An M-Sector, N-Group Behavioral Model of Self-Employment[1]

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ABSTRACT. A behavioral model is developed to determine the number of self-employed individuals in an economy characterized by different production sectors and social groups of which the individuals differ by their risk-aversion, their evaluation of job characteristics other than income, their expected managerial ability, the variance of their managerial ability and their productive performance as employee. The model leads to a linear complementarity problem that can be solved by the Lemke-algorithm. The 1-sector, n-group model and the m-sector, 2-group model are singled out for an extensive comparative static analysis.

1. Introduction

Self-employment receives a lot of attention these days. Among the reasons for this recent interest are the following: (i) after a long period of decline the fraction of the labor force that is self-employed has increased since the mid-1970s in several Western countries (cf. Blau (1987) and Evans and Leighton (1989a)); (ii) self-employed individuals are believed to make a positive contribution to the employment level: by choosing for self-employment they create their own jobs and maybe jobs for others if they engage personnel. The interest of governments for self-employment is apparent from programs in several Western countries to stimulate unemployed workers to begin their own business (cf. Evans and Leighton (1989a)). But also in the academic field there is a growing interest. Apart from the rapidly increasing number of publications on the subject — that will be shortly reviewed below — this is for instance apparent from the establishment of this specialized Journal, the scope of which covers not only empirical work and policy issues, but also theoretical contributions. This paper is intended to provide such a theoretical contribution regarding the determinants of self-employment.

Recently quite a lot of work has been done empirically on the determinants of self-employment. Several studies have been carried out using cross-sectional data on self-employed and wage workers to estimate static models of the choice for self-employment. Examples of studies adopting this approach are Long (1982) and Moore (1983) who investigate (primarily) the influence of taxation, Rees and Shah (1986) who interpret their findings in the light of risk aversion, and De Wit and Van Winden (1989) who (primarily) investigate the influence of childhood ability and family background variables. In part of their paper Evans and Leighton (1989a) also adopt this static approach. However, in the other part they enrich this analysis by investigating the determinants of entry into and exit out of self-employment. Evans and Jovanovic (1989) also investigate the entry into self-employment. Their starting point is the question whether liquidity constraints are important when considering the transition from paid-employment to self-employment. Finally, the studies of Blau (1987) and Evans and Leighton (1989b) are worth mentioning. Applying different methods of analysis, both these studies are concerned with explaining the trends over time in the self-employment rate on a more aggregate level.

In the empirical studies mentioned above the models are kept relatively simple, partly for reasons of estimation, partly because of data limitations. In theoretical studies such considerations are not important, so that different and more

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fundamental questions can be posed. In Lucas (1978) the number of self-employed individuals is determined in a static *general-equilibrium* framework. Thus — contrary to what is encountered in empirical studies — the income received in a job is dependent on the number of individuals that actually chooses for that job. Kanbur (1979, 1981), and Kihlstrom and Laffont (1979) adopt also this static general-equilibrium approach and add the explicit modeling of the risk involved in self-employment. Subsequently they investigate the resulting inefficiency of the equilibrium. In these models the managerial ability of individuals is important when they choose whether or not to become self-employed. Calvo and Wellisz (1980) suggest a dynamical explanation for this managerial ability. They explain the managerial abilities of individuals out of the growth in common technical knowledge, the ability of individuals to acquire that knowledge, and the age of individuals. Finally, the contribution of Jovanovic (1982) is noteworthy. He presents a *dynamical partial-equilibrium* model that focuses on just one production sector. Given the time-path of the demand for the products of this sector and assuming that those who choose to become self-employed learn about their managerial abilities from period to period, entry- and exit-rates of self-employed individuals are determined.

In this paper a behavioral model is developed that determines the number of self-employed individuals within a static general-equilibrium setting. With respect to the existing models mentioned above the following contributions are made:

(i) In the existing models the attractiveness of a job is solely determined by the income that it provides and the variance in that income. In our model other job characteristics, such as the amount of regulation encountered on a job and the status of a job are allowed to be taken into account by individuals when choosing between jobs. This is a useful enrichment, because these factors are widely believed to play an important role when choosing between self-employment and paid-employment.

(ii) In the existing models the economy is simplified to one production sector yielding one homogeneous good. In our model an arbitrary number of sectors is distinguished. This makes it possible to analyze the consequences for the number of self-employed of developments such as shifts in consumer tastes from manufacturing to trade and service sectors (as happened in the past decades).

(iii) In the existing models the job choosing individuals are identical except for one behavioral characteristic (managerial ability) in Lucas (1978) and risk-aversion in Kanbur (1979, 1981) and Kihlstrom and Laffont (1979). In our model an arbitrary number of social groups is distinguished, of which the members differ in as many as five aspects: their risk-aversion, their evaluation of job characteristics other than income, their expected managerial ability, the variance of their managerial ability and their productive performance as employee. This makes it possible to analyze how all these group characteristics interact in the choice for self-employment and determine the "competitive entrepreneurial position" of a social group. Furthermore, the implications for self-employment of schooling of some of the groups, immigration of new groups, or other demographic changes can be analyzed.

(iv) It turns out that the solution of the model can be attained by solving a so-called linear complementarity problem. It is proved that the Lemke algorithm always finds its solution, that (when found) can be written down in analytic form. We want to stress that — given the amount of characterizations and diversifications in the model — it is quite remarkable that a description of the solution is possible at all. In fact, as far as we know , this is the first general equilibrium model in which solutions are obtained, while individuals differ essentially in more than one characteristic.

To enrich the model in the way described while securing an analytic solution, some restrictive assumptions have to be made with respect to in particular the shape of the production and utility functions. Relaxation of these assumptions seems to demand the use of other research methods, such as computer simulation. In this context it is noted, however, that similar specifications as the ones adopted in this paper are even frequently used in (applied) general equilibrium models using simulation techniques (cf. Shoven and Whalley (1984)).

The richness of the model makes the description of it necessarily rather technical. Therefore, the organization of the paper is somewhat different from usual. First, in Section 2, an overview of the paper is given together with a discussion of the results. Subsequently, in Section 3, the general m-sector, n-group model (describing an economy with an abritrary number of m-sectors and an abritrary number of n social groups) is presented in detail. Finally, in Section 4, the 1-sector, n-group model and the m-sector, 2-group model are singled out for an extensive comparative static analysis.

2. Overview and discussion

Roughly, the model works as follows. An arbitrary number of m sectors of production and n groups of identical individuals are distinguished. The sectors are characterized by different production functions. The social groups are characterized by the utility functions, productive performance and managerial abilities of their members. Given these characteristics and the assumption that all individuals seek to maximize their utility, it is determined what profession the individuals will choose. On the one hand they can choose to become self-employed in one of the distinguished sectors, thereby creating a firm (for simplicity each self-employed person runs exactly one firm). On the other hand they can choose to become an employee in the firms created by the self-employed. Simultaneously, other endogenous variables, such as prices, wages and profits are determined in the model.

For analytic reasons the following assumptions are made: (i) the production functions characterizing the technology of the various sectors are of the Cobb-Douglas type; (ii) the utility functions of the members of the various groups are also of the Cobb-Douglas type but nested in a function that establishes constant relative risk aversion; (iii) the preference weights of individuals for products from the various sectors are the same for all social groups; (iv) the distribution functions of managerial ability across the members of the groups are lognormal. Given these assumptions, the solution of the model can be attained by solving a so-called linear complementarity problem. It is proved that — irrespective of the values of the exogenous variables — a solution of the model exists, which can be found by the Lemke algorithm. Moreover, when a solution is found it can be written down in analytic form. The uniqueness of the solution of the model appears in general not to be ensured. However, a sufficient condition is derived to guarantee a unique solution.

The equilibrium solution has the following features. In each sector there are self-employed individuals of at least one group. Furthermore, of each group some members are employee in one of the sectors. However, it is possible that of some groups no individuals are self-employed. Individuals of those groups obtain a higher utility as employee.

To arrive at more specific results the 1-sector, n-group model and the m-sector, 2-group model are analyzed in a comparative static way in Section 4. Because of the many results obtained it is not possible to present them all in this overview. Instead, we confine ourselves to a discussion of only a few interesting ones.

