Appl. Sci. Res. 25 November 1971

A NEW DIE PROFILE WITH HIGH PROCESS EFFICIENCY

S. K. SAMANTA

Carnegie-Mellon Univ., Dept. of Mech. Eng.,
Schenley Park Pittsburg (Penn. 15213) U.S.A.

Abstract

A new technique is used to design die profiles which yield high process efficiency during axi-symmetric extrusion and wire drawing. These profiles have a convex shape.

The upper bound of the average ram pressure is calculated for the practical range of reductions and optimal die lengths. For m = 0,1 where m is the constant friction factor, and up to 55 per cent reduction the reduced extrusion pressure is the same for this convex die and the optimal length cosine die. Above 55 per cent reduction the pressure is slightly higher than that for the cosine die. Up to a reduction of 55 per cent the optimal lengths for this one are slightly shorter than that of the cosine die, while those values for higher reductions are a little higher for this die. The efficiency of the proposed die exceeds those of conventional conical dies.

Nomenclature

$\dot{E}$	$\dot{E}_i + \dot{E}_s + \dot{E}_e$ = total rate of energy dissipation
$\dot{E}_i$	the rate of energy dissipated within the volume V of the deforming zone
$\dot{E}_s$	the rate of energy dissipation due to shear or friction along surfaces of velocity discontinuity
$\dot{E}_e$	the rate of energy dissipated along traction-prescribed boundary
f	flow function
k	yield shear stress in simple shear
L	axial length of the die
m_j	constant friction factor along s_j
$\bar{p}$	mean extrusion pressure
$\bar{p}_d$	mean drawing pressure
p_{en}	mean entry pressure
p_{ex}	mean exit pressure
R	fractional reduction of area $= (R_1^2 - R_2^2)/R_1^2$

R_1, R_2	the initial and final radius of the bar respectively
s, s_j $(j = 1, 2, \ldots)$	boundary surfaces
s_t	the surface of the body on which surface traction T_i is prescribed
T_i	surface traction, prescribed over s_t
v	velocity
$\bar{V}$	volume of the deforming zone
v_r, v_z	components of the velocity fields
v_1, v_2	the entrance and exit velocity respectively
v_j $(j = 1, 2, \ldots)$	magnitude of tangential velocity discontinuities along s_j
Y	yield stress in uniaxial tension or compression $= \sqrt{3}k$ (Mises)
r, θ, z	cylindrical coordinates
$\dot{\varepsilon}_{ij}$, $\dot{\varepsilon}_r$, $\dot{\varepsilon}_\theta$, $\dot{\varepsilon}_z$, $\dot{\gamma}_{rz}$	components of strain rate fields
α	$(R_1/R_2)^2 - 1$
$\bar{\dot{\varepsilon}}$	effective strain rate
η %	percentage efficiency
τ_j $(j = 1, 2, \ldots)$	shear stress or frictional tractions

§ 1. Introduction

Extrusion is one of the youngest of the metalforming processes and was probably first developed at the end of the eighteenth century for the manufacture of lead pipes. In this process a slug or billet of metal is forced to flow, under high pressure, through a die shaped to give the required crossection to the product. In reducing the diameter of a bar through extrusion redundant deformation, in general, can not be prevented. Redundant work of deformation is the extra work required beyond the minimum value to achieve the final dimension of the product. The object of the present work is to obtain an optimum die profile which minimizes this redundant work in axisymmetric extrusion and wire drawing.

Richmond and Morrison [1], using the method of characteristics, have determined streamlined, frictionless wire drawing profiles which are believed to be the shortest possible for each reduction. Another die profile which eliminates as much redundant work as possible was obtained [2] in frictionless axi-symmetric extrusion by the use of the slip-line field technique. Using slip-line field method the present author [3] proposed the solution for plane-strain extrusion through frictionless cosine dies, which almost eliminates redundant work for a rigid/perfectly plastic material. However, the achievement of friction-free die surfaces in actual forming operations is of course impossible. We are therefore led to consider a die profile, which eliminates as much redundant work as possible and which takes into account the effect of friction. This resulted in a convex die with finite entrance and small exit angles.

The material considered in this work is a rigid/perfectly plastic material obeying the von Mises yield criterion and its associated flow rule.

