Euler's "Letters to a Princess of Germany" As an Expression of his Mature Scientific Outlook

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In the past few decades historians and scientists have increased and broadened their study of the Swiss-born mathematician and theoretical physicist LEONHARD EULER (1707–1783). The publication during this time of most of the 74 quarto volumes of his *Opera Omnia* and large portions of his correspondence has prompted and provided a sound base for their research, which has made increasingly clear the pivotal institutional and intellectual role which he played in the history of the exact sciences during the eighteenth century.[1] They have definitely established his leadership, indeed dominance, at mid-century within the St. Petersburg and Berlin Academies of the Sciences (the principal centers of scientific research then in Russia and Prussia respectively).[2] They have also delved deeply into his truly remarkable intellectual achievements. Brilliant and prolific, he wrote over 866 books and articles, which alone account for approximately on third of the entire corpus of research on mathematics, theoretical physics, and engineering mechanics published from 1726 to 1800.[3] In his most significant work he extensively developed the calculus; laid the founda-

[1] See VALENTIN BOSS, *Newton and Russia: The Early Influence, 1698–1796* (Cambridge, Mass.: Harvard University Press, 1972); ERIC G. FORBES, *The Euler-Mayer Correspondence (1751–1755): A New Perspective on Eighteenth-Century Advances in the Lunar Theory* (New York: American Elsevier Publishing Company, Inc., 1971); GEORGE E. OWEN, *The Universe of the Mind* (Baltimore: The Johns Hopkins Press, 1971); C. TRUESDELL, *Essays in the History of Mechanics* (New York: Springer-Verlag Inc., 1968); and EDUARD WINTER (ed.), *Die Deutsch-russische Begegnung und Leonhard Euler* (Berlin: Akademie-Verlag, 1958).

[2] Residency in St. Petersburg 1727–1741 and 1766–1783 (Director of Mathematics Class); residency in Berlin 1741–1766, including Director of Mathematics Section, 1744–1766. See BOSS, *op. cit.*, pp. 93–230; RONALD S. CALINGER, "Frederick the Great and the Berlin Academy of Sciences (1740–1766)", *Annals of Science*, **24**, 3 (1968), pp. 239–251; *Idem*, "The Newtonian-Wolffian Confrontation in the St. Petersburg Academy of Sciences (1725–1746)", *Cahiers d'histoire mondiale*, XI, 3 (1968), pp. 417–436; ALEXANDER VUCINICH, *Science in Russian Culture: A History to 1860* (Stanford, Calif.: Stanford University Press, 1963), pp. 75–247 and 295–329; and EDUARD WINTER (ed.) *Die Registres der Berliner Akademie der Wissenschaften* 1746–1766 (Berlin: Akademie Verlag, 1957), pp. 25ff.

[3] C. TRUESDELL, "Leonard Euler, Supreme Geometer (1707–1783)" in HAROLD E. PAGLIARO (ed.), *Irrationalism in the Eighteenth Century* (Cleveland: The Press of Case Western Reserve University, 1972), pp. 53–54.

tions of analytical mechanics, especially in his *Theory of the Motions of Rigid Bodies* (1765); and conducted fundamental research in hydrodynamics, hydraulics, ship theory, vibrations of fluids and solids, elasticity, acoustics, lunar theory, and the wave theory of light.

In general, recent studies of EULER bear upon but do not resolve the long standing disagreement regarding his mature scientific outlook. At issue is his response to the three major natural philosophies of the eighteenth century: the CARTESIAN, the LEIBNIZIAN (with its WOLFFIAN offshoot), and the NEWTONIAN. This article will examine his scientific outlook as expressed in his *Letters to a Princess of Germany* (three volumes, 1768–1772), the most exhaustive and authoritative treatment of natural philosophy written by a leading scientist in the eighteenth century. At the time of its publication, EULER stood at the peak of his career: he was the director of the mathematics class (division) of the St. Petersburg Academy and an established scholar within the European scientific community. Moreover, although he sent the individual letters between 1760 and 1762, he culled them from articles dating back to 1727. They offer, therefore, an outline of the scientific positions he accepted and the modifications occurring in them.[4]

Through the years EULER's mature scientific outlook has been interpreted in different ways. Very early, the French philosophe VOLTAIRE (1694–1778) wrongly considered him a LEIBNIZIAN;[5] but the dominant view from the late eighteenth century until recently held him to be basically a CARTESIAN – a view his two main biographers of the 1920's, GUSTAV DU PASQUIER and OTTO SPIESS, supported.[6] The EULER, who searched for a mechanical explanation for attraction, who accepted the ether, and who rejected NEWTON's optics, in combination with the mistaken stereotype of him as a pure analyst in the exact sciences largely account for their position. Others have portrayed him as a pre-NEWTONIAN or as generally hostile to NEWTON[7] – views which are consistent with the CARTESIAN appellation, which in a modified form still dominates. As succinctly stated by the Russian historian A.P. YOUSCHKEVITCH, EULER "stood closer to (the) Cartesian natural philosophy than to (the) Newtonian".[8] These CARTESIAN interpretations are not without their critics. Current scholarship describing the NEWTONIAN dimensions to his research has called them into question. For example, the American physicist CLIFFORD TRUESDELL has demonstrated how EULER's articulation of the principle of linear momentum and

[4] His criticisms of the LEIBNIZIAN (WOLFFIAN) monad theory offer an example of a modification in his thought. His comments of 1761 on the monadic doctrine were less acerbic but more thorough than those of 1747. See RONALD CALINGER, "The Newtonian-Wolffian Controversy", *Journal of the History of Ideas,* XXX, 3 (1969), pp. 322–323 and LEONHARD EULER, *Lettres à une princesse d'Allemagne* in *Leonhardi Euleri Opera Omnia*, Third Series, ed., ANDREAS SPEISER, vol. 11, letter CXXV, pp. 295–296.

[5] THEODORE BESTERMANN (ed.), *Voltaire's Correspondence* 107 vols. (Geneva: Institut et musée Voltaire, 1953–1965), vol. 7 (1746) and Voltaire, *Oeuvres Complètes de Voltaire* 52 vols. (Paris: Garnier Frères, 1877–1885), vol. 23, t. 2, pp. 576–578.

[6] L-GUSTAV DU PASQUIER, *Leonard Euler et ses amis* (Paris: Librairie Scientifique J. Hermann, 1927) and OTTO SPIESS, *Leonhard Euler* (Leipzig: Verlag von Huber & Co., 1929).

[7] V. BOSS, *Newton and Russia*, p. 212.

[8] A.P. YOUSCHKEVITCH, "Leonhard Euler" in *Dictionary of Scientific Biography* vol. IV (New York: Charles Scribner's Sons, 1971), p. 481.

his rigorous definition of mass advanced NEWTON's mechanics.[9] Indeed, TRUESDELL has found him a powerful adherent of NEWTON, because he largely carried out the program set in the preface of the *Principia*: to derive the phenomena of nature from mechanical principles by mathematical reasoning. The British historian ERIC FORBES has shown how his study of lunar motion (or more generally the three body problem in astronomy) strengthened the then incomplete NEWTONIAN explanation of long term stability for the operation of the solar system.[10] Such research has reinforced an earlier finding of historian EDUARD WINTER. After investigating the early St. Petersburg and Berlin Acadamies, WINTER portrayed him as an eclectic who had NEWTONIAN leanings in mechanics and who held theology to be the queen of the sciences.[11] Thus the disagreement continues.

As these studies indicate, no conclusive determination of EULER's mature scientific outlook yet exists – to say nothing of the stages in its development, which this paper does not treat. In the past the lack of a thorough understanding of the complexity and changing character of the CARTESIAN, LEIBNIZIAN, and NEWTONIAN natural philosophies seriously hampered such research. This impediment no longer exists. Recent scholarship by improving our understanding has given us a sound base from which to work.[12] Its collective weight shows that a conclusive determination will require a thorough reinvestigation of EULER's thought in mechanics, optics, theory of matter, and epistemology – the principal areas of development in the physical sciences and their methodology in the eighteenth century. This article will examine both the positive and negative content of his thought in these areas as presented in the *Letters*. The *Letters* supports WINTER's eclectic interpretation, but with one important modification: the mature EULER did not consider theology to be the queen of the sciences; rather he held that faith, reason, and experience stood on the same level in leading to truth when applied to problems in their separate spheres of competency in the spiritual and phenomenal aspects of the world.

The Letters

The 234 letters in the collection originated in lessons given to Princess CHARLOTTE LUDOVICA LUISA, a second cousin of FREDERICK THE GREAT. EULER wrote

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[9] C. TRUESDELL, *Essays*, pp. 114–136.

[10] E. FORBES, *Euler-Mayer Correspondence*, pp. 12–21.

