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QUANTUM STATISTICS, IDENTICAL PARTICLES AND CORRELATIONS

ABSTRACT. It is argued that the symmetry and anti-symmetry of the wave functions of systems consisting of 'identical particles' have nothing to do with the observational indistinguishability of these particles. Rather, a much stronger 'conceptual indistinguishability' is at the bottom of the symmetry requirements. This can be used to argue further, in analogy to old arguments of De Broglie and Schrödinger, that the reality described by quantum mechanics has a wave-like rather than particle-like structure. The question of whether quantum statistics alone can give rise to empirically observable correlations between results of distant measurements is also discussed.

1. A SUBJECT WITH A HISTORY

The years 1924–1926 saw a rapid succession of events which culminated in the formulation of wave mechanics by Erwin Schrödinger. Quantum statistics played an important role in this development. In June 1924 Albert Einstein received a manuscript from the Indian physicist S. N. Bose, in which a new derivation of Planck's radiation law was presented. The essential new point in Bose's derivation was its treatment, by means of statistical mechanics, of cavity radiation as an assembly of light-quanta. This had been tried before, but the usual statistical methods, applied to a gas of 'light particles', inevitably lead to Wien's radiation law, which is in conflict with experiment. In fact, this had been an important argument against Einstein's light-quanta hypothesis [1]. Bose himself seemed to be unaware of this previous history of the subject. His use of statistics in the paper was rather opaque, and he appeared to think that he was just applying standard techniques. But Einstein perceived that a radically new hypothesis concerning the equiprobable states of a light-quanta gas constituted the core of Bose's work. He translated Bose's paper from English into German and sent it right away to the *Zeitschrift für Physik* [2]. Even before it had been published there, Einstein presented his own extension of the new statistical method to the case of ordinary matter particles [3]. In a subsequent article on the same subject [4] (the application of what we now call 'Bose–Einstein statistics' to the quantum theory of the ideal gas) he suggested that there might be a connection between the new statistics and De Broglie's recent hypothesis of matter waves.

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That it was not easy to grasp what was at the basis of these developments appears from the fact that Schrödinger – himself an expert in statistical mechanics – at first did not see that a fundamentally new statistical hypothesis was being proposed. In correspondence Einstein, however, pointed out to him the distinctive features of the new statistics – in particular the tendency of particles to group together in the same cells in phase space [5]. This tendency seemed to indicate an "influence of the molecules on each other whose nature is for the present completely mysterious", as Einstein put it. Once he had recognized this point, it did not take long before Schrödinger formed his own opinion on Bose–Einstein statistics. In an article entitled 'On Einstein's Gas Theory' [6], sent to the *Physikalische Zeitschrift* on December 15, 1925, he showed that it was possible to arrive at Einstein's results *without* abandoning traditional statistical methods. In the opening paragraphs of the article he diagnosed the situation as follows.

The following is generally regarded as the essential point in the new theory of the gas worked out by Einstein: a wholly new kind of statistics, the so-called Bose statistics, has to be applied to the motions of the gas molecules. Natural intuition rightly resists considering this new statistics as something primary, incapable of further explanation. On the contrary, it seems to conceal the hypothesis of a certain interdependence or interaction of the gas molecules which is, however, hard to analyse in this form. One is entitled to expect a deeper insight into the real essence of the new theory if one succeeds in preserving the old statistical methods, which have been tested by experience and are logically well-justified, and manages to undertake the change in the foundations at a point where it is possible to do so without 'sacrificium intellectus'.

He then observed that it is possible to derive the correct statistical distribution for radiation, Planck's radiation law, *without* the new statistics. This can be done by applying 'natural' statistics (i.e., the traditional statistics, according to which all states of a given energy are equiprobable) directly to the degrees of freedom of the electromagnetic radiation field. The 'light-quanta' then appear only as *energy levels* of the 'field resonators'. This derivation had first been presented in a famous paper by Debye in 1910 [7] and was well-known to the physics community. It was generally considered as the only consistent application of the quantum hypothesis to the electromagnetic wave field – Planck's original derivation had contained essential elements of classical continuum physics combined with quantum conceptions, and its validity was therefore not beyond doubt. An immediate consequence of Debye's approach is that the quanta lose their individual significance:

only the energy (i.e., the number of quanta) in the various degrees of freedom plays a role in individuating the states of the wave system.

If Debye's derivation of Planck's law is compared with the one given by Bose, the transition from natural statistics to Bose statistics can be seen as the direct result of the change from considering the manifold of energy states to considering the manifold of light-quanta states, when the light-quanta are treated as individual particles. Generalizing this point to the case of the ideal gas, Schrödinger diagnosed the appearance of the new statistics as a consequence of not considering the energy states of the gas as a whole. Natural statistics will therefore lead to the results of Einstein's gas theory if the states of the gas as a whole are treated as excited states of a wave field, instead of as states of a collection of particles. Schrödinger ended his introduction with the following far-reaching conclusion:

> This means nothing else than taking seriously the de Broglie–Einstein wave theory of moving particles, according to which the particles are nothing more than a kind of 'wave crests' on waves constituting the substratum of the world.[1]

Schrödinger's work on quantum statistics, which led him to think that reality is fundamentally wave-like, was a decisive stimulus for him to develop a wave theory [10]. Only six weeks after he submitted 'On Einstein's Gas Theory' for publication, he finished his first article in the series on 'Quantization as a Proper Value Problem' [11]. In it, he proposed what has become known as the Schrödinger equation. Although Schrödinger's introduction of the equation there is highly abstract and mathematical, without direct reference to the wave picture that motivated his discovery, he does remark that he could have presented his results as a generalization of his work on Einstein's gas theory. Further, in connection with his treatment of the hydrogen atom, he states that it is natural to interpret ψ as the description of a *wave phenomenon* in the atom that has more claim to reality than the concept of an electron orbit. In the subsequent articles of the series Schrödinger attempted to elaborate his interpretation of wave mechanics as a description of a wave-like reality. But it soon turned out that there were serious obstacles on the way. The idea that classical material particles be replaced by wave packets eventually foundered in view of the inevitable dispersion of such packets. Furthermore, the waves of the finished

Schrödinger theory are waves in a multi-dimensional configuration-space, and do not, as Schrödinger had hoped, correspond in a one-to-one way with a system of waves in the ordinary, 'real' space of three dimensions. His proposed solution that $e|\psi|^2$ determines a continuous charge distribution in three-dimensional space before long proved untenable. Quantum theory, with its superposition principle, fundamentally operates in configuration space and all attempts at translation in terms of processes in ordinary space unavoidably lose part of the information contained in the original description.

As it became clear that his wave interpretation of quantum mechanics failed, Schrödinger withdrew from contributing further to the physical content of the theory. His later papers on the subject were mostly directed at pointing out alleged difficulties in the Copenhagen interpretation. The great majority of physicists, however, accepted that interpretation, although it seems fair to say that not many have studied its subtle points.

The situation has of course changed in many ways since the days of Schrödinger. Bose–Einstein (B–E) and Fermi–Dirac (F–E) statistics are now related to the symmetry properties of the wave functions. It is generally believed that these symmetry properties are connected with the 'observational indistinguishability' of the particles that are described by the wave function and that in this way a justification of the quantum statistics can be given on the basis of a particle picture. The characteristic differences between quantum statistics and Boltzmann statistics are then regarded as a consequence of the existence of correlations between 'identical' quantum particles.

