO tries to create situations which lead to mathematization, i.e., development of mathematical ideas and concepts. A process-oriented ME must reject the idea of concretization, to be understood as an elementarization of (abstract) mathematical concepts, in favour of a mathematization. But there exists an important asymmetry between these two sorts of activities. To concretize is to give more abstract terms a concrete interpretation and in this way to make them more comprehensible. The activity of concretization is reserved for the planners of the curriculum (compare with the remark of Dieudonné), and as such this activity is removed from the educational process. To mathematize means, in principle, to formulate, to criticize, and to develop ways of understanding; and hence mathematization must take a leading role as an integrated part of the educational process: both the students and the teacher must be involved in the control of this process. However, it is important to note that the control in question has certain limits, because the students have to criticize *inside* a pre-established situation. So, the process-orientation (in the programme of IOWO) builds upon a limited concept of critical competence and not upon the broader concept in CE which ascribes critical competence to the students.

In "Mathematics as an Educational Task" Freudenthal rejects the widespread use of "play-realities" in education. Instead he recommends mathematics fraught with relations:

> When speaking about mathematics fraught with relations, I stressed the relations with a lived-through reality rather than with a dead mock reality that has been invented with the only purpose of serving as an example of application. This is what often happens even in arithmetic teaching. I do not repudiate play-realities. At a low level games may be useful means of motivation. But it is dangerous to rely too much on games. Ephemeral games are no substitutes for lived-through reality. The rules of games that are not daily exercised are as easily forgotten as mathematics or even faster. The lived-through reality should be the backbone which joins mathematical experiences together. However motivating and charming games may be, they can never fill this place. (Freudenthal, 1973, pp. 78–79.)

But according to CE several of the examples developed by IOWO look like play-realities and the concept of problem orientation in the IOWO context is not identical with the corresponding concept of CE. It is as if only a part of the key terms of CE is to be found in the process-orientation of ME as exemplified by Freudenthal and IOWO.

The process-orientation of ME has a rather definite relationship with general educational traditions. Like the Reform Movement, or the Progressivism, which was introduced at the beginning of this century by Ellen Key, Maria Montessori and others, this process-orientation puts the child into the centre of the educational process. The curriculum content should not be imposed upon the child, instead the potentials of the child should be developed in a rich and stimulating milieu. The process-orientation of IOWO also has roots in the tradition of Alexander I. Wittenberg and Martin Wagenschein. Wittenberg connects his philosophy of ME with the "Geisteswissenschaftliche Pädagogik" and Wagenschein stresses the principles of exemplarity. Both relate their ideas to a genetic principle, which has to do with process-orientation. The tradition of Wittenberg and Wagenschein is also related to CE, and the same sort of limitations as mentioned above are found again. For instance it is interesting to note the remarks of Heinz-Dieter Hermann about Wittenberg and Wagenschein concerning ME in the Glocksee-project.[13]

4. WHAT IS THE PROBLEM?

Obviously, the conclusion of the previous section is that there does not exist any integration – nor even any close relationship – between ME and CE. The strongest contradiction is found between structuralism and CE, but neither pragmatism nor process-orientation have much in common with CE. But what is the problem? Why should any such integration exist? Is it important to establish one? Through some theses which are to be formulated in the following I shall argue in favour of the importance of this integration, cf. my postulate A and B.

The first thesis has to do with the relationship between technology and society, and I shall underline an idea, already formulated by the French philosopher of technology, Jacques Ellul, in "The Technological Society". According to Ellul technology has replaced nature as the environment of man. Technology must be conceived as a closed circle around man. In Ellul's words:

> Technique has progressively mastered all the elements of civilisation. . . . man himself is overpowered by technique and becomes its object. The technique which takes man for its

object thus becomes the centre of society: this extraordinary event . . . is often designated of technical civilization. The terminology is exact and we must fully grasp its importance. Technical civilization means that our civilization is constructed by technique (makes a part of civilization only what belongs to technique), for technique (in that everything in this civilization must serve a technical end), and is exclusively technique (in that it excludes whatever is not technique or reduces to technical form). (Ellul, 1964, pp. 127–128.)