[11] E. WINTER, *Die Deutsch-russiche Begegnung*, p. 2.

[12] See E.J. AITON, *The Vortex Theory of Planetary Motions* (New York: American Elsevier Inc., 1972); YVON BELAVAL, *Leibniz critique de Descartes* (Paris: NRF, 1960); I.B. COHEN, *Franklin and Newton* (Philadelphia: The American Philosophical Society, 1956); JEAN EHRARD, *L'idée de nature en France dans la première moitié du XVIII^e^ siècle* (Paris: S.D.V.P.E.N., 1963), C.C. GILLISPIE, *The Edge of Objectivity* (Princeton, N.J.: Princeton University Press, 1960); HENRY GUERLAC, "Newton's Changing Reputation in the Eighteenth Century", in RAYMOND O. ROCKWOOD (ed.), *Carl Becker's Heavenly City Revisited* (Ithaca, N.Y.: Cornell University Press, 1958); ROBERT E. SCHOFIELD, *Mechanism and Materialism* (Princeton, N.J.: Princeton University Press, 1970); WILSON L. SCOTT, *The Conflict between Atomism and Conservation Theory 1644–1860* (London: Macdonald, 1970); and ARNOLD THACKRAY, *Atoms and Powers* (Cambridge, Mass.: Harvard University Press, 1969).

them within a period of two years (1760–1762). The princess was only 15 years old when the correspondence began, and each letter attests to his care in the instruction of his youthful pupil. He had undertaken this assignment at the request of her father, the future Margrave FRIEDRICH HEINRICH VON BRANDENBURG-SCHWEDT, who was also his close friend: the two men shared an interest in music, and EULER sometimes visited the Margrave in his Berlin castle. His ready agreement to teach the girl mathematics, *i.e.* the exact sciences,[13] did not serve to ingratiate him to FREDERICK II, who had earlier exiled the Margrave's spouse to the Kolberg Fortress in Silesia. The Margrave held the noble title to Schwedt, a territory northeast of Berlin, which FREDERICK II wanted to assume for himself. Since the couple had only two young daughters and FREDERICK II forbade visitation rights, the land would ultimately revert to him. This situation may account for the *Letters*' having remained unpublished until after EULER had returned to St. Petersburg from Berlin.

In the *Letters* EULER wrote with insight and clarity on the subjects of cosmography, philosophy, theology, theoretical physics, and technology. Because of his success in gathering into unity immense tracts of experience through mathematical description, he approached the sciences in a unified (or *wissenschaftlich*) manner. Further, he believed that the sciences closely interrelated with other fields of knowledge. The German word *Wissenschaften*, which embraces philosophy and probably reflects the earlier view of "scientia", describes his view of them better than its narrower English counterpart, sciences (*exakte Wissenschaften*), which clearly distinguishes the natural sciences from other fields.

The *Letters* contains three natural divisions: (1) general science: letters 1–79, (2) philosophy: letters 80–133, and (3) physical questions: letters 134–234. After writing section one in terms understandable to a 15-year old, EULER penned more difficult letters (dealing with philosophy) for section two. Both sections are germane to his scientific outlook. In section two, for example, he elucidated his theory of matter, accepted in part LOCKEAN sensationalism in psychology, and rejected the CARTESIAN and LEIBNIZIAN (WOLFFIAN) epistemologies. He also examined the relations between metaphysics and physics and between physics and philosophy. Metaphysics, he held, could not certify physical postulates, laws, and operations; but physics did provide an external test for judging the validity of a philosophy. Consequently, the philosophic letters of section two provide an additional insight into his physical views. In addition, letters eighty-five through ninety-two cover his own thought and are rated highly as a contribution to German philosophy.[14] At the request of his student he discussed only physical questions in section three. In May 1761 EULER had visited the Princess and her father at Magdeburg. She informed him that she could no longer understand his letters completely – during May he had written only on monads. Henceforth, he promised to confine himself exclusively to physical questions – a promise he kept. In section three his search for generalization in the sciences clearly emerged.

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[13] EULER, *Lettres*, p. vii of vol. 11 of the edition cited in footnote 4.

[14] *Ibid.*, p. xiv.

The *Letters* met with prodigious success, as the European reading public quickly recognized its merits.[15] By 1800 it had been translated from the original French into eight other languages – Russian, German, Dutch, Swedish, Italian, English, Spanish, and Danish. By 1840 it ran to over forty editions. A comment of the French encyclopedist DENIS DIDEROT in *Rameau's Nephew* (written in 1760's; 1st edition, 1805) in another connection fully applied: "One needs a profound knowledge of art or science to have a good grasp of their elements. Works of classic rank can only be produced by those who have grown grey in harness. The middle and end illuminate the obscurity of the beginnings".[16] Or again, a comment of VOLTAIRE nicely described EULER's effect: "The ignorant understood, the learned admired him". GOETHE (1749–1832) praised him. EULER had strenuously opposed NEWTON'S optics; so too did GOETHE in his treatise *On the Theory of Colors* (1790–1808). Shortly before his most productive period IMMANUEL KANT (1724–1804) read the *Letters*. Only afterward did he criticize WOLFFIAN philosophy in a manner similar to EULER, whom he ranked second only to NEWTON as a scientist.

The *Letters* stands among the foremost scientific popularizations of the eighteenth century, although they were much more. Their author maintained the high standards set by the French philosopher BERNARD FONTENELLE (1657–1757), who disseminated CARTESIAN scientific thought beginning in the last quarter of the seventeenth century. Indeed, EULER had read FONTENELLE and had discussed his concept of the plurality of worlds.[17] According to the late ALEXANDRE KOYRÉ, the *Letters* may be included among prominent NEWTONIAN popularizations: PEMBERTON's *View of Sir Isaac Newton's Philosophy* (London, 1728), VOLTAIRE'S *Philosophical Letters* (Paris, 1734) and *Elements of Newton's Philosophy* (Amsterdam, 1738), ALGAROTTI'S *Newtonianism for Women* (Naples, 1737), MACLAURIN'S *Account of Sir Isaac Newton's Philosophical Discoveries* (London, 1746), and LAPLACE'S *System of the World* (Paris, 1796).[18] These works presented the intricate mathematico-physical and experimental science of NEWTON in terms understandable to the European reading public.

The *Letters* was, however, not a strictly NEWTONIAN popularization. In it EULER critically and competently examined different scientific theories. Thus his keen mind did not accept NEWTON's natural philosophy *in toto* and attempt only to present it in less abstruse form. Rather he capably analyzed what he considered to be its strengths and weaknesses. By his succinct and cogent comments, he pointed out its lines of influence and development in the late

[15] It may be surmised that EULER really wrote the *Letters* for the European reading public. The range and depth of materials covered went beyond basic instruction for an adolescent. Indeed they taxed the wits of savants. Literary convention combined with his commitment to writing readable educational texts probably determined the format of the *Letters* and its avowed teaching purpose. It followed over two decades of heated debates with the WOLFFIANS, a physiological compulsion apparently motivating him to prepare it as a vehicle to clarify his own scientific thought to the European republic of letters.

[16] JONATHAN KEMP (ed.), *Diderot: Interpreter of Nature* (New York: International Publishers, 1963), p. 260.

[17] EULER, *Lettres*, vol. 11, letter LX, pp. 132–133.

[18] ALEXANDRE KOYRÉ, *Newtonian Studies* (Chicago: The University of Chicago Press, 1968), p. 18.

eighteenth century, when substantial criticisms of NEWTONIAN optics and matter theory existed despite the magisterial eminence of the Cambridge mathematician. He also assessed the competing systems of the CARTESIANS and the WOLFFIANS as well as his own research and that of his contemporaries on such topics as lunar theory and electricity. His exposition of the CARTESIAN system showed the persistence of some of its ideas after its demise as an independent scientific system in the 1740's.

The *Letters* treated all three major natural philosophies in greater depth than did other popularizations. Two major NEWTONIAN works with which it may be compared were VOLTAIRE's *Philosophical Letters* and MACLAURIN's *Account of Sir Isaac Newton's Philosophical Discoveries.* Their critiques of CARTESIAN science were sketchy, their discussion of LEIBNIZIAN thought perfunctory. VOLTAIRE, for instance, asserted at one point that NEWTON invented the calculus and LEIBNIZ deserved no credit for it. As the material below will attempt to show, EULER not only examined in detail the thought of DESCARTES and NEWTON, but also thoroughly and forcefully refuted most LEIBNIZIAN (WOLFFIAN) physical doctrines. He was one of the very few leading scientists who had been well grounded in all three of the major schools of thought and could speak knowledgeably on each.

