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Splat-quench solidification: estimating the maximum spreading of a droplet impacting a solid surface

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This study investigates the mechanisms that contribute to determining the maximum spreading of a liquid droplet impacting a solid surface in connection with splat-quench solidification. This paper defines two domains, the viscous dissipation domain and the surface tension domain, which are characterized by the Weber and the Reynolds numbers, and that are discriminated by the principal mechanism responsible for arresting the splat spreading. This paper illustrates the importance of correctly determining the equilibrium contact angle (a surface tension characteristic that quantifies the wetting of the substrate) for predicting the maximum spreading of the splat. Conditions under which solidification of the splat would or would not be expected to contribute to terminating the spreading of the splat are considered. However, our *a priori* assumption is that the effect of solidification on the spreading of a droplet, superheated at impact, is secondary compared to the effects of viscous dissipation and surface tension.

Nomenclature

a	Thermal diffusivity
C_s	Correction factor
C_v	Correction factor
d	Initial diameter of droplet
D	Final diameter of splat
E_k	Initial kinetic energy at impact
E_s	Rise in surface tension energy
E_v	Viscous energy dissipated
s	Terminal thickness of splat
t_c	Spreading time of splat
u	Velocity of impinging droplet
V	Volume of splat (droplet)
x	Space variable
κ	Madejski's solidification parameter
μ	dynamic viscosity
Φ	Dissipation function
ρ	Density of liquid
σ	Liquid–vapour surface tension
θ_e	Equilibrium contact angle
ξ	D/d (spreading factor)
Pe	ud/a (Péclet number)
Re	$\rho ud/\mu$ (Reynolds number)
We	$\rho u^2 d/\sigma$ (Weber number)

1. Introduction

Much attention has been given to the rapid solidification of melts, particularly alloys, because of its ability to produce new microstructures not encountered in conventional casting or heat-treatment processes. Splat-quench solidification in particular has generated a great deal of interest because of its especially high cooling rates and relative procedural simplicity. There exists a wealth of studies published that have investigated novel microstructures produced by splat-quenching [1–4].

However, relatively little work has been done to investigate the deformation process of the droplet as it impacts the surface, with consideration of its solidification. Because it bears directly on the material quality of the deposition layer, the deformation process is a very important aspect of any splat-quenching application. On a macro-scale, the smoothness of the deposition and the quality of contact between layers is dependent on the degree to which the droplets flatten on impact. The ability of the splat to make good thermal contact in turn affects the microstructure of the deposition as governed by the cooling rate of the splat [5]. Also affecting the cooling rate is the thermal inertia of the splat which is a function of the thickness of the splat [6].

The degree of difficulty introduced by considering the simultaneous effect of solidification within the fluid kinetics of the splat-quenching problem has compelled researchers to uncouple the problem. Theoretical and experimental results have shown this simplification to be justifiable for a variety of conditions [7–9].

Studies that have uncoupled the solidification effects from the fluid kinetics of a liquid droplet impacting the surface attribute viscous energy dissipation and/or surface energy effects to arresting the spreading droplet [10–12]. However, some ambiguity exists as to the extent to which each of these effects individually controls the thinning of the droplet. In one early splat-quenching study the viscous dissipation of energy was described as the dominant factor in the terminal splat thickness [10]. Another splat-quenching study, published recently, asserted that surface tension energy was principally responsible for arresting the spreading droplet [11]. A third study, also published recently, in which n-heptane liquid droplets impacting a hot plate were investigated, combined both viscous energy dissipation and surface tension effects to predict the maximum area covered by the splat [12].

The one splat-quenching study in which the droplet spreading kinetics remained coupled with solidification demonstrated the conditions under which solidification could affect the flattening of the droplet [7]. However, this study, as well as those previously cited, failed to assess the separate contributions of viscous energy dissipation and surface tension on the final flatness of the splat. It is a principal purpose of this paper to assess the extent of these separate contributions. But first it is desirable to review the various existing treatments used in determination of the flattening of a liquid droplet impacting a solid surface.

2. Review of existing models

Since the thinning of the splat is of primary interest in this study, a dimensionless "spread factor" or "degree of flattening" is defined as the ratio of the final splat diameter, D, to the initial droplet diameter, d:

$$\xi = D/d \tag{1}$$

Results of all the studies reviewed here will be expressed in terms of the spread factor so as to facilitate comparisons between them.