§ 2. Method of solution

This theoretical analysis deals with axi-symmetric extrusion of circular bars, whose initial and final radii are R_1 and R_2 respectively, see Fig. 1. To begin with, a detailed analysis for direct extrusion will be given but the same approach may be applied to wire-drawing for calculating the final results. Anslysis can be applied equally well to indirect extrusion of circular rods.

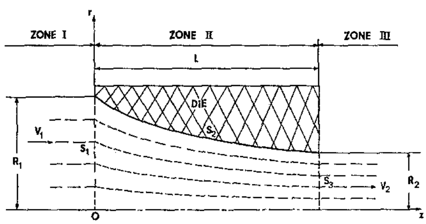

Fig. 1. Proposed die profile and its admissible velocity field.

For the analysis the bar has been devided into three zones (see Fig. 1). Zones I and III are assumed to be rigid and the deformation zone II is plastic. In the rigid zones I and III the velocity is uniform and the incompressibility gives rise to the relationship

$$v_2 = v_1(R_1/R_2)^2.$$

We shall find the kinematically admissible velocity field for which the principal directions of the rate of deformation tensor are the axis of the cylindrical coordinate system, r, θ, z, i.e.

$$\dot{\gamma}_{rz} = \frac{1}{2}\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right) = 0 \qquad (1)$$

The incompressibility condition for the axi-symmetric case is

$$\operatorname{div} \bar{\boldsymbol{v}} = \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0 \tag{2}$$

where v_r and v_z are the r and z components of the velocity, respectively.

The set of equations (1) and (2) is the hyperbolic one. It possesses the following general solution:

$$\begin{aligned} v_r(r, z) = & \int_0^1 \phi(z - r + 2rt) \frac{1 - 2t}{\sqrt{t(1-t)}} \mathrm{d}t + \\ & + \int_0^1 \psi'(z - r + 2rt) \frac{(1-2t) \ln [2rt(1-t)]}{\sqrt{t(1-t)}} \mathrm{d}t - \\ & - \frac{1}{r} \int_0^1 \psi(z - r + 2rt) \frac{\mathrm{d}t}{\sqrt{t(1-t)}} \end{aligned} \tag{3}$$

$$\begin{aligned} v_z(r, z) = & \int_0^1 \phi(z - r + 2rt) \frac{\mathrm{d}t}{\sqrt{t(1-t)}} + \\ & + \int_0^1 \psi'(z - r + 2rt) \frac{\ln [2rt(1-t)]}{\sqrt{t(1-t)}} \mathrm{d}t \end{aligned}$$

where $\phi(x)$ and $\psi(x)$ are arbitrary functions, and $\psi'(x) = \mathrm{d}\psi(x)/\mathrm{d}x$ and $\psi(z - r + 2rt) = \psi(x)|_{x = z - r + 2rt}$.

The above solution can be found in the following way. We shall satisfy trivially the equation (1) looking for velocity field in the form of the flow function f,

$$v_r = -\frac{1}{r} \frac{\partial f}{\partial z}, \quad v_z = \frac{1}{r} \frac{\partial f}{\partial r} \tag{4}$$

Substituting (4) in (1) and changing the independent variables

$$x = z - r, \quad y = z + r \tag{5}$$

we get the following, so called, Euler-Darboux equation

$$\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f/\partial x}{2(x-y)} - \frac{1}{2(x-y)} \frac{\partial f}{\partial y} = 0 \tag{6}$$

This equation has the following general solution [5]

$$(fx, y) = (x-y)^2 \int_0^1 \phi[x + (y-x)t] \sqrt{t(1-t)}\, \mathrm{d}t +$$

$$+ (x-y) \int_0^1 \psi_0'[x + (y-x)t] \frac{(2t-1)}{\sqrt{t(1-t)}} \mathrm{d}t +$$

$$+ (x-y)^2 \int_0^1 \psi_0''[x + (y-x)t] \sqrt{t(1-t)} \ln [t(1-t)(y-x)]\, \mathrm{d}t +$$

$$+ \int_0^1 \psi_0[x + (y-x)t] \frac{1}{\sqrt{t(1-t)}} \mathrm{d}t \tag{7}$$

If we now change the independent variables, perform the operation (4) and substitute $\psi_0' \equiv \psi$, the equation (7) reduces to (3).

