International Tax and Public Finance, 3: 29–44 (1996)
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Imperfect Competition and the Optimal Combination of *ad valorem* and Specific Taxation

GARETH D. MYLES
Department of Economics, University of Exeter, Exeter, EX4 4RJ, U.K.

Abstract

While it has long been recognized that under imperfect competition *ad valorem* and specific taxation differ in their effects, the optimal combination of the two instruments has received little attention. This paper shows how combining the two taxes can eliminate the welfare loss due to imperfect competition by inducing profit-maximizing firms to charge the appropriate Ramsey price.

Key words: imperfect competition, taxation, Ramsey pricing

1. Introduction

Many countries subject some commodities to both *ad valorem* and specific taxation. For example, all member states of the European Union levy both a VAT and an excise duty on tobacco, alcoholic drinks, and petrol. While the excise duty on petrol and alcohol is purely specific, that on cigarettes has both a specific and an *ad valorem* component. The balance between the specific tax and the *ad valorem* tax on cigarettes differs between the northern countries,[1] which favor a role for specific taxation, and the southern countries,[2] which favor *ad valorem* taxation.[3] Present European Union legislation restricts the ratio of specific to total taxation to be in the range of 5 to 55 percent. In the United States, taxation of cigarettes is through specific taxation except for Hawaii, where *ad valorem* taxation is employed. These differing systems of taxation naturally raise the question of whether the choice between *ad valorem* and specific taxation affects the equilibrium outcome and, if it does, what determines the optimal mix of the instruments.

In a competitive environment, *ad valorem* and specific taxation are entirely equivalent. That is, a specific tax can be replaced by an *ad valorem* tax (and *vice versa*) that raises an equal level of revenue and leads to the same consumer and producer prices. As first shown by Wicksell (1896) for the case of constant marginal cost, this equivalence does not apply in the presence of monopoly and, for a given level of revenue, an *ad valorem* tax leads to a lower consumer price and therefore greater output. Since the work of Wicksell, a literature has developed that further explores this nonequivalence. Wicksell's result was extended by Suits and Musgrave (1955) to general cost functions. More recently, Skeath and Trandel (1994) have proved that for any specific tax on a monopolist, there exists an *ad valorem* tax that results in higher consumer surplus, profit, and tax revenue. In an oligopolistic environment, Skeath and Trandel show that this result applies, with linear demand and constant marginal cost, only if the number of firms is small and the taxes are large. Most important for the present paper, Delipalla and Keen (1992) consider the combination of *ad valorem* and specific taxation that maximizes welfare subject to the constraint that both tax instruments are nonnegative and conclude that the nonnegativity constraint is binding on the specific tax.[4]

The restriction in Delipalla and Keen (1992) that both taxes be nonnegative is clearly arbitrary, so that the existing literature, while recognizing the distinct effects of *ad valorem* and specific taxation with imperfect competition, has not yet answered the question of what can be achieved by optimally combining the two instruments. Although *ad valorem* taxation has been shown to dominate specific taxation, this does not preclude the possibility that the simultaneous use of both instruments, allowing negativities, may achieve more than either instrument can achieve individually. It is this simple idea that is explored in the present paper.

The nature of the equilibrium outcome obtained by optimally combining *ad valorem* and specific taxation can best be understood as follows. Consider an economy in which some, or all, productive processes involve fixed costs. Assume that the government sets prices in each industry to maximize social welfare, while collecting a specified level of revenue, subject to the constraint that no lump-sum taxes or subsidies can be employed.[5] The prices that result from this optimization are known as Ramsey prices[6] and represent the second-best given that marginal cost pricing is not feasible in the absence of lump-sum subsidies. The results of this paper prove that the optimal combination of specific and *ad valorem* taxation can achieve these same Ramsey prices in a private ownership economy with imperfect competition. Expressed differently, the use of specific and *ad valorem* taxation is sufficient to eliminate the welfare loss arising from monopolistic behavior, leaving only the welfare loss from the existence of increasing returns.[7] The pair of tax instruments therefore have a degree of effectiveness far beyond that suggested by any of the existing literature.[8]

In interpreting this result it is important to realize that employing profit taxation in addition to the optimal combination of *ad valorem* and specific taxation would not lead to any further gains in welfare. In an imperfectly competitive economy it is not the presence of profits *per se* that leads to allocational distortions but rather the pricing policy that supports those profits. Although a profit tax can raise revenue without adding distortions, it cannot correct the mispricing. The results below show that given the constraint that lump-sum subsidies cannot be given to firms, *ad valorem* and specific taxation lead to the second-best optimal prices. In addition, since profit is zero for each firm at these prices, profit taxation is redundant. It must also be noted that the argument that the first-best can be achieved by a unit subsidy financed by a profit tax is limited at the best of times[9] and does not apply at all with increasing returns,[10] which, of course, is the case of interest with imperfect competition.

Section 2 presents a numerical example of an economy with a monopolistic sector that shows how Ramsey pricing can be generated and illustrates the features of the solution that form the foundation of the formal analysis. The formal results for the single industry case are given in Section 3 with the analysis emphasizing the geometry of the problem. Section 4 discusses how the conclusions can be extended to a general equilibrium economy with any finite number of competitive and imperfectly competitive industries. Conclusions and interpretation are given in Section 5. The appendix contains the proofs of the lemmas and propositions.