In the 1-sector, n-group model it is possible to construct for each social group j a variable C_j that plays a decisive role in determining whether or not this group will supply self-employed individuals in equilibrium. If C_j exceeds a certain threshold (Θ in the model) the members of this social group are suficiently capable/willing to become self-employed so that a certain fraction actually chooses for self-employment, whereas if C_j falls short of this threshold nobody of group j will opt for self-employment. For this reason the quantity C_j can be thought of as resembling the "competitive entrepreneurial position" of a social group. This competitive entrepreneurial position C_j of a social group j is defined as follows:

$$C_j \equiv \frac{N_j\left(1-\sum_{s=1}^{n} \alpha_s\right) E\Theta_j g_j(z_j^s)\left(1+v(\Theta_j)\right)^{-1/2\gamma_j}}{\alpha_j\, g_j(z_j^e)},$$

where N_j denotes the numerical strength of group j, $(1-\Sigma_{s=1}^{n}\ \alpha_s)$ the management elasticity of output in the sector, $E\Theta_j$ the expected managerial ability of members of group j, $g_j(z_j^s)$ the evaluation by members of group j of job characteristics other than income in the case of self-employment, $v(\Theta_j)$ the variance in the managerial ability of members of group j, γ_j the risk aversion of members of group j, α_j the labor elasticity of output for group j, and $g_j(z_j^e)$ the evaluation by members of group j of job characteristics other than income in the case of being employee. So, if the numerical strength of a social group increases, for example, then the competitive entrepreneurial position of this group becomes better. Or, to give another example, if the risk aversion of a group becomes larger, then the competitive entrepreneurial position of a group becomes worse.

The expression of C_j given above shows in compact form the more general nature of the model compared with the existing models in the literature, where only one behavioral aspect determines the ability or willingness to become self-employed. Another important aspect in comparison with the existing models that shows up in the 1-sector, n-group model is the fact that an improvement of the conditions for self-employment for one group does not necessarily imply that the overall level of self-employment increases. Such an improvement does indeed increase the number of self-employed of the group itself, but when this group runs relatively large firms on average (due to relatively high managerial abilities) this increase is more than compensated by the decrease in the number of self-employed individuals of other groups.

The m-sector, 2-group model also produces results that are worth reviewing in the light of the existing models. For example, it is found that an increase in the risk-aversion of a certain group does not necessarily decrease the number of self-employed individuals of that group in each sector. Instead, the self-employed members of the group redistribute themselves from sectors with relatively high uncertainty regarding profits to sectors with relatively low uncertainty in this respect, whereas the number of self-employed summed over all sectors decreases.

Furthermore, the m-sector approach makes it possible to analyze the impact of shifts in consumer tastes on the number of self-employed. It is shown, for example, that when tastes shift to a sector where the scale of production is relatively small (think of the service sector), this results in an increase in the total number of self-employed.

The developed m-sector, n-group model can be used to analyze the impact on the number of self-employed in the economy of a broad range of socio-economic phenomena. For illustration we mention the following:

- the immigration of a new social group into the economy. This can be analyzed by the introduction of a new group j into the model, of which the numerical strength increases from zero upwards;
- the schooling of one or more of the existing groups: affecting, for instance, the labor elasticity, the expectation or the variance of managerial ability in one or more sectors, or a combination of these variables;
- the aging of a society: by identifying some groups as "young" and other groups as "old" and subsequently analyzing the effects of a decrease in number of the first groups accompanied by an increase in number of the latter groups;
- shifts in consumer tastes from manufactured goods to services: by identifying some sectors as manufacturing sectors and others as service sectors and subsequently analyzing the effects of a shift in the preference parameters of the consumers.

Finally, we want to make some comments on the sensitivity of our results with respect to the aforementioned assumptions that we had to make for analytic reasons. As regards the Cobb-Douglas production function, experimentation with another production function with only one labor input — namely, the summed labor inputs of the employees weighted by their (exogenous) efficiency factors — suggests that most qualitative results are not dependent on the particular shape of the production function.[2]

The Cobb-Douglas specification of the utility functions precludes endogenous shifts in preference weights due to changes in income. Thus, the qualitative results of the model may be sensitive with respect to the specification of the utility functions in situations characterized by substantial changes in income. It is noted, however, that if such a situation is analyzed and one has the idea that reality is better described by preference weights that are dependent on income, the results of the present model can nevertheless be useful. For, if one has an idea in what way the preference weights should have changed due to the established changes in income, it is possible to get an impression of the direction in which the result of the present model must be corrected by analyzing separately the effects of these changes in the preference weights.

In the model it is assumed that the preference weights of all social groups are the same. Thus — compared to a specification with preference weights that are different across social groups — the qualitative results of the model may be different in a situation in which substantial shifts in income from one group to another occur. Again, it is noted that if such a situation is analyzed and one has the idea that reality is better described by preference weights that are different across groups, the results of the present model can nevertheless be useful in the way indicated in the last paragraph.

With regard to the lognormal distribution of managerial ability across the members of a social group, it is possible to make other assumptions while retaining analytic solutions. For example, a general distribution function can be assumed, if the indirect utility functions are approximated to the second order. All results remain the same in that case. However, as there is empirical evidence that — given the other assumptions of the model — the distribution of managerial abilities is lognormal (see Subsection 3.1), we have chosen for this specification.

3. Detailed discription of the model

3.1. *Production*

In this subsection the production function for each sector is introduced. This will enable us to determine the profit expected by an individual deciding to become self-employed, as well as its variance. Furthermore, the product supply and labor demand of the firms are derived. In the sequel the following notational convention will be adhered to: sectors are denoted by a subscript i or h while social groups are indicated by a subscript j, k or s.

To obtain analytic solutions the production functions must be specified, so for each sector the following Cobb-Douglas type of production function is assumed to hold:

$$x_{ij} = \Theta_{ij}\left(1 - \sum_{s=1}^{n} a_{is}\right) \prod_{k=1}^{n} l_{ijk}{}^{a_{ik}},$$

$$i = 1, \ldots, m; \quad j = 1, \ldots, n \tag{1}$$

with

- Π : multiplication operator;
- x_{ij} : output level of a firm in sector i run by an individual of group j;
- Θ_{ij}: managerial ability in sector i of the individual of group j running the firm (see below);
- l_{ijk} : number of individuals of group k employed by a firm in sector i run by an individual of group j;
- a_{ik}: given labor elasticity for individuals of group j when employed in sector $i(a_{ik} \geqslant 0, \Sigma_{j=1}^{n} a_{ik} < 1$ for $i = 1, \ldots, m$, and $\Sigma_{i=1}^{m} a_{ik} > 0$ for $j = 1, \ldots, n)$. The latter condition implies that of each group labor is needed in at least one sector. Note that it is allowed that $\Sigma_{j=1}^{n} a_{ik} = 0$ for some sectors: in those sectors only one-person firms occur.

Wages are the only costs distinguished in the model. Thus, profits are given by:

$$\pi_{ij} = p_i x_{ij} - \sum_{k=1}^{n} w_k l_{ijk},$$

$$i = 1, \ldots, m; \quad j = 1, \ldots n \tag{2}$$

with

- π_{ij}: profit made by an individual of group j running a firm in sector i;
- p_i : product price in sector i;
- w_k: wage paid to individuals of group k.

Ex ante an individual is supposed to be uncertain about the profits that will be made when self-employed. It is only after having decided to become self-employed that the true "managerial ability" in this sense will be discovered. The risk involved here is modeled in the following way.

Managerial ability is represented by the positive stochastic variable Θ_{ij} in the production function (1). It is assumed that its distribution is lognormal (an empirical justification for this is given below), characterized[3] by a given expectation $E\Theta_{ij}$ and a given normalized variance $v(\Theta_{ij}) \equiv \text{var}(\Theta_{ij})/(E\Theta_{ij})^2$. This distribution is supposed to be known to the individual. In case the gains from self-employment (i.e., the realization of Θ) turns out to be disappointing, it is assumed that the individual will nevertheless stay self-employed, for example because of the psychic and economic costs of giving up the firm (cf. Kanbur (1979, p. 773)).

Self-employed individuals are assumed to be price-takers in product and factor markets and the labor-hiring decision is made *after* the managerial ability is discovered. Thus, given the existing wage rates, the product price and the discovered managerial ability — and assuming profit maximization — the self-employed individual will recruit employees up to the point where

$$p_i \, \partial x_{ij}/\partial l_{ijk} = w_k$$
$$i = 1, \ldots, m; \quad j, k = 1, \ldots, n. \tag{3}$$

From Eqs (1)—(3) it is obtained that

$$p_i x_{ij} : \pi_{ij} : w_k l_{ijk} = 1 : \left(1 - \sum_{s=1}^{n} \alpha_{is}\right) : \alpha_{ik}$$
$$i = 1, \ldots, m; \quad k = 1, \ldots, n. \tag{4}$$

Using (1) and (4) the output level x_{ij}, profit π_{ij} and the desired number of employees of group k l_{ijk} can be expressed as a function of wage rates, the product price and the managerial ability of the individual running the firm:

$$x_{ij} = x_i(\cdot)\Theta_{ij} \tag{5a}$$

$$\pi_{ij} = \left(1 - \sum_{s=1}^{n} \alpha_{is}\right) p_i x_i(\cdot)\Theta_{ij} \tag{5b}$$

$$l_{ijk} = (\alpha_{ik}/w_k) p_i x_i(\cdot)\Theta_{ij}$$
$$i = 1, \ldots, m; \quad j, k = 1, \ldots, n \tag{5c}$$

with:

$$x_i(\cdot) = \prod_{j=1}^{n} (\alpha_{ij} p_i / w_j)^{\alpha_{ij}/(1 - \Sigma_{s=1}^{n} \alpha_{is})}.$$

Note that $x_i(\cdot)$ can be interpreted as the output level per unit of managerial ability in sector i. Because all firms operating in sector i are assumed to be price-takers, they face the same wage rates and product price. The quantities x_{ij}, π_{ij} and l_{ijk} are, consequently, directly proportional to Θ_{ij}, while their distribution over the sector is the same as that of Θ_{ij}.