All the treatments of a liquid droplet impacting a solid surface reviewed here start with the energy balance

$$E_k = E_v + E_s \tag{2}$$

which is a statement that the initial kinetic energy (E_k) of the droplet impacting the surface is dissipated as viscous energy (E_v) and surface tension energy (E_s). The initial kinetic energy of the droplet is treated identically by all the studies reviewed here and is given as

$$E_k = \frac{\pi}{12} d^3 \rho u^2 \tag{3}$$

where d is the diameter of the droplet before impact, ρ is the density of the molten droplet and u is the impact velocity of the droplet. The viscous energy dissipation and the change in surface tension energy are handled in a variety of manners, as will be discussed.

2.1. Jones' model

Jones [10] evaluated the viscous energy dissipation by modelling the splat as a cylindrical volume of a viscous liquid being flattened between two parallel plates. He followed an earlier theoretical analysis of a plastometer performed by Dienes and Klemm [13] and assumed that the flattening rate of the droplet was a constant, at one-half the impact velocity. Following Jones' assumptions yields for the viscous energy dissipation

$$E_v = \frac{27\pi}{1024} \mu u d^2 \xi^8 \tag{4}$$

where μ is the viscosity of the molten metal.

Jones felt that the surface tension contribution to the termination of the droplet's spreading was negligible. Therefore, combining Equations 2, 3 and 4 yields the resulting expression for the dimensionless spread factor

$$\xi = \left(\frac{4}{3} Re^{1/4}\right)^{1/2} \tag{5}$$

where Re is the Reynolds number given by $Re = \rho u d/\mu$ (the fluid properties are of the liquid droplet).

There are several problems with Jones' model. First, the omission of contributing surface tension effects restricts his solution to a limiting case which will be elucidated later. Second, there are assumptions in Dienes and Klemm's [13] analysis of the plastometer that are prohibitively restrictive to the droplet splatting problem. Perhaps the most inaccurate of the assumptions is that the radial spreading velocities of the splat are slow and steady-state.

Experimental results have failed to support Jones' model, yielding splat thicknesses of about five times greater than Equation 5 would predict [9, 10]. Jones felt that the discrepancy might have arisen from oxidation of the droplets caused by residual oxygen in the inert atmosphere. However, the fact that an independent study yielded the same discrepancy between his model and experimental results would appear to preclude that possibility [9].

2.2. The model of Collings *et al.*

Collings *et al.* [11] developed a model for predicting an upper bound on the final splat diameter. In their model, changes in surface and interfacial energies between the droplet, substrate and ambient gas accounted for the dissipation of the pre-impact kinetic energy of the droplet. The final calculated splat diameter was considered to be an upper bound because of the neglected viscous energy dissipation. Following their model one obtains for the energy dissipated by surface tension

$$E_s = \pi \sigma d^2 \left(\frac{(1 - \cos\theta_e)\xi^2}{4} - 1\right) \tag{6}$$

where σ is the surface tension of the liquid–vapour interface and θ_e is the equilibrium contact angle between the liquid and the substrate. Collings *et al.* [11] assumed that the last term inside the brackets of Equation 6 was negligible. This is equivalent to assuming that the initial surface tension energy of the droplet is negligibly small. Following their assumptions, combining Equations 2, 3 and 6 one obtains for the dimensionless spread factor

$$\xi < \left(\frac{We}{3(1 - \cos\theta_e)}\right)^{1/2} \quad (7)$$

where We is the Weber number given by $We = \rho u^2 d/\sigma$ (the fluid properties are of the liquid droplet).

Collings *et al.* [11] proposed that the viscous energy dissipation was in fact negligible, so that the less-than-equal sign in Equation 7 could be replaced by an equal sign. Furthermore, they suggested that a gas film layer would develop between the advancing liquid and the substrate, causing the contact angle to approach π. Following these assumptions yields the expression

$$\xi = (We/6)^{1/2}. \quad (8)$$

There are several problems with the treatment by Collings *et al.* [11] of the splatting droplet. First, as will be shown later, viscous energy dissipation can be neglected only for highly special cases. Additionally, the initial surface tension energy can typically be 25% ($\xi = 4$, $\theta_e = \pi/2$) of the final surface tension energy, which brings us to question their omission of this term in the equations following Equation 6. Finally, their application of Young's equation in the treatment of the contact angle is questionable. Young's equation, which relates the surface tension energies between the liquid–vapour, liquid–solid and solid–vapour interfaces with the equilibrium contact angle, applies only to equilibrium conditions [14]. Therefore, the equilibrium contact angle must be evaluated after the advancing liquid has come to rest. The advancing contact angle which Collings *et al.* equated to the equilibrium angle has been shown experimentally to be a complicated function of the contact-line speed [15, 16]. It is doubtful that the equilibrium contact angle in the experiment of Collings' *et al.* was as large as π.