2. An example

The purpose of this section is to introduce a numerical example that explores the consequences of combining *ad valorem* and specific taxation. The economy of the example has one consumer, a government and an industry, consisting of a single firm, that produces the consumption good using labor as the only input. The government uses its revenue to purchase labor and labor supply is endogenous. This economy is analyzed in two different institutional settings. The first considers state ownership of the firm and is introduced to characterize the optimal public sector pricing policy when lump-sum tax instruments are not available. The economy is then studied under private-ownership with the government constrained to levy only specific and *ad valorem* taxes. In both cases, the labor required by the firms and government is supplied by the consumer through a competitive market with the wage rate as numeraire.[11] As noted in the introduction, the conclusion will be that the equilibrium outcome is the same in both cases.

When production is controlled by the state, it will choose the optimal Ramsey price for the consumption good by solving the following maximization

$$\max_{\{p\}} V(p, I) \text{ subject to } R + C(X) = p\,X(p),\ I = 0, \tag{1}$$

where $V(p, I)$ is the consumer's indirect utility function, p is the price of the consumption good, I is lump-sum income, R represents the government revenue requirement, $X(p)$ the demand function for the consumption good, and $C(X)$ the cost function. The restriction that lump-sum income is zero is a consequence of the assumption that lump-sum taxes cannot be employed. The formulation of the maximization in (1) captures Ramsey pricing in its simplest setting. The solution to (1), denoted by p^*, characterizes the Ramsey price that gives the maximum attainable welfare without the use of lum-sum taxation. Due to the simple structure of this economy, the maximization is solved by finding the price that satisfies the revenue constraint.

Now consider the same economy under private ownership with the single industry controlled by a profit maximizing monopolist but with the state controlling the level of *ad valorem* and specific taxation. In this case, the market price is obtained as the solution to

$$\max_{\{p\}} \pi = [[1 - t_v]p - t_s]X(p) - C(X(p)), \tag{2}$$

with π the level of profit, t_v the rate of ad valorem taxation, and t_s the level of the specific tax. The optimal price and the maximized level of profit arising from the maximization in (2) will be dependent on the values of t_v and t_s. To capture this dependence, the solution to (2) is denoted by $p = \rho(t_v, t_s)$, the maximized level of profit by $\pi = \zeta(t_v, t_s)$, and the level of consumption $X(\rho(t_v, t_s)) = \xi(t_v, t_s)$.

The Ramsey price identified as the solution to (1) and the government revenue target will both be obtained[12] in the private ownership economy if there exists a combination of *ad valorem* and specific taxes, t_v^*, t_s^* such that the firm's profit is nonnegative, so $\xi(t_v^*, t_s^*) \geq 0$, and

$$\text{(i) } \rho(t_v^*, t_s^*) = p^*, \text{ (ii) } [t_v^* \rho(t_v^*, t_s^*) + t_s^*]\xi(t_v^*, t_s^*) = R. \tag{3}$$

Condition (i) requires that the taxes induce the monopolist to charge the Ramsey price and (ii) that the taxes raise the required revenue. The requirement that profit be nonnegative can be viewed as a participation condition for firms.

To show that there exist economies for which (3) can be satisfied, let the utility function of the consumer be

$$U = aX - \frac{bX^2}{2} - L, \tag{4}$$

and assume the cost function to have constant marginal cost

$$C(X) = F + cX. \tag{5}$$

Two distinct possibilities can arise depending on whether F is zero or positive. The case of positive F, which is the simpler of the two, is treated first.

(i) $F > 0$. Choosing parameter values $R = 200$, $F = 1000$, $a = 100$, $b = 1.1$, and $c = 5$, the solution to (1) gives $p^* = 21.902$ and a utility level of 2772.408. Eqs. (3.i) and (3.ii) are satisfied by $t_v^* = 0.8089$, $t_s^* = -14.8999$. In addition, these tax rates lead to zero profit. The existence of this pair of taxes therefore proves that situations exist in which Ramsey pricing can be sustained in imperfectly competitive markets. It is important to note that this is an interior solution with $t_v^* < 1$.

(ii) $F = 0$. When $F = 0$ the parameter values $R = 1000$, $a = 1000$, $b = 0.8$, and $c = 1$ give $p^* = 1.801$ with corresponding welfare level $V = 622750.13$. The outcomes arising from various combinations of t_v and t_s are reported in Table 1. It is clear by comparing the Ramsey price $p^* = 1.801$ and resulting welfare level $V = 622750.13$ to the entries in Table 1 that the market outcome is tending toward Ramsey pricing as $t_v \to 1$. Hence the combination of specific and *ad valorem* taxation is leading, in the limit, to Ramsey pricing and zero profit.

Table 1.

t_v	t_s	V	p	X	π
0.05	-22.86	474219	488.49	639.38	310695
0.25	-106.08	507754	429.94	712.57	304652
0.5	-166.20	553112	334.80	831.50	276556
0.75	-150.20	597600	201.60	998.00	199200
0.9	-82.48	617504	92.62	1134.22	102917
0.91	-75.83	618415	84.29	1144.64	94334
0.95	-46.07	621288	49.37	1188.28	56480
0.98	-20.14	622491	21.39	1223.26	23941
0.995	-5.93	622730	6.77	1241.54	6165
0.995	-1.50	622749	2.30	1247.12	622
0.99999	-1.01	622750	1.81	1247.73	12

Parameters: $a = 1000$, $b = 0.8$, $c = 1$, $\lambda = 1$, $R = 1000$.