So the size distribution of firms is determined by the distribution of managerial abilities in the model. Because firm size distributions appear to be typically lognormal in reality (Scherer (1980, p. 147)), there exists empirical support for our assumption that the managerial abilities are distributed lognormally.

For the expectation $E\pi_{ij}$ and the normalized variance $v(\pi_{ij})$ one obtains from (5b):

$$E\pi_{ij} = \left(1 - \sum_{s=1}^{n} \alpha_{is}\right) p_i x_i(\cdot) \, E\Theta_{ij} \tag{6a}$$

$$v(\pi_{ij}) = v(\Theta_{ij}), \quad i = 1, \ldots, m; \quad j = 1, \ldots, n. \tag{6b}$$

For convenience the total amount of managerial ability of the self-employed individuals in sector i is labeled Θ_i:[4]

$$\Theta_i = \sum_{j=1}^{n} E\Theta_{ij} n_{ij}, \quad i = 1, \ldots, m, \tag{7}$$

where n_{ij} denotes the number of self-employed individuals of group j in sector i.

The product supply of sector $i(X_i^{\text{sup}})$ and the demand for employees of group $j(L_j^{\text{dem}})$ can be obtained by using (5a) and (5c), respectively:

$$X_i^{\text{sup}} = x_i(\cdot)\Theta_i, \quad i = 1, \ldots, m \tag{8}$$

$$L_j^{\text{dem}} = w_j^{-1} \sum_{i=1}^{n} \alpha_{ij} \, p_i \, x_i(\cdot)\Theta_i, \quad j = 1, \ldots, n. \tag{9}$$

3.2. *Utility functions and risk aversion*

In this subsection the utility functions for the social groups are introduced and attention is paid to the way risk aversion is modeled. For expositional reasons attention will be focused on an individual of group j with a job characterized by an income y that is lognormally distributed, and other characteristics z. In particular, one can think here of the labor intensity of the job, the amount of regulation encountered on the job, and the social status of the job. It is important to note that in this subsection y and z are arbitrary; they will be further specified in the next subsection.

For an individual of group j a direct utility function Φ_j^{dir} of the following type is assumed to hold:

$$\Phi_j^{\mathrm{dir}}(q_1, \ldots, q_m; z) = f_j\left(\prod_{i=1}^{m} q_i^{\beta_i} g_j(z)\right) \quad (10)$$

with

q_i : output of sector i consumed by the individual;

$$f_j(x) = \frac{1}{1-\gamma_j}(x^{1-\gamma_j} - 1), \quad \text{if } \gamma_j \neq 1$$
$$= \ln x, \quad \text{if } \gamma_j = 1,$$

where γ_j is a for each group given real parameter indicating the risk-aversion as shown below;

$g_j(z)$: unspecified, positively valued, utility function of an individual of group j, evaluating the job characteristics other than income;

β_i : given preference weight attached to products of sector $i(\beta_i > 0, \Sigma_{i=1}^{m} \beta_i = 1)$. For simplicity, the β_i's are assumed to be the same for all groups.

Because f_j does not affect the ordinality of the utility function, maximization of the utility function (10) under the budget restriction $y = \Sigma_{i=1}^{m} p_i q_i$ leads to:

$$q_i = \beta_i y / p_i. \quad (11)$$

For later reference the product demand (X_i^{dem}) will be derived at this stage. Because all individuals of all groups have identical preference weights β_i, it follows at once from (11) that:

$$X_i^{\mathrm{dem}} = \beta_i Y / p_i, \quad i = 1, \ldots, m, \quad (12)$$

where Y denotes the income of all individuals together.

As indirect utility function Φ_j^{ind} it is obtained, substituting (11) into (10):

$$\Phi_j^{\mathrm{ind}}(p_1, \ldots, p_m; y; z) = f_j(p^{-1} y g_j(z)) \quad (13)$$

with:

$$p = \prod_{i=1}^{m} (p_i/\beta_i)^{\beta_i}.$$

So the utility obtained by an individual is found to be a function of the different product prices, income and the remaining job characteristics. This utility is a stochastic variable due to the stochastic nature of income y. Our next goal is to find an expression for expected utility.

It now becomes clear what the use is of the function f_j. It is easy to see that, if f_j is absent in (13), expected utility would be linear in Ey but independent of the variance in y. In other words, not using the function f_j — which is equivalent to putting γ_j equal to zero — implies that the model would only apply to individuals that are risk-neutral. Using Eq. (13) it can be shown that $\gamma_j = -y(\partial^2\Phi/\partial y^2)/(\partial\Phi/\partial y)$, which is the Arrow-Pratt measure of relative risk-aversion (e.g., Stiglitz (1969)). An individual is called risk-averse, risk-neutral, or risk-loving, if γ_j is positive, zero, or negative, respectively.

Using the assumption that y is distributed lognormally with expectation Ey and normalized variance $v(y)$, the following expression for expected utility $E\Phi_j^{\mathrm{ind}}(\cdot)$ can be derived:[5]

$$E\Phi_j^{\mathrm{ind}}(\cdot) = f_j(p^{-1} Ey\, g_j(z)\,(1 + v(y))^{-1/2\gamma_j}). \quad (14)$$

Not surprisingly, expected utility will rise, if prices decrease, if expected income increases, or if the utility derived from the job characteristics other than income increases. Furthermore, expected utility is negatively (positively) related to $v(y)$ for risk-averse (risk-loving) individuals, whereas $v(y)$ does not matter if an individual is risk-neutral. Finally, expected utility decreases if people become more risk-averse.

3.3. *Occupational choice*

In this subsection equilibrium conditions are derived from the assumptions with respect to occupational choice.

Individuals have the choice either to become an employee or to become self-employed in one of the sectors. It is assumed that this is a *discrete* choice. If an individual decides to become self-employed in a certain sector, then there is no time left to do other work (cf. Kanbur (1981, pp. 163–164)). So if in equilibrium more than one job appears to be selected by different (identical) members of the same group, then the utility expected from these jobs must have been the same.

Since our model is an equilibrium model, it is assumed that the equilibrium values of w_j and $p_1, \ldots, p_m$ are known to the individual when choosing a job (cf. Kanbur (1979, p. 776)). So the utility derived by members of a particular group j from working as an employee (u_j^e) is known in advance and, using (13), can be evaluated as:

$$u_j^e = f_j(p^{-1} w_j g_j(z_j^e)), \quad j = 1, \ldots, n, \qquad (15a)$$

where the exogenous z_j^e denotes the job characteristics other than income for employees of group j. Note that, for simplicity, it is assumed that for employees these characteristics do not differ across the sectors. However, the utility derived by members of a group j from self-employment in a sector $i(u_{ij}^s)$ is stochastic due to the stochastic nature of π_{ij}. As derived in Subsection 3.1 the distribution of π_{ij} is the same as that of Θ_{ij}, namely lognormal, so (14) can be used to evaluate Eu_{ij}^s:

$$Eu_{ij}^s = f_j(p^{-1} E\pi_{ij} g_j(z_{ij}^s)(1 + v(\pi_{ij}))^{-1/2\gamma_j})$$
$$i = 1, \ldots, m; \quad j = 1, \ldots, n, \qquad (15b)$$

where the exogenous z_{ij}^s denotes the job characteristics other than income for self-employed persons of group j in sector i.

It will be shown now that a positive number of individuals of each group will be employee in equilibrium. Because of the structure of the demand functions (cf. (12)), it is necessary that in every sector there is a positive output. This implies (due to the structure of the production functions (1)) that in all sectors all inputs of which the elasticity is not equal to zero, must be positive as well. The above assertion follows now by the assumption made in Subsection 3.1 that labor of each group is needed for the production in at least one sector.

What precedes can be summarized in the following equilibrium conditions:

$$n_{ij} \cdot (u_j^e - Eu_{ij}^s) = 0 \qquad (16a)$$
$$n_{ij} \geqslant 0; \quad u_j^e - Eu_{ij}^s \geqslant 0$$
$$i = 1, \ldots, m; \quad j = 1, \ldots, n. \qquad (16b)$$

Condition (16a) must be read as follows. If members of group j have chosen to become self-employed in sector i(i.e., $n_{ij} > 0$), then Eu_{ij}^s must equal u_j^e because other (identical) members of the same group have chosen a job as employee. If on the other hand Eu_{ij}^s is smaller than u_j^e, then nobody of group j will become self-employed in sector $i(n_{ij} = 0)$ because this option is not attractive enough. The first part of condition (16b) is obvious. The second part must hold, because if Eu_{ij}^s would exceed u_j^e, then nobody of group j would choose to become an employee (contrary to what is proved above).