The experimental results of Collings *et al.* [11] render a spread factor 100% larger than application of Equation 8 predicts, suggesting an incongruity in their model.

2.3. Chandra and Avedisian's model

Chandra and Avedisian [12] in their boiling study of a liquid droplet impacting a solid surface accounted for both viscous energy dissipation and surface tension effects. The main ingredients of their model also apply to the fluid kinematics of the splat-quench solidification problem. Their treatment of the interfacial energy changes between the droplet, substrate, and ambient gas was essentially the same as that of Collings *et al.*, leading to Equation 6. However, Chandra and Avedisian investigated the viscous energy dissipation with an order-of-magnitude study. They approximated the viscous energy lost to the deformation of a liquid droplet as

$$E_v = \int_0^{t_c}\int_V \Phi \, dV dt \simeq \Phi V t_c \quad (9)$$

where t_c is the spreading time of the splat, V is the volume of the droplet and Φ is the dissipation function given by

$$\Phi = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} \simeq \mu\left(\frac{\partial u_1}{\partial x_1}\right)^2 \simeq \mu\left(\frac{u}{s}\right)^2 \quad (10)$$

where the index 1 refers to the vertical direction and s is the terminal thickness of the splat.

Substituting Equation 10 into Equation 9 and approximating the spreading time by $t_c \simeq d/u$ yields

$$E_v = \frac{3\pi}{8}\mu u d^2 \xi^4 \quad (11)$$

Combining Equations 2, 3, 6 and 11 yields Chandra and Avedisian's [12] relationship for determining the spread factor:

$$\frac{(9/2)\xi^4}{Re} + \frac{3[(1 - \cos\theta_e)\xi^2 - 4]}{We} = 1 \quad (12)$$

Chandra and Avedisian's [12] treatment of the splat process appears sound. However, one would expect their rudimentary treatment of the viscous energy dissipation to adversely affect the results somewhat. Chandra and Avedisian's model for predicting the spread factor yielded results that were 20 to 40% higher than experimentally measured results. It is likely that the discrepancy between experimental and theoretical results arose from the deficient model of the viscous energy dissipation.

The vertical velocity scale of the splatting droplet problem is bounded by zero and u, and the vertical length scale is bounded by s and d. In their order-of-magnitude estimation of the dissipation function given by Equation 10, Chandra and Avedisian evaluated $\partial u_1/\partial x_1$ by characterizing the change in velocity by u (the highest bounding value) and characterizing the change in distance by s (the lowest bounding value). Choosing median scale values for these quantities, however, did not improve the agreement between Chandra and Avedisian's experimental results and the model.

2.4. Madejski's model

The most advanced treatment of the droplet deformation problem arising in splat-quenching was presented by Madejski [7]. His model accounts for viscous energy dissipation, surface tension effects and simultaneous freezing of the splat, without uncoupling the solidification problem from the fluid kinetics problem. Madejski modelled the splat kinetics as a spreading cylindrical cake, assuming that the flow was laminar and that the advancement rate of the freeze in the splat was heat transfer limited (Stefan problem). Madejski's treatment of the surface tension was analogous to that of Collings *et al.* [11] except that he considered only the surface tension energies between the liquid–vapour interfaces, precluding the introduction of the contact angle. Madejski expressed the viscous energy dissipation in terms of the shear stress invoked by the velocity gradient within the spreading splat. However, Madejski had to assume an expression for the velocity distribution within the splat.

Madejski's numerical analysis yielded the degree of flattening as a function of four parameters: *Re*, *We*, *Pe* and κ, where κ is a dimensionless parameter, introduced in Madejski's derivation, that reflects the degree to which the solidification arrests the flattening of the splat. For the case in which κ is zero, the droplet flattens without solidification.