Major Natural Philosophies

A brief account of the central concepts of the CARTESIAN, LEIBNIZIAN (WOLFFIAN), and NEWTONIAN natural philosophies follows, since these are covered at length in the *Letters.*

First in time came the purely rationalistic and mechanistic system of the French philosopher and physicist RENÉ DESCARTES (1596–1650). He placed the structure of science upon an axiomatic foundation universal and self evident serving as an absolute ground for deduction. In his methodology observation and experiment played a subordinate role to reason. Deductive thinking originated in the mind, which was a most fundamental part of his world. Indeed he divided all created existence into *matter* (for him simply extended substance) given motion at the creation and *mind* (unextended thinking substance). For completion, his epistemology required that the disparate parts of this CARTESIAN dualism be connected. His follower ARNOLD GEULINCX added this by making God's ubiquitous presence the link connecting matter and mind (occasionalism).

DESCARTES summarized his thoroughly geometrized physics in his *Principles of Philosophy* (1644). His physics included a mechanistic model of the universe, which was a plenum consisting of a system of vortices (or *tourbillons*) with movement provided by action at contact (impulsion). He defined the "quantity of motion" as the product of the magnitude and speed of a body (or essentially momentum) and asserted its conservation. It should be noted that he had no proper concept of mass and did not treat speed as a vectorial quantity. Thus he did not have the modern concept of momentum. For him the basic particle of matter was the indefinitely divisible, brittle non-atom. In optics he advanced a doctrine of the instantaneous transmission of light, which for him was a pressure, a mechanical property of a luminous body and of a transmitting medium.

The Saxon scholar GOTTFRIED WILHELM LEIBNIZ (1646–1716) set forth another natural philosophy. An integral part of his unified theory of knowledge (or *philosophia perennis*), it embraced the elements of truth from past systems of thought, including the CARTESIAN. Although largely a grand synthesis, it was original in its entirety. It expressed a marked metaphysical and methodological concern for justifying and investigating the world. LEIBNIZ expounded the former concern in his monadic doctrine; the latter, in pure rationalism. These were the two cornerstones of his natural philosphy.

The monadic doctrine dealt with the ultimate nature of substance and provided the foundation for his cosmology. The monads (in Greek *unities*) were metaphysical points of force (energy) which formed a continuum of degrees of awareness. LEIBNIZ had posited them after rejecting the two major theories of matter of his time—DESCARTES' identification of matter with extension and GASSENDI's corpuscular philosophy with its indivisible, impenetrable, passive atoms. The monads differed markedly from both. Since they were metaphysical and not geometrical, the concepts of extension, shape, and divisibility did not apply to them. Further, they contained an elastic force and were living centers of a kind of perception. They differed from each other only in the greater or lesser degree of clarity which they perceived. Infinite aggregates of them gave the appearance of extension. These multiplicities made up corporeal bodies. Over the well-ordered collection of monads existed the absolute clarity, the highest unity, Who was God. The world was His creation. A single law had to govern the world; it was the law of optimism: the real world is the best of all possible worlds. Logically an infinite number of worlds was possible but God selected, or better yet defined, one containing a maximum of goodness (perfection). Associating plenitude with goodness LEIBNIZ depicted the universe as a plenum. Filled with animate monads, it was a dynamic organism endowed with a spirit of "becoming" rather than "being".[19] It had two parts: the phenomenal and the spiritual. Comprehension of that universe required first the teaching of the phenomenal, which was preeminently the subject of physics, and then of the more fundamental spiritual (or monadic) realm, whose truths required metaphysics for their elucidation. The two realms were not disjoint: the contingent truths of the phenomenal were grounded in the absolute truths of the monadic, which provided a moral order base for the sciences. A continuum of knowledge, which he described as a "labyrinth", connected them. Above all, his world was intelligible. It provided clear explanations to the seeker.

Only an organic rationalism could establish and clearly explain the fundamental problems in LEIBNIZ's world. In his *New Essays* (completed in 1707 and published posthumously in 1765) and *Theodicy* (1710), he espoused the primacy of reason for solving the problems of science, metaphysics, and theology. His deductive methodology had two axioms: the principle of contradiction (or

[19] LEIBNIZ's organic universe contrasted sharply with the mechanistic models of DESCARTES and NEWTON. Germanic philosophers had not explained the idea of living nature by the theories of machinery and atomism. Instead, the Neo-platonic tradition, which LEIBNIZ followed, favored an analogy between nature and organism.

identity) drawn from ARISTOTLE's *Posterior Analytics* and *Categories* and the principle of sufficient reason drawn from PLATO's *Timaeus*. He assigned reason a superior epistemic status over experience in his epistemology, which rested on the doctrine of the pre-established harmony. This doctrine held that God had established in the beginning the ideally designed program to coordinate the activities of the soul and body of man. LEIBNIZ used the analogy of two synchronized clocks running totally independently of one another. No direct interaction between mind and matter could occur because the monads were autarkic (windowless) and reflected the entire universe. Learning, therefore, did not proceed primarily through experience (sense perception). All ideas were virtually innate.[20]

LEIBNIZ chiefly employed physics to investigate the phenomenal world because he believed that all natural events were mechanistic. Thus, for him physics was founded on mechanics, which is divided into statics and dynamics. He wrote on the foundations of dynamics, the science dealing with problems of moving bodies. He showed competence in dealing with specifics but not in making generalizations. His approach and his basic concepts were sound. He used the calculus to solve some problems and studied forces by considering the actions of particles not individually but as part of a mechanical system. Following his mentor HUYGENS, he based his dynamics upon two fundamental scalar quantities: *vis viva* (in modern notation mv^2, where m = mass and v = velocity of a particle) and work function (understood as potential energy). He understood neither in their modern sense. As a unifying principle, he posited the law of the conservation of *vis viva.*

LEIBNIZ's natural philosophy lay scattered through a multitude of books, articles, and letters until his chief disciple, the Silesian philosopher CHRISTIAN WOLFF (1679–1754), organized and elaborated it. A far less powerful thinker, he presented a modified version to the European republic of letters in his seven volumes of *Vernünftige Gedanken...* (*Rational Thoughts...*, 1713–1724). Since the two philosophies were not identical,[21] the term LEIBNIZO-WOLFFIAN, which WOLFF abhorred, will not be used.

The Englishman ISAAC NEWTON (1642–1727) produced the third natural philosophy. He presented its core in two major works: the *Principia mathematica* (1687) and the *Opticks* (1704). In the *Principia* he developed a general dynamics by providing sound definitions; creating new concepts; and by organizing, deriving by mathematics, and recasting known but separated phenomena and laws (particularly KEPLER's so-called three laws of planetary motion and GALILEO's laws of free falling bodies). In this work he first formulated the universal law of gravitation (attraction), under which he united celestial and terrestrial dynamics. Attraction operated within a nearly vacuous universe rather than a plenum,

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[20] GOTTFRIED WILHELM VON LEIBNIZ, *New Essays on Human Understanding* trans. by A.G. LANGLEY (Evanston: The Open Court Publishing Company, 1949), p. 46.

[21] For an account of the major philosophical differences between them see CHARLES A. CORR, "Christian Wolff and Leibniz", *Journal of the History of Ideas* XXXVI, 2 (1975), pp. 241–263. A good introduction to LEIBNIZ's original writings is GOTTFRIED WILHELM LEIBNIZ, *Philosophical Papers and Letters*, trans. and ed. by LEROY E. LOEMKER (Dordrecht, Holland: D. Reidel Publishing Co., 1970).

although he did make some allowance for an ether.[22] He based his dynamics upon two fundamental quantities: momentum (*mv*) and force (*F*). He associated mass with inertia and set forth his three axioms of motion. He developed the quantitative relations between the forces acting on an object and the change in the object's motion. But he neither analyzed nor specified the properties of systems of forces in general. The universe he portrayed as a clocklike machine. Stressing rigor, he employed synthetic geometrical proofs but with an approach different from the classical Greek style. Like HUYGENS, he based his proofs on a theory of limits (an embryonic stage of the calculus). From the *Principia* emerged a consistent (non-contradictory) system of equations, which he recognized might have a restricted field of application, *i.e.,* his might be a closed theory, which indeed it is.[23]

NEWTON delineated his optics, matter theory, and methodology in his second major work, *Opticks.* It contained his corpuscular (emission) theory of light with an allowance in the queries for a possible wave theory. Here he also implied the nutshell theory of matter, which held that matter occupies but a small portion of space and is extremely tenuous. The hard, indivisible, passive atom was his basic particle of matter. His particulate (atomistic) view of the universe contrasted with LEIBNIZ's projected continuum. In methodology NEWTON adopted critical empiricism, which he called the method of analysis and synthesis.[24] Therein he directly anchored his theory in experience.

