

AN ANALYSIS OF CRITICAL SIMULTANEOUS GAS/LIQUID FLOW THROUGH A RESTRICTION AND ITS APPLICATION TO FLOWMETERING

by N. C. J. ROS

Kon./Shell Exploratie en Produktie Laboratorium (Shell Internationale Research
Maatschappij), Delft, Netherlands

Summary

A theoretical analysis is presented of the mechanism of simultaneous flow of gas and liquid through a restriction at critical speed. The study shows that a relationship exists between the mass flows of gas and liquid, restriction size and upstream pressure. Comparison of this relationship with measurements showed a reasonable agreement. It has therefore been concluded that a restriction can be used as a gas/liquid flowmeter with reasonably high accuracy, provided that the flow is a critical one. Further increase of accuracy seems possible by using a restriction especially designed for the purpose.

§ 1. *Introduction.* Restrictions through which gas/liquid mixtures flow at critical speed are widely used in oil field practice in the form of well head restrictions. Gilbert ¹⁾ presented an empirical correlation, based on oil field data, linking liquid production rate, gas/liquid ratio, upstream restriction pressure and restriction size. Such a correlation offers the possibility of using a restriction as a flowmeter; from its pressure readings the gas/liquid ratio can be calculated, once the liquid flow rate is known, and vice versa. Tangren *et al* ²⁾ presented an analysis of the behaviour of an expanding gas/liquid system. In this analysis a "homogeneous" mixture has been assumed, in the sense that the gas bubbles are so small and uniformly distributed in the liquid that the velocities of both phases may be considered equal during expansion. This assumption seems reasonable for systems where the liquid phase is continuous, occurring when the volumetric gas/liquid ratio is lower than about 1. However, when this ratio exceeds 1, the opposite situation is likely to occur. Then, the liquid phase being

the dispersed one, a difference in velocity of the phases is to be expected, causing energy losses due to slip. In accordance with his assumptions, these energy losses have been ignored in Tangren's study. The intention of the present analysis is to extend Tangren's study to gas/liquid systems with a continuous gas phase and to develop a flowmeter formula for these systems.

The analysis is based on the well-known energy balance which says that, for a fluid flowing uniformly and under stationary conditions between any two points, the sum of expansion energy, potential energy, kinetic energy and irreversible energy must be constant ³). Thus, with the usual symbols

$$\int_1^2 (V dp + g dh + d(\frac{1}{2}v^2) + dW) = 0.$$

For a flowmeter, the value of the total mass flow is of interest. This mass flow follows from the area of the restriction throat, the throat density and the throat velocity. The latter can be determined after integration of the above equation from the kinetic energy term, once the other terms are known. The integration boundaries 1 and 2 then apply before the restriction and in the restriction throat respectively.

For single-phase flow, when the fluid velocity may be considered uniform within each cross-section, this procedure offers no particular difficulties and results in the well-known flowmeter formula ³). Application of the procedure to the case of gas/liquid flow with a continuous gas phase, however, seems impossible, since generally the throat velocity of both liquid and gas will not be uniform, which means that the integration of the kinetic energy term of the above equation cannot be performed. Furthermore, the relation between V and p is not known, while the contribution of the irreversible energy term is uncertain on account of the slip losses. Thus, the integration necessary for the development of a flowmeter formula cannot be performed for the general case of gas/liquid flow.

Fortunately, the procedure can still be applied in certain cases, among others those for which it can be shown that:

1. the throat velocity v_2 may be considered uniform;
2. the gas expands polytropically, resulting in a known V/p relationship;

3. the potential energy term and part of the irreversible energy term, namely surface energy and wall friction, can be ignored;
4. the remaining part of the irreversible energy term, namely the slip losses, can be determined.

In the next section criteria will be derived and the correctness of the above points will be demonstrated for a specific case, characterized by table I, representing average oil field conditions.