The example reveals two forms of solution: one interior and the other found as a limit. The common property of these solutions is that the combination of tax instruments is succeeding in entirely eliminating the welfare loss due to monopoly power. Geometrically, the structure of these solutions can be easily understood. The curves in Figures 1a and 1b labeled $p = p^*$ and $R = const.$ are given by the loci of pairs $\{t_v, t_s\}$ that solve $\rho(t_v, t_s) = p^*$ and $[t_v \rho(t_v, t_s) + t_s]X(\rho(t_v, t_s)) = R$, respectively. These are sketched, for the linear demand case, in Figure 1a for $F > 0$ and in 1b for $F = 0$. The $p = p^*$ curve is upward sloping in this case since its gradient is given by $a - 2p^* > 0$. Point A represents the use of specific taxation alone ($t_v = 0$) and point B only *ad valorem* taxation ($t_s = 0$). As shown in Section 3, a simultaneous solution to (3.i) and (3.ii), which occurs at the intersection of the two curves, also implies that profit is zero. The important properties of Figure 1 are that when $F > 0$ the two curves intersect at a value of t_v less than 1 and that when $F = 0$ the two curves converge as $t_v \to 1$, the gradient of $p = p^*$, in terms of dt_s/dt_v, being greater than that of $R = const.$ for a given value of t_v. The role played by fixed costs in distinguishing these two cases is discussed in Section 3. By definition, pairs of taxes on the $p = p^*$ curve generate the price that maximizes welfare but do not generally achieve the required level of revenue. Those on the $R = const.$ curve satisfy the revenue requirement but, generally, at an inefficient price. However, the closer the $R = const.$ curve gets to the $p = p^*$ curve, the closer the market price becomes to the efficient price. Therefore, loosely speaking, welfare is raised by any tax change that moves toward the $p = p^*$ curve. This observation underlies all the results that follow.

In summary, when there are fixed costs, the optimal combination of *ad valorem* and specific taxes can generate Ramsey pricing at an interior solution with the curves $p = p^*$ and $R = const.$ crossing at a value of $t_v < 1$. When fixed costs are zero, Ramsey pricing can be sustained in the limit. The welfare loss due to the imperfect competition is eliminated in both cases. The intuition behind these results is quite simple. The *ad valorem* tax has the effect of reducing the gradient of the marginal revenue curve so that the perceived influence of the monopoly on price falls. In contrast, the specific tax can be treated as an addition to marginal cost (actually a reduction since it is negative at the optima). Together,

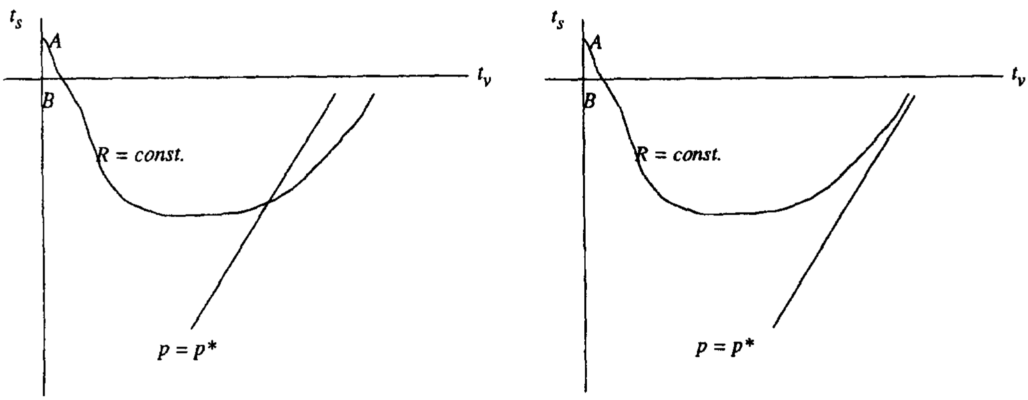

Figure 1. (a) $F > 0$. (b) $F = 0$.

these provide enough flexibility to manipulate the intersection of marginal cost and marginal revenue to the point where the desired quantity and price are obtained. That profit is also zero at this point is just a consequence of the accounting identities; this is shown below. The importance of fixed costs is due to the change from constant returns to scale to increasing returns that they cause; this will be discussed further below.

3. A single industry

Section 2 has shown numerically that Ramsey pricing can be achieved in a monopolistic economy by combining *ad valorem* and specific taxation. The purpose of this section is to show this conclusion can be extended to an oligopolistic economy and to demand and cost functions more general than those of the example. Sufficient conditions for the existence of a pair of taxes that lead to Ramsey pricing are derived and the optimal taxes are characterized.