Apart from finding that technology completely closes around man, Ellul finds that the evolution of technology is fully determined by technology itself, and that society and the way of living is fully determined by this evolution; therefore Ellul's view on technology is deterministic – and pessimistic too. However, I do not intend to discuss the different kinds of determination between technology and society, only to stress the postulate about the total integration of society into technology and vice versa. Let me emphasize it in the following:

> *Thesis of technology (Ellul's thesis)*:
> Technology is the dominating feature of civilization, and man is completely immersed in technology.

This thesis deals with power and power-relations, because via technology it is possible to establish and/or to increase power-relations. A corollary of the above thesis is that man is located in a civilization with power-relations determined by and integrated in a technological structure.[14]

The next thesis deals with education, especially the curriculum. It is my opinion, and a general one too, that power-relations can be embedded into a specific curriculum organization.[15] I shall give a single formulation:

The original introduction of prepacked material was stimulated by a specific network of political, cultural, and economic forces, originally in the 1950s and 1960s in the United States. The views of academics that teachers were unsophisticated in major curriculum areas 'necessitated' the creation of what was called teacher-proof material. The cold war climate (created and stimulated by the State in large part) led to a focus on the efficient production of scientists and technicians as well as a relatively stable work-force; this, the 'guaranteeing' of this production through the school curriculum became of increasing import." (Apple, 1982, p. 150.)

In this formulation we have interwoven at least implicitly the following two propositions: (1) The contents of the curriculum are determined, not primary by factual causes having to do with the logical structure of the curriculum, but with economic and political forces having to do with the power-relations of the society; and (2) the curriculum can function as an extension of the existing social relations. It must be noted that (1) does not imply that for instance the specific contents in the structuralistic curriculum are not derived from logical analysis of mathematics, but (1) implies that the structuralistic reform is made possible because a structuralistic curriculum seems to be in accordance with dominating economic and political interests: the interests in a stable work-force with technical skills without regard to political, humanistic and moral reflections.

Summarizing the above we state the following thesis:

> *Thesis of curriculum*:
> The fundamental structuring principles of the curriculum are derived from or are in accordance with the dominant power-relations of the society.

After formulating the two general theses we shall now look more closely at education in mathematics. In this connection I find two different aspects important. First, how does ME serve to introduce a particular way of thinking, acting, evaluating etc. in a technological society? To what extent does ME in the educational system work as an instrument for technological interests? These questions naturally deal with the problems about the hidden curriculum in ME.[16]

The next aspect which I find important concerns the students and the relationship among the students. In an educational system the structure of knowledge (in a narrow sense) of the students is developed and transformed, but also the structure of social relations among the students and the students' experience of it are changed. Several investigations indicate that ME plays a part in reorganizing the world around the students. They take up an attitude towards the technological society. They learn that some people can manage technological problems, and that some people cannot. And consequently the "incapable" students learn to become servile to technological questions, and to be servile to those who can manage such questions.

I shall state my idea about ME as follows:

> *Thesis about mathematical education*:
> In the educational system ME serves as the most significant introduction to the technological society. It is an introduction which both endows (a part of) the students with relevant technical skills, and at the same time endows (all) the students with a "functional" attitude towards the technological society. ("Functional" is seen from the perspective of the dominating power-structures.)

The basic axiom in CE is that education must not serve as a passive reproduction of existing social relations and power relations. This axiom makes sense in the talk about critical competence, critical distance, and critical engagement. Education has to play an active role in identifying and combating social disparities. Naturally education does not play any important role in social and technological changes – such changes do not follow educational enterprises; but education must strive to play an active role parallel to other critical social forces.