TABLE I

		Liquid (index <i>l</i>)	Gas (index <i>g</i>)	Unit
Upstream density	ρ	0.8	10×10^{-3}	g/cm ³
Throat density	ρ^*	0.8	5×10^{-3}	g/cm ³
Heat conductivity	λ	7×10^{-4}	7×10^{-5}	cal/cm s °C
Specific heat at constant pressure	C_p	0.58	0.56	cal/g °C
Specific heat at constant volume	C_v		0.43	cal/g °C
Critical velocity	v_c		36 000	cm/s
Mass flow per unit mass flow of mixture	m	0.85	0.15	—
Surface tension	σ	30		dyne/cm
Upstream volumetric gas/liquid ratio	R	$= \frac{m_g \rho_l}{m_l \rho_g} = 14$		
Volumetric gas/liquid ratio in throat	R^*	$= \frac{m_g \rho_l}{m_l \rho_g^*} = 28$		
Throat length $L = 1$ cm				
Throat diameter $D \sim 1$ cm				
Gas phase continuous				

§ 2. *Flow mechanism in a restriction.* 2.1. Flow pattern. The high velocities occurring with critical simultaneous flow of gas and liquid will cause dispersion of the liquid and result in mist flow. A part of the liquid, however, will travel as a film along the restriction wall. The thickness of such a film δ has been investigated by van Rossum⁴), who has obtained a Weber correlation

$$We = \rho_g^* v_c^2 \delta / \sigma \sim 30.$$

This correlation is valid for systems with a constant gas velocity. Assuming that the correlation also holds for cases where the gas phase accelerates, substitution of the values of table I results in $\delta = 1.5 \times 10^{-4}$ cm. The velocity of the liquid in this thin layer

will be very small, so that the liquid transport will be negligibly small. All of the liquid may thus be reckoned to be transported as a mist.

The maximum possible droplet diameter in this mist D_{\max} can be estimated using another Weber correlation, given by Hinze⁵⁾:

$$We = \rho_g^* v_c^2 D_{\max} / \sigma \sim 20.$$

The average droplet diameter \bar{D} , according to Mugele and Evans⁶⁾ is $\sim 0.3 D_{\max}$, so that

$$\bar{D} \sim 6\sigma / \rho_g^* v_c^2. \quad (1)$$

Substitution of the values of table I gives

$$\bar{D} \sim 3 \times 10^{-5} \text{ cm.} \quad (2)$$

The liquid thus flows through the restriction predominantly in the form of extremely small droplets.

2.2 Mechanics in restriction throat. The dispersion of the liquid takes place at the entry of the restriction. After dispersion the liquid droplets will have at first a relatively low velocity and will be accelerated by the kinetic head of the gas. This will mainly occur in the restriction throat where both the velocity and the density of the gas are more or less constant.

The friction force F exerted on a droplet accelerates it and thus reduces the velocity difference $v_g - v_l$:

$$F = \frac{\pi}{4} \bar{D}^2 \frac{1}{2} \rho_g^* (v_g - v_l)^2 C_w = -\frac{\pi}{6} \bar{D}^3 \rho_l \frac{d(v_g - v_l)}{dt},$$

the friction coefficient C_w having a value of 0.43 or higher³⁾. If 0.43 is substituted and ρ_g^* and v_g are considered constant, the equation can be integrated:

$$t = \frac{4}{3} \frac{\rho_l}{\rho_g^*} \frac{\bar{D}}{0.43} \left[\frac{1}{(v_g - v_l)} - \frac{1}{(v_g - v_l)_0} \right],$$

$(v_g - v_l)_0$ being the initial velocity difference.

The time required for passage of the gas t_p is equal to L/v_g . Assume the initial liquid velocity to be zero; then, with $v_l/v_g = \zeta$ and $t/t_p = \tau$ the above equation becomes after solving for ζ :

$$\zeta = \frac{\tau}{\tau + a}, \text{ with } a = \frac{4}{3} \frac{1}{0.43} \frac{\rho_l}{\rho_g^*} \frac{\bar{D}}{L}. \quad (3)$$

With the data from table I and equation (2) for \bar{D} , the value of a becomes: $a = 0.015$.

2.2a. *Mechanical equilibrium*.* The velocity difference at the end of the throat $v_g - v_l$ is small, while the relationship between the mass flow and the kinetic energy of the mixture is practically unaffected by this remaining velocity difference. This appears as follows.

