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VAGUE OBJECTS FOR THOSE WHO WANT THEM*

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In a recent paper, "Another Argument Against Vague Objects, "[1] F. J. Pelletier constructs an argument which, if it were sound, would establish a very strong metaphysical conclusion. The structure of his argument is the following: (a) one who is committed to the existence of vague objects (i.e., one who is committed to an *ontological* "vagueness-in-reality" as opposed to a *semantic* "indeterminacy-of-reference") is thus committed to the acceptance of a many-valued logic of a certain sort. (b) But a reasonable many-valued logic with sufficient resources for representing such an ontological vagueness will entail a contradiction on the assumption of the existence of a vague object *a* (more specifically, on the assumption that some identity formula '*a* = *b*' is assigned an 'indeterminate' semantic value). Therefore, (c) we must "deny that there is any such thing as vagueness-in-reality" (p. 492).

We think that there is some reason, in general, to be suspicious of arguments that purport to derive such a strong metaphysical conclusion from premises that are essentially logical or 'formal'. Although it perhaps might not survive a deeply probing examination, premise (a) has at least a *prima facie* ring of plausibility. So we will accept it for the sake of argument. It is premise (b) which we wish to dispute. The idea behind premise (b) is that only many-valued logics of a certain sort validate certain cherished intuitions that we have concerning very general logical principles — especially, Leibniz' Law. And, Pelletier argues, such a logic entails a contradiction on the supposition of the existence of a vague object. We see two problems here. The first is determining exactly 'how much' of cherished logical intuitions remain intuitively compelling when we move from classical to non-classical (e.g., many-valued) contexts. The second is determining what well-

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formed formulae best express the remaining intuitive core of general logical principles in a non-classical context.

In more specific terms, we shall suggest that the appropriate well-formed formula for representing what remains compelling about Leibniz' Law within the context of one of Pelletier's many-valued logics is not the same formula that is customarily used to represent it in classical, two-valued predicate logic. We believe that is important to emphasize that our aim is not to argue that there are vague objects. Rather, it is to argue that there is a perfectly reasonable many-valued logic of the sort considered by Pelletier which does not entail a contradiction on the assumption of the existence of vague objects.

Pelletier points out that "the present logic is completely extensional. . . . In general, if *a* and *b* are names in this language and '$a = b$' is true, then any predicate that can be formulated in the language will apply truly to *a* just in case it applies truly to *b*" (p. 487). If the formal systems sketched by Pelletier *merely* exemplified this sort of extensionality, the vagueness-in-reality theorist would not have any obvious response to Pelletier's claim that an attempt to avoid his conclusion (c) by restricting lambda-abstraction or Leibniz' Law is not, in the present case, defensible. But, in fact, Pelletier's systems go much further. Principles that he enunciates entail that if '$a = b$' is *indeterminate*, then *Fa* is definitely true (definitely false) if and only if *Fb* is definitely true (definitely false), where '*F*' is any predicate of a sort we shall call 'semantically specific' and which we shall shortly explain in greater detail. We take it that this feature renders Pelletier's systems unreasonable, at least from the perspective of the vagueness-in-reality theorist. That is, we conjecture that few such theorists will want to agree that, from the fact that '$a = b$' is indeterminate and the fact '*Fa*' is definitely true (where '*F*' is any semantically specific predicate), it follows that '*Fb*' is definitely true as well.

Must the vagueness-in-reality theorist then give up something 'logically reasonable', such as Leibniz' Law? No: we suggest a very natural modification of the principle Pelletier uses to represent Leibniz' Law in his system, a modification that restricts its application to *definitely true* identities. This modification *preserves* what is intuitively compelling about Leibniz' Law within the context of a many-valued logic of the sort considered by Pelletier while *blocking* Pelletier's derivation of a contradiction from the assumption of the existence of a vague object.

PELLETIER'S SYSTEMS

Pelletier does not specify *a* (particular) many-valued logical system which he claims the vagueness-in-reality theorist must accept. Rather, his claim seems to be that each of a (fairly extensive) range of such systems — some *one* of which must be accepted by the vagueness-in-reality theorist — leads to paradox on the assumption of the existence of vague objects. His characterization of this range of logical systems is as follows:

(I) Any such system must contain 'semantic-value' or 'J' operators that allow the "vagueness-in-nature" theorist to "speak about" vagueness. In the particular 3-valued system that Pelletier uses as an example (value 1 [definite truth], value 2 [indeterminate], value 3 [definite falsity]), there are three such operators:

> '$J_1\Phi$' is 1 if Φ is 1, 3 otherwise; '$J_2\Phi$' is 1 if Φ is 2, 3 otherwise; '$J_3\Phi$' is 1 if Φ is 3, 3 otherwise.