Since all individuals that do not choose to become self-employed are available as employees, the (dependent) labor supply (L_j^{sup}) can now be formalized as follows.

$$L_j^{sup} = L_j \equiv N_j - \sum_{i=1}^{m} n_{ij}, \quad j = 1, \ldots, n, \qquad (17)$$

where the exogenous $N_j(>0)$ denotes the number of individuals belonging to group j.

3.4. *Solution of the model*

The model is solved in the following way. First all the n_{ij}'s (the number of self-employed individuals) are thought to be fixed. Demand and supply in the various product and labor markets are then only dependent on prices and wages, so that the prices and wages that clear these markets, can be determined in the usual way.

However, the n_{ij}'s are not fixed in this model. Therefore, actually the clearing prices and wages are determined as a function of the n_{ij}'s. The utility derived from being an employee and the utility expected from self-employment (dependent on wages and prices) are also determined as a function of the n_{ij}'s now. Thus, the equilibrium conditions (16) can be stated solely in terms of the n_{ij}'s. The resulting expressions form a linear complementarity problem that can be solved by the Lemke-algorithm.

The number of self-employed individuals fixed. With fixed n_{ij}'s, prices and wages can be determined by the clearing conditions: $X_i^{dem} = X_i^{sup}$ $(i = 1, \ldots, m)$ and $L_j^{dem} = L_j^{sup}(j = 1, \ldots, m)$. The resulting expressions can be written compactly, if first the equilibrium values of the output per sector (X_i) and of total income (Y) are presented.[6]

$$X_i = \Theta_i\left(1 - \sum_{s=1}^{n} \alpha_{is}\right) \prod_{j=1}^{n} \left(\frac{\beta_i\alpha_{ij}}{\sum_{h=1}^{m} \beta_h\alpha_{hj}} L_j\right)^{\alpha_{ij}}$$

$$i = 1, \ldots, m. \qquad (18)$$

So input in sector i increases, for example, if the managerial ability in the sector (Θ_i) or the (dependent) labor force L_j increases. Notice that X_i depends on the n_{ij}'s through Θ_i and L_j (cf. (7) and (17)). Output also increases, if the propensity to consume products of sector $i(\beta_i)$ increases relatively to the β's in sectors in which the same labor inputs are needed for production. Changes in the labor elasticities in sector i (the α_{ij}'s) have ambiguous effects on X_i.

Taking p (defined below (13)) as the numéraire, one obtains for Y:

$$Y = \prod_{i=1}^{m} X_i^{\beta_i} p. \qquad (19)$$

Now equilibrium prices and wages can be expressed as:

$$p_i = \beta_i Y/X_i, \quad i = 1, \ldots, m \qquad (20)$$

$$w_j = \sum_{i=1}^{m} \beta_i\alpha_{ij} Y/L_j, \quad j = 1, \ldots, n. \qquad (21)$$

So the share of total income for all the employees of group j together (defined as w_jL_j/Y) is solely determined by the preference weights β_i referring to the sectors in which group j is needed for production and the labor elasticities of group $j(\alpha_{ij})$ in these sectors. Consequently, an increase (decrease) in the number of employees of a group decreases (increases) the wage for this group relatively to Y.

Analogous results hold for the equilibrium value of expected profits $E\pi_{ij}$ (obtained by substituting (20) into (6a)):

$$E\pi_{ij} = \beta_i\left(1 - \sum_{s=1}^{n} \alpha_{is}\right) E\Theta_{ij}/\Theta_i$$

$$i = 1, \ldots, m; \quad j = 1, \ldots, n. \qquad (22)$$

The share of total income for all the self-employed in sector i together (defined as $\Sigma_{j=1}^{n} n_{ij}E\pi_{ij}/Y$) is solely determined by the preference weight β_i referring to sector i and the management elasticity of output $1 - \Sigma_{s=1}^{n} \alpha_{is}$ in this sector. Consequently, an increase (decrease) in the number of self-employed in a sector decreases (increases) the individual expected profits in this sector relatively to Y.

The linear complementarity problem. Before describing how the solution is found in the general m-sector, n-group model, attention will be focused on an economy with just one sector and one social group for illustrative purposes. Substituting the equilibrium values for the wage rate (21) and expected profits (22) into the expressions for utility (15), gives in the 1-sector, 1-group model (dropping subscripts):

$$u^e = f\{\alpha\, E\Theta^{1-\alpha}\,(N/n - 1)^{\alpha-1}\, g(z^e)\} \qquad (23a)$$

$$Eu^s = f\{(1-\alpha)E\Theta^{1-\alpha}\,(N/n - 1)^{\alpha} \times \times g(z^s)\,(1 + v(\Theta))^{-1/2\gamma}\}. \qquad (23b)$$

Note the positive relationship between u^e and n and the negative relationship between Eu^s and n. Because equilibrium conditions (16) simplify to $u^e = Eu^s$ in the 1-sector, 1-group model, equilibrium is reached as indicated in Figure 1.

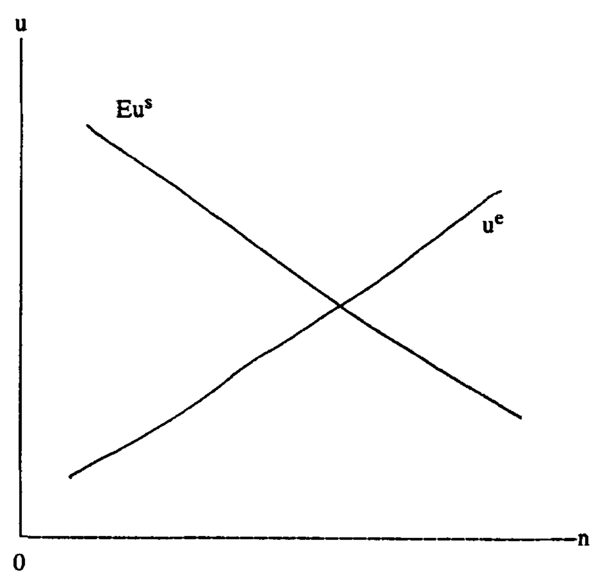

Fig. 1. Determination of the number of self-employed individuals n in the 1-sector, 1-group model.

The m-sector, n-group model is solved analogously. Again the equilibrium values for wages (21) and expected profits (22) are substituted into the expressions for utility (15) in order to get the utilities as a function of the n_{ij}'s. If the resulting expressions are substituted into the equilibrium conditions (16), one obtains[7]

$$n_{ij} \cdot \left(c_{ij} \sum_{k=1}^{n} E\Theta_{ik} n_{ik} + \sum_{h=1}^{m} n_{hj} - N_j \right) = 0 \qquad (24a)$$

$$n_{ij} \geqslant 0; \quad c_{ij} \sum_{k=1}^{n} E\Theta_{ik} n_{ik} + \sum_{h=1}^{m} n_{hj} - N_j \geqslant 0$$

$$i = 1, \ldots, m; \quad j = 1, \ldots, n, \qquad (24b)$$

where c_{ij} is defined as:

$$c_{ij} \equiv \frac{\sum_{h=1}^{m} \beta_h \alpha_{hj} g_j(z_j^e)}{\beta_i \left(1 - \sum_{s=1}^{n} \alpha_{is} \right) E\Theta_{ij} g_j(z_{ij}^{\prime}) (1 + v(\Theta_{ij}))^{-1/2\gamma_i}}. \qquad (25)$$

Expressions (24) can be identified as a linear complementarity problem in the variables n_{ij}. It is further analyzed in De Wit (1989). It is proved there that the Lemke-algorithm finds a solution to this problem irrespective of the values of the exogenous variables.[8]

The solution found by the Lemke-algorithm appears to the non-unique for certain values of the exogenous variables. An example will make clear that this could have been expected from the outset. Consider an economy with only two groups and two sectors. Furthermore suppose that the two groups have the same characteristics in every respect. In this case the model can determine how many members of each group become self-employed in the two sectors together or how many members of both groups together become self-employed in each sector. However, the separate n_{ij}'s cannot be determined, because there is no difference whatsoever between the members of the groups.

So, apparently, if the exogenous variables are such that two groups "resemble" each other too much, the model cannot distinguish any more between them. This idea is made more precise in De Wit (1989), where an exact (sufficient) condition on the exogenous variables is established to guarantee uniqueness.

4. Comparative statics

To arrive at more specific results the 1-sector, n-group model and the m-sector, 2-group model will be analyzed in a comparative static way. In the first model the effects of demographic changes (through immigration, for example) or schooling can be analyzed in a relatively simple framework, whereas in the second model the impact of, for example, a shift in consumer tastes from goods of one sector to goods of another sector can be investigated most easily.