However, in the following I shall argue that much of Schrödinger's (and De Broglie's) original argumentation still has force. It can still be said that quantum statistics shows the 'unnaturalness' of a particle picture and the 'naturalness' of a field-theoretical interpretation. As will be demonstrated, it is not possible to derive the symmetry (or asymmetry) of wave functions on the basis of a demand of observational indistinguishability of particles. Although arguments with similar conclusions occur in the literature, their validity and implications seem not to have been widely appreciated. The symmetry properties of the quantum mechanical wave functions correspond to something much stronger than observational indistinguishability: they show that even a conceptual distinction between different 'particles' is dubious. We are thus led back to a 'wave picture' instead of a 'particle picture', although a much more abstract one than originally envisaged by Schrödinger. The abstract states of quantum field theory now assume the role that Schrödinger intended for his waves in three-dimensional space. Quantum statistics is the immediate result of the 'natural', uniform, distribution over these states of quantum field theory. There are no correlations between the states according to this statistical distribution. The impression of a correlation between states is only created if individual particle states are artificially added to the theoretical framework of quantum field theory. At the end of the paper I shall finally consider the question to what extent quantum statistics can be said to entail *empirically observable* correlations, not between states, but between measurement outcomes. I shall argue that quantum statistics by itself cannot be expected to give rise to correlations between the results of measurements performed at a distance from each other. If such correlations are found the existence of a 'common background' to the outcomes has to be supposed.

2. STATISTICS IN THE WAVE AND PARTICLE PICTURES

In the second section of his article on Einstein's gas theory Schrödinger compared and contrasted two representations of an ideal gas. According to the usual representation the gas consists of n molecules, each of which has a value for its energy from the energy spectrum $\epsilon_1, \epsilon_2, \ldots, \epsilon_s, \ldots$.

In the alternative approach, the one preferred by Schrödinger, each one of the above energy values corresponds to a degree of freedom of the total system. These degrees of freedom have the characteristics of the one-dimensional harmonic oscillator. That means that the sth degree of freedom is capable of having the energies $0, \epsilon_s, 2\epsilon_s, \ldots, n_s\epsilon_s, \ldots$ (corresponding to $0, 1, 2, \ldots, n_s, \ldots$ molecules in the state characterized by ϵ_s, according to the usual point of view). In this approach the total system is treated as an assembly of linear oscillators (analogous to the case of electromagnetic radiation in a cavity).

Standard methods of statistical mechanics can be applied to such a system of oscillators. The partition sum is given by

$$\sum e^{-1/kT(n_1\epsilon_1+n_2\epsilon_2+\cdots+n_s\epsilon_s+\cdots)}.$$

The sum must be taken over nonnegative integral values for the $\{n_s\}$.

In the case of cavity radiation there would be no further restriction on the values of the $\{n_s\}$. The characteristic feature of the gas system is that the $\{n_s\}$ must also satisfy the condition that $\Sigma_s n_s = n$; according to the usual, corpuscular, point of view this expresses the fact that the number of particles in the gas is fixed. With this condition the standard techniques of statistical mechanics lead to the same results that were found by Einstein via the application of B–E statistics to the system of gas molecules.

It is not difficult to see the basic point in Schrödinger's approach. According to the particle picture a state of the total gas in which there are n_s molecules with energy ϵ_s can be realized in $n!/\Pi_s n_s!$ ways. This multiplicity of the state is a consequence of the individuality of the particles: it makes a difference whether molecule i or molecule j is in the one-particle state with energy ϵ_s. By contrast, there is only *one* state of the system of oscillators with a specific set of 'occupation numbers' $\{n_s\}$. B–E statistics thus attributes to a particle system the multiplicity of states appropriate to a system of oscillators.

The above can readily be generalized to the case of F–D statistics. If we add to the restrictions on the set of occupation numbers $\{n_s\}$ that each one of them take only the values 0 or 1, the states that are considered as equiprobable in the Fermi–Dirac distribution result.

Of course modern quantum mechanics does not give us a picture of a gas as a collection of three-dimensional matter waves, with 'wave crests' representing particles. Schrödinger's original interpretation has convincingly been shown to be untenable. But the *formal structure* of the quantum mechanics of many-particles systems is almost identical to the one inherent in Schrödinger's treatment of a gas. This is most clearly seen in the formalism of quantum field theory. The basic states there are characterized by the *occupation numbers* of the various 'field modes'; they have the form

$$|n_1, n_2, \ldots, n_s, \ldots\rangle,$$

where n_s indicates the number of quanta in the sth mode. These states, which span the Fock space, can be obtained by repeated application of creation operators on the vacuum state. The characteristic difference between bosons and fermions is in the commutation relations between creation and annihilation operators; for the states of the Fock space the resulting difference is that in the case of fermions the occupation numbers can only be 0 or 1 – for bosons there is no such restriction.

It is clear that there is a one-to-one relation between the Fock states of an 'n-particle system' consisting of bosons on the one hand and the Schrödinger gas states on the other hand. Also the typical wave property of superposition is preserved: the Fock states, in their abstract multi-dimensional space, obey a superposition principle just as the three-dimensional Schrödinger waves did. But from the point of view of statistics the most significant fact is that there are exactly as many basic states in the treatment of the 'n-particle system' according to quantum field theory as in the Schrödinger wave treatment.

The formalism of quantum field theory, with its Fock space, is equivalent to what we get from elementary quantum theory if we follow the prescription of symmetrizing (bosons) or anti-symmetrizing (fermions) the wave functions for many-particles systems. Consider, for simplicity, a two-particle system of identical bosons. The states of this system are contained in the space that is the tensor product of two one-particle Hilbert spaces, $\mathcal{H}_1$ and $\mathcal{H}_2$. Suppose that $\{|\psi_i^1\rangle\}$ and $\{|\psi_j^2\rangle\}$ are orthonormal bases of $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively. According to the symmetrization postulate the space of two-particle states is not the full space spanned by the state vectors $|\psi_i^1\rangle \otimes |\psi_j^2\rangle$, but rather the subspace spanned by the vectors

$$\sqrt{\tfrac{1}{2}}\{|\psi_i^1\rangle \otimes |\psi_j^2\rangle + |\psi_j^1\rangle \otimes |\psi_i^2\rangle\}, \qquad i \neq j, \text{ and } |\psi_i^1\rangle \otimes |\psi_i^2\rangle.$$

There is consequently exactly *one* total state in which the one-particle states $|\psi_i\rangle$ and $|\psi_j\rangle$ are occupied. In the general case there is analogously exactly *one* symmetrical total state corresponding to a given set $\{n_s\}$ of occupation numbers, instead of the $n!/\Pi_s n_s!$ states if the full tensor product of one-particle Hilbert spaces were used. Completely analogous results hold for the case of fermions.

The formalism of quantum mechanics therefore accommodates B–E and F–D statistics in a very natural way. These two kinds of statistics directly follow from applying standard, classical, statistical methods to the available quantum states of the system. B–E and F–D statistics consequently emerge as the 'natural' statistics, given the quantum mechanical description. However, if not the quantum mechanical description, but instead a classical particle description is taken as basic, the quantum statistics seem to appear as the consequence of some 'mysterious correlation' between the particles, as Einstein already observed. Suppose that we consider a two-boson system such that the basic states in the individual Hilbert spaces can be labelled H ('heads') and T ('tails'). According to quantum theory there are three possible states in the combined space: $|2, 0\rangle$, $|0, 2\rangle$ and $|1, 1\rangle$, where the first number inside the ket represents the occupation number of the 'H-mode' and the second one the occupation number of the 'T-mode'. With the usual statistical hypotheses, each state gets a probability 1/3. But the natural statistics for a classical particle system would of course assign probabilities 1/4 to the states $\{H, H\}$ and $\{T, T\}$ and 1/2 to the case in which one particle is in state H and the other in state T, because the latter case can be realized in two different ways (i.e., by two different two-particle states). Therefore, quantum statistics seems to be a consequence of a tendency of the particles to 'flock together' which makes the probabilities of the states $\{H, H\}$ and $\{T, T\}$ greater than what had to be expected.