The path x_l , travelled by the droplet in a time t , is equal to

$$\begin{aligned} x_l &= \int_0^t v_l dt = \int_0^\tau v_g \zeta t_p d\tau = L \int_0^\tau \zeta d\tau = L \int_0^\tau \frac{\tau}{a + \tau} d\tau = \\ &= L \left[\tau - a \ln \left(1 + \frac{\tau}{a} \right) \right]. \end{aligned}$$

At the end of the throat $x_l = L$ and thus $\tau - a \ln(1 + \tau/a) = L$. With $a = 0.015$, τ takes the value 1.06 and the velocity ratio $v_l/v_g = \zeta = 0.987$.

At complete equilibrium $\zeta = 1$. Any deviation of ζ from unity is of interest only in so far as the relationship between mass flow and kinetic energy per unit mass is affected, since it is the latter which will be determined from the energy balance.

With Φ_m for the total mass flow, the volumetric flows are $m_l \Phi_m / \rho_l$ and $m_g \Phi_m / \rho_g^*$ for liquid and gas respectively. The cross-sectional area of the total flow thus is

$$A = \left(\frac{m_l}{\rho_l v_l} + \frac{m_g}{\rho_g^* v_g} \right) \Phi_m.$$

The kinetic energy per unit mass is

$$E_k = \frac{1}{2} m_l v_l^2 + \frac{1}{2} m_g v_g^2.$$

From these two equations, with $\zeta = v_l/v_g$, it follows that

$$\Phi_m = \frac{\rho_l A}{m_l} \sqrt{\frac{2E_k}{m_l}} \frac{1}{\left(\frac{1}{\zeta} + \frac{m_g}{m_l} \frac{\rho_l}{\rho_g^*} \right) \sqrt{\frac{m_g}{m_l} + \zeta^2}}.$$

At complete mechanical equilibrium $\zeta = 1$, and the equation reduces to

$$\Phi_m = \frac{\rho_l A}{m_l} \sqrt{\frac{2E_k}{m_l}} \frac{1}{\left(1 + \frac{m_g}{m_l} \frac{\rho_l}{\rho_g^*} \right) \sqrt{\frac{m_g}{m_l} + 1}}.$$

With the data from table I and $\zeta = 0.987$ the discrepancy between the two equations becomes 1% and may be ignored. In other words, the throat velocity v_2 in the energy balance may be considered uniform.

2.2b. *Slip losses.* The energy withdrawn from the gas in order to accelerate the droplet is equal to

$$E_g = \int_0^t F v_g dt.$$

Likewise, the energy supplied to the droplet

$$E_l = \int_0^t F v_l dt.$$

The energy dissipation due to slip is thus equal to

$$E_s = E_g - E_l = \int_0^t F(v_g - v_l) dt.$$

The reduced energy dissipation, being the ratio between E_s and E_l , now appears nearly equal to one:

$$\begin{aligned} \frac{E_s}{E_l} &= \frac{\int_0^t F(v_g - v_l) dt}{\int_0^t F v_l dt} = \frac{\int_0^t (\pi/4) D^2 \frac{1}{2} \rho_g^* (v_g - v_l)^2 C_w (v_g - v_l) dt}{\int_0^t (\pi/4) D^2 \frac{1}{2} \rho_g^* (v_g - v_l)^2 C_w v_l dt} = \\ &= \frac{\int_0^\tau (1 - \zeta)^3 d\tau}{\int_0^\tau (1 - \zeta)^2 \zeta d\tau}. \end{aligned}$$

With $\zeta = \tau/(\tau + a)$ (see (3)), E_s/E_l becomes equal to $1 + 2a/\tau$, with $2a/\tau \ll 1$, thus $E_s \approx E_l$. The slip losses occurring with the acceleration of a certain amount of liquid are thus nearly equal to the kinetic energy of that amount of liquid.

2.3 Surface energy and wall friction. During the dispersion of the liquid the surface area between both phases will increase, for which energy is required. This surface energy E amounts to

$$E = \sigma A_a,$$

A_a being the surface area of the droplets in the throat. (Before the restriction this area is negligibly small.)