From his analysis, Madejski was able to produce analytical expressions for some special cases, as well as a graphical solution to the most general case. For the case in which the flattening of the splat is arrested by the surface tension only, $\kappa = 0 = Re^{-1}$, Madejski found that

$$\xi_s = (We/3)^{1/2} \tag{13}$$

if $We > 100$. This is recognized to be equivalent to Equation 7 with $\theta_e = \pi/2$.

For the case in which the flattening of the splat is arrested solely by viscous dissipation of energy within the splat, $\kappa = 0 = We^{-1}$, Madejski found that

$$\xi_v = 1.2941(Re + 0.9517)^{1/5} \tag{14}$$

Alternatively, for $Re > 100$, Equation 14 can be approximated as

$$\xi_v = 1.2941\,Re^{1/5} \tag{15}$$

For the more general case in which both surface tension and viscous dissipation contribute to terminating the flattening of the splat, but freezing does not, $\kappa = 0$, Madejski found that

$$\frac{3\xi^2}{We} + \frac{1}{Re}\left(\frac{\xi}{1.2941}\right)^5 = 1 \tag{16}$$

provided that $Re > 100$ and $We > 100$.

An independent study [9] on the formation of thermally sprayed alumina in which extremely high values of *We* (10^3 to 2×10^4) prevailed showed Equation 16 to yield "excellent agreement" with experimental results. Under the conditions of this experiment, the first term in Equation 16, representing the surface tension effects, can be neglected, seeming to indicate that the remaining term represents a credible account of the viscous energy dissipated in the splat.

Madejski's treatment [7] of the freezing aspect of the droplet deformation process assumes that the temperature of the droplet at the time of impact is the solidification temperature of the metal, and that the onset of solidification is simultaneous with the time of impact. Furthermore, Madejski assumes that the freezing front advances through the melt in the near equilibrium limit controlled by the rate at which latent heat is transported away from the melt–freeze interface (commonly known as the Stefan problem). Madejski's assumptions are highly idealized and not very compatible with accepted rapid solidification theory.

It will be the premise of this work that solidification does not contribute significantly to terminating the spreading of the splat. However, one noteworthy condition, in which the effect of solidification can be expected to contribute substantially, is the case in which undercooling of the droplet (cooling below the freezing temperature) occurs prior to impact. Under this circumstance, nucleation can be expected to coincide closely with the time of impact, but nucleation sites will not be limited to the splat–substrate interface, allowing for simultaneous freezing across the thickness of the melt not accounted for in Madejski's model. Additionally, the crystal growth within the undercooled melt will not be heat transfer limited, allowing for much faster crystal growth kinetics than predicted by the Stefan problem incorporated in Madejski's model.

Frequently, the impinging droplet is in a state of superheat (above the freezing temperature). If this is the case, then freezing cannot commence until much of the superheat is removed (assuming predominantly Newtonian cooling). Additionally, sluggish nucleation encourages undercooling of the melt after impact, which further increases the time lapse between impact and the start of solidification. Consequently, in a more physically realistic system than that depicted by Madejski's model, the effect of solidification on arresting the spreading of a superheated droplet is likely to be secondary compared to the effects of viscous dissipation and surface tension.

Jones [10] made the same observation: that solidification considerations could be neglected, by noting that the freezing velocities (approximated by the Stefan problem) were more than two orders of magnitude less than typical impact velocities encountered in his experiments. While the rapid solidification that is typical of splat-quenching can produce much faster crystal growth kinetics than predicted by the Stefan problem, consideration of the undercooling encountered in rapid solidification dictates that the reduction in freezing time due to higher solidification speed is offset by the delay in nucleation.

The preceding review of models for predicting the flattening of a liquid droplet impacting a solid surface has illuminated several similarities and differences in the existing treatments with regard to surface tension and viscous dissipation of energy. All treatments of surface tension effects were fundamentally the same, with differences arising from varying degrees of simplifying assumptions. The splat energy dissipated through surface tension effects was shown to be proportional to the spread factor squared plus the initial droplet surface tension (when considered). The treatments of viscous energy dissipation, however, were all dissimilar. The various models yielded viscous energy dissipation as proportional to the spread factor raised to the fourth, fifth, and eighth power.