4.1. *The 1-sector, n-group model*

Consider an economy with only one product sector and an arbitrary number of n social groups. Because there is only one sector, subscripts to denote sectors can be dropped. For ease of reference the set of groups with self-employed members in equilibrium is called S.

Description of the solution. Conditions (24) simplify for the 1-sector, n-group model to:

$$n_j \cdot (c_j \Theta + n_j - N_j) = 0 \qquad (26a)$$

$$n_j \geqslant 0; \quad c_j \Theta + n_j - N_j \geqslant 0, \quad j = 1, \ldots, n, \qquad (26b)$$

where θ denotes the total amount of managerial ability of the self-employed individuals as defined in (7). From (26a) it follows that for $j \in S$ (so $n_j > 0$): $N_j/c_j > \Theta$, whereas from (26b) it follows that for $j \notin S$ (so $n_j = 0$): $N_j/c_j \leqslant \Theta$. So only the group(s) with N_j/c_j larger than Θ will supply self-employed individuals in equilibrium. In other words, if the n social groups are arranged in such a way that the quantity N_j/c_j is a decreasing function of the index j, the set S can only be of the form $\{1, \ldots, b\}$ will $1 \leqslant b \leqslant n$.[9]

The actual number of groups in S can be determined by the Lemke-algorithm: it can vary from only one group to all n. It is proved in De Wit (1989) that this set S is unique.[10] Given the set S, Eqs. (26) can be solved analytically:

$$\begin{aligned} n_j &= 0, \quad \text{if } j \notin S \\ &= N_j - c_j \Theta, \quad \text{if } j \in S \end{aligned} \qquad (27)$$

with:

$$\Theta = \sum_{k \in S} E\Theta_k N_k \bigg/ \left(1 + \sum_{k \in S} E\Theta_k c_k \right).$$

Analysis of the solution. When an exogenous variable is gradually increased or decreased, two successive regimes regarding the impact of this on the number of self-employed individuals of each group ("the n_j's") can generally be distinguished:

1. first, the set S is not affected: the n_j's outside S remain zero and only the n_j's within S are influenced. The direction in which these latter n_j's are influenced is indicated by the sign of the partial derivative of the n_j's as determined from (27);
2. next, the set S changes. In what way S changes can be determined by analyzing the impact of a change in an exogenous variable on Θ (below (27)) and the various N_j/c_j's.

After the set S is changed, the analysis can be repeated with the new S.

The impact of changes in the various exogenous variables is shown in Table 1. For expositional reasons, it is assumed that $n \geqslant 3$ and $S = \{1, \ldots, b\}$ with $2 \leqslant b \leqslant n - 1$. The extension to other values of n and b is straightforward.[11]

First, changes in the number of individuals (N_j), which may be due to immigration or other demographic developments, are analyzed. Consider an increase in N_k where k is a group with self-employed members (so $k \in S$). If all n_j's would remain constant, the utility derived by group k from being an employee u_k^e would become less than the utility expected from self-employment Eu_k^s because of the decrease of the wage w_k relatively to the expected profits $E\pi_k$ (cf. (21) and (22)). Consequently, a larger number of individuals of group k will decide to become self-employed, causing the expected profit-wage ratio $E\pi/w$ to decrease for group k as well as all other groups (cf. (22) and (7)). As a result, fewer individuals of other groups with self-employed members will decide to become self-employed, whereas the number of self-employed members outside S remains zero. What will happen to the total number of self-employed (see the column "summed n_j's" in the Table)? This depends on the average scale of production of group k with respect to the other groups with self-employed members. If the average scale of production, that is determined by the managerial ability (cf. (5)), of group k is small (large) compared to that of the other groups, then the increase in relatively small (large) firms run by members of group k invokes the disappearance of relatively few large scale (many small-scale) firms run by members of other groups, resulting in an overall increase (decrease) in the number of self-employed individuals. The precise condition is given in Table I.

In what way will the set S be influenced? As remarked above, an increase in N_k will cause the number of self-employed of groups other than k to decrease. If N_k grows sufficiently large, the number of self-employed of the group with the lowest N_j/c_j in $S\backslash\{k\}$ will be the first to become zero. In other words, if $k \neq b$, group b will be the first to leave S, otherwise group $b - 1$.

If N_k decreases, obviously the effects will be opposite: n_k decreases, for $j \in S\backslash\{k\}$ n_j increases, and for $j \notin S$ $E\pi/w$ increases. If N_k becomes sufficiently small, S will change in one or both of the following ways. Either n_k decreases to zero, so group k is removed from S, or Eu_j^s becomes equal to u_j^e for the group with the highest N_j/c_j outside S (i.e., $b + 1$), causing group $b + 1$ to join S.

Now consider an increase in N_r where r is a group with no self-employed members (so $r \notin S$). This will not influence the ratio $E\pi/w$ of the other groups (cf. (21) and (22)), so the n_j's of these groups will not change. However, the wage w_r will decrease relatively to $E\pi_r$. If N_r grows sufficiently large, u_r^e will become equal to Eu_r^s and group r joins S.

If N_r decreases, obviously at first the effects on the n_j's will be zero as above. However, further decreases will also have no effects, because the concomitant decrease in $E\pi_r/w_r$ will only confirm the members of group r in their choice to become employee.

It is not difficult to understand now that changes in utility derived by the self-employed from job characteristics other than income $g_j(z_j^s)$, expected managerial ability $E\Theta_j$, or variance of managerial ability $v(\Theta_j)$ if the group is risk-loving ($\gamma_j < 0$), have the same effects as changes in N_j. On the other hand, changes in risk-aversion γ_j, utility derived by employees from job characteristics other than income $g_j(z_j^e)$, or variance of managerial ability $v(\Theta_j)$ if the group is risk-averse ($\gamma_j > 0$), have the opposite effects of changes in N_j. However, in case of a change in expected managerial ability of a group k with self-employed members ($E\Theta_k$), the expression determining the

TABLE I
Impact of changes in exogenous variables

Exogenous variables	Sign of derivative of: Summed n_j's	Individual n_j's $j = k$	Individual n_j's $j \in S\backslash\{k\}$	Possible change in S if an exogenous variable: Increases	Possible change in S if an exogenous variable: Decreases
$N_k, g_k(z_k^s), v(\Theta_k)$ if $\gamma_k < 0$	$s(x_k)$	+	−	$b \blacktriangleleft S$	$k \blacktriangleleft S, b+1 \blacktriangleright S$
$N_r, g_r(z_r^s), v(\Theta_r)$ if $\gamma_r < 0$	0	0	0	$r \blacktriangleright S$	·
$\gamma_k, g_k(z_k^e), v(\Theta_k)$ if $\gamma_k > 0$	$-s(x_k)$	−	+	$k \blacktriangleleft S, b+1 \blacktriangleright S$	$b \blacktriangleleft S$
$\gamma_r, g_r(z_r^e), v(\Theta_r)$ if $\gamma_r > 0$	0	0	0	·	$r \blacktriangleright S$
$v(\Theta_j)$ if $\gamma_j = 0$ $(j = 1, \ldots, n)$	0	0	0	·	·
$E\Theta_k$	$s(y_k)$	+	−	$b \blacktriangleleft S$	$k \blacktriangleleft S, b+1 \blacktriangleright S$
$E\Theta_r$	0	0	0	$r \blacktriangleright S$	·
α_k if $a_k > 0$	−	−	−	$k \blacktriangleleft S, b \blacktriangleleft S$	$b+1 \blacktriangleright S$
α_k if $a_k = 0$	−	−	0	$k \blacktriangleleft S$	·
α_k if $a_k < 0$	n.c.	−	+	$k \blacktriangleleft S, b+1 \blacktriangleright S$	$b \blacktriangleleft S$
α_r	−	−	−	$b \blacktriangleleft S$	$r \blacktriangleright S, b+1 \blacktriangleright S$

1. Assumed is $n \geqslant 3$; $S = \{1, \ldots, b\}$ with $2 \leqslant b \leqslant n-1$.
2. k refers to a group out of $S\backslash\{b\}$.
   r refers to a group out of $\{1, \ldots, n\}\backslash S$.
   Changes in exogenous variables referring to group b have the same impact as that of changes in exogenous variables referring to other groups in S, with the exception that $b \blacktriangleleft S$ changes into $b-1 \blacktriangleleft S$ in the last two columns.
3. Explanation of symbols used in the last two columns:

   $j \blacktriangleright S$: j joins S;
   $j \blacktriangleleft S$: j leaves S;
   · : S does not change;
   ..,.. : one or both changes are possible.