Per unit mass of mixture the liquid volume V_l is given by

$$V_l = m_l/\rho_l.$$

For an average droplet diameter \bar{D} the surface area then becomes

$$A_d = \frac{V_l}{\frac{1}{6}\pi\bar{D}^3} \pi\bar{D}^2 = \frac{6m_l}{\bar{D}\rho_l}.$$

Substitution of (1) for \bar{D} gives $A_d = \rho_g^* v^2 m_l / \sigma \rho_l$, so that $E = \sigma A_d = \rho_g^* v^2 m_l / \rho_l$. Comparing E with the kinetic energy $\frac{1}{2}v^2$:

$$\frac{E}{\frac{1}{2}v^2} = \frac{2\rho_g^* m_l}{\rho_l}.$$

With the values of table I the value of $E/\frac{1}{2}v^2$ becomes about 0.01. The kinetic energy thus far exceeds the surface energy, so that the influence of the latter on the energy balance can be ignored.

The effect of wall friction can also be ignored if the restriction is short enough. With single-phase flow the pressure drop due to wall friction ΔP is given by $\Delta P = 4f\frac{1}{2}\rho v^2 L/D$, L and D being the length and diameter of the throat and $4f$ being the friction coefficient. At very high velocities (with correspondingly high values of the Reynolds number) $4f$ becomes very small, of the order of 10^{-2} (ref. 3)), so that with a short restriction ($L \sim D$) ΔP will be only about one per cent of $\frac{1}{2}\rho v^2$. The same will also hold for the case of gas/liquid flow, so that the influence of wall friction may be neglected.

2.4. Thermal equilibrium and gas expansion law. In the restriction entry a rapid decrease in pressure occurs. In the first instance the liquid temperature will not be affected, while the gas temperature decreases adiabatically. Thus a temperature difference arises, causing a heat flow from liquid to gas. This heat flow is so intense that the temperatures of liquid and surrounding gas may be considered equal during the expansion while the gas expansion law becomes a polytropic one. This appears as follows.

The heat exchange in the restriction may be approximated by the following model: the spherical liquid droplets with uniform diameter \bar{D} are distributed in the gas phase according to a hexagonal equidistant pattern. The droplets are then surrounded by gas bodies, the shape of which is roughly spherical. Suppose that complete expansion takes place in an infinitesimally short time.

The gas sphere then will attain a uniform temperature ΔT_0 lower than the uniform liquid temperature, and heat flow by conduction starts. At the interface the discontinuity in temperature disappears instantaneously, so that the temperature difference ΔT_l between the centre of the liquid sphere and the interface is at maximum and likewise ΔT_g between interface and outer surface of the gas. The time required for these temperature differences ΔT to disappear may be found from the theory of heat conduction for this model ⁷).

Any reduced temperature difference $\Delta T/\Delta T_0$ is a function of the Fourier number $\bar{\tau}$, defined as

$$\bar{\tau} = \lambda t / C_p \rho R^2.$$

Practical equilibrium $\Delta T/\Delta T_0 = 10^{-2}$ is attained for $\bar{\tau} \sim 1$ for each phase.

For the liquid phase the equilibrium time t_l becomes, with sphere radius $R_l \sim 1.5 \times 10^{-5}$ cm (see (2)) and the values of table I for λ_l , C_{pl} and ρ_l :

$$t_l \sim 2 \times 10^{-7} \text{ s.}$$

For the gas phase, the sphere radius R_g is given by:

$$R_g \sim R_l \sqrt[3]{R^* + 1}; \text{ with } R_l \sim 1.5 \times 10^{-5} \text{ cm,}$$

and with the values of table I for λ_g , C_{pg} , ρ_g and R^* the value of t_g becomes:

$$t_g \sim 10^{-7} \text{ s.}$$

Whichever time is longer, t_l or t_g , determines the time t_m required to attain practically uniform temperature throughout, thus:

$$t_m \sim 2 \times 10^{-7} \text{ s.}$$

On the other hand, the passage time is given by $t_p \sim L/v_c$. With throat length $L \sim 1$ cm and $v_c \sim 36\,000$ cm/s, t_p is in the order of 3×10^{-5} s, which far exceeds the thermal equilibrium time t_m of the model. The mixture will thus have a uniform temperature in the throat and most probably also during the expansion before.

This uniform mixture temperature, however, will decrease during the expansion. This expansion is governed by the thermodynamics of irreversible processes in mixtures. The latter is rather complex and will not be used here. Instead, a simplified description of the expansion of the mixture will be given, showing that the gas

expansion can be approximated by a polytropic expansion law. To that end the mixture expansion process, including effects of irreversible energy losses and velocity differences between the phases, is enclosed between two imaginary processes, i.e. a process where both irreversible energy losses and differences in velocity are assumed to be zero, and a second one where both irreversible energy losses and velocity differences are assumed to have their maximum possible value.