3. Selection of a working model

To pursue the objective of evaluating the relative roles of viscous energy dissipation and surface tension effects on the final flatness of the splat it becomes necessary to select the "best" treatment of the problem at hand. This is not difficult with regard to surface tension effects because of the apparent consensus in modelling this aspect of the problem. However, in consideration of the viscous dissipation of energy there is no such accord. Because it provides the most theoretically sound treatment of viscous energy dissipation, and has the most published experimental support, Madejski's treatment of the problem will be exploited in this investigation. Madejski's description of the surface tension contribution to the spread factor given by Equation 13, however, effectively ignores the initial surface tension energy of the droplet prior to impact. To theoretically improve Madejski's model we will refine his treatment of the surface tension effects.

From Madejski's model [7] we can extract for the viscous energy dissipation

$$E_v = \frac{\pi}{12}\mu u d^2\left(\frac{\xi}{1.2941}\right)^5 \tag{17}$$

for $Re > 100$.

Madejski's treatment of the surface tension can be refined by using the treatment of Collings *et al.* [11] (as well as that of Chandra and Avedisian [12]), as given by Equation 6. Combining Equation 17 with Equations 2, 3 and 6 yields

$$\frac{(\xi/1.2941)^5}{Re} + \frac{3[(1-\cos\theta_e)\xi^2 - 4]}{We} = 1 \tag{18}$$

In their study of an n-heptane liquid droplet impacting a surface, Chandra and Avedisian [12] investigated the effect of heating the substrate to cause boiling of the splat as it spreads, an intriguing reversal of the solidification problem. In their handling of this complexity, they photographically studied the change in contact angle as a function of substrate temperature for evaluating Equation 12. They found substantial changes in the contact angle as the heating of the plate created a vapour film at the splat–substrate interface. The ensuing effect on the spread factor was substantial. Chandra and Avedisian found that as the contact angle changed from 30 to 180° the spread factor was reduced by 40%.

The equilibrium contact angle reflects a state of minimum surface free energy associated with the liquid, solid and gas interfaces that characterize the splat before solidification. It delineates a unique value that is independent of other influences which may dictate a non-equilibrium contact angle. In consideration of a spreading splat, one would expect the dynamics of the "rolling" liquid flow near the moving contact line to have a large effect on the contact angle. This type of compounding effect must be accounted for when assessing the equilibrium contact angle.

Some dynamic effects do have a legitimate impact on the equilibrium contact angle. For example, gas entrapment between the solid substrate and the advancing liquid front of the splat will have an influence on the surface free energy associated with the "liquid–solid" interface. In Chandra and Avedisian's study [12], the droplet boiling on the substrate introduced a vapour film between the liquid and solid interfaces that clearly influenced the equilibrium contact angle. However, it is unclear to what extent the contact angles measured in Chandra and Avedisian's study were dictated by the vapour film effect as opposed to other dynamic effects.

Chandra and Avedisian's study [12] clearly demonstrates that changes in the equilibrium contact angle have a significant influence on the spread factor. To further investigate the impact of the equilibrium contact angle on the spread factor, Equation 18 is solved in Fig. 1, for various Weber numbers, as a function of the equilibrium contact angle. Fig. 1 illustrates that for decreasing Weber number, the equilibrium contact angle has an increasing effect on the spread factor. If the droplet does not "wet" the surface (θ_e is large) the impact of the Weber number on the spread factor becomes large. Fig. 1 demonstrates that incorrectly measuring the equilibrium contact angle by one radian can easily introduce an error of more than 20% in the spread factor.

4. Viscous dissipation and surface tension domains

For the purpose of illustrating the relative contributions of viscous energy dissipation and surface tension of terminating the spreading of the splat, a typical value for the equilibrium contact angle of a liquid metal on solid metal will be assumed. Taken the equilibrium contact angle in Equation 18 as $\pi/2$ yields

$$\frac{(\xi/1.2941)^5}{Re} + \frac{3(\xi^2 - 4)}{We} = 1 \tag{19}$$

In the extreme case in which $We^{-1} = 0$ (surface tension effects become negligible) Equation 19 becomes the same as Equation 15. In the extreme case in which $Re^{-1} = 0$ (viscous energy dissipation becomes negligible) Equation 19 becomes