Denoting by n the number of active firms, the Ramsey price p^* is defined, by analogy to (1), as the solution to

$$p^* X(p^*) = R + nC(n^{-1}X(p^*)). \tag{6}$$

As above, this is the price that a central planner, who was unable to employ lump-sum transfers, would adopt. It should be noted that the price p^* is optimal given that the existing number of firms is maintained; changes in the number of firms are not considered.[13]

To define the equilibrium in the private ownership economy, the firms are indexed by $i = 1, \ldots, n$. Letting x_i be the output of firm i, the inverse demand function facing the industry is given by $p = p(X)$, $X = \Sigma_{i=1}^{n} x_i$, with first derivative $p_X < 0$. Each firm has the cost function $C = C(x_i)$ where $C(0) \equiv F \geq 0$ and $C_x > 0$. The assumption of identical cost functions imposes symmetry between firms and is employed for simplicity. To give the model the widest possible interpretation, the conjectural framework of Seade (1985) is employed. The common conjecture of the firms on the value of dX/dx_i is denoted by λ, with $0 < \lambda \leq n$, so that $\lambda = 1$ leads to the Cournot equilibrium and the Bertrand equilibrium is approached as λ tends to 0.

The first-order condition for profit maximization of a typical oligopolistic firm is

$$[1 - t_v]p - C_x - t_s + [1 - t_v]\gamma p_X X = 0, \tag{7}$$

where $\gamma = \lambda/n$, $0 < \gamma \leq 1$. Since the symmetry implies that $x_i = x = X/n$, all $i = 1, \ldots, n$, equation (7) can be solved to express the equilibrium level of output as a function of the tax rates. As in Section 2, the output function is denoted

$$X = \xi(t_v, t_s). \tag{8}$$

Substituting (8) into the inverse demand function then determines the equilibrium price conditional on the taxes. Hence

$$p = p(\xi(t_v, t_s)) = \rho(t_v, t_s). \tag{9}$$

Finally, using (9) and the profit identity

$$\pi_i = [[1 - t_v]p(X) - t_s]\frac{X}{n} - C\left(\frac{X}{n}\right), \; i = 1, \ldots, n, \tag{10}$$

determines the equilibrium profit level as

$$\pi_i = \pi = \zeta(t_v, t_s), \text{ all } i = 1, \ldots, n. \tag{11}$$

It is assumed that $\xi(t_v, t_s)$, $\rho(t_v, t_s)$ and $\zeta(t_v, t_s)$ are continuously differentiable.

Following the discussion in Section 2, Ramsey pricing can be sustained if the tax rates satisfy (3) and give each firm a nonnegative profit. In fact, the latter condition is redundant since if (3.i) and (3.ii) are satisfied each firm actually makes zero profit. To see this, note that

$$\begin{aligned}\pi_i &= [[1 - t_v]p - t_s]\frac{X}{n} - C,\\ &= \left[\rho - \frac{R}{\xi}\right]\frac{\xi}{n} - C, \text{ using (3.ii)}\\ &= \frac{1}{n}[\rho\xi - R - nC],\\ &= 0 \text{ using (3.i) and (6).}\end{aligned}$$

Therefore, as noted above Figure 1, the satisfaction of (3.i) and (3.ii) implies zero profit so that in the analysis of the optimal tax scheme the participation condition need not be explicitly considered.

The formal results now establish conditions that determine when the optimal taxes will be an interior solution that generates Ramsey pricing or a limit solution. Defining, as above, the $p = p^*$ and $R = const.$ curves as the loci of solutions to $\rho(t_v, t_s) = p^*$ and $[t_v\rho(t_v, t_s) + t_s]\xi(t_v, t_s) = R$,[14] respectively, Lemma 1 relates the relative positions of the curves to the profit levels of the firms and Lemma 2 determines the change in profit along the $p = p^*$ curve. Proposition 1 then characterizes the conditions under which there is an interior solution.

The following assumptions are maintained throughout.

Assumption 1. *There exist t_v^0 and t_s^0 such that* $[t_v^0\rho(t_v^0, 0)]\xi(t_v^0, 0) > R$ *and* $t_s^0\xi(0, t_s^0) > R$.

Assumption 2. *For all pairs of taxes $\{t_v, t_s\}$ that solve* (3.i), $[t_vC_x + t_s]/[1 - t_v] < -[1 - \gamma]p_XX$.

The interpretation of Assumption 1 is that the revenue requirement is sufficiently small that it can be achieved by use of either of the tax instruments alone and is adopted in order to give the problem some content. Assumption 2 is also very weak since the right side of the inequality is always positive, whereas only values of t_v satisfying $t_v \leq 1$ need be considered and t_s is generally negative.[15] An alternative interpretation of Assumption 2 can be obtained by using (7) to write the inequality as $p + Xp_X < (1 - t_v)p + (1 - t_v)\gamma Xp_X - t_s$. This requirement is equivalent to true marginal revenue being below perceived marginal revenue. Since the aim of the policy is to reduce the equilibrium price to the Ramsey level, this will be achieved by encouraging an expansion in output that is exactly what follows from raising perceived marginal revenue above actual marginal revenue.

Lemmas 1 and 2 are now given.

Lemma 1. *If for* $t_v = \hat{t}_v$, $\bar{t}_s$ *and* $\tilde{t}_s$ *are defined by* $\rho(\hat{t}_v, \bar{t}_s) = p^*$, $[\hat{t}_v\rho(\hat{t}_v, \tilde{t}_s) + \tilde{t}_s]\xi(\hat{t}_v, \tilde{t}_s) = R$, *then*

(i) $\bar{t}_s < \tilde{t}_s$ *if* $\zeta(\hat{t}_v, \tilde{t}_s) > 0$,

and

(ii) $\bar{t}_s > \tilde{t}_s$ *if* $\zeta(\hat{t}_v, \tilde{t}_s) < 0$.