The first process is adiabatic since, the passage time being very short ($t_p \sim 3 \times 10^{-5}$ s, see above), heat transfer between wall and mixture must be negligibly small, while furthermore irreversible energy losses according to the assumptions are zero. Then, the first law of thermodynamics becomes

$$dQ = dU + p dV = m_l(dU_l + p dV_l) + m_g(dU_g + p dV_g) = 0.$$

With $dU_l = C_{vl} dT$, $dU_g = C_{vg} dT$ and $dV_l = 0$ (incompressible liquid), the above equation becomes

$$dT(m_l C_{vl} + m_g C_{vg}) + m_g p dV_g = 0. \quad (4)$$

The equation of state for the gas phase may be approximated by

$$pV_g = RT = (C_{pg} - C_{vg})T.$$

Differentiation of the last expression, elimination of dT with (4) and integration then leads to a relation between p and V_g :

$$pV_g^n = \text{constant, with } n = 1 + \frac{m_g(C_{pg} - C_{vg})}{m_l C_{vl} + m_g C_{vg}}.$$

This expression has already been found by other investigators²⁾. The value of n becomes, with the data from table I:

$$n = 1.035.$$

The second process with liquid velocity zero and irreversible energy losses at maximum, i.e. kinetic energy also zero, is in fact a throttling process of the gas flowing at low velocity through a packed bed of non-moving liquid droplets. Then, the enthalpy of the gas phase remains constant; that is, with the usual symbols

$$dH = d(U_g + pV_g) = C_{vg} dT + d(pV_g) = 0.$$

This equation, together with the differentiated equation of state

for the gas phase, gives $dT = 0$; thus $T = \text{constant}$ (isothermal expansion), that is $n = 1$.

The difference between the two expansion coefficients is very small. Thus, it would seem reasonable to assume polytropic gas expansion for the real process, with the expansion coefficient following from linear interpolation on the basis of dissipated energy. The latter equals the liquid's kinetic energy (see § 2.2*b* and § 2.3), which for equal gas and liquid velocity is the fraction m_l of the mixture's kinetic energy and the fraction $m_l/(1 + m_l)$ of the expansion energy (see § 3.1). The result is (see fig. 1)

$$n = 1 + \frac{1}{1 + m_l} \frac{m_g(C_{pg} - C_{vg})}{m_l C_{vl} + m_g C_{vg}}.$$

With the values of table I, n becomes $n = 1.019$.

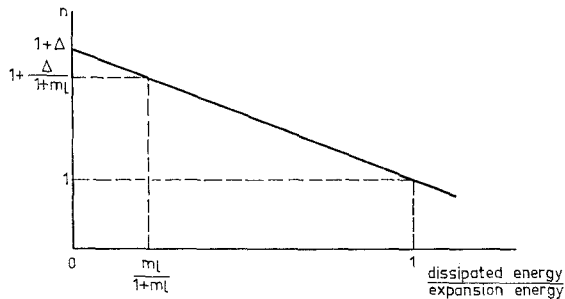


Fig. 1. The exponent n in the relation $pV_g^n = \text{constant}$ as a function of the fraction of expansion energy dissipated.

§ 3. *Development of formula.* 3.1. General formula. It has now been shown that of the irreversible energy term dW only the slip losses have some influence on the energy balance. These losses appeared to be equal to the kinetic energy of the liquid, so that

$$\int_1^2 dW = \int_1^2 m_l d(\frac{1}{2}v^2),$$

m_l being the mass of liquid per unit mass of mixture*). Furthermore,

*) This relationship has been derived on the basis of a continuous gas phase, since it had been shown that this situation occurs with flow through a restriction for the conditions of table I, comprising a high gas/liquid ratio R . At low R -values ($R < 1$), however, the opposite situation will occur. Then a chain of discussions can be presented, similar

at critical flow conditions, the potential energy term $g dh$ is negligibly small compared with the other terms, so that the energy balance simplifies to

$$\int_1^2 [V dp + d(\frac{1}{2}v^2) + m_l d(\frac{1}{2}v^2)] = 0,$$

from which the uniform throat velocity v_2 can be established.

