Appl. Sci. Res. **29** Januari 1974

BARKHAUSEN NOISE IN FLUXGATE MAGNETOMETERS

J. L. M. J. VAN BREE*, J. A. POULIS*, and F. N. HOOGE**

Technological University Eindhoven
P.O. Box 513, Eindhoven, THE NETHERLANDS

Abstract

Barkhausen noise is one of the limiting factors for the sensitivity of fluxgate magnetometers. In order to set up calculations about Barkhausen noise, the theory of Bittel and Storm [1] had to be modified. As a result an equation for the signal-to-noise ratio is given, which gives criteria to increase the sensitivity of fluxgate magnetometers.

§ 1. Introduction

The interest in the measurement of small magnetic fields has increased considerably in the past twenty years. This stimulated the development of fluxgate magnetometers, which have already been commercially available for some time. At the Technological University of Eindhoven these fluxgate magnetometers are used for the measurement of magnetic susceptibilities of diamagnetic samples. The utmost sensitivity is required in these measurements. Among the limiting factors of this sensitivity the Barkhausen noise may be expected to play an important role together with the Nyquist noise of the circuitry. The theory of Barkhausen noise given by Bittel and Storm [1] has been generalised and applied to the fluxgate magnetometers.

§ 2. Theory

It is the aim of this paper to discuss the Barkhausen noise in a fluxgate magnetometer according to principles which have been introduced by Förster [2, 3] and Wurm [4].

* Dept. of Physics
** Dept. of Electrical Eng.

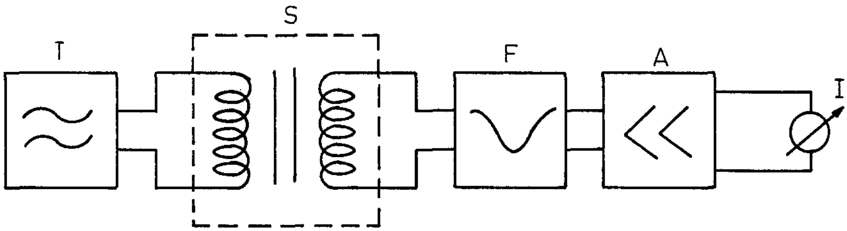

Fig. 1. Schematic of a single fluxgate magnetometer [2, 3].

T transmitter
S probe
F filter for the double frequency
A phase sensitive amplifier
I indicator

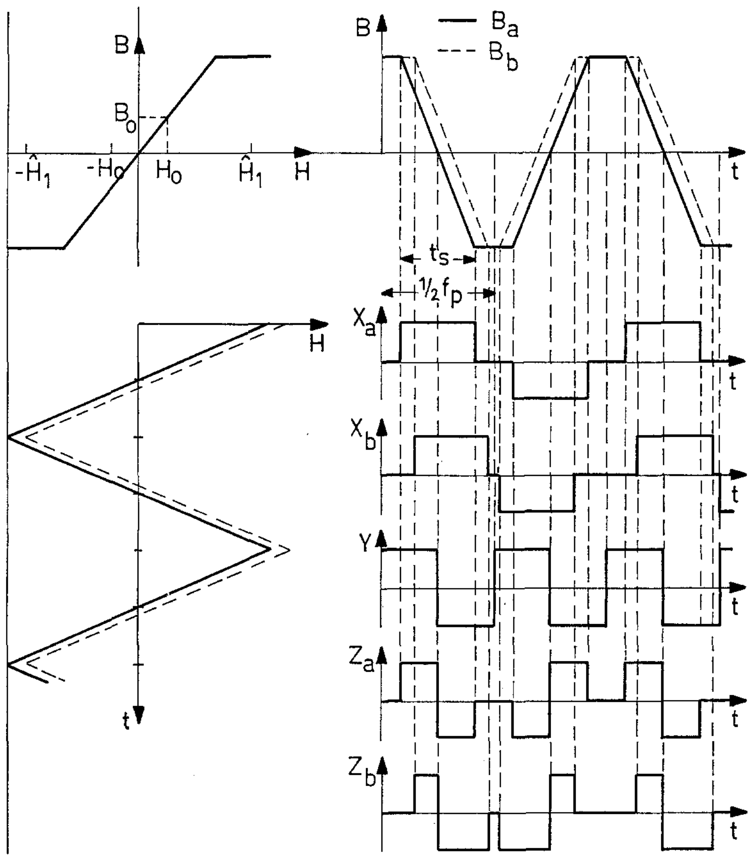

Fig. 2. The principles of the fluxgate method for meaning of the symbols, see text.