$$\xi_s = \left(\frac{We}{3} + 4\right)^{1/2} \tag{20}$$

In general, if the degree of flattening is predicted by surface tension considerations while neglecting viscous dissipation, the prediction will be higher than the actual value. Let ξ represent the degree of flattening predicted by consideration of both viscous energy dissipation and surface tension energy, as given by Equation 19. Then a correction factor can be introduced to Equation 20 in order to compensate for the neglected viscous energy dissipation:

$$\xi = C_s\xi_s = C_s\left(\frac{We}{3} + 4\right)^{1/2} \tag{21}$$

Eliminating ξ from Equations 21 and 19 yields an equation relating the Reynolds number to the Weber number and correction factor:

$$Re = \frac{\{C_s[(We/3) + 4]^{1/2}/1.2941\}^5}{(1 - C_s^2)[1 + (12/We)]} \tag{22}$$

Similarly, the degree of flattening predicted by viscous energy dissipation considerations, while neglecting surface tension effects, can be corrected by introducing

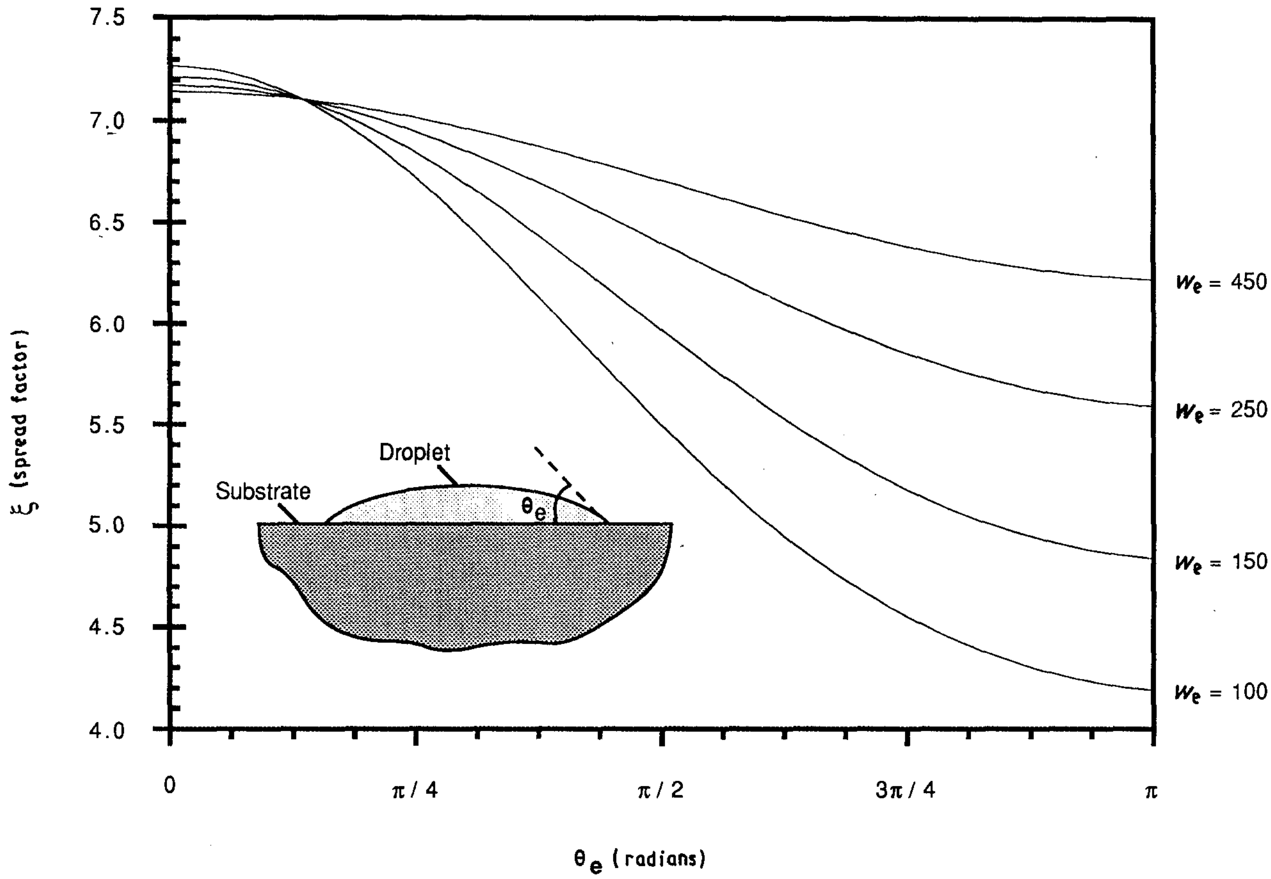

Figure 1 Influence of the equilibrium contact angle on the spread factor. $Re = 5000$ for all curves.