The brief accounts above highlight major areas of thought to which EULER responded, They are obviously not exhaustive. NEWTON's interest in alchemy and natural magic and LEIBNIZ's interest in teleology and apologetics, for example, are not explored. Moreover, EULER did not restrict his critiques to the thought of DESCARTES, LEIBNIZ, and NEWTON. He recognized that each of them had advanced a dynamic system of thought. By the mid-eighteenth century the partisans of each had elaborated, refined, and sometimes corrupted their respective natural philosophies. EULER reacted at times to these developments. Finally, he judged doctrines on the basis of their own merits and not the reputation of their authors. He particularly sought to find which doctrines offered the best available methods for solving given problems. As will be shown, he always required that the scientific ideas and positions he accepted be consistent with the basic laws and assumptions of NEWTON's mechanics.

The Scientific Outlook of Euler

In the *Letters* EULER accepted the NEWTONIAN vision of the universe, except for its optical explanations. He wrote: "...the system of Newton made at first a great noise, and with good reason, as no one before had made such a fortunate discovery, and which diffused at once so much clear light over every branch

[22] Sir ISAAC NEWTON, *Sir Isaac Newton's Mathematical Principles of Natural Philosophy and his System of the World* trans. by ANDREW MOTTE with notes by FLORIAN CAJORI (Berkeley, Calif.: University of California Press, 1934), vol. II, pp. 674–675.

[23] *Ibid.*, vol. II, p. 400.

[24] Sir ISAAC NEWTON, *Opticks* (1730) the version of EINSTEIN, WHITTAKER & COHEN (New York: Dover Publications, Inc., 1952), pp. 404–405.

of science".[25] He called the law of universal gravitation an "established" and "incontestable fact",[26] a fact that he believed was principally established by the following research: the confirmation of the oblate spheroid shape of the Earth, a proof of which NEWTON had given; and studies showing that attraction, not the action of vortices, controlled the orbits of comets and the movements of tides.[27] (It should be borne in mind that after the members of the Paris Academy of Sciences debated these topics in the 1720's and 1730's, they generally accepted NEWTONIAN science in the 1740's. In Paris and with EULER the NEWTONIAN position triumphed in each instance.)[28]

In examing these topics EULER advanced well-conceived empirical explanations and criticized inadequate empirical and strictly deductive ones. In discussing the shape of the earth, for example, he rejected the view that the earth was a prolate ellipsoid (lemon-shaped), which the French astronomer J.D.CASSINI (1677–1756), a CARTESIAN spokesman, had advanced in his book *On the Size and Figure of the Earth* (1722). This finding, which was based on the measurements of arcs of meridian in the north and the south of France, contradicted NEWTON's (and HUYGENS') physics, which projects an oblate spheroid (tomato-shape). Since CASSINI's measurements were not sufficiently separated in latitude, the Paris Academy had outfitted geodetic expeditions to Peru (1735–1744) and to Lapland (1736–1737) to resolve the issue. Many had accepted as conclusive the return of MAUPERTUIS from Lapland in 1737 with measurements confirming NEWTON's view. But EULER felt that more was needed. He also appealed to the research of the French physicist ALEXIS CLAIRAUT (1713–1765) in fluid mechanics, notably his *Theory of the Figure of the Earth* (1743), which correctly projected a modified oblate spheroid (orange-shape), and to the precise measurements of the expedition to Peru.[29] His discussion of comets followed in the same methodological vein. He noted that the return of HALLEY's comet in 1759 corroborated NEWTON's theories—another reference to the research of CLAIRAUT, this time as expressed in *Theory of the Motion of Comets* (c. 1760) and *Research on the Comet* (1762). EULER greatly admired CLAIRAUT and found his conclusions regarding comets convincing. Thus he held that NEWTON's explanation of the elongated orbits of comets was correct and that future "exact observations" would conclusively verify it.[30]

On the subject of tides, he praised DESCARTES' observation that they were regulated by the motion of the moon, but criticized his fallacious claim that they were caused by pressure from the moon on the air of the atmosphere

[25] EULER, *Lettres*, letter LIII, p. 118. In another letter he offered a general appraisal of NEWTON: "Newton was without doubt one of the greatest geniuses who ever lived. His profound knowledge and his penetration into the most hidden mysteries of nature will always be the most brilliant subject of our admiration and that of our posterity. But the errors (*i.e.*, optics) of this great man should serve to humble us and to point out the weakness of the human spirit, which after having risen to the highest degree possible of which humans are capable, risks nevertheless plunging into the most gross errors"; letter XVIII, p. 44, *Note:* The above and following translations are my own from the French original.

[26] *Ibid.*, letter LIV, p. 120.

[27] *Ibid.*, letters XLV–LXIX, pp. 102–149.

[28] EULER praised the Paris Academy and observed that the chief advances in mechanics came through its biennial prize competitions. *Ibid.*, letter LVII, pp. 127–128.

[29] *Ibid.*, letters XLVIII, pp. 107–110.

[30] *Ibid.*, letter LIX, pp. 130–131.

and, thus, by pressure on the water.[31] He considered the NEWTONIAN attractionist explanation of the tides directly contrary to the CARTESIAN one. He maintained that experience must decide which to accept.[32] Simple observation of the moon at its zenith and nadir provided the necessary proof; the absence of a corresponding change in the tides disproved the CARTESIAN position and strengthened the NEWTONIAN explanation, which he adopted.[33]

EULER struggled to find a precise mechanical explanation for attraction. NEWTON had not presented one. Empirical evidence outweighed both theological and semantic arguments. Thus, he rejected the assertion of British NEWTONIANS that God had endowed bodies with this property, because he felt this amounted to a most improbable "perpetual miracle".[34] Here he stood closer to NEWTON than did his British followers, for in his letters to the Anglican cleric and philologist RICHARD BENTLEY (1662–1742) NEWTON argued that gravity was not an inherent property of bodies.[35] EULER also dismissed the continuing charge of the CARTESIAN impulsionists that attraction was no more than an "occult quality", one of those unsubstantiated verbal categories with which decadent scholasticism abounded.[36] To resort to an occult quality was to beg an issue. To say, for example, that opium produces sleep, because it has a sleep inducing (occult) quality was in his opinion no explanation at all. Attraction, however, acted on matter[37] and produced an effect that could be measured. It was, therefore, not an occult but what the ARISTOTELIANS called a manifest quality. Only its cause remained to be discovered.[38]

In search of a precise definition for attraction, EULER apparently turned to the scholium of section XI of the second edition of the *Principia*. Here NEWTON explained that the terms "attraction" and "impulse" have no definite meaning but are mathematical forces. NEWTON wrote: "I here use the word attraction in general for any endeavor whatever, made by bodies to approach each other, whether that endeavor arises from the action of the bodies themselves, as tending to each other or agitating each other by spirits emitted; or whether it arises from the action of the ether or of the air, or of any medium whatever, whether corporeal or incorporeal, in any manner impelling bodies placed therein towards each other."[39] In examining if either of these

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[31] *Ibid.*, letter LXII, pp. 138–139. He rejected as "ridiculous" the KEPLERIAN explanation that the earth was a living organism and its respiration caused the tides.

[32] *Ibid.*, letter LXVI, p. 143.

[33] *Ibid.*, letters LIII, p. 118 and pp. 139–141. He had shared the prize of the Paris Academy in 1740 for his paper generalizing the NEWTONIAN theory of the tides. EULER also accepted NEWTON's lunar theory and praised the precise work of the Göttingen astronomer TOBIAS MAYER (1723–1763) in confirming and extending it. Letter LXI, p. 135.

[34] *Ibid.*, letter LXVIII, p. 148.

[35] I.B. COHEN (ed.), *Isaac Newton's Papers and Letters on Natural Philosophy* (Cambridge, Mass.: Harvard University Press, 1958), letter III, p. 302.

[36] EULER, *op. cit.*, letter LXVIII, pp. 148–149.

[37] *Ibid.*, letter XLV, pp. 102–103.

[38] NEWTON, *Opticks*, p. 401.