The apparatus, shown in simplified form in Fig. 1, consists of a primary and a secondary coil which are coupled by a core of high permeability. We approximate the magnetic behaviour of this core by a very simplified B–H curve shown in Fig. 2. We base the calculations on a primary current which is triangularly dependent upon time, of frequency f_p (see Fig. 2, curve H_a), resulting in an induction, the time-dependence of which is given by curve B_a. This leads to an inductive voltage in the secondary coil as shown in curve X_a. In order to obtain phase detection the signal X_a is multiplied by a square wave of unit height of the double frequency (curve Y). This multiplication yields curve Z_a, which is integrated over a time interval t_m. In order to obtain a time-average value of Z_a the integral is divided by t_m. Without external magnetic field the value of this integral amounts to zero (see curve Z_a, Fig. 2).

Let us now consider the influence of an external magnetic field H_0. Together with the field of the current through the primary coil, this delivers for the total induction curve B_b. The inductive voltage is now given by curve X_b, which after multiplication with the curve Y results in curve Z_b. Contrary to Z_a, Z_b is not symmetric and therefore delivers a non-zero value upon integration. The time average of Z_b is taken as a measure of the magnetic field H_0.

The effect of the Barkhausen noise that accompanies the magnetization will be discussed by analyzing its consequences on the time average of Z_a. For this description let us follow the usual procedure [1] of subdividing the core into N_B equal Barkhausen volumes, each with a flux ϕ_B related to the total saturation flux ϕ_s as:

$$\phi_s = N_B \cdot \phi_B. \tag{1}$$

The magnetization of the Barkhausen volumes are supposed to be either parallel or antiparallel to the field of the primary current. Because of this current the core periodically switches its magnetization from $-\phi_s$ to $+\phi_s$, and vice versa. The time interval necessary for one switch from $-\phi_s$ to $+\phi_s$ we shall call t_s (see Fig. 2).

t_s is related to the frequency by:

$$t_s = \gamma \cdot \frac{1}{2f_p}, \tag{2}$$

where γ is a dimensionless constant with value between 0 and 1, depending upon the time interval during which the core is saturated. For the sake of simplicity we shall in the following restrict ourselves to cycles with very short saturation times, so we shall use $\gamma = 1$.

During the switching the average number of Barkhausen volumes jumping per second will amount to N_B/t_S.

For the description of the Barkhausen noise we introduce a characteristic time interval τ. This τ represents the stochastic character of the moment of jumping of individual Barkhausen volumes. To show this let us consider the situation at t seconds after the beginning of the switching time interval t_S. We represent the number of Barkhausen volumes that have jumped unquestionably – in spite of Barkhausen noise – by:

$$\frac{t - \frac{1}{2}\tau}{t_S} \cdot N_B, \tag{3}$$

where the factor $\frac{1}{2}$ is chosen for the sake of elegance of the following equations. The number of Barkhausen volumes that unquestionably did not jump amounts to:

$$\frac{t_S - t - \frac{1}{2}\tau}{t_S} \cdot N_B. \tag{4}$$

There remains the number of Barkhausen volumes:

$$\frac{2 \cdot \frac{1}{2}\tau}{t_S} \cdot N_B \tag{5}$$

the magnetization of which is unpredictable because of the Barkhausen noise. The number that will have jumped between $t - \frac{1}{2}\tau$ and t is $\frac{1}{2}\tau.\ N_B/t_S$ on the average. The fluctuations in this number give the fluctuations in the number of jumps until the time t.

$$\{\langle(\Delta N)^2\rangle_{X,t}\}^{\frac{1}{2}} = \left(\frac{1}{2}\,\frac{\tau}{t_S} \cdot N_B\right)^{\frac{1}{2}} = (f_D \cdot \tau \cdot N_B)^{\frac{1}{2}}. \tag{6}$$

Apart from (6) we could also give a second interpretation of the quantity τ, as being related to the highest magnetic field frequency the probe can follow with its magnetization.

Returning to the measurent procedure, we want to know the error in the number of jumps between $t = 0$ and $t = \frac{1}{2}t_S$ and hereto we can use (6). In addition we are interested in this error in the number of jumps for the time interval between $t = \frac{1}{2}t_S$ and $t = t_S$. The number of jumps between $\frac{1}{2}t_S$ and t_S is not independent of the number of jumps between 0 and $\frac{1}{2}t_S$ since the two numbers must add up to N_B so that the error in the number of jumps over half a period of the X signal becomes zero. For the Z signal things are different.

