

## Rapid Communication

# A Simple Laser Linac

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**Abstract.** When a laser is focused by a lens or mirror array the field acquires a longitudinal component. Under certain conditions this longitudinal laser field could be used to accelerate an injected particle to high energies.

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The linear accelerator (linac) has a rich history [1] predating modern quantum physics. In 1924 Iring proposed the concept, which was later put into practice by Widerøe in 1928. In today's linacs, charged particles are injected into a traveling-wave guide containing a TM mode which has a longitudinal electric field. Particles (e.g., electrons, which we shall focus on in this communication) traveling with the phase velocity of the electromagnetic wave can then be continually accelerated by the longitudinal field just as if it were constant in time. Typical field strengths in today's linacs are in the megavolt per meter range and so a kilometer-long device would yield GeV energies; the Los Alamos Meson Facility, for example, is ca. 800 m long and yields particles in the 800 MeV range.

Recently, various proposals have been made suggesting the use of laser fields instead of RF fields in a linac. For example, the inverse free-electron laser [2], evanescent-wave concepts [3], inverse Cherenkov acceleration [4], laser-plasma wave interactions [5] and other schemes [6].

The present proposal is based on the fact that focused beams have a longitudinal as well as a transverse component. For example, even a weakly focused field will develop a longitudinal component of magnitude [7]

$$E_z(x, y, z) = \frac{i}{k} \nabla_{\perp} \cdot \mathbf{E}_{\perp}(x, y, z) \quad (1)$$

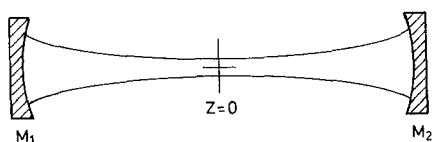
where the field is propagating in the  $z$ -direction with wave vector  $\mathbf{k}$ , and  $\nabla_{\perp}$  and  $\mathbf{E}_{\perp}$  denote the transverse gradient and field, respectively.

In order to facilitate the discussion in a simple closed form, we first consider a weakly focused laser beam in a confocal resonator and/or a lens waveguide array, and two Gedanken laser linacs are suggested. We conclude with the case of a strongly focused beam.

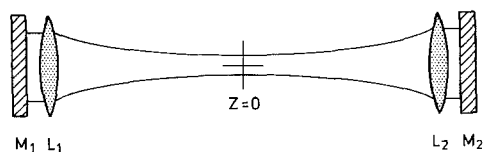
For the purposes of the present discussion, it is useful to consider the longitudinal component associated with an intercavity stable resonator laser field as in Fig. 1.

As is well known, the laser field in a stable confocal resonator may be written in terms of Hermite Gaussian modes [8]. For example the  $x$ -polarized 1,0 mode is given by

$$\begin{aligned}
 E_{\perp}(x, y, z) = & -\hat{x}E_0 \left[ \frac{2R^2}{R^2 + 4z^2} \right] \exp \left[ -k \frac{x^2 + y^2}{R + 4z^2/R} \right] \\
 & \times H_1(x\sqrt{k/R}) H_0(y\sqrt{k/R}) \quad (2) \\
 & \times \exp \left[ -ik \left( z + \frac{R}{2} + \frac{2z(x^2 + y^2)}{R^2 + 4z^2} \right) - 2i\Phi \right]
 \end{aligned}$$



**Fig.1b.** Confocal stable resonator consisting of flat mirrors  $m_1$  and  $m_2$  together with lenses  $L_1$  and  $L_2$



**Fig.1a.** Confocal stable resonator consisting of mirrors  $m_1$  and  $m_2$  having curvature  $R$

where  $E_0$  is the field amplitude,  $R$  the mirror curvature,  $k$  the propagation vector  $H_0(\xi) = 1$ ,  $H_1(\xi) = 2\xi$  and the phase  $\Phi$  is defined by

$$\Phi = \tan^{-1}[(R - 2z)/(R + 2z)]. \quad (3)$$

From (1) and (2) we have the on-axis longitudinal field component

$$E_z(0,0,z) = \frac{-iE_0}{\sqrt{kR}} \left[ \frac{2R^2}{R^2 + 4z^2} \right] \times \exp[-ik(z + \frac{1}{2}R) - 2i\Phi] \quad (4)$$

and we see that  $E_z \sim E_0/\sqrt{kR}$  which for  $R \sim 1$  and  $10 \mu\text{m}$  radiation, i.e.  $k \sim 10^6$ , says that  $E_z \sim 10^{-3}E_0$ .