It follows from these definitions, of course, that any formula prefixed by a J operator can have only a *classical* semantic value (1 or 3).

(II) Sentential connectives and "whatever principles the [vagueness-in-reality] theorist uses must agree with classical logic on the classical values" (p. 487). This restriction would seem to leave the theorist a wide range of many-valued logics at his/her disposal. However, as we shall see, another principle postulated by Pelletier considerably restricts the choices.

(III) Finally, Pelletier imposes some additional principles "governing the use of J operators, Leibniz's law, the reflexivity of identity, and the interaction of J operators with quantifiers" (p. 487). These principles he takes to be correct, noting that "anyway most 'vagueness-in-nature' theorists believe them" (p. 487).

Since these are crucial both to our argument and to that of Pelletier, we briefly rehearse them, adding a few comments where it seems appropriate to do so.

1: (USV) This is a principle governing the J operators "saying that every formula takes at exactly one of the three semantic values" (p. 488):

(USV) $(\exists! i)\mathrm{J}_i\Phi$, for $1 \leqslant i \leqslant 3$.

2: (J_1) states that any conditional with antecedent asserting that Φ is determinately true and consequent asserting simply Φ is an axiom:

(J_1) $\mathrm{J}_1\Phi \rightarrow \Phi$.

3: (E) is a principle stating that if a biconditional is semantically valid, formulas obtained by distributing the J operators over the biconditional are also valid:

(E) If $(\Phi \leftrightarrow \Psi)$ is semantically valid, then so is $(\mathrm{J}_i\Phi \leftrightarrow \mathrm{J}_i\Psi)$, for any $1 \leqslant i \leqslant 3$.

Actually, a slightly stronger principle is probably required:

(E′) If Φ, Ψ are equivalent in the sense that, for each interpretation, Φ and Ψ have the same semantic value, then $\mathrm{J}_i\Phi$, $\mathrm{J}_i\Psi$ are also equivalent in this sense, for each i.

The latter principle is stronger because, in a number of many-valued logics, one formula can have the same semantic value as another, for each interpretation, without its being the case that the corresponding biconditional is valid.[2]

4: (refl′) states the reflexivity of the identity relation. Each instance of the following is an axiom:

(refl′) $\mathrm{J}_1(\alpha = \alpha)$.

5: (LL) is Pelletier's version of Leibniz' Law:

(LL) $a = b \leftrightarrow (\forall F)(Fa \leftrightarrow Fb)$

In effect, Pelletier proposes that any "reasonable" many-valued logical system must have (LL) as an axiom if that system is to capture what is intuitive about Leibniz' Law. We shall argue, however, that Leibniz' Law is correctly (and harmlessly) represented in a many-valued logic by a weaker principle, which we call (ClassicalLL), which has the J_1 operator distributed over the biconditional (LL). Pelletier's stronger (LL), in conjunction with (E), entails not only our (ClassicalLL) but also theorems in which the J_2 and J_3 operators are distributed over the biconditional. It is precisely here, we shall argue, rather than with the assumption of the existence of vague objects, that Pelletier's systems get into trouble.

6: ($J_2 - \forall$) is the one principle that Pelletier specifies that pertains to the "interaction" of J operators and quantifiers.

($J_2 - \forall$) $J_2(\forall x)Fx \rightarrow \sim(\exists x)J_3Fx$.[3]