Because of the change of sign in the Y signal at $t = \frac{1}{2}t_S$ we can distinguish between positive and negative numbers in the Z signal. The error in the algebraic number of jumps covered by the integration of the Z signal over half a period becomes:

$$\{\langle(\Delta N)^2\rangle_{Z,t_s}\}^{\frac{1}{2}} = 2\{\langle(\Delta N)^2\rangle_{X,t}\}^{\frac{1}{2}} = 2(f_p\cdot\tau\cdot N_B)^{\frac{1}{2}} = \left(2\frac{\tau}{t_S}\cdot N_B\right)^{\frac{1}{2}}. \quad (7)$$

The R.M.S. value of the total error in the number of jumps during the integration time t_m amounts to:

$$\{\langle(\Delta N)^2\rangle_{Z,t_m}\}^{\frac{1}{2}} = 2f_p\cdot(2\tau\cdot t_m\cdot N_B)^{\frac{1}{2}} = \left(2\frac{\tau}{t_S}\cdot\frac{t_m}{t_S}\cdot N_B\right)^{\frac{1}{2}}. \quad (8)$$

The error in the flux caused by this error in the number of jumps amounts to:

$$\{\langle(\Delta\Phi)^2\rangle_{t_m}\}^{\frac{1}{2}} = 4f_p\cdot\Phi_S\cdot\left(\frac{2\tau\cdot t_m}{N_B}\right)^{\frac{1}{2}}. \quad (9)$$

From (9) we get for the final expression of the Barkhausen noise:

$$\{\langle(\Delta V)^2\rangle_{t_m}\}^{\frac{1}{2}} = n\cdot\frac{1}{t_m}\{\langle(\Delta\Phi)^2\rangle_{t_m}\}^{\frac{1}{2}}, \quad (10)$$

where n is the number of windings of the secondary coil. This leads to:

$$\langle(\Delta V)^2\rangle_{t_m} = 32n^2\cdot f_p^2\cdot\frac{\tau\cdot\Phi_S^2}{N_B\cdot t_m}. \quad (11)$$

§ 3. Comparison and discussion

Equation (11) is to be compared with a similar relation given by Bittel and Storm [1]. Their relation (eq. 3.6/16 [1]) reads (in our symbols):

$$(\Delta V)^2 = f_p\cdot 8n^2\cdot\Phi_S^2\cdot\frac{1}{N_B}\cdot\frac{1}{t_m}. \quad (12)$$

Here we used relation 3.6/11 (ref. [1]) and our definition (1) and took into account that their relations were originally formulated for one hysteresis loop. Comparing eqs. (11) and (12) one notes a difference of a factor of $4f_p\tau$. For a discussion of this factor we compare the error in the number of jumps in the Förster measuring procedure and in the measuring procedure of Bittel and Storm. Also the consequences of our introduction of τ as a characteristic noise parameter will explicitely be discussed. The factor of $4f_p\tau$ will be seen to be partly due to difference in model and partly to difference in measuring procedure. Here we use the following notation.

N_{t_i}: Average number of jumps in time interval t_i
$\langle(\Delta N)^2\rangle_{t_i}$: Average square of the error in the number of jumps in the time interval.

We take $\gamma = 1$, thus $1/(2f_p) = t_s$.

The consequences of applying different models are as follows:
Bittel-Storm model: for $t_i \ll t_s$

$$\langle(\Delta N)^2\rangle_{t_i} = N_{t_i} = \frac{t_i}{t_s} N_B \tag{13}$$

Our model: We assume that the error over a time interval t_i is given by the error in small intervals $\frac{1}{2}\tau_1$ and $\frac{1}{2}\tau_2$ at the beginning and at the end of t_i

$$\langle(\Delta N)^2\rangle_{t_i} = \langle(\Delta N)^2\rangle_{\frac{1}{2}\tau_1} + \langle(\Delta N)^2\rangle_{\frac{1}{2}\tau_2} \tag{14}$$

with $\tau < t_i < t_s$.