The polytropic expansion of the gas gives

$$p(V - V_l)^n = \text{Constant}, \quad (5)$$

the term V_l being the liquid volume, incorporated in one unit of mass of mixture.

Elimination of V from the above equations and integration give

$$\begin{aligned} \frac{n}{n-1} (V_1 - V_l) p_1 + V_l p_1 + \frac{1}{2}v_1^2(1 + m_l) = \\ = \frac{n}{n-1} (V_2 - V_l) p_2 + V_l p_2 + \frac{1}{2}v_2^2(1 + m_l). \end{aligned} \quad (6)$$

The restriction throat area is usually much smaller than the area upstream of the restriction so that $v_2 \gg v_1$ and the latter may be neglected.

The total mass flow Φ_m follows from the throat velocity v_2 , the throat area A , the mixture density in the throat $1/V_2$ and the discharge coefficient C :

$$\Phi_m = ACv_2/V_2, \quad (7)$$

C being a correction for all neglected phenomena.

Combination of (5), (6) and (7) gives

$$\varphi = \frac{\Phi_m}{AC} \sqrt{\frac{V_l(1 + m_l)}{2p_1}} = \frac{\sqrt{R \frac{n}{n-1} (1 - x^{(n-1)/n}) + (1-x)}}{Rx^{-1/n} + 1}. \quad (8)$$

In this formula *) the dimensionless total mass flow φ defined above

to those given above and with the same results, except that in this case the slip losses are equal to the kinetic energy of the gas phase. Thus,

$$\int_1^2 dW = \int_1^2 m_g d(\frac{1}{2}v^2).$$

Since $R < 1$, m_g will be very small compared with unity and the term dW can be ignored in the energy balance. Then (8) will also change: the term $1 + m_l$ must be omitted (see also ref. 2)).

*) See previous foot-note.

is related to the pressure ratio $x = p_2/p_1$, the volumetric gas/liquid ratio before the restriction $R = (V_1 - V_l)/V_l$ and the polytropic expansion coefficient n .

3.2 Critical pressure ratio. When for a given mixture composition and upstream restriction pressure p_1 the total mass flow Φ_m is determined as a function of the throat pressure p_2 using (8), it appears that Φ_m first increases with decreasing p_2 and subsequently decreases. This behaviour is illustrated in fig. 2, where the dimensionless mass flow φ is plotted against the pressure ratio x . The maximum value φ_c of φ occurs at an x -value which is defined as the critical pressure ratio x_c . Experiments with single phase gas

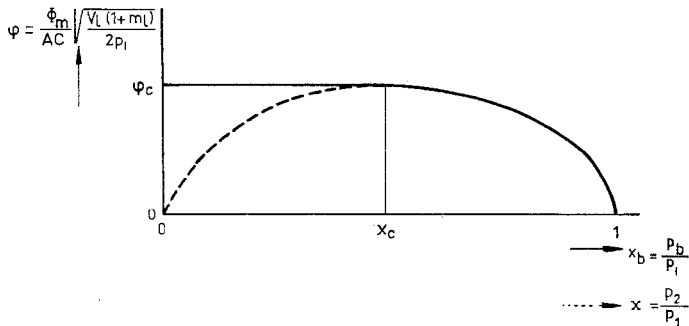


Fig. 2. Dimensionless mass flow φ against pressure ratios x and x_b for given R and n values.

flow ($R = \infty$), however, showed that with decreasing back pressure p_b the value of φ closely follows (8) for $p_b > x_c p_1$, but that for $p_b \leq x_c p_1$, φ is constant and equal to φ_c . Apparently the throat pressure p_2 is equal to the back pressure p_b for $p_b > x_c p_1$, while p_2 remains equal to $x_c p_1$ for $p_b \leq x_c p_1$. In the latter situation the throat velocity is equal to the velocity of sound ²⁾ and apparently independent of the value of the back pressure p_b . Hence, the unbroken horizontal line in fig. 2 represents the φ/x_b relationship, where x_b refers to p_b/p_1 .