Our problem is to determine the kinematically admissible velocity field v_r and v_z, i.e. by satisfying the following boundary conditions:

$$\begin{aligned} v_z &= v_1 && \text{for } z = 0 \\ v_z &= v_1(R_1/R_2)^2 && \text{for } z = L \\ v_r &= 0 && \text{for } r = 0 \\ v_r/v_z &= \mathrm{d}r_{\mathrm{D}}/\mathrm{d}z && \text{for } r = r_{\mathrm{D}}(z) \end{aligned} \tag{8}$$

where $r = r_{\mathrm{D}}(z)$ is the equation of the die. It can be shown that for a given die profile the conditions (8) are sufficient to determine the unique velocity field in the zone II, Fig. 1. We shall try to find such die profile for which we can assume $\psi = 0$, i.e. we assume that $\psi = 0$ in the whole region II.

Taking into account that the boundary conditions for $z = 0$ and $z = L$ are constant we shall try to find the equation of the die assuming

$$\phi(x) = Gx + H \tag{9}$$

in the deforming zone II, Fig. 1.

Equation (3) with the aid of (9) and the condition $\psi \equiv 0$ leads to

$$v_r = -\tfrac{1}{2}\pi r G, \quad v_z = \pi(Gz + H) \tag{10}$$

From the equation (10) and the first two boundary conditions we have

$$G = v_1\alpha/\pi L, \quad H = v_1/\pi$$

where $\alpha = (R_1/R_2)^2 - 1$.

Thus

$$v_r = -v_1\alpha\frac{r}{2L}, \quad v_z = v_1\left(1 + \frac{z}{L}\alpha\right) \tag{11}$$

Hence we get the equation of the stream lines

$$\frac{\mathrm{d}r}{\mathrm{d}z} = -\frac{r\alpha}{2(z\alpha + L)}$$

or

$$r^2 = q(z\alpha + L)^{-1} \tag{12}$$

where q is a constant. Now substituting $r = R_1$ for $z = 0$ we get $q = R_1^2 L$ and the equation of our proposed die profile in the meridian plane is

$$r_{\mathrm{D}}(z) = \frac{R_1}{\left(1 + \dfrac{z\alpha}{L}\right)^{\frac{1}{2}}} \tag{13}$$

The kinematically admissible strain-rate components in the deformation region are

$$\dot{\varepsilon}_r = -\frac{v_1}{2L}\alpha, \quad \dot{\varepsilon}_\theta = -\frac{v_1}{2L}\alpha, \quad \dot{\varepsilon}_z = \frac{v_1}{L}\alpha$$

and $\dot{\gamma}_{rz} = 0$.

The effective strain-rate $\bar{\dot{\varepsilon}}$, is

$$\bar{\dot{\varepsilon}} = (\tfrac{2}{3}\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})^{\frac{1}{2}} = \frac{v_1}{L}\alpha \tag{14}$$

§ 3. Upper-bound for average extrusion pressure

Mathematically consistent solutions of axi-symmetric plastic flow problems are very difficult even for rigid-perfectly plastic materials. Approximate solutions are thus important [4]. Of all approximate methods, the upper bound method [6] is most powerful.

For a von Mises isotropic, rigid-perfectly plastic material, which also obeys Levy-Mises stress-strain rate relation, the rate at which work has been performed may be represented by [7]:

$$\dot{E} = \dot{E}_{\mathrm{i}} + \dot{E}_{\mathrm{s}} + \dot{E}_{\mathrm{e}} = \int_{\bar{V}} Y\bar{\dot{\varepsilon}}\,\mathrm{d}\bar{V} + \int_{s_j} \tau_j v_j^*\,\mathrm{d}s_j - \int_{s_t} T_i v_i^*\,\mathrm{d}s_t \tag{15}$$

and for direct extrusion [6]

$$\bar{p}v_1\pi R_1^2 = \int Y\bar{\dot{\varepsilon}}\,\mathrm{d}\bar{V} + \int_{s_j} kv_j^*\,\mathrm{d}s_j + \pi v_2 R_2^2\,p_{\mathrm{ex}} \tag{16}$$

where p_{ex} stands for the average prescribed pressure at the exit.