The difference in measuring procedure has its consequences too. In the Förster procedure many hysteresis loops are completed in t_m. The application of the Y-wave has its special consequences on the noise. In t_m there are t_m/t_s complete switches from $-\Phi_s$ to $+\Phi_s$ or the reverse.

$$\langle(\Delta N)^2\rangle_{t_m} = \frac{t_m}{t_s} \langle(\Delta N)^2\rangle_{t_s}. \tag{15}$$

One must first calculate $\langle(\Delta N)^2\rangle_{0-\frac{1}{2}t_s}$, which will depend on the model. Assuming $\langle(\Delta N)^2\rangle_{0-\frac{1}{2}t_s}$ to be known, one proceeds as follows:

$$(\Delta N)_{0-\frac{1}{2}t_s} = -(\Delta N)_{\frac{1}{2}t_s-t_s} \tag{16}$$

which yields $(\Delta N)_{0-t_s} = 0$ as is required since N_B volumes must jump in the time interval t_s. But since the Y-wave changes sign at $t = \frac{1}{2}t_s$ one gets for the effect in the product Z_a

$$(\Delta N)_{0-t_s} = 2(\Delta N)_{0-\frac{1}{2}t_s} \tag{17}$$

$$\langle(\Delta N)^2\rangle_{0-t_s} = 4\langle(\Delta N)^2\rangle_{0-\frac{1}{2}t_s}. \tag{18}$$

Substitution into (15) gives

$$\langle(\Delta N)^2\rangle_{t_m} = \frac{4t_m}{t_s}\langle(\Delta N)^2\rangle_{0-\frac{1}{2}t_s}. \tag{19}$$

This is the expression for the error in the Förster procedure, whereas for the Bittel-Storm procedure we must simply take

$$\langle(\Delta N)^2\rangle_{t_m} \tag{20}$$

since here $t_m \ll t_s$.

The origin of the factor of $4f_p\tau$ now can easily be seen.

a). *Förster Procedure, our model.*

Starting from complete saturation there is no error at the beginning of the time interval $0 - \frac{1}{2}t_s$. Relation (14) yields

$$\langle(\Delta N)^2\rangle_{0-\frac{1}{2}t_s} = \langle(\Delta N)^2\rangle_{\frac{1}{2}\tau} = N_{\frac{1}{2}\tau} = \frac{\tau}{2t_s} N_B. \tag{21}$$

Substitution into (19):

$$\langle(\Delta N)^2\rangle_{t_m} = \frac{2t_m}{t_s}\cdot\frac{\tau}{t_s}\cdot N_B. \tag{22}$$

b). *Bittel-Storm procedure, our model.* (See Fig. 3.)

The error in each time interval is given by the error in τ at the beginning and in τ at the end of the interval. Relation (14) gives:

$$\langle(\Delta N)^2\rangle_{t_m} = 2\langle(\Delta N)^2\rangle_{\frac{1}{2}\tau} = 2N_{\frac{1}{2}\tau} = \frac{\tau}{t_s} N_B. \tag{23}$$

c). *Bittel-Storm procedure, their model.* (See Fig. 3.)

Expressions (20) and (13) give

$$\langle(\Delta N)^2\rangle_{t_m} = N_{t_m} = \frac{t_m}{t_s} N_B. \tag{24}$$

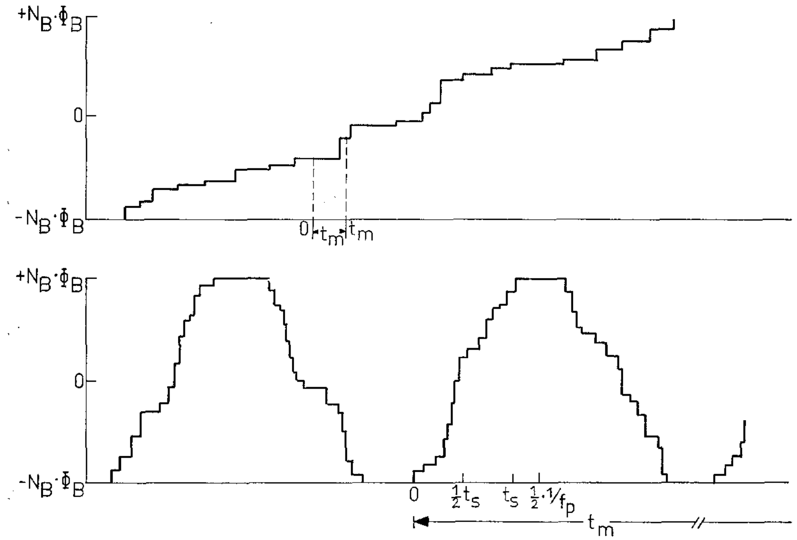

Fig. 3. Comparison of the appearance of Barkhausen noise in a noise experiment after Bittle and Storm and in a fluxgate magnetometer.