4. Explanation of other symbols:

   n.c. : not clear;
   $s(\cdot)$: sign$(\cdot)$;
   $a_k \equiv g_k(z_k^s)(1 + v(\Theta_k))^{-1/2\gamma_k} - g_k(z_k^e)$;

   $x_k \equiv 1 + \sum_{j \in S} c_j(E\Theta_j - E\Theta_k)$;

   $y_k \equiv \sum_{j \in S} c_j(E\Theta_j N_j/c_j - E\Theta_k N_k/c_k)$.

direction of change in the total number of self-employed individuals is slightly different. This can be understood as follows. As explained above this expression is dependent on the scale of production (determined by the managerial ability) of the various groups. Thus, if the scale of production of group k itself changes, an additional effect has to be taken account of.

If a group is neutral towards risk ($\gamma_j = 0$), it is obvious that the variance of managerial ability has no effects whatsoever.

Finally, we discuss the effects of increases in the labor elasticity α_j (caused by schooling, for example). Consider an increase in α_k with $k \in S$. With fixed n_j's, two effects will occur:

1. $E\pi/w$ decreases for all groups j, because the management elasticity $1 - \Sigma_{s=1}^{n} \alpha_s$ decreases;

2. w_k increases relatively to $E\pi_k$, because the higher α_k results in a relatively higher wage for group k.

Both effects for group k are neutralized by a decrease of n_k, causing $E\pi/w$ to increase for group k as well as all other groups. So, for all groups except k, $E\pi/w$ is influenced in the following two ways:

1. negatively, by the decrease in $1 - \Sigma_{s=1}^{n} \alpha_s$ (proportional to $g_k(z_k^s)\ (1 + v(\Theta_k))^{-1/2\gamma_k}$ as it appears);
2. positively, by the decrease in n_k (proportional to $g_k(z_k^e)$ as it appears).

There are three possibilities now:

1. the first effect dominates, so $E\pi/w$ decreases. To neutralize this the n_j's of the groups in $S\backslash\{k\}$ will decrease. If the increase in α_k is large enough, either group b (or $b - 1$, if $k = b$) will leave S, or group k itself;
2. both effects cancel each other, so $E\pi/w$ remains the same. Then also the n_j will not change. A large increase in α_k can only cause group k to leave S;
3. The second effect dominates, so $E\pi/w$ increases. The n_j's of the groups in $S\backslash\{k\}$ must increase to offset this. If the increase in α_k is large enough, either group $b + 1$ enters S, or group k leaves S.

Next an increase in α_r, with $r \notin S$, is considered. With fixed n_j's the aforementioned two effects occur. However, because $r \notin S$, the decrease in $E\pi_r/w_r$ does not evoke any neutralizing actions of that group. So $E\pi/w$ of the other groups is only influenced negatively through the decrease of $1 - \Sigma_{s=1}^{n} \alpha_s$, which calls forth a decrease of all the n_j's in S. If the increase in α_r is large enough, group b will leave S.

If an exogenous variable is gradually increased or decreased, several successive changes in S can occur. These successive changes can be analyzed by using Table I repeatedly. For example, if the increase in N_r of a group with no self-employed members grows sufficiently large, the following will happen. First, nothing happens. Next, members of group r start to become self-employed. Subsequently, the members of other groups are one by one competed out of self-employment until, finally, only members of group r are self-employed.

4.2. *The m-sector, 2-group model*

Consider now an economy with an arbitrary number of m sectors and two social groups. For ease of reference, the set of sectors in which members of group 1 (2) are self-employed in equilibrium is called S^1 (S^2).

Description of the solution. The following statements are straightforward from results derived in De Wit (1989):[12]

1. there are no sectors without self-employed individuals in equilibrium;
2. $c_{i1}/c_{i2} \leqslant L_1/L_2$ if $i \in S^1$; $c_{i1}/c_{i2} \geqslant L_1/L_2$ if $i \in S^2$;
3. a solution with both groups self-employed in more than one sector is not unique.

From these statements it is clear that if the sectors are arranged in such a way that the quantity c_{i1}/c_{i2} is an increasing function of the index i, only the following three types of solution are possible:

1. a unique solution with no sector in which both groups are self-employed. S^1 is empty or of the form $\{1, \ldots, b\}$ with $1 \leqslant b \leqslant m$, whereas $S^2 = \{1, \ldots, m\} \setminus S^1$;
2. a unique solution with only one sector in which both groups are self-employed. S^1 is of the form $\{1, \ldots, b\}$ with $1 \leqslant b \leqslant m$, whereas $S^2 = \{b, \ldots, m\}$;
3. a non-unique solution.

Which type of solution prevails can be determined by the Lemke-algorithm. If type 1 is found, Eqs. (24) can be solved analytically, leading to:

$$
\begin{aligned}
n_{ij} &= 0, \quad \text{if } i \notin S^j, \quad j = 1, 2 \\
&= d_{ij} N_j \Big/ \left(1 + \sum_{h \in S^j} d_{hj}\right), \quad \text{if } i \in S^j, \; j = 1, 2,
\end{aligned} \tag{28}
$$

where $d_{ij} \equiv 1/(c_{ij} E\Theta_{ij})$. If type 2 is found, (24) can again be solved analytically, producing:

$$
\begin{aligned}
n_{ij} &= 0, \quad \text{if } i \notin S^j, \quad j = 1, 2 \\
&= d_{ij} c_{bj} \Theta_b, \quad \text{if } i \in S^j \backslash \{b\}, \quad j = 1, 2
\end{aligned}
$$

$$
n_{bj} = N_j - \left(1 + \sum_{h \in S^j \backslash \{b\}} d_{hj}\right) c_{bj} \Theta_b, \quad j = 1, 2 \tag{29}
$$

with:

$$\Theta_b = \frac{E\Theta_{b1}N_1 + E\Theta_{b2}N_2}{1 + \left(1 + \sum_{h \in S^1 \setminus \{b\}} d_{h1}\right) \Big/ d_{b1} + \left(1 + \sum_{h \in S^2 \setminus \{b\}} d_{h2}\right) \Big/ d_{b2}}$$

Solutions of type 3 are not further analyzed in this paper.

Analysis of the solution. The impact of changes in exogenous variables will be investigated in exactly the same way as in the 1-sector, n-group model. The partial derivatives of the n_{ij}'s can be determined from (28) and (29). The changes in the S^j are determined by analyzing the impact of a change in an exogenous variable on L_1/L_2 and the various c_{i1}/c_{i2}'s.

To avoid unnecessary repetitions, only solutions of type 1 are analyzed, characterized by $m \geqslant 3$ and $S^1 = \{1, \ldots, b\}$, $S^2 = \{b+1, \ldots, m\}$ with $2 \leqslant b \leqslant m-1$. Furthermore, in view of the symmetry, only changes in exogenous variables referring to group 1 are discussed. The results are presented in Table II.

TABLE II
Impact of changes in exogenous variables referring to group 1

Exogenous variables		Sign of derivative of: Summed n_{i1}'s	Individual n_{i1}'s: $i=h$	$i \in S^1 \setminus \{h\}$	Possible change in S^1 or S^2 if an exogenous variable: Increases	Decreases
N_1		+	+	+	$b+1 \blacktriangleright S^1$	$b \blacktriangleright S^2$
$g_1(z_1^c)$		−	−	−	$b \blacktriangleright S^2$	$b+1 \blacktriangleright S^1$
γ_1		−	$s(g_{i1})$	$s(g_{i1})$	n.c.	n.c.
$v(\Theta_{h1})$ if	$\gamma_1 > 0$	−	−	+	$b+1 \blacktriangleright S^1, h \blacktriangleright S^2$	$b \blacktriangleright S^2$
	$\gamma_1 = 0$	0	0	0	·	·
	$\gamma_1 < 0$	+	+	−	$b \blacktriangleright S^2$	$b+1 \blacktriangleright S^1, h \blacktriangleright S^2$
$v(\Theta_{r1})$ if	$\gamma_1 > 0$	0	0	0	·	$r \blacktriangleright S^1$
	$\gamma_1 = 0$	0	0	0	·	·
	$\gamma_1 < 0$	0	0	0	$r \blacktriangleright S^1$	·
$g_1(z_{h1}^s)$		+	+	−	$b \blacktriangleright S^2$	$b+1 \blacktriangleright S^1, h \blacktriangleright S^2$
$g_1(z_{r1}^s)$		0	0	0	$r \blacktriangleright S^1$	·
$E\Theta_{h1}$		0	0	0	·	$h \blacktriangleright S^2$
$E\Theta_{r1}$		0	0	0	$r \blacktriangleright S^1$	·
α_{h1} if	$\alpha_{h1} > 0$	−	−	+	$b+1 \blacktriangleright S^1$	$b \blacktriangleright S^1$
	$\alpha_{h1} = 0$	−	−	0	·	·
	$\alpha_{h1} < 0$	−	−	−	$b \blacktriangleright S^2$	$b+1 \blacktriangleright S^1$
α_{r1}		−	−	−	$b \blacktriangleright S^2$	$b+1 \blacktriangleright S^1$
β_h if	$s_{h1} + e_{h1} > e_{h2}$	$s(b_{h1})$	+	−	$b \blacktriangleright S^2$	$b+1 \blacktriangleright S^1$
	$s_{h1} + e_{h1} = e_{h2}$	$s(b_{h1})$	+	−	·	·
	$s_{h1} + e_{h1} < e_{h2}$	$s(b_{h1})$	+	−	$b+1 \blacktriangleright S^1$	$b \blacktriangleright S^2$

Legenda:
1. Assumed is: $m \geqslant 3$; $S^1 = \{1, \ldots, b\}$, $S^2 = \{b+1, \ldots, m\}$ with $2 \leqslant b \leqslant m-1$.