a correction factor to Equation 15:

$$\xi = C_v \xi_v = C_v 1.2941 Re^{1/5} \quad (23)$$

Eliminating ξ from Equations 23 and 19 yields a second equation relating the Reynolds number to the Weber number and correction factor:

$$Re = \left(\frac{[(We/3)(1 - C_v^5) + 4]^{1/2}}{1.2941 C_v} \right)^5 \quad (24)$$

The relationships between the Weber number, Reynolds number and the correction factors, as given by Equations 22 and 24 are plotted in Fig. 2.

Several comments are in order in assessing the meaning of Fig. 2. The first is that the We–Re domain can be divided into two regions. In one region the degree of flattening of the splat is dominated by viscous energy dissipation and in the other region the degree of flattening is dominated by surface tension effects. The border between these two domains is marked by the bold-face curve representing $C_s \simeq C_v \simeq 0.816$. The value of the correction coefficients near the border implies that even if the droplet spreading is described by only viscous energy dissipation or only surface tension effects, if the dominating influence is correctly selected, the error introduced by the simplification will be no more than approximately 20% (according to the model).

The second comment is that even well into the viscous dissipation domain the effects of the surface tension are still significant, while in the surface tension domain the effects of viscous dissipation disappear far more rapidly as one moves away from the borderline.

The results of this analysis allow us to state the condition under which surface tension effects dominate the termination of the spreading of the splat:

$$We < 2.80 Re^{0.457} \quad (25)$$

This result is obtained by curve-fitting the bold-face border curve. The condition given by Equation 25 is stated as such because the viscous energy dissipation contribution to the spread factor rapidly declines within the surface tension-dominated region. The alternative inverse of the condition given by Equation 25 was not stated for the simple reason that significant residual surface tension effects extend well into the viscous energy dissipation region.

With the insight gained from the above analysis we can now assess the assumptions about the relative magnitude of viscous energy dissipation and surface tension effects made by Jones [10] and Collings *et al.* [11] in their respective experimental studies. Jones' [10] experiment is characterized by $Re \simeq 110$ and $We \simeq 360$, which is well into the viscous dissipation domain of Fig. 2. From Equation 24 we can determine that the spread factor should be about 98% correctly predicted from viscous energy dissipation alone. Therefore, it is apparent that the failure of Jones' model to predict the spread factor was not the result of neglected surface tension effects.