Lemma 2. *Profit is monotonically decreasing along the* $p = p^*$ *curve as* t_v *increases.*

The implication of Lemma 1 is that the $p = p^*$ curve lies outside the $R = const.$ curve whenever the firms earn positive profits on the $R = const.$ curve. Hence if there are points on the $R = const.$ curve such that the firms earn negative profits then the two curves must cross at some point prior to this and, from Lemma 2, they must only cross once. From these observations follows Proposition 1.

Proposition 1. *If there exist* t_s^n, t_v^n *such that* $t_v^n < 1$, $[t_v^n\rho(t_v^n, t_s^n) + t_s^n]\xi(t_v^n, t_s^n) = R$ *and* $\zeta(t_v^n, t_s^n) < 0$, *then there exists a unique pair* $\{t_v^*, t_s^*\}$ *with* $t_v^* < 1$ *that generates Ramsey pricing.*

Proposition 1 has demonstrated that Ramsey pricing can be generated when the firms become unprofitable at some combinations of taxes that satisfy the revenue requirement. This provides the first possible form of optimal tax policy. The economic reasoning lying behind this result is that the use of an *ad valorem* tax reduces the perceived market power of the imperfectly competitive firms by reducing the effect of output changes on marginal revenue while the specific tax can be targeted as a subsidy towards covering fixed costs.

The requirement of the proposition, that profit become negative at some point on the $R = const.$ curve, was seen in the example of Section 2 to occur when F, $\equiv C(0)$, was positive. In the general case it has not proved possible to isolate a sufficient condition. A perspective on the role of fixed costs[16] can be obtained by substituting from (7) into (10) to write maximized profits as

$$\pi = -[1 - t_v]\gamma p_X nx^2 - [C - C_x x]. \tag{13}$$

As the first term on the right of (13) in nonnegative, profit can only become negative when $C - C_x x > 0$. With increasing ($C_{xx} > 0$) or constant marginal cost ($C_{xx} = 0$), this can only occur if fixed costs are positive. With decreasing marginal cost ($C_{xx} < 0$), fixed costs are not strictly necessary. Furthermore, (13) also shows that with constant returns to scale profit tends to zero as $t_v \to 1$. This is what lies behind the findings of the numerical results. For the case of constant marginal cost the necessity of positive F for an interior solution can also be seen by solving (7) and (10) for t_s and t_v. Doing this, the level of t_s and t_v are characterized implicitly by

$$[1 - t_v] = - \frac{Fn}{\gamma p_X X^2}, \; t_s = - n \left[\frac{F + cX/n}{X} + \frac{F}{\gamma p_X X^2} \right], \tag{14}$$

where X and p_X are evaluated at the Ramsey price and quantity. From (14), t_v can only be less than 1 when F is positive.

The optimal policy when there is no pair of tax rates on the $R = const.$ curve that lead to negative profits is described in proposition 2. To prove this, it is first necessary to establish a third lemma.

Lemma 3. *Tax revenue is monotonically increasing along the $p = p^*$ curve as t_v increases.*

Application of this lemma now gives Proposition 2.

Proposition 2. *If there does not exist any pair t_s^n, t_v^n such that $t_v^n < 1$, $[t_v^n \rho(t_v^n, t_s^n) + t_s^n]\xi(t_v^n, t_s^n) = R$ and $\zeta(t_v^n, t_s^n) < 0$, then the optimal policy is to let $t_v \to 1$ with t_s determined by the $R = const.$ curve. If $\lim \pi = 0$ as $t_v \to 1$ along $R = const.$ then Ramsey pricing is generated in the limit.*

The general superiority of *ad valorem* over specific taxation demonstrated by Delipalla and Keen (1992) follows from noting that assumption 1 implies that the $R = const.$ must pass through points A and B as shown in Figure 1. The relative positions of A and B, in conjunction with Lemma 1, then provides a simple illustration of the Delipalla-Keen result for imperfect competition since B is closer to $p = p^*$ than A.

4. Generalization

The previous section has demonstrated the conditions under which Ramsey pricing can be generated in a single imperfectly competitive market. The purpose of this section is to indicate how this argument can be generalized to a general equilibrium economy with M final goods. As in Section 2, the economy will first be analyzed under state control in order to define the optimal set of Ramsey prices. This will be followed by an analysis of the private ownership equivalent.

The set of available final goods is partitioned into two subsets. The goods in the first subset, indexed $j = 1, \ldots, m_1$, are produced with constant returns to scale. In the private ownership economy, these will represent the goods produced by competitive industries. The remaining goods, $j = m_1 + 1, \ldots, M$, are those that will be produced by imperfectly competitive industries and the production technology for these is not restricted to be constant returns to scale. All productive processes employ only labor and the wage rate is taken as numeraire.