Comparing (22) and (24) explains the factor of $2\tau/t_s \approx 4f_p\tau$, where a factor of $2t_m/t_s \approx 4f_pt_m$ is due to difference in measuring procedure, and a factor of τ/t_m is due to difference in model.

For the application of (11) to practical situations we shall calculate the signal-to-noise ratio. Thereto we must compare (11) with the signal V produced by H_0 (see Fig. 2) which is given by:

$$V = 4n \cdot \mu \cdot A \cdot f_p \cdot H_0, \tag{25}$$

where μ stands for the permeability and A for the cross section area of the core. It follows for the signal-to-noise-ratio:

$$\left(\frac{V}{\Delta V}\right)^2 = N_B \cdot \frac{\mu \cdot H_0^2}{B_s^2/\mu} \cdot \frac{t_m}{\tau} = \frac{H_0^2}{H_s^2} \cdot \frac{t_m}{\tau} \cdot N_B. \tag{26}$$

For estimating purposes we shall use the following specific values: $t_m = 1 s$; $B_s = 2$ tesla; $\mu = 10^{-2}$ V·s/A·m; $\tau = 10^{-6}$ s, corresponding with an upper limit of the frequency of 1 MHz; for N_B we shall use two different values, $N_B = 10^{10}$ and $N_B = 10^4$, corresponding to the two limits Bittel and Storm give for the number density of Barkhausen volumes (p. 188, ref. [1]). In addition we use for the core volume V_c the value $3 \cdot 10^{-7}$ m³.

When we define the minimum observable magnetic field $H_{0_{\min}}$ by the condition $V/\Delta V = 1$, equation (26) gives:

$$H_{0_{\min}} = 2 \cdot 10^{-6}\left(\frac{\mathrm{A}}{\mathrm{m}}\right) \quad \text{or} \quad H_{0_{\min}} = 2 \cdot 10^{-3}\left(\frac{\mathrm{A}}{\mathrm{m}}\right), \tag{27}$$

corresponding respectively with the values of N_{B} given above.

The value $H_{0_{\min}} = 10^{-4}$ A/m as found experimentally [5, 6] lies between the limits given in (27). This implies that possibilities do exist that Barkhausen noise is reduced by carefully selecting the core material.

Eq. (26) can be used to get the optimal choice of the material parameters μ, τ, N_{B} and H_{s}.

In addition to the Barkhausen noise there is thermal noise, given by the Nyquist relation:

$$(\Delta V)^2 = 4kT \cdot \mathrm{Re}\{Z\} \cdot \Delta f. \tag{28}$$

Because of the measurement procedure applied this yields:

$$(\Delta V)^2 = 8\pi kT \cdot f_{\mathrm{p}} \cdot \frac{1}{Q} \cdot \frac{n^2 \cdot A}{l} \cdot \mu \cdot \frac{1}{t_{\mathrm{m}}} \tag{29}$$

where l stands for the length of the coil.

Combining this with (25) the signal-to-noise ratio is:

$$\left(\frac{V}{\Delta V}\right)^2 = \frac{2}{\pi} \cdot Q \cdot \frac{\mu \cdot H_0^2}{kT/V_{\mathrm{C}}} \cdot f_{\mathrm{p}} \cdot t_{\mathrm{m}}. \tag{30}$$

Assuming $Q = 100$ and $f_{\mathrm{p}} = 10^{-4}$ Hz the minimum detectable magnetic field at room temperature is $H_{0_{\min}} = 1 \cdot 4 \cdot 10^{-9}$ A/m, which is negligible compared with the limit set by the Barkhausen noise. Our conclusion is that eq. (26) is the key to reduction of the noise in the fluxgate magnetometers, the sensitivity of which is unaffected by the existance of thermal noise.

Received 29 September 1972

REFERENCES

[1] BITTEL, H. and L. STORM, Rauschen, Springerverlag Berlin, Heidelberg, New York, 1971.

[2] FÖRSTER, F., Z. Metallkde. **46** (1955) 358.

[3] FÖRSTER, F., Z. Metallkde. **32** (1940) 184.

[4] WURM, M., Z. Ang. Physik **2** (1950) 210.

[5] WEIJTS, A. G. L. M., C. H. MASSEN, J. A. POULIS: Appl. Sci. Res. **24** (1971) 203.

[6] HARDY, H. J. J., C. v.d. STEEN, C. H. MASSEN, and J. A. POULIS, Appl. Sci. Res. **27** (1972) 129.