In education, theory as well as practice, CE is the most important movement which tries to negate the thesis of curriculum. The intention of CE is to unmask the dominating structuring principles of the curriculum as historical and accidental. In the light of Ellul's thesis the most important school subjects must be "technologies" (in a broad sense), with respect to ME this is underlined by the thesis about mathematical education. So, these were the general ideas on which I have based my postulate (A): that it is necessary to increase the interaction between ME and CE, if ME is not to degenerate into one of the most important ways of socializing students into the technological society.

5. ANY SOLUTIONS?

There are examples of CE carried out in practice; in Denmark both in the primary and in the secondary schools, but also at university level, particularly at the University Centre of Roskilde and at the University Centre of Aalborg. Two different strategies are used when a CE-practice is developed: *thematization* or *projectorganization*. While thematization is widely used in the primary school and (perhaps to less extent) in the secondary school, the projectorientation is found at university level, particularly at the above mentioned University Centres.

It is possible to use the strategy *thematization* even if the educational structure of the school system is rather traditional. By integrating different school subjects and by cooperation between the teachers it could be possible to eliminate the demarcations between the school subjects and to put the time-table out of function, to make room for long and continuous periods of work. The strategy *projectorganization* is more demanding, in that it (as developed at university level) requires not only a special organization of study programme, but also a special arrangement of the university building. When elaborating projects the students have to work together in groups, and to make their work much like "real" research work it is important that every project-group has a room of their own at the university, where to work, to collect material, to receive advice, etc. Much experience in connection with these strategies has been gained through the last 10 years, and, evidently, much thematized and projectorganized education is developed without a close relation to CE. But from the perspective of CE these two strategies are still considered to be "reasonable" (if not necessary), but they are in no way sufficient as strategies. Problemorientation has to play an important role.

CE has had almost all its influence on school subjects in the humanities and the social sciences. As previously indicated there does not exist much influence on the technical subjects – with exceptions naturally, for instance at the above mentioned university centres. However, this does not imply that recommendations do not exist for such an integration of CE. We have an example in the article "Plädoyer für einen problemorientierten Mathematikunterricht in emanzipatorischer Absicht" by Dieter Volk.

These and other such recommendations I could summarize in some criteria for the selection of the problem in a problem-oriented ME as follows:

(1) It should be possible for the students to realize that the problem is of importance. That is, the problem must contain subjective relevance for the students. It must have to do with situations connected to the experience of the students.
(2) The problem must have to do with important processes in society.
(3) Somehow and to some extent the engagement of the students in the problem-situation and in the process of problem-solving should prepare a basis for (later) political and social engagement.

These criteria constitute an explicit specification of the intentions behind CE, but such types of criteria do not play any important role in the discussion of the contents in ME. The criteria normally found in ME are in some way related to mathematics itself: to the logic of mathematical structures as in structuralism, to the applicability of mathematics as in pragmatism, or to the mathematical way of thinking as in the process-orientation.

It seems to be more and more difficult to realize a CE the closer the concerned section of the educational system is to technology. This has to do with Ellul's thesis and the thesis of the curriculum. To preserve the functionalism of the educational system it is necessary, particularly for the technological subjects (and for mathematics), to maintain a demarcation between the "factual" and the "normative" including all ethical questions, questions about relevance etc. This demarcation is intimately connected with the postulate of the social neutrality of the criteria for selection of curriculum content. This postulate is asserted in structuralism in the most explicit way, and negated in CE.

The whole idea of CE is to interpret the curriculum and education as a normative structure; and if the conceptualization of the (from a sociological point of view) most important normative curriculum structures is to be effected in practice, CE has to be explicitly integrated into the technical sciences and into ME. CE cannot do without such an integration. That is the background for my postulate (B): that is is important for CE to interact with the technical sciences and mathematics to prevent CE fading away into an unimportant and uncritical educational theory.