Finally we note that the phase velocity [8] for the field is

$$v_0 = \omega/[k - 4R/(R^2 + 4z^2)] \simeq c \quad (5)$$

since  $k \sim 10^6$  is much larger than  $R^{-1} \sim 1$ .

It should also be noted that the phase variation upon crossing the cavity will be small if we confine the electron beam to a region about the origin such that  $z < R$ ; refer to Fig.3 for a possible configuration satisfying this condition. There we see a race-track microtron type set up [9]. In this configuration the particle round-trip time would be designed to coincide properly with the laser pulse so as to ensure further acceleration, and the particle would thus receive a boost each time it passes through the optical cavity. We conclude this portion of the discussion with expressions and numerical examples for the beam waist (focal radius) and longitudinal field strength.

The beam width is given by [10]

$$w(z) = w_0 \sqrt{1 + \left[ \frac{\lambda z}{\pi w_0^2} \right]^2}, \quad (6a)$$

where the beam radius  $w_0$  at the center of the resonator is

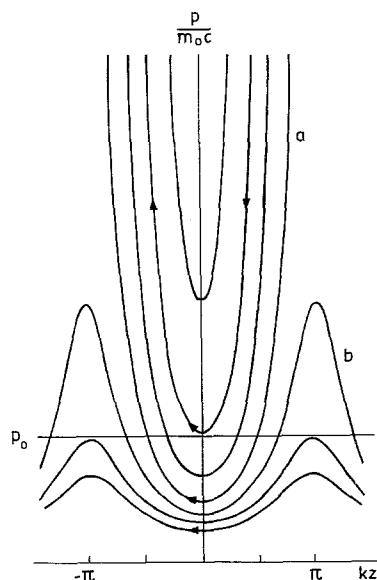
$$w_0^2 = \frac{\lambda}{2\pi} R. \quad (6b)$$

The interaction length  $z_1$  is clearly less than  $R$  and from (3) and Fig.2 we reasonably take  $z_1 \sim 10$  cm when  $R = 1$  m, say. Then, from (6a), we see that the beam width  $w(z_2)$  will be  $\sqrt{2}w_0$  when  $z_2 = \pi w_0^2/\lambda$ . This tells us that, for  $w_0$  in the mm range, and  $\lambda$  around  $1 \mu\text{m}$ ,  $z_2 \sim 1$  m; that is, the laser beam will not expand much in its travel between the mirrors. Alternately, if  $\lambda \sim 0.5 \cdot 10^{-6}$  m (optical) and we choose  $z_2 = z_1 = 10$  cm then we have a focal area  $w_0^2 \sim 10^{-7} \text{ m}^2$ . Finally we note that if we focus to  $w_0 \sim \lambda$  then  $z_2$  (which is essentially the Rayleigh length) will likewise be of the order  $\lambda$ .

For a laser system having an intercavity power  $P$  per pulse, the longitudinal field goes as

$$E_z \sim \frac{1}{\sqrt{kR}} \sqrt{P/\epsilon_0 c w_0^2}. \quad (7)$$

Hence we see that for  $kR \sim 10^6$  and  $w_0^2 \sim 10^{-7} \text{ m}^2$  the field strength is of order  $E_z \sim 30\sqrt{P}$  V/m where



**Fig.2.** Phase space plot following Slater [10]. Net acceleration is possible when the  $p_0$  line exceeds the minimum of curve  $a$

P is the power in watts. Naturally we could make the field much stronger by focusing to a spot of order  $\lambda^2$ , but then the "Rayleigh length" will be foreshortened. Such trade-offs will be discussed elsewhere.