Reversing the argument, the fact that we find B–E and F–D statistics realized in nature, together with the fact that there is in those cases no indication of a dynamic origin of any correlation between particles, could be taken as evidence that the concept of individual particles is not appropriate and that the quantum mechanical states should literally be construed as descriptions of the different states the world can be in. This would lead to a view similar to that of Schrödinger in his article on Einstein's gas theory, but with the difference that the "waves constituting the substratum of the world" would now be the highly abstract states of quantum field theory.

Before discussing this argument in more detail, it is useful to look at the relations between the symmetry properties of the wave functions and the "indistinguishability of identical particles". There are conflicting and sometimes confused statements on this point in the literature. An analysis of the situation will help to make it clearer what exactly is the status of 'identical particles' in quantum theory.

3. IDENTICAL PARTICLES AND INDISTINGUISHABILITY

There are two main positions concerning the status of the symmetrization prescriptions represented in the literature. According to the first of them, the symmetry properties of the wave functions have to be taken as a brute fact of experience [12]. Admittedly there is the connection, in quantum field theory, between spin and symmetry or antisymmetry of the states, as first pointed out by Pauli [13]. But Pauli's result only shows that particles with non-integer spin values cannot consistently be quantized with symmetrical states, and that particles with integer spin cannot be quantized with anti-symmetrical states. From this it does not follow that symmetrical and anti-symmetrical states are the only possible ones. Pauli's result does show that if symmetrical and anti-symmetrical states are the only possible ones, particles with non-integer spin should be fermions and particles with integer spin bosons. But the point at issue here is whether the existence of only symmetrical and anti-symmetrical states can be derived from some deeper principle.

The second position often found is that the symmetry properties of the wave functions follow from the "indistinguishability of identical particles". In this connection, two particles are said to be identical if all their intrinsic properties, like mass, spin and charge, are exactly the same. It follows that two identical particles are indistinguishable in the following sense: it does not make any difference for the properties of a physical system if the two particles are interchanged. No experiment can distinguish between the situation in which particle 1 is in state A and particle 2 is in state B, and the situation that particle 1 is in state B while particle 2 is in state A. The claim now is that the demand that predictions for the outcomes of experiments are indeed insensitive to the permutation of two or more particles in the theoretical expressions, automatically leads to the result that the wave functions for many-particle systems are either symmetrical or anti-symmetrical.

The most sophisticated defences of this claim were presented by Kaplan [14] and Sarry [15]. The starting point of Kaplan's argument is the requirement that the expectation values of all one-particle operators O_i should be independent of the particle index i. If $|\psi\rangle$ represents the state of the many-particle system, this requirement can be put in the form that $\langle\psi|O_i|\psi\rangle$ should be independent of i for all one-particle operators O. The requirement is satisfied for symmetrical and anti-symmetrical states $|\psi\rangle$, and not generally satisfied for arbitrary $|\psi\rangle$ with different symmetry properties. Sarry subsequently showed that there was a small lacuna in Kaplan's argument which made invalid the conclusion that *only* strictly symmetrical and anti-symmetrical $|\psi\rangle$ are allowed. Sarry's own derivation, intended to fill this lacuna, proceeds along the same lines as the one by Kaplan. Suppose that Q is an arbitrary observable of the many-particles system. The requirement of observational indistinguishability in its most general form can be formulated thus:

$$\langle P\psi|Q|P\psi\rangle = \langle\psi|Q|\psi\rangle,$$

or equivalently

$$\langle\psi|P^{-1}QP|\psi\rangle = \langle\psi|Q|\psi\rangle,$$

where the unitary operator P represents a permutation of two or more particles. As this must hold for arbitrary states $|\psi\rangle$ in the state space of the many-particles system, Sarry concludes that the following equality between operators must hold:

$$P^{-1}QP = Q.$$

In other words, *all* observables of the system must commute with *all* permutation operators. Making one specific choice for Q (namely, a particular non-symmetrical operator) Sarry infers that the permutation operators P, which all have to commute with the chosen operator, must form a one-dimensional representation of the permutation group. This in turn implies that all states $|\psi\rangle$ must be either symmetrical or antisymmetrical under permutations.

The above argument does not, however, prove what it is meant to prove. It overlooks the fact that not all operators can be observables for a system consisting of identical particles. It follows from the definition of identical particles, at the beginning of this section, that no experiment can distinguish between situations that differ only in that the roles of two or more particles have been interchanged. It has to be stressed that this is so on quite general physical grounds relating to the nature of the interactions which can be used for the purposes of detection. The argument does not depend on anything peculiar to the formalism of quantum mechanics and is equally valid in classical mechanics. The point is that interactions with a measuring device depend only on the dynamic states of the particles and their intrinsic properties. Particle indices by themselves play no role: it doesn't make any difference whether particle i or particle j in a given dynamical state enters a detector. All interactions remain the same, regardless of the index of the particle that is involved in the interaction. As a result, all measurable properties of the system are independent of particle indices. For instance, for a two-particle system $\mathbf{r}_1 - \mathbf{r}_2$ is *no* observable, but the mutual distance $|\mathbf{r}_1 - \mathbf{r}_2|$ *is*. This fact, that all observables for a system of identical particles are symmetrical on account of the dynamical irrelevance of particle indices, is certainly not new and has been explicitly pointed out in some quantum mechanics textbooks [16]. But its significance for the discussion about the derivability of the symmetry properties of quantum states seems not to have been generally noted.[2]

The basic idea in the proofs reviewed above was that the required symmetry in the expectation values of certain operators leads to restrictions on the possible forms of the wave functions. But it should be clear that this idea can only work if non-symmetrical operators are considered, like the one-particle operators O_i used by Kaplan or the specific Q chosen by Sarry. If only symmetrical operators are considered, the symmetry of the measurement outcomes is automatically guaranteed, and no restrictions on the possible forms of $|\psi\rangle$ can be derived. In particular, Sarry's requirement that $QP = PQ$ is trivially satisfied for all symmetrical operators Q, and nothing can be inferred about the representation of the permutation group to which the operators P belong.

As observables are always symmetric in the case of identical particles, no restrictions on the states of the particles can be derived from observational indistinguishability. It has already been mentioned that this point is quite general, and has in itself nothing to do with quantum mechanics. Also in classical mechanics, identical particles (with the same definition as in quantum mechanics) are indistinguishable in the sense that an interchange of particles has no empirical consequences. Suppose that we have two identical classical particles. At $t = 0$, let particle 1 be in the state $\{\mathbf{r}_0^a, \mathbf{p}_0^a\}$, and particle 2 in the state $\{\mathbf{r}_0^b, \mathbf{p}_0^b\}$. Because the charge, mass, and other intrinsic properties of the particles are the same, there is no observational difference with the situation in which particle 2 is put in the state $\{\mathbf{r}_0^a, \mathbf{p}_0^a\}$, and vice versa. This is of course still true at any later moment, when the two states have evolved in accordance with the laws of motion.