However, as described, CE has not influenced ME in any important respect. Moreover, most of the advocates of CE who have developed its special aspects, do not show any interest in ME. (Exceptions like Volk are involved in ME.) The CE-ignorance of ME is built into the conceptual framework of CE, and in my opinion this ignorance is a catastrophe for CE, because it will destroy its critical power.

One of the explanations of this CE-ignorance is found in the philosophy of technology of Critical Theory, which concept of technology is based on the assumption that technology can be identified with the natural sciences. In Jürgen Habermas' philosophy the knowledge-constituting interest of the natural sciences is technical and quite different from the emancipatory interest of the social sciences; so obviously, an emancipatory enterprise as worked out in CE can ignore technology, including mathematics.

This conception of technology is, however, strongly criticized. It does not establish a basis for analysing the function of technology in society. The philosophy of technology in Critical Theory can not be used if the intention is to influence the technological structures themselves.

Therefore, one of the main jobs for a CE is to develop a more adequate philosophy of technology, so that CE can manage and interpret technical education, so CE and ME can become integrated, so ME can become a critical education, and so CE again can become critical.

NOTES

[1] The neglect of CE is obvious for instance in Hans-Georg Steiner's introduction in Steiner (1978).

[2] In the following books (all in Danish) I have discussed relations between ME and CE:
Skovsmose, O.: 1980, *Forandringer i matematikundervisningen*, Gyldendal, Copenhagen.
Skovsmose, O.: 1981a, *Matematikundervisning og kritisk pædagogik*, Gyldendal, Copenhagen.
Skovsmose, O.: 1981b, *Alternativer i matematikundervisningen*, Copenhagen.
Skovsmose, O.: 1984, *Kritik – undervisning og matematik*, Lærerforeningernes Materialeudvalg, Copenhagen.

[3] For a further exposition of Critical Theory see for instance Jay (1973) and Held (1980).

[4] For an exposition and examination of the philosophy of Habermas see McCarthy (1978).

[5] Also Habermas relates the Critical Theory to the hermeneutics of Dilthey; see the discussion in Habermas (1968a).
[6] See for instance Nohl (1970).
[7] The two most important books (both in Danish) are as follows:
Illeris, K.: 1974, *Problemorientering og deltagerstyring*, Munksgaard, Copenhagen.
Illeris, K.: 1981, *Modkvalificeringens pædagogik*, Unge Pædagoger, Copenhagen.
[7a] Identifying these alternatives, primarily I am concerned with ME in Europe during the last, say, twenty years. I believe that a quite different conceptual framework has to be introduced if trends and problems related to the countries of the 3rd World are to be discussed.
[8] A basis for an applied oriented ME is found in Pollak (1973).
[9] The introduction to "Mathematics Applicable" is:
Mathematics Applicable: Teaching Mathematics Applicable, Introductory Guide, School Council Sixth Form Mathematics Project, Heinemann Educational Books for the School Council, London 1979.
[10] For a general exposition of the 'genetic' trend in ME see Schubring (1978).
[11] See Brouwer (1948).
[12] See "Five Years IOWO".
[13] Detailed information about the Glocksee-project is found in the Danish journal *Kontext*, No. 35, 1978, which contains articles by Henning Schultz, Hannelore Simonsen, Anna Krovoza, Inge Negt, Oskar Negt, Thomas Ziehe, Peter Weigelt and Heinz-Dieter Hermann.
[14] For more details about philosophy of technology see for instance Rapp (1981).
[15] See for instance the article 'An Approach to the Study of Curricula as Socially Organized Knowledge', by M. F. D. Young in Young (1971, pp. 19–46). Also B. Bernstein's conceptions are of special importance in this relation.
[16] An important analysis and discussion of the hidden curriculum is found in:
Broady, D.: 1981, *Den dolda läroplanen*, Symposion Bokförlag, Stockholm. (Swedish).
See also Bauersfeld (1979).

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