Let us consider next the dynamics of the accelerated particles. Following Slater [11], we write the particle Hamiltonian as

$$H = \sqrt{m_0^2 c^4 + p^2 c^2} - p v_0 - e E_z v_0 / \omega \cos(\omega z' / v_0) \quad (8a)$$

where  $m_0$  is the rest mass of the particle,  $p$  its momentum,  $E_z$  the longitudinal field strength,  $\omega$  the laser frequency, and  $z' = z - v_0 t$ . In the present case of phase velocity  $v_0 \sim c$ , (6a) becomes

$$H \approx \sqrt{m_0^2 c^4 + p^2 c^2} - p c - e E_z \frac{\lambda}{2\pi} \cos k z' . \quad (8b)$$

Phase-space plots of  $p$  vs  $z'$  (Fig.2) provide insight into the particle dynamics implied by (8b). In trajectories such as curve *a* particle energy grows as  $z'$  decreases. However, in the case of particles on curve *b* no net energy is transferred to the particle. Thus we would like to inject particles with sufficient energy to ensure phase-space trajectories of type *a*.

The minimum injection momentum  $p_0$ , necessary to accomplish this, is found to be [12]

$$p_0 c = -[(e E_z \lambda / \pi)^2 - m_0^2 c^4] / 2 e E_z \frac{\lambda}{\pi} , \quad (9)$$

and using  $mc^2 = \sqrt{p_0^2 c^2 + m_0^2 c^4}$ , yields a minimum injection energy

$$(mc^2)_{\text{inject}} = m_0 c^2 \frac{m_0 c^2}{2 e E_z \lambda / \pi} + e E_z \frac{\lambda}{\pi} . \quad (10)$$

This shows that in a laser linac with  $\lambda \simeq 10 \mu\text{m}$  say, as compared to a RF linac with  $\lambda \simeq 1 \text{cm}$ , either the injection energy must be larger than in the RF case or the field strength  $E_z$  of the laser must be larger than the RF field strength, or both. For example if we consider a pulse of  $10^3 \text{J}$  energy and 1 ps duration, then from (7) and the subsequent discussion we have  $E_z \sim 30\sqrt{P} \sim 10^9 \text{V/m}$  and if  $\lambda \sim 10 \mu\text{m}$  then  $e E_z \lambda \sim 10^4 \text{eV}$ ; in such a case (10) implies

$$(mc^2)_{\text{inject}} \sim 25 \text{MeV} . \quad (11)$$

We conclude the present "weak focus" discussion with a couple of schemes designed to (hopefully)

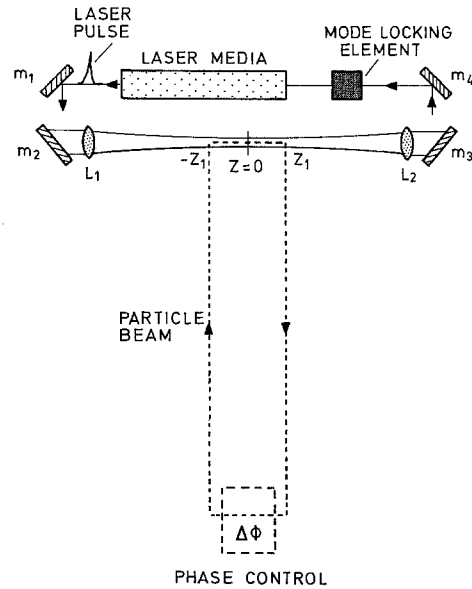


Fig.3. Race-track microtron configuration. The particle beam interacts with the laser pulse over a distance  $2z_1$  which is small compared to the curvature parameter

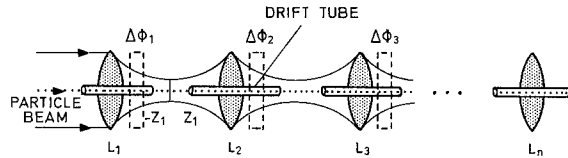


Fig.4. Laser linac via lens guiding array. Drift tubes and "phase shifters"  $\Delta\Phi_1, \Delta\Phi_2$ , etc. are indicated only as possible ways of dealing with conceivable problems

convince ourselves that such accelerators are at least in principle possible.

In Fig.3 we see a particle beam guided by appropriate steering magnets into the region  $\pm z_1$  about the origin. As discussed earlier, this region could be such that  $z_1 \ll R$  so that the phase given by (3) is constant over the interaction volume. Taking  $\lambda \sim 10 \mu\text{m}$  and  $P \sim 10^{14} \text{W}$ , as in the previous example, the field  $E_z \sim 10^9 \text{V/m}$  and after a modest number of laps around the "race track" we would have added a few GeV of energy to the particle. Naturally the particle could pass through the laser cavity many more times with the attendant increase in energy.