The rate of energy dissipated within the volume of the deforming zone is given by

$$\dot{E}_{\mathrm{i}} = 2\pi Y v_1 R_1^2 \ln (R_1/R_2) \tag{17}$$

Now if $\dot{E}_{\mathrm{s}} = \dot{E}_{\mathrm{e}} = 0$, equations (15), (16) and (17) lead to

$$\bar{p}/Y = 2 \ln (R_1/R_2)$$

which is the same as in the homogeneous frictionless ideal case.

The rate of energy dissipations due to the velocity discontinuity along s_1 is given by,

$$\dot{E}_{s_1} = k\iint v_{s_1}^*\,\mathrm{d}s_1 = \frac{\pi Y v_1 \alpha R_1^3}{3\sqrt{3}\,L} \tag{18}$$

and along s_3

$$\dot{E}_{s_3} = k\iint v_{s_3}^*\,\mathrm{d}s_3 = \frac{\pi Y v_1 \alpha R_2^3}{3\sqrt{3}\,L} \tag{19}$$

Assuming with Shield [8] that the shear stress is proportional to the friction stress, i.e. $\tau = mk$, where $0 \leqslant m \leqslant 1$, the rate of energy dissipation due to friction along the material-die interface s_2 has been calculated as

$$\dot{E}_{s_2} = \iint mkv_{s_2}^*\,\mathrm{d}s_2 = \\ = \frac{4\pi Y v_1}{3\sqrt{3}}\,mR_1\,\frac{L}{\alpha}\left[\left(\frac{R_1^3}{R_2^3} - 1\right) - \frac{R_1^2\alpha^2}{4L^2}\left(\frac{R_2^3}{R_1^3} - 1\right)\right] \tag{20}$$

The total rate of energy dissipation is given by,

$$\dot{E} = \pi Y v_1 R_1^2 \left\{ 2 \ln \left(\frac{R_1}{R_2} \right) + \frac{\alpha}{3\sqrt{3}\, L R_1^2} (R_1^3 + R_2^3) + \right.$$
$$\left. + \frac{4mL}{3\sqrt{3}\, R_1 \alpha} \left[\left(\frac{R_1^3}{R_2^3} - 1 \right) - \frac{R_1^2 \alpha^2}{4L^2} \left(\frac{R_2^3}{R_1^3} - 1 \right) \right] \right\} + \pi v_2 R_2^2 P_{\mathrm{ex}} \quad (21)$$

so that the relative mean extrusion pressure $\bar{p}/Y$ is

$$\frac{\bar{p}}{Y} = 2 \ln \left(\frac{R_1}{R_2} \right) + \frac{\alpha}{3\sqrt{3}\, L R_1^2} (R_1^3 + R_2^3) +$$
$$+ \frac{4mL}{3\sqrt{3}\, R_1} \left[\left(\frac{R_1^3}{R_2^3} - 1 \right) - \frac{R_1^2 \alpha^2}{4L^2} \left(\frac{R_2^3}{R_1^3} - 1 \right) \right] + \frac{P_{\mathrm{ex}}}{Y} \quad (22)$$

The first term of equation (22) is attributed to the ideal energy of deformation, the second term is attributed to the redundant energy of deformation due to velocity discontinuities near the entrance and the exit and the last term is due to the effects of friction stresses $m_j k$ between the rigid die and the extruded material.

§ 4. Results and discussion

The optimum length of the die is obtained by minimising the rate of energy dissipation, $\partial \dot{E} / \partial L = 0$. The optimum length of the die is given explicitly by

$$L = \frac{\alpha R_2^{\frac{3}{2}}}{2 R_1^{\frac{1}{2}}} \left\{ 1 + \frac{1}{m} \left(\frac{R_1^3 + R_2^3}{R_1^3 - R_2^3} \right) \right\}^{\frac{1}{2}} \quad (23)$$

A finite optimum length exists at which the extrusion pressure is minimum. This is shown in Fig. 2 for $m = 0.1$ which is a reasonable value for extrusion and wire drawing.

The optimal die length for various reductions are compared in Fig. 3 with the same for the cosine dies [4]. It seems for the above results that for $m = 0.1$ and up to a reduction of 55 per cent the optimal lengths for the proposed die are slightly shorter than that for the cosine die, while those values for greater reductions ($R \geqslant 55\%$) for the proposed die are a little higher. If friction could be neglected, the required extrusion pressure is equal to its minimum value of ideal deformation as $L/R_1 \to \infty$.