Legenda (Continued)

---

2. h refers to a sector out of $S^1\backslash\{b\}$.
   r refers to a sector out of S^2.
   Changes in exogenous variables referring to sector b have the same impact as that of changes in exogenous variables referring to other sectors in S^1, with the following two exceptions:
   $b \blacktriangleright S^2$ changes into: $b-1 \blacktriangleright S^2$;
   $b+1 \blacktriangleright S^1$, $h \blacktriangleright S^2$ changes into: $b \blacktriangleright S^2$.
3. The signs of derivatives of the n_{i2}'s are not tabled. They are all zero except:
   $\text{sign}(\partial n_{r2}/\partial \alpha_{r1}) = -$, $r \in S^2$;
   $\text{sign}(\partial n_{i2}/\partial \alpha_{r1}) = +$, $r \in S^2, i \in S^2\backslash\{r\}$;
   $\text{sign}(\partial n_{i2}/\partial \beta_h) = -$, $h \in S^1, i \in S^2$.
4. The effects of changes in $\beta_r(r \in S^2)$ are not tabled, because, mutatis mutandis, they are the same as that of changes in $\beta_h(h \in S^1)$.
5. Because of the constraint $\Sigma_{i=1}^{m} \beta_i = 1$, changes in β_r are compensated: see Note 13.
6. Explanation of symbols: (see also Table I)

$$g_{i1} \equiv \sum_{h \in S^1} d_{h1}\{ln(1+v(\Theta_{h1})) - ln(1+v(\Theta_{i1}))\} - ln(1+v(\Theta_{i1}))$$

$$a_{h1} \equiv g_1(z^s_{h1})(1+v(\Theta_{h1}))^{-1/2\gamma_1} - g_1(z^e_1)$$

$$b_{h1} \equiv A_1/B_1 - \{\alpha_{h1}g_1(z^e_1)\} \Big/ \left\{\left(1 - \sum_{s=1}^{n} \alpha_{hs}\right) g_1(z^s_{h1})(1+v(\Theta_{h1}))^{-1/2\gamma_1}\right\}$$

$$s_{h1} \equiv \left(1 - \sum_{s=1}^{n} \alpha_{hs}\right) g_1(z^s_{h1})(1+v(\Theta_{h1}))^{-1/2\gamma_1}/(A_1+B_1)$$

$$e_{hj} \equiv \alpha_{hj}g_j(z^e_j)/(A_j+B_j),$$

where:

$$A_j \equiv \sum_{i=1}^{m} \beta_i \alpha_{ij} g_j(z^e_j)$$

$$B_j \equiv \sum_{i \in S^j} \beta_i \left(1 - \sum_{s=1}^{n} \alpha_{is}\right) g_j(z^s_{ij})(1+v(\Theta_{i1}))^{-1/2\gamma_1}.$$

An increase in the size of group 1 N_1 causes — with n_{ij}'s fixed — w_1 to decrease relatively to all $E\pi_{i1}$'s. Consequently, n_{i1} increases in the sectors in which group 1 is self-employed. If N_1 grows sufficiently large, u^e_1 will become equal to Eu^s_{i1} in the sector with the smallest c_{i1}/c_{i2} outside S^1 — i.e., $b+1$ —, causing sector $b+1$ to join S^1.

A decrease in N_1 has just the opposite effect: for $i \in S^1$ the n_{i1}'s decrease. So total managerial ability Θ_i decreases in these sectors and with that the expected profits-wage ratio $E\pi_{ij}/w_j$ increases for *both* groups (cf. (22)). If N_1 decreases sufficiently, the sector in S^1 with the highest c_{i1}/c_{i2} — i.e., sector b — will be the first for which Eu^s_{i2} becomes equal to u^e_2, inciting b to join S^2.

The impact of changes in $g_1(z^e_1)$, that is the utility derived by employees from job characteristics other than income, is exactly opposite to the impact of changes in N_1, so it needs no further comment.

An increase in the risk-aversion of group 1 (γ_1) has — with fixed n_{ij}'s — the obvious effect of decreasing Eu^s_{i1} in all sectors (proportional to $ln(1+v(\Theta_{i1}))$ as it appears). This calls forth a change in the n_{i1}'s in two ways:

1. first, through a redistribution within S^1 of self-employed individuals of group 1 from sectors with a relatively high $v(\Theta_{i1})$ to sectors with a relatively low $v(\Theta_{i1})$, equalizing the Eu^s_{i1}'s ($i \in S^1$). (This effect corresponds to the first part of the expression for g_{i1} in Note 6 of the legenda of Table II.)
2. Second, through a decrease of the summed n_{i1}'s, equalizing the Eu^s_{i1}'s ($i \in S^1$) with u^e_1. (This effect corresponds to the second part of the expression for g_{i1} in Note 6 of the legenda of Table II.)

So an arbitrary n_{i1} will only increase if the variance of managerial ability of group 1 in this sector is sufficiently small for effect 1 to be stronger than effect 2.

If the increase in γ_1 is large enough, S^1 or S^2 will change. Because there are no simple expressions to give for the way in which these sets will change, these changes are not further analyzed here.

Obviously, decreases in γ_1 give effects opposite to those mentioned above.

Next, changes in $v(\Theta_{i1})$ are investigated in the case that group 1 is risk-averse ($\gamma_1 > 0$). Consider an increase in $v(\Theta_{h1})$ with $h \in S^1$. With Fixed n_{ij}'s, this causes Eu^s_{h1} to decrease relatively to u^e_1, enticing fewer individuals of group 1 into self-employment in sector h (n_{h1} decreases). Those discouraged will partly decide to become employee (the summed n_{i1}'s decrease), and partly to become self-employed in one of the other sectors in S^1 (n_{i1}'s increase for $i \in S^1\backslash\{h\}$).

If $v(\Theta_{h1})$ grows sufficiently large, the sets S^j will change in one or both of the following ways:

1. the decrease in n_{h1} causes Θ_h to decrease. Thus, $E\pi_{hj}/w_j$ increases in sector h for both groups (cf. (22)). If $E\pi_{h2}/w_2$ is risen so much that Eu^s_{h2} reaches u^e_2, h joins S^2;
2. the increase in the number of employees of group 1 causes w_1 to decrease relatively to all $E\pi_{i1}$'s. If it decreases sufficiently, u^e_1 will become equal to Eu^s_{i1} in the sector with the smallest c_{i1}/c_{i2} outside S^1 — i.e., $b + 1$ —, causing sector $b + 1$ to join S^1.

A decrease in $v(\Theta_{h1})$ has opposite effects, so n_{i1} decreases for $i \in S^1\backslash\{h\}$, which causes Θ_i to decrease in these sectors and thereby $E\pi_{i2}/w_2$ to increase. If the decrease in $v(\Theta_{h1})$ is large enough $Eu^s{}_{i2}$ becomes equal to u^e_2 in the sector with the highest c_{i1}/c_{i2} in $S^1\backslash\{h\}$. In other words, if $h \neq b$, sector b joins S^2, otherwise sector $b - 1$.

An increase in $v(\Theta_{r1})$ with $r \in S^2$ — however large — has no influence on the n_{ij}'s, because it decreases Eu^s_{r1}. Individuals of group 1 are only confirmed in their choice not to become self-employed in sector r. On the other hand, a sufficiently large decrease in $v(\Theta_{r1})$ will equalize Eu^s_{r1} and u^e_1, causing r to join S^1.

Obviously, there is no impact of changes in $v(\Theta_{i1})$ if group 1 is risk-neutral, whereas the impact is exactly opposite to that described above if group 1 is risk-loving. This latter result also holds for changes in $g(z^s_{i1})$ and $E\Theta_{i1}$ with the following exception.

Changes in $E\Theta_{h1}$ with $h \in S^1$, do *not* influence $E\pi_{h1}/w_1$ (cf. (22)). This is due to the fact that total profits in sector h as a share of Y (equal to $\beta_i(1 - \Sigma^n_{s=1} \alpha_{is})$) are not influenced, while the share that individuals of group 1 that are self-employed in sector h get from the total profits in sector $h(E\Theta_{h1}/\Theta_h)$ is also not affected because only individuals of group 1 are self-employed in the sector. Only if a decrease in $E\Theta_{h1}$ is large enough, $E\Theta_{h1}$ has an impact. For, a decrease in $E\Theta_{h1}$ causes Θ_h to decrease, and thus $E\pi_{h2}/w_2$ to increase, equalizing eventually Eu^s_{h2} and u^e_2, so that h joins S^2.