With all productive processes under state control, the Ramsey pricing problem for this economy is[17]

$$\max_{\{p_1, \ldots, p_M\}} V(p_1, \ldots, p_M, I), \tag{15}$$

subject to

$$I = 0,$$

and

$$\sum_{i=1}^{M} p_j X^j(p_1, \ldots, p_M) = R + \sum_{j=1}^{M} n^j C^j \left[\frac{X^j(p_1, \ldots, p_M)}{n^j} \right], \tag{17}$$

where $X^j(\cdot)$ is the demand function of industry j, n^j the number of firms in the industry and $C^j(\cdot)$ the cost function for each firm in that industry. The solution to this problem is denoted by the Ramsey price vector $p^* = p_1^*, \ldots, p_M^*$

In the private ownership economy, the first m_1 goods are produced by competitive industries. The constant returns to scale assumption with labor as the only input implies that the pretax price of each of these goods is just a multiple of the wage rate. The theory of optimal commodity taxation (for example, Diamond and Mirrlees, 1971) then shows that the Ramsey prices $p_1^*, \ldots, p_{m_1}^*$ can be sustained in these industries by the use of either specific or *ad valorem* taxation regardless of the behavior of the rest of the economy (although these prices will only be optimal when the prices determined by the imperfectly competitive industries are also set at the Ramsey level).

In the imperfectly competitive industries, the construction in Myles (1989) shows that for each industry, j, the first-order conditions for profit maximization and the profit identities for the firms in that industry can be used to express the market clearing price, output level, and the profit level of firms in industry j as functions of the tax rates facing industry j and the prices, p_{-j}, on other markets. This argument is a simple extension of that following (7) above. Given the vector of Ramsey prices p^*, define for each $j = 1, \ldots, m_1$ the revenue level of R_j^* that is collected from industry j by the optimal set of Ramsey prices.[18] Obtaining Ramsey pricing is then equivalent to showing that (3.i) and (3.ii) can be solved for all industries simultaneously for price p_j^* and level of revenue R_j^*.

To show that this can be done,[19] Assumptions 1 and 2 are adopted[20] and it is also assumed for each industry that there exists the pair of taxes defined in Proposition 1 that yield negative profit. The proof then proceeds as follows. Fix all taxes and prices in industries other than j. For industry j this gives a situation equivalent to Section 3 and Proposition 1 shows that (3) can be solved. It is then argued that the tax rates that solve (3) for industry

j are continuously dependent on the tax rates levied on other industries. The tax rates for industry j solving (3) can therefore be treated as arising from a continuous reaction function which has as its arguments the tax rates on other industries. Moreover, the reaction function takes values in a compact, convex set. Application of Brouwer's theorem then proves that there is at lest one fixed point where the reaction functions are satisfied simultaneously and the tax rates solve (3) for all industries.

The formal theorem can be summarized as follows.

Theorem 1. *There exists a tax system* $\{t_s^{1*}, \ldots, t_s^{M*}, t_v^{1*}, \ldots, t_v^{M*}\}$ *such that Ramsey pricing is generated in all industries.*

Theorem 1 assumes that, whatever prices rule elsewhere in the economy, there exists a set of tax rates on the $R = const.$ locus for each imperfectly competitive industry that make profit negative. When this does not hold but, as in Proposition 2, profit can be driven to zero in the limit, the following theorem can be given.

Theorem 2. *There exists a tax system* $\{t_s^{1+}, \ldots, t_v^{M+}\}$ *such that (i) for* $j = 1, \ldots, m_1$, $\rho^j(\cdot) - p_j^* < \epsilon$ *for any* $\epsilon > 0$, *(ii)* $p_j = p_j^*$ *for* $j = m_1 + 1, \ldots, M$ *and (iii)* $[t_v^{j+} \rho^j + t_s^{j+}]\xi^j = R_j^*$ *for* $j = 1, \ldots, M$.

As a final consideration, in the case in which Ramsey pricing cannot be achieved at an interior point nor approached in the limit, it will not necessarily be optimal in the many-good case to let the *ad valorem* tax tend to 1 in contrast to the single-good result. Although doing so would lead each price to be as close as possible to its Ramsey level, standard considerations of second-best theory show that if one price does not attain the Ramsey level, it may not be optimal to try and attain Ramsey pricing on other markets.

5. Conclusions

The paper has taken as its starting point the literature contrasting *ad valorem* and specific taxation. Although this literature has investigated the differing effects of the two tax instruments in the presence of imperfect competition, it has not provided a satisfactory analysis of their optimal combination. The results obtained in the present paper show that *ad valorem* and specific taxation can achieve the same outcome as a central planner who controls production but cannot make lump-sum transfers. Since this outcome eliminates all welfare losses due to imperfect competition, the combination of instruments have an effectiveness far in excess of what can be achieved by the use of either instrument alone.

The mechanism at work behind this striking result is easily explained. Consider the case of a zero revenue requirement for which the optimal combination of instruments will consist of a positive *ad valorem* tax and a negative specific tax.[21] The role of the *ad valorem* tax is to reduce the perceived market power of the firms, and it achieves this by reducing the firm's influence upon marginal revenue. In contrast, the negative specific tax can be interpreted as a *constant* price per unit that cannot be influenced by the firm. To see the role of these two factors most clearly, let fixed costs be zero. The optimal policy is then an *ad valorem* tax rate of 100 percent and a specific subsidy equal to marginal cost. Marginal revenue for a firm faced with this tax system is given by the specific subsidy, which, of course, cannot be affected by changes in the firm's level of output. The firm therefore sees itself as facing a fixed price that is independent of output. This results in it producing at the competitive output level with price at marginal cost. The same general reasoning is also correct when fixed costs are positive, though in this case the firm must be permitted to generate a surplus in order to cover fixed costs. In consequence, the optimal *ad valorem* tax rate is less than 100 percent in order to leave the firm with some limited monopoly power.