[39] NEWTON, *Principles*, vol. I, p. 192; see also *Opticks*, p. 376. For himself EULER believed that "impulsion" and "attraction" correctly understood had the same effect. He felt that there were sometimes useless "battles of words" involving these terms. With the ambiguity of the CARTESIAN impulsionists and the precision of NEWTON, he largely adhered to the term attraction in the *Lettres*. Letter LIV, p. 120.

two causes were correct, EULER was drawn into the realm of metaphysics, where he believed that research on the causes of attraction properly fell.[40] For once he agreed with LEIBNIZ.[41] While the effects of attraction were measurable, its cause(s) were not subject to quantifiable study. Thus mathematics could not provide the answer. Since the answer lay in metaphysics, he concluded that the prospects for success in his investigation were slim. Nevertheless, he again repudiated the above contention that attraction might be an intrinsic property of matter, for he never accepted action-at-a-distance or animate matter.[42]

After thus rejecting the first section of NEWTON's statement, he adopted its second (CARTESIAN) explanation. It was consistent with his belief that the ether filled all of space.[43] He wrote: "It is always better to believe that what one calls attraction is a force contained in the subtile matter (the ether) which fills the whole space of the heavens; though we do not know how."[44] In discussing the lodestone he had given one example of this when he stated: "One cannot doubt, however, that there is very subtile, though invisible, matter (the ether), which produces this effect (attraction) by actually impelling the iron towards the lodestone."[45] Still, he held that attraction, not impulsion, was the valid physical explanation of such phenomena. He believed that the members of the Paris Academy on the expedition to Peru had proved this when they "observed nearby a very high and prodigious mountain the effect of a slight attraction, by which the body of the mountain attracts neighboring bodies. Thus, in adopting the system of attraction, one does not have to fear that it will lead us to false consequences: one can always be assured in advance of their verity."[46] The empirical verification of the NEWTONIAN theories on the shape of the Earth, the tides, and comets had helped confirm the verity of attraction rather than impulsion.

EULER included in the defense of NEWTONIAN mechanics a discussion of its first law of motion with its concept of inertia. It stated that "every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it."[47] He called it "the principal law of motion and rest" and maintained that it provided "the foundation of the entire science of motion, which one calls mechanics".[48] After equating its certainty with that of a geometrical truth,[49] he asserted that all who had made great discoveries in mechanics agreed unanimously that

[40] EULER, *Lettres*, letter LXVIII, p. 147.

[41] LEIBNIZ believed that the fundamental truths of physics were grounded in metaphysics.

[42] EULER, *Lettres*, letter LXXV, p. 164.

[43] *Ibid.*, letter LXVIII, p. 148.

[44] *Ibid.*, letter LXXV, p. 164. He continued: "We must accustom ourselves to acknowledge our ignorance on a number of other important subjects." Like the Italian humanist JULIUS CAESAR SCALIGER (1484–1558) in his *Exotericarum exercitationum liber* (1537), he viewed nature as having an extreme (if not infinite) subtlety. He believed that there were limitations to the power of human intelligence to unravel the arcanae (secrets of nature).

[45] *Ibid.*, letter LV, p. 122.

[46] *Ibid.*, letter LIV, p. 122.

[47] NEWTON, *Principles*, p. 13.

[48] EULER, *Lettres*, letters LXVIII–LXIV, pp. 158–164.

[49] *Ibid.*, letter LXXI, p. 154.

their researches were founded solely upon this principle.[50] He then turned to the positions of different scientific philosophies on this law. The CARTESIANS supported it since DESCARTES had essentially presented it in his three laws of motion[51] in book two of the *Principles*, providing a background for NEWTON. Indeed EULER expressed this law as a combination of the first two laws of DESCARTES.[52]

The chief opposition to the law of inertia, which came from the ARISTOTELIANS and WOLFFIANS, drew his criticism. He joined the large number of critics who challenged the ARISTOTELIAN theory that rest is the natural state of bodies. Believing that this allegation stemmed from a mistaken interpretation of experiments on motion, he cited an experiment with a billiard ball.[53] According to the ARISTOTELIANS the ball, after being set in motion, would stop due to its intrinsic propensity to rest. EULER disagreed; he found that external causes, namely friction and air resistance, accounted for it. Having disposed of the ARISTOTELIANS, he proceeded to the "more formidable" opposition of the WOLFFIANS. They expressed great respect for NEWTON's first law of motion (inertia) but simultaneously advanced a monad theory, which held that bodies seek to change their own state. Since inertia — the resistance to change — contradicted the monad theory, he felt the WOLFFIANS subverted their physics.[54] Again, he believed that physics had to be consistent or, put another way, without internal contradictions. Choices had to be made between opposing doctrines. In this instance he accepted NEWTON's law of inertia, since it had empirical verification which the monadic doctrine lacked. Further, he rejected WOLFFIAN physics since it embraced contradictory doctrines.[55]

Although EULER accepted NEWTON's mechanics, he opposed NEWTONIAN optics.[56] Stressing this opposition, his treatment of optics preceded that of mechanics in the *Letters*. He presented three general criticisms of the NEWTONIAN emission theory of light and instead adopted the wave theory. First, he believed that the sun would soon be exhausted if it emitted floods of light corpuscles in all directions with their prodigious velocity.[57] Second, he felt transparent bodies could not be as porous as the NEWTONIAN "nut-shell" theory had indi-

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[50] *Ibid.*, letter LXIII, p. 158.

[51] 1. A body must continue in its state of rest or motion until some external influence produces a change. 2. The direction of motion is rectilinear, and a body never changes direction of itself. 3. A body in motion meeting another of greater force is reflected without losing any part of its first motion; but when meeting a body moving with less force, it carries it along and loses as much motion as is transferred to it. See RENÉ DESCARTES, *Principia Philosophiae*, I, *Oeuvres de Descartes*, edited by C. ADAM & P. TANNERY (Paris: M. Cerf, 1897–1913), t. VIII, pp. 62–65.

[52] EULER, *Lettres*, letter LXIII, p. 160.

[53] *Ibid.*, letter LXIII, pp. 158–159.

[54] *Ibid.*, letter LXIII, pp. 159–160. The WOLFFIAN position here demonstrates that competing scientific systems did not necessarily argue in terms of the total error of others.

[55] *Ibid.*, letter LXXIV, pp. 161–162.

[56] EULER also rejected DESCARTES' theory of light, especially since its concept of instantaneous transmission had been disproven by the measurement of the time it takes light from the sun to reach the earth. Interestingly, he believed that NEWTON's corpuscular theory of light derived from DESCARTES, who held that the second element (round globules) of matter transmit light. *Ibid.*, letter XVII, pp. 40–41.

[57] *Ibid.*, letter XVII, p. 41.

cated with its doctrine of the extreme tenuity of matter. Since light rays are transmitted freely through transparent bodies in all directions, these bodies must "contain (an infinite number of) pores, disposed in straight lines",[58] if the corpuscular theory of light were true. EULER asked for proof of this. Paradoxically, his third and major criticism arose from his acceptance of the NEWTONIAN void.[59] He asked whether the corpuscular theory was consistent with "NEWTON's major doctrine, which requires an absolute vacuum in the heavens".[60] But light corpuscles from the sun and other stars would fill the universe as completely as the CARTESIAN and LEIBNIZIAN plenums. He, therefore, found the emission theory "self-contradictory" and rejected it. He then defined light as "an agitation or concussion of the particles of the ether".[61] In his mind, the scientific reputation of NEWTON alone, not empirical evidence or sound reasoning, accounted for the dominance of the corpuscular theory.[62] It also made difficult the task of advancing opposing theories, such as his.[63]

EULER developed a concise theory of matter which relied heavily upon his acceptance of two NEWTONIAN concepts: absolute space and the void. He presented it in two different places in the *Letters* – once in the section on general science and again in the philosophical section. It involved his mechanical and optical thought and was inseparable from them. For him as for NEWTON, the elements of matter shared three basic characteristics: extension, inertia, and impenetrability.[64] He considered DESCARTES' definition that extension alone constitutes matter "not sufficient".[65] Otherwise space, which was a receptacle for matter, would be an element of matter. The second necessary characteristic, inertia, required that the elements of matter be passive – not active like the WOLFFIAN monads, Thus, the cause of mobility came from without. Space remains unchanged, while bodies move in it. Still, extension and motion alone did not constitute a body of matter. According to the superstitious, specters had these two attributes. But they clearly lacked the third attribute of matter, impenetrability. He defined impenetrability as "what a vacuum lacks in order to be a body".[66] An element of matter became "an impenetrable extension".[67] His optical theories required that his matter be less tenuous than NEWTON's. He used the example of a sponge to explain tenuity or porosity. Finally, he regarded the notion that two bodies of matter could not penetrate each other

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[58] *Ibid.*, letter XVII, p. 42.

[59] He used the definition of a vacuum that NEWTON had offered in Query 18 of the *Opticks*. It stated that after the air is drawn out of a vacuum the ether remains. *Ibid.*, letter LX, p. 131.

[60] *Ibid.*, letter LVIII, p. 43.

[61] *Ibid.*, letter XX, p. 47.

[62] As EULER put it: "...you will no doubt be astonished that this (corpuscular) system could have been devised by so great a man and embraced by so many philosophers. But CICERO has long ago remarked that nothing so absurd can be imagined as to find no support among philosophers." *Ibid.*, letter XVII, p. 42.

[63] He advanced the wave theory of light, as had the Dutch physicist CHRISTIAN HUYGENS (1629–1695), and laid the groundwork for the studies of the English physicist THOMAS YOUNG on interference phenomena early in the nineteenth century.