The critical x -value is determined by $\partial\varphi/\partial x = 0$ for $x = x_c$. Differentiation of (8) gives with $x = x_c$ and $\partial\varphi/\partial x = 0$,

$$\frac{n}{2} x_c^{(n-1)/n} (R + x_c^{1/n})^2 = R \left[R \frac{n}{n-1} (1-x_c)^{(n-1)/n} + (1-x_c) \right]. \quad (9)$$

3.3. Critical dimensionless mass flow φ_c . To determine the critical value φ_c of φ , x_c should be solved from (9) and the result substituted in (8). This procedure, however, cannot be performed analytically because of the implicit character of (9). Fortunately, it is possible to derive fairly accurate explicit approximations for φ_c .

3.3a. $R > 0.6$. When (9) is evaluated, it appears that for a given n the x_c -value is practically constant in the range $R > 0.6$, see fig. 3. Thus, it is possible to approximate x_c by a constant value x_n chosen between the extreme values of x_c encountered in the R -range.

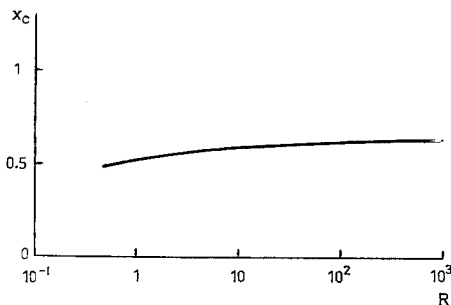


Fig. 3. Critical pressure ratio x_c against volumetric gas/liquid ratio R for a constant n value.

Substitution of x_n in (8) now introduces only a small error, since the φ -value is nearly independent of x for x -values around x_c ($\partial\varphi/\partial x = 0$ for $x = x_c$). The resulting expression then has the form

$$\varphi_c = \frac{C_1 \sqrt{R + C_2}}{R + C_3},$$

in which the values of C_1 , C_2 and C_3 depend on n . These dependences, however, are nearly linear ones and thus can in turn be approximated by: $C_1 = a + bn$, etc. The resulting φ_c -approximation becomes

$$\varphi_c = \frac{(0.2905 + 0.138n) \sqrt{R + (0.6903 + 0.065n)}}{R + (0.4778 + 0.076n)}. \quad (10)$$

The discrepancy between this expression and (8) is smaller than 0.7%.

3.3b. $R < 0.6$ *). In this range, the mass of gas per unit mass of mixture m_g will usually be very small, resulting in an n -value nearly

*) See previous foot-notes.

equal to one. Then φ_c will be a function of R only. This relationship can be approximated by

$$\varphi_c = 1 - \sqrt{2R} + (0.777 - 0.488 \log R)R. \quad (11)$$

Comparison of (11) with (8) shows a discrepancy smaller than 0.04%.

§ 4. *Discussion.* The flowmeter formula (8) has been checked on a number of oil field data dealing with critical gas/liquid flow through well-head restrictions. Any discrepancy between reality and the assumptions on which the theoretical formula is based will show up in the value of the discharge coefficient C . According to (8)

$$C = \frac{\Phi_m}{A\varphi_c} \sqrt{\frac{V_l(1 + m_l)}{2\phi_1}},$$

φ_c being given by (10). Substituting values, deduced from the basic and field data, C appeared to have a value of: $C = 1.06 \pm 0.12$ for $R > 1$. For the restriction shape in question, i.e. a cylindrical channel with a slightly rounded edge (see fig. 4), a C -value of about 0.95 is to be expected. Comparison of this figure with the measured C -values shows the discrepancy to be rather small so that the theoretical analysis would seem fairly justified. The standard deviation of 0.12 can probably partly be ascribed to inaccuracy in the field data.

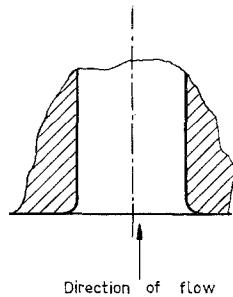


Fig. 4, Restriction shape.

This standard deviation, however, is too big to allow application of (8) to flowmetering. Possibly the accuracy of the flowmetering procedure can be increased by making tests using a restriction specially designed for the purpose. Experimental results of other

investigators point in this direction: Tangren²⁾ investigated a gas/water mixture flowing at critical speed through a nozzle. From the results a C -value of 1.04 ± 0.02 can be deduced, of which the standard deviation of 0.02 is remarkably small.

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