It is often maintained that there is still a decisive difference between the quantum mechanical case and the classical one in that classical particles follow well-defined trajectories through space – the particles can be identified by means of their paths. There is a good deal of truth in this remark, but as it stands it can nevertheless be misleading. The crucial difference between quantum and classical mechanics – with regard to our present discussion – does not reside in the fact that continuous, identifiable, trajectories exist in classical mechanics. Also in quantum mechanics it is possible to have situations with two wave packets that are widely separated in space and that define 'paths' by their evolution in time. The existence of the paths is not the decisive factor in such cases, but rather the fact that if the two paths are occupied by identical particles, the formalism of quantum mechanics yields only *one* state to describe the situation, whereas classical mechanics envisages *two* possible states. The point is that in classical mechanics an individual state, associated with one of the trajectories, is assigned to each one of the particles, whereas there are no different individual states in the quantum case, as we shall shortly see (Section 4). Thus, in classical mechanics by themselves unobservable particle indices are associated with the distinct paths. Although there are no corresponding observable differences, a distinction is then made between the two cases that result from each other by an interchange of indices. There is no such association between indices and paths in quantum mechanics, not even in the situation where quantum mechanics allows a description by means of well-defined paths. However, in the case of widely separated wave packets it can very easily happen that the state gets a double statistical weight in quantum mechanics also; see Section 6. In such situations quantum statistics yields the same probability distribution as Boltzmann statistics, in spite of the fact that there are no individual particle indices in the formalism.

Because quantum mechanics does not operate with unobservable indices, its state attribution stays closer to what is empirically observable than classical mechanics. This point of difference would in principle remain the same even if classical particles would not follow continuous trajectories but would move by making discontinuous leaps determined by an irreducibly stochastic process, so as to make it impossible to experimentally follow an individual path. A classical mechanics modified along these lines could still work with individual particle states (as in the theory of Brownian motion). Going one step further, one could even speculate about variations on classical mechanics in which there is no identity over time of individual particles – theories in which there is no 'genidentity'. In such theories it would no longer be possible to identify two particle states at two different moments as belonging to the history of one and the same particle. Even then the theory could be such that at each moment there are n individual particle states (more or less like the situation in modern formulations of Newtonian mechanics, where there are individual space points at each moment of time but where there is no connection between space points at different moments; so-called neo-Newtonian spacetime). There would still be a big difference between such a theory and quantum mechanics in which there are no different individual particle states (Section 4).

Summing up, the crucial difference between the classical and the quantum mechanical treatment of systems of identical particles is the addition, in the classical theory, of unobservable particle indices to the empirically distinguishable states. Although these indices are themselves not empirically observable, they make their presence felt through the multiplicity of states which is important in statistical considerations.

4. QUANTUM MECHANICS: CONCEPTUAL INDISTINGUISHABILITY OF 'IDENTICAL PARTICLES'

We have seen that the symmetry properties of the quantum mechanical wave functions cannot be derived from observational indistinguishability of identical particles. That only symmetrical or anti-symmetrical wave functions are allowed for the description of identical-particle systems must be considered to be an independent postulate which expresses something that is stronger than mere observational indistinguishability. Indeed, in a system consisting of identical 'particles' – described by a symmetrical or anti-symmetrical wave function – the individual 'particles' are all in *literally the same state*, to the extent that one can speak of well-defined one-particle states at all. In the preceding section we emphasized that quite generally there is no observational difference connected with a permutation of particle indices over different one-particle states; but in quantum mechanics there even are no different one-particle states that could serve as 'carriers' of indices and thus of the individuality of particles.

Let us consider, for simplicity, a symmetrical two-particle state built from orthonormal states $|\phi\rangle$ and $|\psi\rangle$:

$$|\Psi\rangle = \sqrt{\tfrac{1}{2}}\{|\phi^1\rangle \otimes |\psi^2\rangle + |\psi^1\rangle \otimes |\phi^2\rangle\}.$$

The superscripts 1 and 2 refer to the Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively, whose tensor product contains the two-particle states. The spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ can be thought of as isomorphic copies of one single-particle space $\mathcal{H}$, in which $|\phi\rangle$ and $|\psi\rangle$ are part of an orthonormal basis. Now, since $|\Psi\rangle$ is not a simple product of two one-particle states, it is not possible to associate pure states in $\mathcal{H}_1$ and $\mathcal{H}_2$ with $|\Psi\rangle$. The best one can do if one wishes to extract one-particle states from $|\Psi\rangle$ is to form the partial traces of $W \equiv |\Psi\rangle\langle\Psi|$:

$$W_1 \equiv \sum \langle n|\Psi\rangle\langle\Psi|n\rangle, \qquad W_2 \equiv \sum \langle m|\Psi\rangle\langle\Psi|m\rangle,$$

with $\{|n\rangle\}$ and $\{|m\rangle\}$ orthonormal bases of $\mathcal{H}_2$ and $\mathcal{H}_1$, respectively. This leads to states in $\mathcal{H}_1$ and $\mathcal{H}_2$ which are not pure, but are represented by *mixtures*; and it is easy to see that these two mixtures are *identical*:

$$W_1 = W_2 = \tfrac{1}{2}|\phi\rangle\langle\phi| + \tfrac{1}{2}|\psi\rangle\langle\psi|.$$

The same result would have been obtained if we had started from an anti-symmetrical state for the two-particles system.

We can conclude that in a symmetrical or anti-symmetrical two-particles state generally no pure states can be assigned to individual particles. The only exception is the case that $|\Psi\rangle$ has the symmetrical form $|\phi\rangle \otimes |\phi\rangle$; then of course the one-particle states are exactly the same. In the general case only *mixtures* are defined in the one-particle spaces; and again these are exactly the same. So in all cases it is not possible to differentiate between 'particles' on the basis of the properties of their states.[3] Consequently, there is nothing in the quantum mechanical description to associate particle indices with; neither intrinsic properties nor dynamical states individuate particles. The indices in the expressions only refer, through a conventional numbering, to the various Hilbert spaces, as mathematical objects. At this point it is clear that the appropriateness of the particle-concept itself becomes doubtful. If the quantum mechanical description is considered complete, 'particles' are not only observationally indistinguishable; they are conceptually indistinguishable.

This result can without difficulty be generalized to the n-particle case. Here also we find that symmetrical and anti-symmetrical wave functions in the tensor product space $\mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3 \cdots \otimes \mathcal{H}_n$ yield precisely the same reduced states in each one of the spaces $\mathcal{H}_i$. The reverse is also true. If we ask what states in the tensor product space give exactly the same reduced states in all spaces that are k-fold ($k < n$) tensor products of individual Hilbert spaces (the individual spaces themselves are special cases), the answer is that only the completely symmetric and the completely anti-symmetric states will do so. For the proof of this it is sufficient to inspect again Sarry's above-mentioned argument. If the reduced states are the same in all spaces of the form $\mathcal{H}_{i_1} \otimes \mathcal{H}_{i_2} \ldots \otimes \mathcal{H}_{i_k}$ all k-particle operators must have expectation values that are independent of the values and order of the k particle-indices (because such expectation values can be calculated in the reduced state). Note that this must hold for all operators, not just for symmetrical observables. As a consequence, Sarry's proof now goes through, and all permutation operators must be represented by multiplication by either +1 or −1. In other words, the state $|\Psi\rangle$ must be either completely symmetric or completely anti-symmetric under permutations.

Absence of different particle states on the one hand and symmetry or anti-symmetry of the wave function on the other are therefore strictly equivalent. But if there are no different individual particle states with which particle indices can be associated, the very essence of the particle idea seems to be lost. A way out of this dilemma would of course be to maintain that the quantum mechanical description is incomplete, and that there are underlying individual particle states in a more complete description. Something more will be said about this in the following section. But if the quantum mechanical description is accepted as complete, it turns out that the situation very nearly mirrors the one considered by Schrödinger in 1925. There we had a proposed formalism for the treatment of a gas in which occupation numbers of "modes of vibration" defined the total state of the gas. Here, in the context of quantum mechanics, we have the symmetric or anti-symmetric states of the total system which are analogously individuated by occupation numbers. In Schrödinger's gas theory the question as to the individual behaviour of quanta does not even arise; as far as it makes sense at all to assign dynamical states to quanta, all quanta are in exactly the same state. In the quantum theory of 'many-particle systems' the question of how the particles behave individually of course does arise naturally. This is, however, a consequence of the way the subject is commonly introduced, namely as a generalization for many degrees of freedom of the quantized version of a classical one-particle system. If one starts from the finished theory with its symmetrical or anti-symmetrical states, or equivalently from the formalism of quantum field theory, which is perfectly tailored to the situation, there is no reason to conceive of particles with individual states; formally the state of affairs is the same as in Schrödinger's gas theory.