[64] *Ibid.*, letter LXIX, pp. 149–151; letter CXII, pp. 288–290.

[65] *Ibid.*, letter LXIX, p. 150.

[66] *Ibid.*, letter LXIX, p. 150.

[67] *Ibid.*, letter LXX, p. 151.

or occupy the same place simultaneously as a fundamental law of nature and the real source of observable motions.[68] He called it the mainspring of the laws of motion.

In making his point regarding impenetrability, EULER referred to the MAUPERTUIS-KÖNIG controversy at the Berlin Academy from 1751 to 1752. In this dispute, the French Newtonian PIERRE MAUPERTUIS (1698–1759), the President of the Academy, claimed originality and priority for the principle of least action. The Dutch mathematician JOHANN SAMUEL KÖNIG (1712–1757), however, maintained that LEIBNIZ had stated it much earlier. After an extensive investigation, the Academy recognized MAUPERTUIS' priority.[69] The principal antagonists of MAUPERTUIS had been the WOLFFIANS, who supported KÖNIG and held that their studies of the collision of elastic bodies led to the principle of least action. EULER discussed it only in terms of impenetrability (hard body collisions).[70] He thus reaffirmed MAUPERTUIS' discovery of it. Clearly, in the section of the *Letters* on general science, EULER's views on the nature of matter shared much with those of NEWTON. But in the section on philosophy the situation changed somewhat. As will be seen, here he repudiated the indivisibility of NEWTON's atom.

His general acceptance of the NEWTONIAN natural philosophy entailed a concomitant rejection of much of the CARTESIAN and LEIBNIZIAN (WOLFFIAN) scientific systems. His views on them differed sharply, however. Like VOLTAIRE in his *Philosophical Letters* (1734)[71] and D'ALEMBERT in his *Preliminary Discourse to the Encyclopedia of Diderot* (1751)[72], he considered CARTESIAN physics an intermediary stage between NEWTON and the ancients. He did not judge so favorably the natural philosophy of LEIBNIZ, which WOLFF had organized and modified. He criticized at length its core: the monad theory and the doctrine of the pre-established harmony. He bluntly called the doctrine of the best-of-all-possible-worlds "artificial" and "insufficiently developed" in its existing form[73] and dismissed the organismic view of the universe as "ridiculous".[74] In mechanics he did accept the principle of the conservation of *vis viva* and the concept of a continuum. These were fundamental notions of LEIBNIZ and WOLFF, but he derived neither of them from their metaphysical system. Instead, he took the principle of the conservation of *vis viva* from HUYGENS' dynamics; the concept of a physical or spatial continuum, mainly from his mentor JOHANN BERNOULLI (1667–1748), whose work in general dynamics and hydraulics served as a basis for his research on continuous media. He profoundly opposed LEIBNIZ and WOLFF and sought not to give them undeserved credit in the sciences.

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[68] *Ibid.*, letter LXX, p. 152.

[69] For a synopsis of this dispute see CALINGER, "The Newtonian-Wolffian Controversy," pp. 324–328.

[70] EULER, *Lettres*, letter LXXVIII, p. 169–170.

[71] Trans. by ERNEST DILWORTH ("Library of Liberal Arts": New York: The Bobbs-Merrill Company, Inc., 1961), p. 61.

[72] Trans. by RICHARD N. SCHWAB ("Library of Liberal Arts": New York: The Bobbs-Merrill Company, Inc., 1963), pp. 78–82.

[73] EULER, *Lettres*, letters LX, pp. 132–133 and LXXXIX, p. 171.

[74] *Ibid.*, letter LXIII, p. 138. It should be noted that he accepted the principle of sufficient reason but attributed it to modern philosophers without giving LEIBNIZ particular credit for it.

In narrating his conflict with WOLFF over the monad theory, EULER maintained that a unity exists between scientific inquiry and philosophical knowledge. But he rejected the LEIBNIZIAN position that physics was grounded in metaphysics. He contended that the converse is true. Since the NEWTONIAN laws of physics were based on careful observation and since they were controlled and useful, he believed one could no longer doubt them.[75] It followed that if philosophical thought contradicted these physical laws, it must be false.

Using this approach he refuted the monad theory, the central core of LEIBNIZ's natural philosophy. According to LEIBNIZ the rules of monadology underlay all knowledge, including mechanics. Again, his monads were dynamic.[76] Their unifying factor was a non-mechanical force which was the logical and ontological principle accounting for every organizational pattern in the phenomenal world. Their elasticity and their imperceptible psychological gradations of perception confirmed his crucial continuity principle (*lex continui*). For instance, in his economical mechanical system the monads accounted for his law of the conservation of *vis viva.*

EULER evaluated the eight tenets of the monad theory, which WOLFF had posited in his *Rational Thoughts on God, the World, the Soul of Man, and All Things in General* (1719). They were:

(1) Experience shows us that all bodies are perpetually changing their state.
(2) Whatever is capable of changing the state of a body is called a force.
(3) All bodies, therefore, are endowed with a force capable of changing their state.
(4) Each body, therefore, is making a continual effort to change.
(5) This force belongs to a body only in so far as it contains matter.
(6) It is therefore a property of matter to be continually changing its own state.
(7) Matter is a compound of a multitude of parts, denominated the elements of matter (monads).
(8) Therefore, the compound can have nothing but what is founded in the nature of its elements, every elementary part must be endowed with the power of changing its own state.[77]

EULER noted that the elements of matter (tenet seven) could not be elastic monads. They had to be impenetrable bodies, since hard body collisions were the real cause of the laws of motion.[78] He then rebutted the monad theory with the aid of the law of inertia. This empirically verified law held that bodies of matter cannot set themselves in motion. It proved tenet three, whereby bodies are endowed with a force capable of changing their state, to be "equivocal and altogether false".[79] The law of inertia also contradicted tenet four and the following four tenets, which were successively founded upon it, since it

[75] *Ibid.*, letter LXXVI, pp. 164–165 and letters CXXV–CXXXII, pp. 294–310.
[76] LEIBNIZ wrote: "Quod non agit non existit." (What is not active does not exist.)
[77] EULER, *Lettres*, letter LXXVI, p. 165.
[78] *Ibid.*, letter LXX, pp. 151–153.
[79] *Ibid.*, letter LXXVI, pp. 165–166.

held that each body tends to preserve its state, not to change it.[80] Since the WOLFFIAN theory of matter contradicted the law of inertia, EULER found the WOLFFIAN philosophy built upon it false.

This argument alone did not satisfy EULER. He sought a more complete refutation of the monadic doctrine and its metaphysical foundation. His investigation of the essence of matter in the section on philosophy of the *Letters* provided the means. Again, LEIBNIZ had depicted the monad as a metaphysical point, which was dimensionless. He had also maintained that infinite aggregates of the monads constituted seemingly extended corporeal bodies. In order to reach these conclusions he had to make certain assumptions. For instance, he altered EUCLID's definition of a point from that which has no parts (*Punctum est cuius pars est nulla*) to that which is dimensionless, an alteration he adopted from the *De cive* (1650) of the English political theorist THOMAS HOBBES (1588–1679). Although he sometimes reverted to EUCLID's position, he generally followed HOBBES here. In addition, he accepted the approach of the Italian mathematician BONAVENTURA CAVALIERI (c. 1598–1647), which recognized the point, line, surface, and solid as relations within a system. This allowed for the generation of a world of dimensions from dimensionless monads. Since the monads were metaphysical points, the geometrical concept of divisibility (*ad infinitum*) did not apply to them. This meant that LEIBNIZ's matter was irreducible to extension and consequently his mechanics was not altogether reducible to geometry.

EULER diametrically opposed the above basis of the monadic doctrine. He followed EUCLID and stressed the fundamental nature of geometry. For him non-extended monads could not be at the base of the real world which we observe. Contending that they were ciphers, he observed that ciphers added to ciphers (whether they be geometrical or metaphysical) give ciphers.[81] Thus, the LEIBNIZIAN and WOLFFIAN world could have no extension. Observations, however, showed that the opposite was true. There was real extension in the world. This extension was the object of geometry as well as mechanics. The more general, geometrical abstractions relating to extension applied also to mechanics.[82] He upbraided metaphysicians, LEIBNIZ and WOLFF included, for declaring that geometers could not necessarily make such connections. He asserted that general ideas form the foundation of all knowledge.[83] Divisibility *ad infinitum* was a geometrical property of extension. He demonstrated this by dividing a line segment into as many parts as he wished.[84] One could continue this process of subdivision to infinity, for, according to EUCLID, between every two points on a line lies a third point. When EULER applied to mechanics the geometrical property of infinite divisibility, arguments supporting a basic, indivisible element of matter became "absolutely untenable".[85] This explains

[80] *Ibid.*, letter LXXIV, p. 160.
[81] *Ibid.*, letter CXXII, p. 291.
[82] *Ibid.*, letter CXXII, pp. 288–289. He himself had soundly and successfully integrated physics into the mathematical sciences.
[83] *Ibid.*, letter CXXIV, p. 293.
[84] *Ibid.*, letter CXXII, p. 290.
[85] *Ibid.*, letter CXXIII, p. 293.

his careful avoidance of the word atom in discussing the elements of matter. By definition the atom of the ancient Greeks was indivisible.[86]

EULER buttressed his anti-monadic reasoning by condensing the above argument into the following ARISTOTELIAN syllogism:

All bodies are extended.