Let us, next, consider an increase in α_{i1} (a decrease has opposite effects). An increase in α_{h1} with $h \in S^1$ and n_{ij}'s fixed, will produce three effects:

1. w_1 increases relatively to all other incomes;
2. because $1 - \Sigma^n_{s=1} \alpha_{hs}$ decreases, $E\pi_{h1}$ decreases relatively to all other incomes;
3. the same as effect 2, but for $E\pi_{h2}$.

Because no members of group 2 are self-employed in sector h effect 3 does not evoke any reactions. The first two effects, however, cause the summed n_{i1}'s and n_{h1} to decrease. The influence on the n_{i1}'s with $i \in S^1\backslash\{h\}$ is dependent on which effect dominates. If the first effect dominates, the n_{i1}'s with $i \in S^1\backslash\{h\}$ decrease and if the increase in α_{h1} is large enough, b joins S^2. If both effects cancel each other, an increase in α_{h1} — however large — has neither an influence on the n_{i1}'s with $i \in S^1\backslash\{h\}$, nor on S^1 or S^2. If the second effect dominates, the n_{i1}'s with $i \in S^1\backslash\{h\}$ increase and if the increase in α_{h1} is large enough, $b + 1$ joins S^1.

If α_{r1} increases in a sector in which group 2 is self-employed (so $r \in S^2$), the aforementioned three effects occur again. The first effect causes all n_{i1}'s with $i \in S^1$ to decrease. The second effect does not evoke any reactions now, because group 1 is not self-employed in sector r. The third effect causes n_{h2} to decrease, the remaining n_{i2}'s in S^2 to increase and the summed n_{i2}'s to decrease. It should be clear by now that if the increase in α_{r1} is large enough, b joins S^2.

Changes in the preference weights for products of sector $i(\beta_i)$ are discussed now. Since $\Sigma^m_{i=1}$

$\beta_i = 1$, a change in β_i can only be investigated in relation to changes in other β_i's. We have chosen here to investigate situations in which an increase (decrease) in β_i is compensated by relatively equal decreases (increases) in the other β_i.[13] Attention will be restricted to a relative increase in β_h with $h \in S^1$, because a relative decrease in β_h has exactly the opposite effects, whereas changes in β_r with $r \in S^2$ have, *mutatis mutandis*, the same effects as changes in β_h.

As may be expected, a relative increase in β_h causes n_{h1} to increase at the cost of a decrease in the other n_{i1}'s ($i \in S^1\backslash\{h\}$). The question then arises in what way the total number of self-employed individuals of group 1 will change. This appears to depend on the mean scale of production in sector h or, more precisely, the mean employee — self-employed ratio for group 1 in sector h. If this ratio (equal to the second term of the expression for b_{h1} in Note 6 of the legenda of Table II as can be derived from (28)) is larger (less) than the ratio for all sectors together (equal to the first term in the same expression) then the total number of self-employed individuals of group 1 decreases (increases), whereas it remains constant if the two ratio's are equal. The number of self-employed individuals of group 2 is only effected if labor of group 2 is an input in sector $h(\alpha_{h2} \neq 0)$. In that case the number of self-employed of this group decreases due to the extra demand for employees in sector h.

Finally, changes in the sets S^j are considered. An increase in β_h draws people to sector h in three ways: (1) members of group 1 to become self-employed there (proportional to s_{h1} defined in Table II), (2) members of group 1 to become employee there (proportional to e_{h2} defined in Table II), (3) members of group 2 to become employee there (proportional to e_{h2} defined in Table II). Effects (1) and (2) both decrease the n_{i1}'s in the sectors other than h, so they stimulate that group 2 becomes self-employed in sector b. However, effect (3) decreases the n_{i2}'s, so it stimulates the reverse, namely that group 1 becomes self-employed in sector $b + 1$. So if the combined effects (1) and (2) exceed effect (3) in strength, then b joins S^2, if they cancel nothing happens, and if the reverse holds $b + 1$ joins S^1.

As with the 1-sector, n-group model, a gradual change in an exogenous variable can evoke several successive changes in the sets S^1 and S^2. Formally, these successive changes can not be analyzed solely by using Table II, because alternately solutions of type 1 and 2 will occur (excluding for convenience solutions of type 3). However, by now the successive changes should be clear without derivation.[14]

Notes

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[2] Some exceptions are obvious. For example, the result that of all groups some members are employee in equilibrium is of course directly related to the specification of the production function that is used in the paper.

[3] For convenience, the lognormal distribution is characterized by $E\Theta$ and $v(\Theta)$ instead of $LN(\mu, \sigma^2)$. As is well known, the connection between the two sets of parameters is given by (e.g., Aitchison and Brown (1957)).

$$E\Theta = \exp(\mu + \tfrac{1}{2}\sigma^2)$$
$$v(\Theta) = \exp(\sigma^2) - 1.$$

[4] Of course, Θ_i is only the *expected* amount of managerial ability in sector i. However, for large numbers of n_{ij} it is also the *realized* amount of managerial ability in sector i.

[5] (14) is derived using the expressions in Note 3 and the following property of the lognormal distribution that is also derived in Aitchison and Brown (1957). If $y \sim LN(\mu, \sigma^2)$ and $g(y)$ denotes the related distribution function, then:

$$\int y^{\alpha} g(y)dy = \exp(\alpha\mu + \tfrac{1}{2}\alpha^2\sigma^2).$$

[6] Expressions (18) through (21) can be derived as follows. Substituting (12) and (8) into $X_i^{dem} = X_i^{sup}$ ($i = 1, \ldots, m$) and (9) and (17) into $L_j^{dem} = L_j^{sup}$ ($j = 1, \ldots, m$) gives:

$$\text{(I)} \quad \beta_i Y/p_i = x_i(\cdot)\Theta_i, \quad i = 1, \ldots, m$$

$$\text{(II)} \quad 1/w_j \sum_{i=1}^{m} \alpha_{ij} p_i x_i(\cdot)\Theta_i = L_j, \quad j = 1, \ldots, n.$$

Now (20) is found by rewriting (I) and using $X_i = x_i(\cdot)\Theta_i$; (21) is found by substituting (I) into (II) and rewriting it; (18) is found by substituting (20) and (21) into the expression for

$x_i(\cdot)$ (below (5)) and using $X_i = x_i(\cdot)\Theta_i$; (19) is found by combining (I) with the definition of the numéraire below (13).

[7] To arrive at (24) use is made of the following. The expressions for u^e and Eu^s are both of the form $f_j(\cdot)$. Because f_j is a strictly increasing function, conditions (16) hold not only for u^e and Eu^s, but also for the respective arguments of f_j. Therefore, only the arguments of $u^e = f_j(\cdot)$ and $Eu^s = f_j(\cdot)$ need to be substituted into (16).

[8] Only a *real number* solution for the n_{ij}'s is guaranteed to exist, whereas the n_{ij}'s are integers in the theoretical model. For large numbers (guaranteed by large N_j's), however, this does not make much difference.

[9] Note that the quantity N_j/c_j has been identified earlier in Section 2 as the "competitive entrepreneurial position" C_j.

[10] The uniqueness follows from De Wit (1989, proposition 6.1), combined with the fact that a matrix of dimensions $1 \times n$ is necessarily *nondegenerate*, a concept that is defined in De Wit (1989, p. 10).

[11] If $S = \{1\}$, sign $(\partial n_1/\partial E\Theta_1)$ is unexpectedly zero instead of +. The reason for this will be discussed when analyzing the m-sector, 2-group model.

[12] The first statement follows De Wit (1989, Proposition 5.1). The second statement follows from De Wit (1989, Eq. (13)). The third statement follows from De Wit (1989, Corollary 5.1 (i)).

[13] To define a "partial derivative with respect to βh" consistent with $\Sigma_{i=1}^{m} \beta_i = 1$, the β_i are thought to depend on a variable x in the following way: $\beta_h(x) = (\beta_h + x)/(1 + x)$ and $\beta_i(x) = \beta_i/(1 + x)$ for $i \neq h$. So for $x = 0$ the following holds:

1. the β_i's have their original values;
2. $\partial\beta_h/\partial x = 1 - \beta_h$; $\partial\beta_i/\partial x = -\beta_i$ for $i \neq h$;
3. $\partial \Sigma_{i=1}^{m} \beta_i/\partial x = 0$.

The "partial derivative with respect to β_h" is now defined as the partial derivative with respect to x for the value $x = 0$.

[14] For example, starting from a situation in which only members of group 2 are self-employed, a gradual increase in N_1 will evoke the following changes. First, nothing happens. Then, members of group 1 become self-employed in sector 1. Gradually the members of group 2 are competed out of sector 1 until only members of group 1 are self-employed in this sector. Next, the same happens in sector 2 and so on until eventually only members of group 1 are self-employed.

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