Appendix

Proof of Lemma 1

Since p^* is derived as the solution to (6), define

$$G(t_s\ ;\ \hat{t}_v) \equiv p(\xi(\hat{t}_v, t_s))\ \xi(\hat{t}_v, t_s) - R - nC(n^{-1}\xi(\hat{t}_v, t_s)).$$

The first point to establish is that $\partial G/\partial t_s > 0$. By definition,

$$\frac{\partial G}{\partial t_s} = [p - C_x]\frac{\partial \xi}{\partial t_s} + Xp_X\frac{\partial \xi}{\partial t_s} = \frac{\S y}{\partial t_s}[p - C_x + Xp_X].$$

Since $\partial\xi/\partial t_s < 0$, it is only necessary to show that $[p - C_x + X p_X] < 0$. From (7)

$$[p - C_x] + Xp_X = \frac{t_s + t_v C_x}{1 - t_v} + [1 - \gamma]p_X X.$$

The restriction that $[t_v C_x + t_s]/[1 - t_v] < -(1 - \gamma)p_X X$ then implies $[p - C_x + Xp_X] < 0$ and hence $\partial G/\partial t_s > 0$.

Since $\partial G/\partial t_s > 0$ it is clear that if $G(t_s\ ;\ \hat{t}_v)$ evaluated at $\{\tilde{t}_s, \hat{t}_v\}$ is positive then the solution to $G(t_s\ ;\ \hat{t}_v) = 0$ is reached by reducing t_s and hence $t_s < \tilde{t}_s$. The converse holds if $G(t_s\ ;\ \hat{t}_v)$ is negative. Now note that if $[\hat{t}_v p + \tilde{t}_s]X = R$, then

$$G = pX - R - nC = pX - nC - X[t_v p + t_s] = [[1 - t_v]p - t_s]X - nC = n\pi.$$

Therefore, $G(\tilde{t}_s\ ;\ \hat{t}_v) > 0$ if $[[1 - t_v]p - t_s]X - nC = n\pi = n\zeta(t_v, t_s) > 0$ and $G(\tilde{t}_s; \hat{t}_v) < 0$ if $n\pi = n\zeta(t_v, t_s) < 0$. This proves lemma 1. □

Proof of Lemma 2

Writing profit as

$$\pi = [1 - t_v]p(\xi(t_v, t_s))\xi(t_v, t_s) - C(\xi(t_v, t_s)) - t_s\xi(t_v, t_s),$$

the envelope theorem gives

$$\left.\frac{d\pi}{dt_v}\right|_{p=p^*} = -p\xi(t_v, t_s) - \xi(t_v, t_s)\left.\frac{dt_s}{dt_v}\right|_{p=p^*},$$

or

$$\left.\frac{d\pi}{dt_v}\right|_{p=p^*} = -x\left[p + \left.\frac{dt_s}{dt_v}\right|_{p=p^*}\right]$$

From (6)

$$\left.\frac{dt_s}{dt_v}\right|_{p=p^*} = -\frac{[p - C_x]\dfrac{\partial \xi}{\partial t_v} + X\dfrac{\partial \rho}{\partial t_v}}{[p - C_x]\dfrac{\partial \xi}{\partial t_s} + X\dfrac{\partial \rho}{\partial t_s}}$$

The inverse demand function, $p = p(X)$, implies $\partial\rho/\partial t_i = p_X(\partial\xi/\partial t_i)$, $i = v, s$. In addition, total differentiation of (7) provides the expressions

$$\frac{\partial \rho}{\partial t_s} = \frac{1}{[1 - t_v][1 + \gamma\eta]},\ \eta = 1 - \frac{C_{xx}}{\lambda[1 - t_v]p_x} + \frac{p_{xx}X}{p_x},$$

$$\frac{\partial \rho}{\partial t_v} = \Phi\frac{\partial \rho}{\partial t_s},\ \Phi = \frac{C_x + t_s}{1 - t_v}.$$

Using these gives

$$\left.\frac{dt_s}{dt_v}\right|_{p=p^*} = -\frac{\dfrac{\partial \rho}{\partial t_v}}{\dfrac{\partial \rho}{\partial t_s}} = -\frac{C_x + t_s}{1 - t_v}.$$

Hence

$$\left.\frac{d\pi}{dt_v}\right|_{p=p^*} = -x\left[p - \frac{C_x + t_s}{1 - t_v}\right] = -\frac{x}{1 - t_v}\,[[1 - t_v]p - C_x + t_s] < 0$$

from (7). □

Proof of Proposition 1

The statements in the proposition imply, by Lemma 1, that the $p = p^*$ curve must lie to the right of the $R =$ *const.* curve at t_s^n, t_v^n. Therefore, since Assumption 1 implies $\pi \geq 0$ at B, the point where the $R =$ *const.* curve crosses the t_v – axis, there must be some point at which they cross and at this point $t_v < 1$. The crossing point therefore determines the taxes that generate Ramsey pricing. The crossing point is also unique since if the curves intersected more than once Lemma 2 would be violated. □