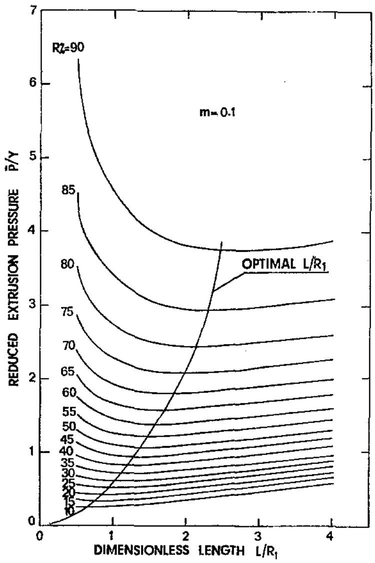

Fig. 2.

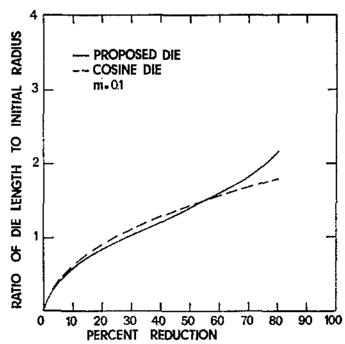

Fig. 3.

Fig. 2. The effect of die length and percent reduction on reduced extrusion pressure, $\bar{p}/Y$, in extrusion through proposed die, $p_{ex} = 0$, $m = 0.1$.

Fig. 3. Comparison of die length for cosine and proposed die.

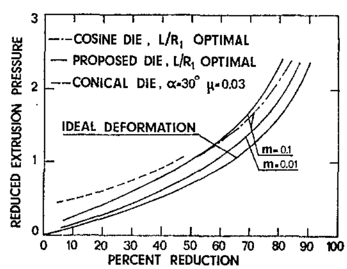

Fig. 4. The effect of reduction and friction on reduced extrusion pressure, $\bar{p}/Y$, in extrusion through proposed die, $p_{ex} = 0$.

Effect of friction on extrusion pressure. The reduced mean extrusion pressure for $m = 0.1$ and $p_{ex} = 0$ are shown in Fig. 4. For $m = 0.1$ and up to 55 per cent reduction $\boldsymbol{p}/Y$ is same for this die and the optimum cosine die. For higher reductions the optimum length cosine die yields slightly better results. The results for $m = 0.01$

and for ideal deformation is also included in Fig. 4 for comparison. The available solutions for conical dies cover only the effect of cone angle and reduction on reduced extrusion pressure as computed by Avitzur [9] for $\mu = 0.03$ and semi-cone angle 30° is shown in Fig. 4. The results show that the proposed die is better than the conical die within the range of available solution.

§ 5. Upper bound for average drawing stress

Our previous discussion was restricted to extrusion only, although it holds for wire drawing too. For wire drawing, the rate of energy dissipation along the boundary on which the surface tractions are prescribed is that incident to the stress exerted on the wire at the entrance of the die, while for extrusion it is that incident to the pressure exerted on the extrusion at the exit of the die.

Hence the relative mean drawing stress, $\bar{p}_d/Y$ is

$$\frac{\bar{p}_d}{Y} = \frac{p_{en}}{Y} - \left\{2 \ln\left(\frac{R_1}{R_2}\right) + \frac{\alpha}{3\sqrt{3}\, L R_1^2}(R_1^3 + R_2^3) + \right.$$
$$\left. + \frac{4mL}{3\sqrt{3}\, R_1 \alpha}\left[\left(\frac{R_1^3}{R_2^3} - 1\right) - \frac{R_1^2\alpha^2}{4L^2}\left(\frac{R_2^3}{R_1^3} - 1\right)\right]\right\} \tag{24}$$

Acknowledgements

The author is grateful to Dr. B. Raniecki and Mr. J. Mägi for helpful comments on this work. Acknowledgements is also made to Prof. F. F. Ling and Dr. C. T. Chen of Rensselaer Polytechnic, New York for supplying the numerical values for the cosine dies.

This work is part of a programme of reserach on the mechanics of extrusion which is supported by the Swedish Board for Technical Development and C–B Nathhorsts Science Foundation to which the author is indebted.

Received 19 May 1970
In final torm 21 April 1971

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