Extension is divisible *ad infinitum.*

Therefore, every body must be divisible *ad infinitum.*[87]

In addition, if non-extended monads formed a world without extension, geometry, which deals with it, would be pure speculation. But geometry was beyond contradiction one of the most useful sciences. Its endeavors could not be pure chimera.[88] Again EULER rejected the monads and the metaphysical system which had spawned them. This time he had reinforced his mechanical counter-arguments with geometrical ones.

EULER had changed the basis for but not the intensity of his criticism of the monadic doctrine over the years. Initially, in the late 1730's he charged that it led to atheism.[89] He believed then that the eternal, living monads shared the eternal nature of God and removed the need for Him.[90] This theological argument may partially account for historian EDUARD WINTER's assertion that he considered theology to be the queen of the sciences. In attempting to strike down the monadic doctrine, which he considered dangerous to religion, his arguments became emotional at times. He characterized support for it as feeble, chimerical, and most stupid.[91] As noted above, his primary arguments in this disputation by the 1760's were mechanical and geometrical. In discussion of the sciences he had by then not only discarded theological arguments but also had repudiated their use. He wrote: "to reproach one's adversaries with atheism or idolatry is a villainous manner of arguing."[92] In explaining celestial motions he had likewise written: "...it is absurd to employ, as some persons do, passages of the Holy Scriptures to prove that the earth is at rest, and the sun in motion."[93] Prominent among his concluding statements upon the monad theory was his criticism of LEIBNIZ and WOLFF for basing it upon purely and exclusively rational concepts. He called it "a remarkable monument of the aberration into which

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[86] EULER objected to the punctual or indivisible characteristic of the atom but not to its other properties. Indeed, his fundamental elements of matter, which were infinitely divisible and comprised the physical continuum, shared all properties of atoms save for their punctual characteristic. Put another way, his theory of matter stressed the continuous without denying all of atomism. Thus, he did not admit as reasonable only what is now called continuum physics, the chief subject of his treatises on physics. He also advanced his own brand of atomism, which his pioneer work in the kinetic theory of gases best illustrates. His theory of matter, therefore, did not dictate excluding molecular theories from physics.

[87] EULER, *Lettres*, letter CXXV, p. 296.

[88] *Ibid.*, letter CXXV, p. 296.

[89] A letter of November 3, 1738 to physicist GEORG BERNHARD BILFINGER (1693–1750), EDUARD WINTER (ed.), *Die Registres der Berliner Akademie der Wissenschaften 1746–1766* (Berlin: Akademie-Verlag, 1957), p. 31.

[90] EULER, *Lettres*, letter CXXXII, pp. 310–312.

[91] *Ibid.*, letters CXXV ff, pp. 140ff.

[92] *Ibid.*, letter CXXII, p. 310.

[93] *Ibid.*, letter LXXI, p. 153.

the spirit of philosophizing may fall".[94] Again, the importance to him of empirical verification emerged.

EULER clarified his scientific stance further with his epistemology, which embraced a modification of LOCKEAN sensationalism. LOCKEAN thought had provided a philosophical basis for NEWTONIAN critical empiricism. EULER based his epistemology upon a CARTESIAN dualism of matter and spirit. An intimate union connected the body and soul of man. But it was not known "how to study thoroughly how this (union) is produced",[95] since it was "the greatest mystery of divine Omnipotence".[96] It constituted the essence of life.[97] EULER disavowed the unitary positions of the spiritualists (such as the WOLFFIANS) and the materialists (such as LA METTRIE). His antimonadic arguments based upon geometrical extension and impenetrability contradicted the spiritualists (or dogmatic Idealists as he also called them).[98] It was absurd that the materialists had endowed matter with the faculty of thought.[99] In addition, matter had the attributes of extension, inertia, and impenetrability; spirit had those of intelligence, will, and liberty.[100] It was invalid to ascribe the characteristics of one to the other.

A two-faculty theory accounted for learning within this dualism. Learning occurred through the interaction of the five senses and the soul.[101] The soul acted in the brain at the juncture of all the nerve endings. EULER referred to this portion of the brain as the "callous body".[102] He spoke the language of brain tissue and nerve ganglia. In France the LOCKEAN Abbé ETIENNE CONDILLAC (1715–1780) had presented a similar theme in his *Treatise on Sensations* (1754). His successors DESTUTT COMTE DE TRACY and PIERRE CABANIS (1757–1808) followed the same approach as EULER,[103] who considered the soul as more than the strict LOCKEAN passive mirror or *tabula rasa*.[104] It also had a limited power of acting and influencing part of the brain. After it received sense impressions, it derived first ideas. Combinations of these formed judgments, reflections, reasoning, and whatever was necessary to perfect knowledge.[105]

In arriving at his epistemology, EULER rejected two prominent past ones, CARTESIAN occasionalism and the LEIBNIZIAN pre-established harmony. During the late seventeenth century the CARTESIANS had developed occasionalism. In its crudest form, that of GEULINCX and CORDEMOY, divine ubiquity accounted for the union between body and soul. Through a perpetual miracle God provided this union within each individual.[106] The CARTESIAN leader NICHOLAS MALE-

[94] *Ibid.*, letter CXXX, p. 305.
[95] *Ibid.*, letter XCVII, p. 219.
[96] *Ibid.*, letters LXXX, p. 183 and XCVIII, p. 221.
[97] *Ibid.*, letters LXXXII, p. 186 and XCIII, p. 210.
[98] *Ibid.*, letter XCVII, pp. 219–221.
[99] *Ibid.*, letters LXXX, p. 181 and XCVI, pp. 220–221.
[100] *Ibid.*, letter XCIII, pp. 209–211.
[101] *Ibid.*, letter LXXXI, pp. 183–185.
[102] *Ibid.*, letters LXXXI, p. 184 and XCIV, p. 212.
[103] A. GOODWIN (ed.), *The New Cambridge Modern History* (Cambridge: at the University Press, 1965), vol. VIII, p. 156.
[104] EULER, *Lettres*, letters LXXXI, p. 183 and XCIV, p. 212.
[105] *Ibid.*, letter LXXXI, p. 184.
[106] *Ibid.*, letter LXXXII, p. 186.

BRANCHE (1638–1715) later replaced divine ubiquity with rules which God followed in the creation and continual recreation of substance. Most of the European learned community had discarded this cumbersome theory and so did EULER. Many had adopted instead LEIBNIZ's solution for how one learns, his hypothesis of the pre-established harmony, Indeed, within the German principalities it attained such prominence that, for a time, to question it "was to incur the imputation of ignorance or bigotry".[107] LEIBNIZ and, in turn, the WOLFFIANS believed that this hypothesis synthesized the thought of DESCARTES, MALEBRANCHE, and LOCKE on the learning process. The WOLFFIANS made it doctrine. EULER found it false and contradictory to the intended synthesis.