Proof of Lemma 3

Since revenue is given by $[t_v \rho(t_v, t_s) + t_s]\xi(t_v, t_s)$, it follows that a long the $p = p^*$ curve

$$\left.\frac{dR}{dt_v}\right|_{p=p^*} = \left[\rho + t_v \frac{\partial \rho}{\partial t_v} + t_v \frac{\partial \rho}{\partial t_s}\left.\frac{\partial t_s}{\partial t_v}\right|_{p=p^*} + \left.\frac{\partial t_s}{\partial t_v}\right|_{p=p^*}\right]\xi + [t_v \rho + t_s]\left[\frac{\partial \xi}{\partial t_v} + \frac{\partial \xi}{\partial t_s}\left.\frac{\partial t_s}{\partial t_v}\right|_{p=p^*}\right].$$

Using the results of Lemma 2 to simplify gives

$$\left.\frac{dR}{dt_v}\right|_{p=p^*} = \left[\rho + t_v \frac{\partial \rho}{\partial t_v} - t_v \frac{1}{\Phi}\frac{\partial \rho}{\partial t_v}\Phi - \Phi\right]\xi + [t_v \rho + t_s]\left[\frac{1}{p_X}\frac{\partial \rho}{\partial t_v} - \frac{1}{p_X}\frac{\partial \rho}{\partial t_s}\Phi\right],$$

or

$$\left.\frac{dR}{dt_v}\right|_{p=p^*} = [\rho - \Phi]\xi > 0.$$

using the first-order condition (7). □

Proof of Proposition 2

Lemma 1 has shown that when profit is positive, for any level of *ad valorem* tax the specific tax on the $p = p^*$ curve is less than that on the $R = const.$ curve. Tax revenue is therefore less on the $p = p^*$ curve than the $R = const.$ curve for a given level of the *ad valorem* tax. Since Lemma 3 shows that tax revenue is increasing along the $p = p^*$ curve, the two curves must converge as the *ad valorem* tax increases. Hence the optimal policy is always to let t_v tend to 1 with t_s determined by the $R = const.$ curve. The second part of the proposition follows from lemma 1. □

Acknowledgments

Thanks are due to Martin Cripps, Norman Ireland, the two anonymous referees of this journal and seminar participants at Bristol, Exeter, and Warwick. Special thanks are due to Mick Keen for his guidance in revising the paper.

Notes

1. Denmark, Germany, Ireland, the Netherlands, and the United Kingdom.
2. Belgium, France, Greece, Italy, Luxembourg, Portugal, and Spain.

3. Further discussion of these issues can be found in Delipalla (1994).
4. Some of these issues have also been investigated in the context of international trade theory (see Helpman and Krugman, 1989).
5. This implies that each industry must, at least, break-even.
6. Bös (1985) clearly elucidates the major principles of Ramsey pricing and provides a number of alternative characterizations of the relevant optimality conditions. For a discussion from an industrial organization perspective see Waterson (1988).
7. Guesnerie and Laffont (1978) show that a similar outcome can be achieved when lump-sum taxation is employed. Given the difficulties inherent in the implementation of lump-sum taxes (see Mirrlees, 1986; Myles, 1995), the present result can claim greater applicability.
8. A partial exception is Dillén (1994), who approaches a similar problem from a different perspective but considers only the case of zero fixed costs.
9. If the monopoly price when the subsidy is equal to marginal cost (so net marginal cost is zero) is greater than marginal cost (which may arise when marginal cost is low), the subsidy that supports marginal cost pricing must lead to the firm facing a negative net marginal cost for each unit of output. If demand is unbounded at a price of zero and marginal cost is constant, it will then be optimal for the firm to give the product away rather than charge marginal cost.
10. With increasing returns, profit is negative at the marginal cost pricing outcome. The scheme cannot, then, be self-financing.
11. This could be formally justified by introducing additional competitive industries into the example. These would not alter the conclusions (see Section 4) but would simply lead to an increase in complexity.
12. Given the instruments under its control, the government cannot do better than to obtain the Ramsey price.
13. Free-entry oligopoly is also not considered. This is for two reasons. First, given the number of active firms the free-entry equilibrium achieves average cost pricing without any intervention (given n, it solves (1) for $R = 0$). Second, when variation in the number of firms is incorporated the free-entry model is of more interest in conjunction with an analysis of product differentiation. Optimization of the tax system in this context has been analyzed elsewhere (see Kay and Keen, 1987; Delipalla, 1994).
14. If this equation has multiple solutions, the solution with the lowest value of the specific tax (for a given value of *ad valorem* tax) is chosen. This ensures maximal welfare and, given continuity, that the outcome is on the "right" side of the Laffer curve with tax revenue being an increasing function of the specific tax.
15. Recall that the inequality is evaluated at pairs of taxes that generate the Ramsey price. This requires a positive *ad valorem* tax (to reduce perceived market power) and a specific subsidy to reduce the posttax price. In addition, when t_v is close to zero, t_s must be large (in absolute value) in order to achieve the reduction of price to p^*.
16. This argument was suggested by Mick Keen.
17. This is the direct generalization of the optimization in (1). The interpretation given there applies again here.
18. It follows that $\Sigma_{j=1}^{M} R_j^* = R$.
19. The formal proof is available from the author on request.
20. Both being defined for the revenue level R_j^*.
21. Since $[t_v p + t_s]X = 0$ when $R = 0$, it follows that $t_s = - t_v p < 0$.

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