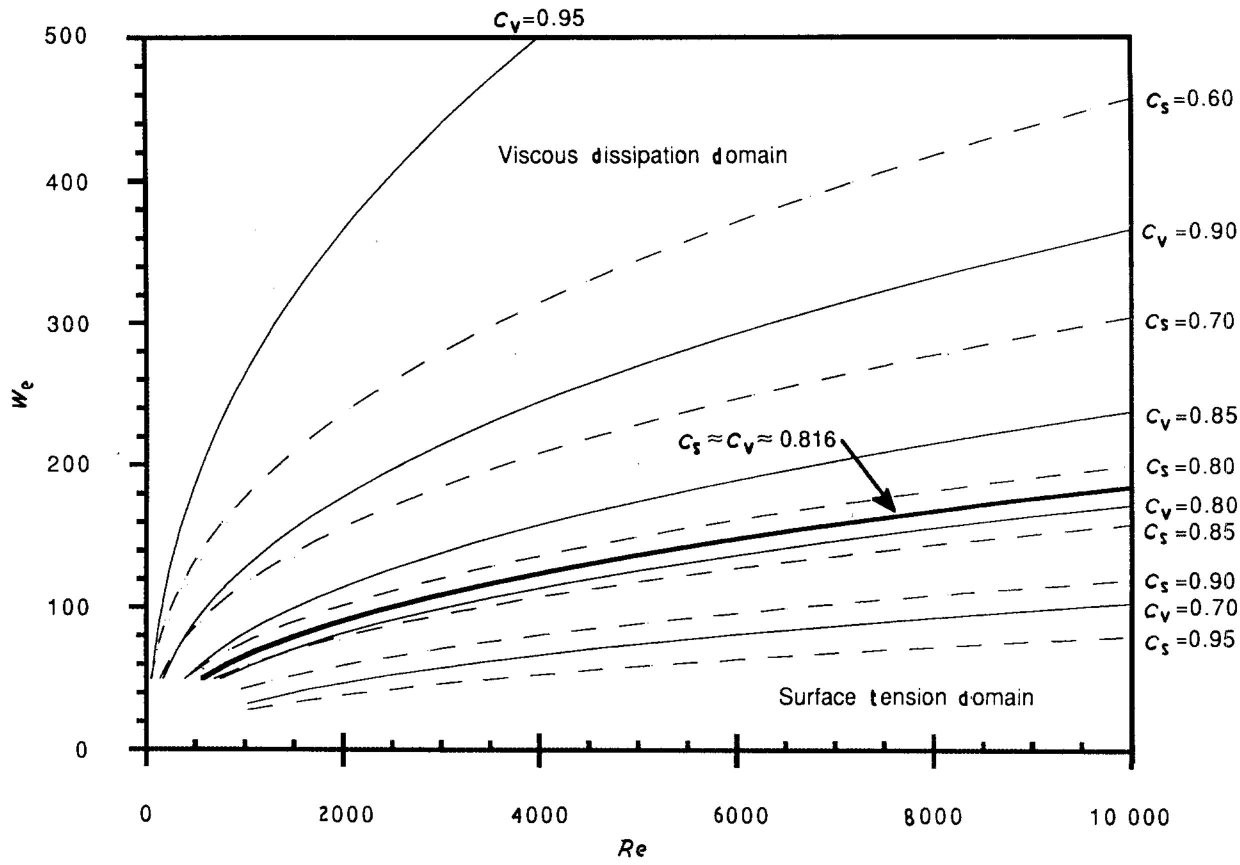

Figure 2 Domains of influence on the spread factor.

The experiment of Collings *et al.* [11] is characterized by $Re \simeq 4.5 \times 10^4$ and $We \simeq 92$, which is well into the surface tension domain. From Equation 22 we can determine that the spread factor should be about 98% correctly predicted from surface tension effects alone. However, the model of Collings *et al.* which was based on surface tension effects alone predicted a spread factor 100% too small. The grossly large discrepancy indicates that their treatment of the equilibrium contact angle was incorrect. Use of Equation 19 (which assumes the value of the contact angle to be $\theta_e = \pi/2$) predicts a spread factor 28% smaller than what Collings *et al.* obtained experimentally. The improved prediction provided by Equation 19 demonstrates the importance of contact angle consideration as well as inclusion of the initial droplet surface tension in the model.

5. Conclusion

The results of this study have clarified the extent to which the separate contributions of viscous energy dissipation and surface tension effects determine the terminal thickness of a liquid droplet impacting a solid surface. This paper has defined two domains, characterized by the Weber and Reynolds numbers, which are discriminated by the principal mechanism responsible for arresting the splat spreading. In doing so, we have introduced the condition, stated by Equation 25, under which surface tension effects dominate the termination of the spreading of the splat. We have also shown that surface tension is significant to the spread factor even well into the viscous dissipation domain, while the effect of viscous dissipation rapidly disappears in the surface tension domain (shown in Fig. 2).

This paper has illustrated the importance of correctly determining the equilibrium contact angle for predicting the spread factor of a splat (Fig. 2). Improper treatment of the equilibrium contact angle can easily introduce significant error in predicting the spread factor. Special attention has been given to the equilibrium condition imposed on the contact angle measurement, resulting from application of Young's equation in the modelling of surface tension effects.

Conditions under which solidification of the splat would or would not be expected to contribute to terminating the spreading of the splat were considered in this paper. We illuminated the inadequacies of modelling the freezing kinetics found in the splat-quenching problem with a simple Stefan problem approach. However, our *a priori* assumption was that the effect of solidification on the spreading of a droplet, superheated at impact, is most often secondary compared to the effects of viscous dissipation and surface tension.

Our review of the various treatments used in the theoretical determination of the spread factor, discloses a need for more experimental studies to further evaluate existing treatments of the splat problem, as well as to suggest new refinements of these models. We exploited facets of existing models which we believe were the most sound; however, we appreciate the need for further refinement of existing theories regarding splat kinetics.

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