5. 'NATURAL' STATISTICS

Let us return to statistics. The central point in Schrödinger's argumentation for the reality of his matter waves was the observation that B–E statistics for particles could be replaced by 'natural' statistics for the states of a matter field, and that there is therefore reason to believe that these latter states represent the actual states of the world. By 'natural' statistics in this connection is meant the statistics that gives equal weight to all states belonging to a particular value of the energy. Of course, one of the ideas behind the adjective 'natural' is that we have here a principle that is very familiar from classical statistical mechanics. Giving equal probabilities to all states of a particular energy is at the basis of the micro-canonical ensemble and consequently at the basis of the whole of classical statistical mechanics. But is there also an objective reason, something different from familiarity, why attributing equal probabilities to states would be 'more natural' than other assignments of probabilities? Why shouldn't we just accept B–E or F–D statistics, when these are found to be empirically adequate, as the basic statistics for particle states, without any further explanation [18, 19, 20, 21]? Schrödinger wrote that this would be against 'natural intuition', but surely that can be no conclusive argument. Perhaps it is just a brute fact of nature that some particle states occur more often than other ones.

The just-mentioned point of view would be very plausible if there would be no connection between the probability distributions used in statistical mechanics and the physical characteristics of the systems that are described. We would then not be in a position to justify, on theoretical grounds, the choice of one distribution over another. But as a matter of fact, there *are* physical grounds on which it is possible to base a preference for 'natural' statistics. In classical statistical mechanics the traditional justification of the micro-canonical ensemble does not consist in an appeal to some dubious a priori Principle of Indifference, but relies on physical arguments related to ergodic theory in one form or another. It was especially Einstein who, at the beginning of this century, urged that statistical physics should not start by just *postulating* a probability distribution over states, but should always relate an assignment of probabilities to the dynamical behaviour of the system under consideration [22]. Einstein in particular advocated equating probabilities of states with times of sojourn in those states, a motive for much research in subsequent decades. Although not all problems in the foundations of statistical mechanics have been solved, there has been important progress in ergodic theory and connected areas since those days. It has been established that many-particle systems as a rule exhibit very irregular, pseudo-random, behaviour (chaos). This makes statistical concepts applicable. Further, the 'mixing' property possessed by such systems ensures that, in the long run, subsets of the energy hypersurface are visited with frequencies that are equal to their measures in phase space. Finally, in cases with very many degrees of freedom it also plays a role that the values of macroscopic quantities are very insensitive to the exact position on the energy hypersurface representing the microstate. These factors ensure that microscopic details are almost always unimportant for the macroscopic properties of a system. The fact that the micro-canonical ensemble 'works' thus is a symptom of the physical, dynamical, irrelevance of parameters other than the energy for macroscopic quantities. Jaynes' way of identifying suitable ensembles for all kinds of situations, by means of the so-called maximum-entropy principle, can be seen as a systematic method of discriminating between those parameters that are macroscopically relevant and those that are not. In this method the appropriate ensemble is determined as the one giving equal weights to all states that do not differ in characteristics that are dynamically relevant at a macroscopic scale.

Generally then, the attribution of equal probabilities to states has a physical background. The equiprobable states are physically equivalent with respect to the things that can be calculated with the probability distribution. In cases where only a limited number of degrees of freedom are involved, this physical equivalence usually consists in the occurrence of chaotic, uncontrollable, transitions between the states. Research during the last decades has shown that the occurrence of such chaotic phenomena is virtually inevitable and has to be expected in even the simplest systems. Given the ubiquity of chaotic behaviour, an empirically found adequateness for predictions of the uniform distribution can be taken as tentative evidence that the states in question are the only relevant ones for the calculations in which we are interested, and that they are physically equivalent in the sense that chaotic transitions between these states occur.

It turns out that the core of the above remains valid in quantum mechanics. If a classical system is chaotic, then its quantum counterpart is extremely sensitive to external perturbations, and shows the characteristics of chaos and mixing [23]. It is therefore possible to argue as follows in the cases of B–E and F–D statistics. We know that a uniform distribution over symmetrical (or anti-symmetrical) states gives the right statistical results. This is an indication that those states are the only ones which are relevant for the magnitudes we are interested in, and are equivalent in the sense that there are no physical mechanisms favouring one of them over the others. Conversely, seen from the theoretical side, this is in agreement with what quantum mechanics itself teaches us about available states and their dynamical evolution. As discussed before, the symmetrical (or anti-symmetrical) states are the only ones in the Hilbert space of the system; they can to some extent be compared with phase points in the phase space of a classical system and show the same type of chaotic behaviour.

By contrast, if we assume that a symmetrical state

$$\frac{1}{\sqrt{2}}\{|\phi^1\rangle \otimes |\psi^2\rangle + |\psi^1\rangle \otimes |\phi^2\rangle\}$$

really corresponds to two particle states, one in which particle 1 is in state $|\phi\rangle$ and particle 2 in state $|\psi\rangle$, and one with the particles interchanged, we also have to assume that these two individual states have a probability that is only half as great as the probability that both particles are in state $|\phi\rangle$, or both in state $|\psi\rangle$. In view of the omnipresence of chaotic behaviour leading to equal probabilities of states (not just as an empirical fact, but also as something that has to be expected on theoretical grounds), it is justified to ask what specific mechanisms are responsible for the different behaviour here. But there is no known physical mechanism which could account for the apparent lack of equivalence. Hence we come back to the possibility that we should stop at this juncture, and content ourselves with the observation that the particle states just have different probabilities, apparently as a basic fact of nature [18, 19, 20, 21].[4]

Although this position cannot be rejected on grounds of logic alone, the above considerations show that it is methodologically weak. Many-particle systems governed by known interactions, and even almost all systems governed by (analytical) hamiltonians existing only on paper, exhibit the kind of dynamical behaviour that leads to chaos. For these systems it has to be expected that a uniform distribution over states is appropriate in statistical considerations; deviations from a uniform distribution must have an identifiable cause in physical processes favouring one case over another. The connection with dynamics thus makes it possible to understand in what situations the statistical distribution will be uniform and in what situations this will not be the case. By contrast, if the connection with dynamics is not taken into account, it can only be said that a uniform distribution sometimes is, and sometimes isn't appropriate; it then seems justified to accept any empirically found distribution as a basic fact of nature. Consideration of the link with dynamics allows a deeper analysis. In that analysis it is *not* justified to accept any given distribution. It has to made clear what special circumstances are responsible for non-uniform statistics.

The principle that a physical process that is responsible must be supposed to exist if one physical state occurs significantly more often than another one, if this is not expected on dynamical grounds, thus naturally fits in with modern physical theory. It is therefore justified that this principle plays a pivotal role in scientific methodology in the way it actually does. Quantum mechanics interpreted as a complete theory whose states are *basic* descriptions – not corresponding to multiples of underlying particle states – is in complete harmony with this principle.