EULER first explained and then scathed the doctrine of the pre-established harmony. He employed social arguments, since only these applied to the world of the spirit (human affairs). Again, this doctrine assumed that God had established a logical program, which foresaw the proper organization needed between the body and soul of man.[108] Individual reactions were but different realizations of this general programming. LEIBNIZ had described the relation of body to soul as that of two synchronized clocks running totally independently of one another. They, therefore, could not interact. EULER called this assumption "destitute of proof".[109] He contended that there is limited interaction between body (matter) and soul (spirit). Indeed, God was a spirit, and his influence upon the physical world could not be denied. In addition, without interaction all knowledge must derive from innate ideas, since the senses could not contribute to the knowledge of the soul. He found this idea "patently absurd".[110]

EULER's main criticism of the pre-established doctrine of harmony, however, arose because it was "utterly destructive of human liberty".[111] Like the French Huguenot scholar and exile PIERRE BAYLE (1647–1706), he believed that it was a mechanism fatal to free choice.[112] He held that liberty was to the spirit what extension and impenetrability were to matter.[113] This meant that the analysis of problems in the realm of the spirit (human affairs) was concerned with voluntary effects, not the necessary ones of physics. In other words, mechanistic thought, which could explain natural events, could not explain human actions, or what he denoted as moral events. Ironically LEIBNIZ had similarly stressed the importance of liberty (freedom of the will) and the voluntary nature of human actions.[114] But he had endeavored to reconcile them with his doctrine of the pre-established harmony, the cardinal problem of his *Theodicy* (1710), which was written in rebuttal to BAYLE. He derived his theistic arguments of divine foreknowledge largely from BOETHIUS' *De consolatione philosophiae* (Book V) and maintained that divine infallibility does not produce neces-

[107] *Ibid.*, letter LXXXIII, p. 187.
[108] *Ibid.*, letter LXXXII, p. 187.
[109] *Ibid.*, letter LXXXIII, p. 188.
[110] *Ibid.*, letter LXXXII, p. 189.
[111] *Ibid.*, letter LXXXV, p. 192.
[112] *Ibid.*, letter LXXXV, p. 192.
[113] *Ibid.*, letters LXXXV–LXXXVII, pp. 192–198.
[114] See GEORGE HENRY RADCLIFFE PARKINSON, *Leibniz on Human Freedom:* Sonderheft 2 of *Studia Leibnitiana* ed. by KURT MÜLLER & WILHELM TOTOK (Wiesbaden/Germany: Franz Steiner Verlag GmbH, 1970).

sity, since actions taken derive from an infinity of possible courses of action existing in the divine mind. EULER concurred that divine prescience does not preclude liberty. But for him God cooperated in human actions by furnishing motives not by operating within a doctrinal system. He held that there can be no constraints on liberty, not even on the part of God.[115] Thus EULER spoke the language of Roman *libertas* (freedom from bondage) and rejected LEIBNIZ's Stoic concept of a system for a moral order.

EULER found the true foundation for liberty (or free will) in motives. Once again his premise agreed with LEIBNIZ, who held in replies 4 and 5 of his *Correspondence* with CLARKE and in chapter 21 of his *New Essays* that the concept of motive was basic to his theory of will. He wrote: "a mere will without a motive is a fiction".[116] But motives were quite different for the two men. For EULER a motive was an inclination, tendency, or proneness, which involved a personal decision, a responsibility. With this decision came the committing or suspending of an action. For LEIBNIZ motive was something within man's nature—within the autarkic monads. EULER believed that the monad theory provided an insurmountable obstacle to liberty; for since the monads had mechanistic properties, how could they have the mutually exclusive properties of spirit including liberty? Mechanism was apparent in LEIBNIZ's view of motives. Following ST. JOHN DAMASCENE, he regarded them as consequent volitions. This meant that the relations between judgment, will, and act were causal; ultimately LEIBNIZ spoke of motive in terms of final, not efficient, causes. EULER found this theory of will contradictory, for it involved explaining human affairs in terms of mechanical causes not the teleological ones, which LEIBNIZ sought.

The strength of EULER's opposition to LEIBNIZ's pre-established harmony doctrine and its attendant theory of will may be judged by his condemnation of mechanical explanations in human affairs. He noted that without free will and with only the past and the environment to control human behaviour, a criminal would not be guilty of his crime.[117] Moreover, a human history based upon predetermined, mechanical operations would not be worthy of the Creator. He held that liberty, "which is absolutely essential to the human spirit, has a very great influence upon the events of the world".[118] Thus he felt that the deterministic consequences of the pre-established harmony made its falsity dangerous.

In EULER's methodology each individual problem demanded its best means of solution. He divided all human knowledge into three classes: (1) Sensible (or physical), based upon the senses; (2) intellectual (or logical), based upon ratiocination; and (3) historical (or moral) truths, based upon faith.[119] Adopting a broad approach to methodology, he aptly added LOCKEAN empiricism to faith and reason as an equal path to truth within its realm of application.[120]

[115] EULER, *Lettres*, letter LXXXVI, pp. 194–195 and letter CXI, pp. 205–207.
[116] LEIBNIZ, *Philosophical Papers* (Reply 4), p. 687.
[117] EULER, *Lettres*, letter LXXXIV, p. 190.
[118] *Ibid.*, letter LXXXVII, p. 197.
[119] *Ibid.*, letters CXV, pp. 273–274; CXVI, pp. 275–276; CXIX, pp. 281–283.
[120] *Ibid.*, letters CXVII, pp. 277–278 and CXX, pp. 283–285.

Following in the earlier tradition of Christian PLATONISM and of the Italian humanist THOMAS CAMPANELLA (1568–1639), he wrote: "proofs of these three classes of truth are very different, but if they are sound, each in its kind, they must equally produce conviction",[121] and "it cannot be affirmed that the truths of any one order are better founded than those of another".[122] For him, therefore, the CARTESIAN and LEIBNIZIAN (WOLFFIAN) deductive methodologies, whose unlimited use he eschewed, did not offer the sole means for obtaining scientific truths. Neither did he consider theology to be the queen of the sciences as WINTER holds. For him the truths reached by experience and reason were equally valid with those of faith. His discussion of the best way to attain the "highest degree of perfection"[123] closely parallels the critical empirical (hypothetico-deductive) method advanced by NEWTON in the *Opticks*.[124] His methodology, therefore, was close to that of NEWTON.

Conclusion

The *Letters to a Princess of Germany* is a rich source of the scientific thought of EULER.[125] To borrow a phrase from GOETHE, it is a "creative mirror", which reflects what he had learned in the sciences in addition to his original speculations. The *Letters* is particularly useful in ascertaining his mature scientific outlook, for it includes his comprehensive response as a mature, established scholar to the CARTESIAN, LEIBNIZIAN (WOLFFIAN), and NEWTONIAN natural philosophies. It demonstrates that the all-too-simple CARTESIAN appellation is unacceptable. While EULER's scientific thought is not without CARTESIAN elements, especially in questions of form, there are major differences in content. He regarded DESCARTES highly; he sought a mechanical explanation for attraction; and he continued the trend to mathematicize the physical sciences (a trend not limited to the CARTESIANS). But he never accepted the whole of the CARTESIAN natural philosophy. EULER differed with DESCARTES and his followers on the cause of the tides, the orbits of comets, the basic laws of mechanics, optics, matter theory, and epistemology – differences which can not be ignored. It is time to abandon the picture of EULER as CARTESIAN. Not only is it erroneous for what it says about the CARTESIAN influence, but it also lends support to the mistaken notions that EULER was pre-NEWTONIAN or generally hostile to NEWTON. The *Letters* disproves these views. Perhaps the most serious defect in applying a CARTESIAN label is that it impoverishes EULER's scientific thought. The originality and complexity are lost.

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[121] *Ibid.*, letter CXV, p. 274.

[122] He continued: "Each class is liable to error which may mislead us, but there are likewise careful precautions, which carefully observed, provide us with nearly the same degree of conviction." *Ibid.*, letter CXVI, p. 275.

[123] *Ibid.*, letter CXIX, p. 283.

[124] NEWTON, *Opticks*, pp. 404–405.

[125] While the sections cited in this article from the *Letters* deal with EULER's mature scientific outlook, they only indicate the scope and substance of his research. Further sections on electricity, magnetism, lunar theory, methods of determining longitude, lenses, telescopes, and celestial distances show him to be in the forefront of research in the physical sciences in his time.

EULER emerges from the *Letters* as both an original and a synthetic thinker who exhibited a spirit of critical enquiry when examining the intricate and changing CARTESIAN, LEIBNIZIAN, and NEWTONIAN natural philosophies. He neither accepted nor rejected any *in toto*. Again, like VOLTAIRE and D'ALEMBERT he regarded DESCARTES as an intermediary between NEWTON and the ancients. He strongly criticized most of the natural philosophy of LEIBNIZ and WOLFF, especially its core of the monad theory and the pre-established harmony. While he respected LEIBNIZ's brilliance, he scorned the WOLFFIANS and their dogmatic idealism. What appears to be LEIBNIZIAN in his thought, the conservation of *vis viva* doctrine and the concept of a physical continuum, he derived from HUYGENS and JOHANN BERNOULLI. From NEWTON he accepted the mechanics and critical empirical methodology but rejected the optics and modified the matter theory. He advanced the wave theory of light in place of the corpuscular theory; he held matter to be less tenuous than NEWTON's nutshell theory implied; and he posited the infinitely divisible point element as the basic particle of matter instead of the atom. In searching for a precise mechanical explanation for attraction and in developing his wave theory of light, he appealed to the ether for answers. The ether, a subject NEWTON had left open in the *Principia*, was central to his natural philosophy. Thus, in the *Letters* EULER deftly combined original speculations with selected doctrines and concepts from the leading schools of scientific thought in order to produce his own consistent (non-contradictory) theory. Above all, he was an eclectic who embraced and articulated the fundamental mechanical and methodological thought of NEWTON.

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