In Fig.4 the particles are injected into a lens guiding array which focuses the laser light in many successive acceleration regions. The phase of the laser radiation will be constant (independent of  $z$ ) only over a given region,  $\pm z_1$ , as indicated in Fig.4. Therefore the "drift tubes" are envisioned as one means of shielding the particle beam from the effects of this phase variation and the phase control indicated in Fig.4 is another. Other approaches to this problem will be discussed elsewhere.

We turn now to the case of a Gaussian laser field focused to a small spot. It is intuitively obvious that the weak-focus result,  $E_z \sim E_0/\sqrt{kR}$ , underestimates the longitudinal field for a well-focused laser. In that case we find that for an x-polarized Gaussian beam of the form

$$E_x(x, y, z) = E_0 \exp[-(x^2+y^2)/2w_0^2] f(z), \quad (12a)$$

the longitudinal field is given by [7]

$$E_z(x, y, z) = \frac{i}{k} \frac{\partial}{\partial x} E_x(x, y, z) \quad (12b)$$

and (12a, b) imply that

$$E_z(x, y, z) = -\frac{ix}{kw_0^2} E_x(x, y, z). \quad (12c)$$

Of course, the on-axis field vanishes, but at  $x = w_0$  the longitudinal field goes as  $E_z \sim (P/\epsilon_0 c w_0^2)^{1/2} \times (kw_0)^{-1}$ . If we now assume  $w_0 \sim \lambda$ , and take  $\lambda = 10 \mu\text{m}$  as before, we find  $E_z \sim 10^5 \sqrt{P}$ ; and if the pulse is again such that  $P = 10^{15} \text{ W}$  we now find  $E_z \sim 3 \cdot 10^{12} \text{ V/m}$ .

Clearly the present discussion leaves open many questions of practice, e.g. particle beam stability [13], optical breakdown [14], etc. However, we hope that the present report has convinced the reader that a simple lens guiding array is worthy of further study.

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7. The fact that a focused light beam has a longitudinal component was shown for example by W. Carter: J. Opt. Soc. Am. 62, 1195 (1972); Expression (1) was derived by Louisell using a perturbation analysis in *Phys. of Quant. Elec.* Vol. III, ed. by S. Jacobs, M. Scully, M. Sargent, C. Cantrell, p. 378. See also M. Lax, W. Louisell, W. McKnight: Phys. Rev. A 41, 3727 (1975). In their paper they concentrated on the importance of the longitudinal components of a laser beam interacting with a plasma to accelerate electrons. In the present paper we consider the case of fast electrons interacting with a laser pulse. Equation (12b) follows from the works of Carter and of Cicchitelli et al. by using the asymptotic expression for Bessel functions in the limit  $kz \gg 1$ , which is the case in our problem
8. See for example M. Sargent, M. Scully, W. Lamb: *Laser Physics*. The phase velocity associated with (2) is very slightly greater than  $c$  (by an amount  $c\lambda/R$ ). This will cause no serious problem and can be compensated for by, for example, the phase shifters of Fig.4
9. S. Humphries: *Principles of Charged Particle Acceleration* (Wiley, New York 1986)
10. See Eqs.(23) and (48) of H. Kogelnik, R. Li: Appl. Opt. 5, 1550 (1966)
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12. For a good discussion of this point see R. Helm, R. Miller: In *Linear Accelerators*, ed. by P. Lapostolle, A. Septier (North-Holland, Amsterdam 1970)
13. Transverse instabilities are not a problem in RF linacs due to the extreme relativistic nature of the particle. The same would tend to be true in the present problem. But the transverse variation in the laser could lead to new beam instabilities, and this point will be discussed elsewhere. We note, however, that there are many well-known ways to compensate for transverse perturbation, e.g. the application of a longitudinal magnetic field
14. The field strength at a lens would be much smaller (where the beam is spread over a large area) than at the focus. Nevertheless it may prove advantageous to use mirrors instead of lenses