The view that not the quantum mechanical states but instead states of individual particles are fundamental has consequently at least two methodological problems to face.[5] First, in problems where statistical mechanics plays no role, quantum mechanics is able to do without an individual particle picture, and is empirically well-supported. The onus of proof is therefore with the supporters of a particle picture that the features that must be added to accommodate individual particles do not just constitute superfluous excess metaphysical baggage. The problem is actually more serious than that, because there are well-known problems for hidden-variables theories in even *reproducing* well-confirmed quantum mechanical predictions (Bell-inequalities), something they must be able to do before new predictions – demonstrating that the additional variables are not superfluous – are in order. But secondly, there is the problem that the particle states cannot receive equal weights in statistical calculations, even if there are no known dynamical factors that can discriminate between them. A 'particle interpretation' of quantum mechanics should in a non-ad-hoc way make it clear why the usual mechanisms that cause chaotic transitions between states are not effective in the case of the basic particle states.

6. QUANTUM CORRELATIONS

It has already been emphasized that the 'naturalness' of the statistics in which all accessible independent states get equal probabilities is a consequence of the equivalent dynamical roles played by these states. All considerations concerning chaotic behaviour, ergodicity, etc., relate to the dynamics of the system, and all conclusions drawn with respect to equivalence of states consequently apply to the dynamical states only. Now, in quantum mechanics there is, unlike in classical mechanics, no simple one-to-one connection between measurement results and dynamical states. The arguments for the uniform distribution over states therefore do not simply carry over to the distribution over possible measurement outcomes. Indeed, the fact that the basic quantum mechanical states are – by virtue of their symmetrical or anti-symmetrical form – in general not products of one-particle states, already shows that probability distributions for joint measurements will often be non-factorizable so that correlations between measurement outcomes may result.

There are two possible sources of such 'quantum correlations' between results of measurements. First, measurement results in a single (anti-)symmetrical state may show statistical correlations, as just indicated. That is, if an experiment is repeated with each time the same initial 'many-particle' quantum state, there need not be statistical independence between measured one-particle attributes. This can be so even if there is no interaction term in the Hamiltonian of the system. Consider, for instance, two momentum eigenstates in one dimension. In the position representation these states are proportional to $\exp(ipx/\hbar)$ and $\exp(ip'x'/\hbar)$, respectively. A simple product state proportional to $\exp(ipx/\hbar)\cdot\exp(ip'x'/\hbar)$ would give rise to independent uniform probability distributions for the positions of the two particles. However, the symmetrized states proportional to

$$\exp(ipx/\hbar)\cdot\exp(ip'x'/\hbar) \pm \exp(ip'x/\hbar)\cdot\exp(ipx'/\hbar)$$

lead to more complicated expressions for the probability that one detector is triggered at position x_1 and another one at position x_2. This probability is proportional to

$$1 \pm \cos\{(p-p')(x_1-x_2)\}/\hbar\}$$

with the plus and minus sign for the symmetrical and anti-symmetrical case, respectively. It is clear from this expression that the chance that 'bosons' are found in each other's vicinity is greater than has to be expected on the basis of statistical independence, whereas the reverse is true for 'fermions'.

The second possible source of correlations comes from natural, uniform, statistics applied to the symmetrized quantum states. Suppose that there are two possible states for a simple boson system: $|H\rangle$ ('heads') and $|T\rangle$ ('tails'). There are then three possible states for a composite system ("two identical particles"), namely $|H, H\rangle$, $|T, T\rangle$, and $|H, T\rangle$, and each one of these receives a probability of 1/3. This leads to the probability 1/2 for the measurement outcomes 'heads' and 'tails', $P(H) = P(T) = 1/2$, whereas the joint probability for 'heads' and 'tails' is 1/3, $P(H\&T) = 1/3$. Clearly, $P(H\&T) > P(H)P(T)$, so that there is a positive statistical correlation between the outcomes 'heads' and 'tails'.

The distribution over states also entails non-classical results in cases where the (anti-)symmetrical form of the wavefunctions has a consequence for the energy. This leads (via the Boltzmann factor) to statistical weights for the states that differ from the ones that would be applicable if the states were simply product states. In this way it can happen that states that exhibit a particular type of correlation get additional statistical weight. Such effects are important in the theory of magnetism, where correlations between spins can sometimes be accounted for on the basis of the lower Coulomb energy of the correlated spin state. For instance, if it is the symmetrical spatial wave functions that have the lowest Coulomb energy, electron spins will tend to be anti-parallel, in virtue of the anti-symmetry of the total wave function.

It is therefore true that the formalism of (anti-)symmetrical states, and natural statistics applied to them, can lead to classically unexpected correlations, and that this has important physical effects as in the theory of the periodic system and the theory of magnetism. However, it is *not true*, in contradistinction to what has sometimes been suggested in the literature [24], that the formalism naturally leads to the appearance of correlations between distant measurement outcomes without there being a common origin to the events or an interaction term in the Hamiltonian of the system.

Consider again the case of a composite 'two-particle' state. A first remark, often made in this context, is that there will be no correlation between results of measurements of a wide class of observables if the two one-particle states from which the composite state has been formed do not overlap in space. Contemplate the state

$$C\{|\phi^1\rangle \otimes |\psi^2\rangle + |\psi^1\rangle \otimes |\phi^2\rangle\}$$

with C a normalization constant. The expectation value of a symmetrical observable Q in this state differs from its value in the unsymmetrized state $|\phi^1\rangle \otimes |\psi_2\rangle$ by the 'interference term' $2|C|^2\langle\phi^1\psi^2|Q|\psi^1\phi^2\rangle$. If $|\phi\rangle$ and $|\psi\rangle$ can be represented by spatial wave packets that do not overlap (the case corresponding to the classical case of distant particles) this interference term vanishes for observables Q which are local functions of the position operators.

But more important is that even if the wave functions *do* overlap, as in the case of the momentum eigenfunctions mentioned above, there is in general no reason to expect a correlation between the results of measurements on the two 'particles'. The correlation inherent in the (anti-)symmetrical quantum state only shows itself experimentally, as an actual correlation in a series of measurement outcomes, if there is a relation between the successive initial states with which the experiment is performed. In the example that we discussed above, the (approximately) same difference between the values of p and p' must be realized in each individual experiment. If p and p' vary freely and independently from experiment to experiment, we have to average the term $\cos\{(p-p')(x_1-x_2)/\hbar\}$ over the p and p' values, with the result that no correlation remains. Now, it is a fact of experience in quantum mechanics no less than in classical mechanics, that in order to ensure that there is a relation between p and p' in successive experiments, there must either be a suitable past interaction that is the same in repetitions of the experiment (as in the Einstein–Podolsky–Rosen case), *or* there must be a selection mechanism that picks out systems with momenta that have a more or less constant relation to each other. In the first case the systems have a common history of interaction. In the second case two kinds of situations can be distinguished. It may be that p and p' vary from experiment to experiment and that some device is responsible for a constant difference between them. In that case the question as to the origin of the correlations is shifted to the question of how the mechanism is able to produce correlated values of p and p' again and again. In its essence this situation is not different from the one in the first case; experience teaches that such persistent correlations do not occur unless eventually a region in the intersection of the past light cones of the two measurement events is identifiable in which there is a local interaction that is responsible for the correlations. There might for instance be an experimenter who locally prepares the two-particle system and sets a knob on an apparatus in a certain position.

There remains a second kind of situation. Here p and p' are selected independently, for instance at widely separated locations, but in such a way that the same values of p and p' are chosen in successive experiments. In this situation there are separate selection mechanisms for p and p'. They must possess a memory, so that there is a correlation between the values chosen in successive experiments, but there need not be a relation between the distant mechanisms. To schematize the case, assume that $|\alpha\rangle$ and $|\beta\rangle$ are the states in which the selection devices are left behind after they have selected $|p\rangle$ and $|p'\rangle$, respectively. We then have: $\langle\alpha|\beta\rangle = 0$. The combined system of particles and selection devices is represented by a ket-vector of the following form:

$$\frac{1}{\sqrt{2}}\{|\alpha p\rangle|\beta p'\rangle \pm |\beta p'\rangle|\alpha p\rangle\}.$$

In this state description it has been made explicit that the selector states $|\alpha\rangle$ and $|\beta\rangle$ have become correlated to $|p\rangle$ and $|p'\rangle$, respectively. The important point is that any observable that pertains only to the states $|p\rangle$ and $|p'\rangle$ will have an expectation value in the above states that is equal to the expectation value it has in a simple product state of $|p\rangle$ and $|p'\rangle$. This is so because the interference terms contain $\langle\alpha|\beta\rangle$ or $\langle\beta|\alpha\rangle$ as a factor, which makes them vanish. But the typical quantum correlations occur as a consequence of just these interference terms, so that they will not be detectable under the described conditions. It may be added that the constant relation between p and p' which is detectable (the same values of p and p' will be found in succesive experiments) is not what normally would be called a *correlation* between p and p'. For a correlation we would require that there is a constant relation between p and p' in spite of variations in these quantities.

We can conclude that in the discussed case an empirically detectable correlation between measurement results will appear only if there is a non-empty intersection of the past lightcones of the two measurement events, with a physical state that can appropriately be called the common origin of the correlation between the measurement outcomes.

The same kind of reasoning applies to more complicated cases. An important example is furnished by two-fermion systems with spin. There are anti-symmetrical wave functions that can give rise to typical quantum correlations between spin measurements, e.g., those wave functions that can be factorized in a spatial part and a spin part of the following form:

$$|s_a^1\rangle \otimes |s_b^2\rangle - |s_b^1\rangle \otimes |s_a^2\rangle.$$

As before, correlations only materialize in actual experiments if the same relations between $|s_a\rangle$ and $|s_b\rangle$ are reproduced over and over again. If $|s_a\rangle$ and $|s_b\rangle$ vary independently no correlation will survive. A constant relation can result if there is some interaction that is responsible for a repetition of the same total spin state, as in the Einstein–Podolsky–Rosen case (Bohm's spin version). An additional possibility in a fermion system is that the spin correlations are directly (i.e., without the intervention of any interaction) attributable to the symmetry properties of the total wave function. This occurs if the spatial parts of the one-particle wave functions coincide exactly; the spins must then be anti-parallel. Finally, it can be that a spatial wave function of a particular symmetry type is advantageous energetically, if an inter-particle interaction influencing the energy of the system is effective. This can in turn lead to spin correlations (as in the case of magnetism). In these cases either an interaction or a coincidence of spatial parts of wave functions is a necessary condition for the occurrence of empirically observable correlations.

Quite generally, statistics in and over quantum states only leads to the appearance of empirical correlations at a distance if there is, or has been in the past, an overlap of spatial wave functions or an interaction.

Let us discuss in somewhat more detail an important mechanism that is operative here. The applicability of probabilistic considerations to states has a physical, dynamical, background (Section 5) and the appropriateness of one or another statistical distribution therefore depends on the physical characteristics of the situation. To see the relevance of this remark consider the following situation, which is analogous to the 'heads and tails example' discussed in the beginning of this section. Suppose that there are two one-particle boson states represented by wave packets that do not overlap and that have also always been widely separated in the past; let $|L\rangle$ (left) and $|R\rangle$ (right) denote these two states. Now it is important to note that the two-particle states $|L, L\rangle$, $|R, R\rangle$ $|L, R\rangle$ are not as a matter of course dynamically equivalent under the stated conditions. There will not automatically be ergodic or chaotic behaviour connecting the three states, which are far apart from each other in Hilbert space. The appropriate statistical distribution therefore entirely depends on how exactly the three states are actually produced. In particular, it is not justified to posit an a priori probability 1/3 for each one of the states. Now assume, to add the lacking information in a realistic way, that the three states are randomly prepared by a locally operating device. It is of course possible that the device operates in such a way that a probability 1/3 for each one of the three two-particle states results. For this, we should have three equivalent states of the device, which are chosen with equal probabilities by an experimentator, or between which uncontrollable transitions occur. This latter situation could be found if the selection process is located in one spatially limited region (and the result subsequently transmitted to the regions occupied by the wave packets). However, if it is independently decided by random processes in the two distant regions whether or not a particle will be prepared in the state $|L\rangle$ or $|R\rangle$, different values of the probabilities should be used. In this case it is not difficult to see that the two one-particle states acquire individual characteristics. For instance, they will be produced shortly after one another, so that we get the following possible states:

$$|L_1,L_2\rangle,\ |R_1,R_2\rangle,|\ L_1,R_2\rangle,\ |L_2,R_1\rangle,$$

where the subscripts 1 and 2 refer to the times at which the states have been created. Natural statistics applied to these states, i.e., a uniform distribution over them, yields the classically expected probabilities for 'left' and 'right' without correlation between them. Furthermore, if a preparation mechanism works this way, by associating individual characteristics with one-particle states, all effects of interference between these states are destroyed, as we discussed before for the two momentum eigenstates; the particle states will become correlated with orthogonal states of the preparation device. This mechanism will obliterate the difference between classical and quantum statistics in all cases where independently operating selection mechanisms are involved. In such situations the classical particle concept becomes applicable.

The general point is that in situations where there has not been an overlap, interaction or common origin of wave functions, there is no reason to expect the appearance of correlations between distant measurement results on the basis of quantum statistics alone.

In summary, there are in quantum mechanics sometimes correlations when we do not expect their presence on the basis of classical theory. The following principle, although not representing an a priori truth, remains however empirically well supported as a central point of methodology, also in quantum mechanics. If correlations are found between physical magnitudes measured at distant points in space, it is justified to seek an 'explanation' for these correlations. Such an 'explanation' can consist in the identification of a common origin, an interaction between the regions in the past lightcones of the measurements, or in the demonstration that there have been coinciding spatial wave functions such that the correlation is a direct consequence of the symmetry requirements. Distant correlations cannot be accounted for solely on the basis of the symmetry properties of the wave functions, or merely on the basis of the applicability of B-E or F-D statistics.

7. CONCLUSION

I have attempted to show that taking the states of quantum field theory seriously, as corresponding to the different states the world can be in, results in a satisfactory and consistent scheme. The introduction of individual particle states is empirically superfluous and moreover leads to conceptual, methodological and technical difficulties.

As an additional example of this, consider the following question which has sometimes been asked: if an electron is created in a distant galaxy, how does it know that it has to anti-symmetrize its state with respect to the electrons in our neighbourhood, and how is it able to do so? Such questions clearly have as their background a picture of individual particles, in which the quantum mechanical states only serve to describe (statistical) regularities in their behaviour. By contrast, if the states of the quantum field theory are accepted as giving the complete spectrum of physical states of the world, such questions do not arise. Every creation process is automatically described as the transition from one (anti-)symmetrized state to another one. Something similar can be said with regard to the issue of "how particles can be individuated" in quantum theory. We have seen that all 'one-particle states' are identically the same in the formalism of completely symmetrical or anti-symmetrical states. It has therefore often been queried how reference to individual particles is possible in the theory; how the relation of reference between particle-index i and the corresponding denoted particle can be established. Again, the question does not arise if the quantum field states are taken as basic. These states are individuated in an unproblematic fashion by their sets of occupation numbers. Individual particle indices play no role in the finished formalism. They only occur in the construction of the states from the states in 'single-particle Hilbert spaces'. In that context, the indices can be taken to refer to the individual Hilbert spaces, as mathematical objects, and *not* to individual particles. There is no problem with reference in this case either.

The concept of individual particles is fraught with difficulties in quantum mechanics. In a realistic interpretation of quantum mechanics (in the philosophical sense) the abstract states of quantum field theory must therefore be considered to be the prime candidates for providing descriptions of the world. In this sense, one could say that the abundant empirical evidence that supports the validity of the general characteristics of the quantum formalism also supports the thesis that individual particles do not exist in nature. Of course, there are well-known problems in giving a realistic interpretation to quantum mechanics. The focus of these difficulties is in the relation between the formalism and measurement results; this is connected with the notorious measurement problem of quantum mechanics. Further, there is the question of how the states of quantum field theory, defined as they are in an abstract mathematical space, can be descriptive at all of anything in the real world. The discussion of these issues would take us too far afield here. Let it suffice here to say that I think they can be satisfactorily dealt with; they are the subject of other papers [25].

NOTES

[1] De Broglie had in fact given a similar argument in the statistical chapter of his thesis [8]. Commenting on the correlations between particles carried by the same "accompanying wave", he stated "we can no longer take the single atoms as 'objects' of the general theory, it is the elementary stationary phase waves which must play this role". For more on de Broglie's reasoning and its historical background, see [9].

[2] It should be noticed that the preceding argument for the symmetry of observables only depends on the equality of particle properties relevant for physical interactions. Observational indistinguishability can then be *derived*. On this point our argument differs from similar ones in the literature [12, 18] in which a principle of observational indistinguishability is taken as the starting point. If the latter line is followed it remains unclear whether indistinguishability should be interpreted as entailing a restriction on *states* or on *observables*.

[3] This point has been noted in the literature; see [17, 20].

[4] As Redhead remarks [18, 19], it is then sufficient to postulate as an *initial condition*

that only symmetrical or anti-symmetrical states occur. In view of the symmetry of the Hamiltonian the symmetry character of the states is conserved in time.

[5] That an interpretation of quantum field theory in terms of individual particles can *logically speaking* always be upheld is stressed by Redhead in [18]. In a sequel to this article [19] he however favours a field interpretation, on grounds independent from the ones adduced here. He mentions in particular the difficulties that arise if it is attempted to give the vacuum state of quantum field theory an interpretation in terms of particles and if "virtual particles" are treated as really existing entities.

REFERENCES

[1] Ehrenfest, P.: 1911, 'Welche Züge der Lichtquantenhypothese spielen in der Theorie der Wärmestrahlung eine wesentliche Rolle?', *Ann. d. Phys.* **36,** 91; Ehrenfest, P. and H. Kamerlingh Onnes: 1914, 'Simplified Deduction of the Formula from the Theory of Combinations which Planck Uses as the Basis of his Radiation Theory', *Proc. Amsterdam Acad.* **17,** 870; also in M. J. Klein (ed.): 1959, *Paul Ehrenfest, Collected Scientific Papers*, North-Holland, Amsterdam.

[2] Bose, S. N.: 1924, 'Plancks Gesetz und Lichtquantenhypothese', *Z. Phys.* **26,** 178; English translation, 1976, *Am. J. Phys.* **44,** 1056.

[3] Einstein, A.: 1924, 'Quantentheorie des einatomigen idealen Gases', *Sitz. ber. Preuss. Akad. Wiss.*, 261.

[4] Einstein, A.: 1925, 'Quantentheorie des einatomigen idealen Gases; 2. Abhandlung', *Sitz. ber. Preuss. Akad. Wiss.*, 3.

[5] Hanle, P. A.: 1979, 'The Schrödinger–Einstein Correspondence and the Sources of Wave Mechanics', *Am. J. Phys.* **47,** 644.

[6] Schrödinger, E.: 1926, 'Zur Einsteinschen Gastheorie', *Phys. Z.* **27,** 95.

[7] Debije, P.: 1910, 'Der Wahrscheinlichkeitsbegriff in der Theorie der Strahlung', *Ann. d. Phys.* **33,** 1427.

[8] de Broglie, L.: 1924, 'Recherches sur la théorie des quanta', *Ann. de Phys.* **3,** 22.

[9] Bergia, S.: 1983, 'Who Discovered the Bose–Einstein Statistics?', *Conf. Proc.* '*Symmetries in Physics* (1600–1980)', San Feliu de Guixols; S. Bergia, C. Ferrario, and V. Monzoni: 1985, 'Side Paths in the History of Physics: The Idea of Light Molecule from Ishiwara to De Broglie', *Riv. Stor. Sci.* **2,** 71.

[10] Wessels, L.: 1979, 'Schrödinger's Route to Wave Mechanics', *Stud. Hist. Phil. Sci.* **10,** 311.

[11] Schrödinger, E.: 1926, 'Quantisierung als Eigenwertproblem, 1. Mitteilung', *Ann. d. Phys.* **79,** 361.

[12] A seminal paper here is Messiah, A. and O. W. Greenberg: 1964, 'Symmetrization Postulate and Its Experimental Foundation', *Phys. Rev.* **B136,** 284.

[13] Pauli, W.: 1940, The Connection Between Spin and Statistics', *Phys. Rev.* **58,** 716.

[14] Kaplan, I. G.: 1975, 'The Exclusion Principle and Indistinguishability of Identical Particles in Quantum Mechanics', *Sov. Phys. Usp.* **18,** 988.

[15] Sarry, M. F.: 1979, 'Permutation Symmetry of Wave Functions of a System of Identical Particles', *Sov. Phys. JEPT* **50,** 678.

[16] See, for instance: Dirac, P. A. M.: 1958, *The Principles of Quantum Mechanics*,

Clarendon Press, Oxford, p. 207; C. Cohen-Tannoudji, B. Din, and F. Laloë: 1973, *Mechanique Quantique*, Hermann, Paris, p. 1382.
[17] French, S. and M. Redhead: 1988, 'Quantum Physics and the Identity of Indiscernibles', *Br. J. Phil. Sci.* **39**, 233.
[18] Redhead, M.: 1983, 'Quantum Field Theory for Philosophers', in *PSA* 1982, vol. 2, P. Asquith and T. Nickles (eds.), Philosophy of Science Association, East Lansing, Michigan.
[19] Redhead, M.: 1988, 'A Philosopher Looks at Quantum Field Theory', in H. Brown and R. Harré (eds.), *Philosophical Foundations of Quantum Field Theory*, Clarendon Press, Oxford.
[20] van Fraassen, B.: 1984, 'The Problem of Indistinguishable Particles', in J. Cushing, C. Delaney, and G. Gutting (eds.), *Science and Reality*, University of Notre Dame Press, Notre Dame, Indiana.
[21] van Fraassen, B.: 1972, 'Probabilities and the Problem of Individuation', *Probabilities, Problems and Paradoxes*, S. Luckenbach (ed.), Dickenson, Encino, p. 121.
[22] Einstein, A.: 1909, 'Zum gegenwärtigen Stand des Strahlungsproblems', *Phys. Z.* **10**, 185.
[23] Peres, A.: 1988, 'Schrödinger's Immortal Cat', *Found. Phys.* **18**, 57; and references contained therein.
[24] van Fraassen, B.: 1985, 'Salmon on Explanation', *J. Phil.* **82**, 639.
[25] Dieks, D.: 1988, 'The Formalism of Quantum Theory: An Objective Description of Reality?', *Ann. d. Phys.* **45**, 174; Dieks, D.: 1989, 'Quantum Mechanics and Realism', *Conceptus* **23**, 31; Dieks, D.: 1989, 'Quantum Mechanics Without the Projection Postulate and its Realistic Interpretation', *Found. Phys.* **19**, 1395.

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