The rationale behind this principle is that if it is indeterminate whether *everything* is *F*, then it cannot be the case that there is something of which it is definitely false that it is *F*. We note, however, that acceptance of this principle is legitimate only if certain sorts of matrices are adopted for conjunction and disjunction. We assume that the universal quantifier will be treated as a "generalization" of conjunction and the existential quantifier as a generalization of disjunction. (Otherwise, the equivalence of quantified formulas and their standard "truth-functional expansions" for finite domains will not hold.) If this assumption is granted, then ($J_2 - \forall$) will hold only if the universal quantifier is a generalization of an account of conjunction such that a conjunction has the maximal value of its conjuncts (i.e., a conjunction "is precisely as false as the most false of its conjuncts"). Then, in order to preserve the duality relation, existential quantification will be a generalization of an account of disjunction such that a disjunction has the minimal value of its disjuncts (i.e., a disjunction "is precisely as true as the most true of its disjuncts"). In the case of 3-valued logics, not all matrices "which agree with classical logic on classical values" satisfy this condition. Kleene's "strong" matrices do. But his "weak" matrices (i.e. those of Bochvar) do not: they make any truth-fnctionally compound formula indeterminate just in case at least one of its constituents is indeterminate. Consequently, the generalization of Bochvar-conjunction and -disjunction to the univeral and existential quantifiers, respectively, will fail to validate ($J_2 - \forall$).[4]

7: Finally, Pelletier states a "lemma" (which follows from the account of the J-operators and constraint (I) above requiring that connectives "agree with classical logic on the classical values") concerning the J_2 operator: "If each sentential part of a formula Φ is already in the scope of some J operator, prefixing a J_2 operator to Φ will yield the value 3 [definite falsity]; that is, its negation will yield the value 1" (pp. 489–90).

(Lemma) $\sim J_2\Phi$ (equivalently, $J_3J_2\Phi^5$), if all sentences in Φ are already in the scope of some J operator.

In fact, a slightly stronger lemma holds:

(Lemma') '$\sim J_2\Phi$' (equivalently, '$J_3J_2\Phi$') is definitely true (has value 1), if each of the *atomic* formulas contained in Φ falls within the scope of some J operator or other.

We term a predicate or sentential function in which each atomic formula falls within the scope of a J operator a *semantically specific* predicate or sentential function.

Having sketched Pelletier's system — or, rather, range of systems — we turn to our assessment of the flaw that we believe has been incorporated into his account.

THE PROBLEM WITH PELLETIER'S SYSTEMS

As Pelletier argues, the following derivation schema can be constructed from the preceding principles.

1.	$J_2(a=b)$	Assumption
2.	$J_2(a=b) \leftrightarrow J_2(\forall F)(Fa \leftrightarrow Fb)$	Theorem, from applying (E) to (LL)
3.	$\sim(\exists F)J_3(Fa \leftrightarrow Fb)$	1, 2, $\leftrightarrow$ elim. and (J_2-$\forall$)
4.	$(\forall F)[J_1(Fa \leftrightarrow Fb) \vee J_2(Fa \leftrightarrow Fb)]$	3, (USV) and quantifier duality
5.	$J_1([\lambda x)(J_1(a=x)(a) \leftrightarrow (\lambda x)(J_1(a=x)(b)] \vee J_2[(\lambda x)J_1(a=x)(a) \leftrightarrow (\lambda x)(J_1(a=x)(b)]$	4, instantiation using '$(\lambda x)J_1(a=x)$'
6.	$J_1[J_1(a=a) \leftrightarrow J_1(a=b)] \vee J_2[J_1(a=a) \leftrightarrow J_1(a=b)]$	5, lambda conversion
7.	$\sim J_2[J_1(a=a) \leftrightarrow J_1(a=b)]$	(Lemma) and semantic specificity of '$J_1(a=x)$'

8. $J_1[J_1(a=a) \leftrightarrow J_1(a=b)]$	6, 7, disj. syl.
9. $J_1(a=a) \leftrightarrow J_1(a=b)$	8, (J_1)
10. $J_1(a=b)$	9, (refl′) and $\leftrightarrow$ elim.
11. $\sim J_2(a=b)$	10, (USV)

It should be emphasized that we can apply rules of conditional proof and *reductio ad absurdum* only when semantically specific assumptions are employed. Semantically specific assumptions can have only classical semantic values (1 or 3); and since the rules we employ preserve truth, we know that conditional proof and *reductio* will then function as they do in a classical context. That is, we know that if we derive, by truth-preserving rules, a formula Ψ from a formula Φ that must have a classical truth value, then the corresponding conditional must be true. And if we derive a formula Ψ that must, for all interpretations, be false from a formula Φ that must have a classical value, then Φ must be false ($\sim \Phi$ must be true).

With respect to Pelletier's proof, we wish to point out that analogues of steps 5 through 8 can be applied using *any* property F issuing in a semantically specific sentential function (with one free variable) at step 6, with conditional proof applied after step 8. This process yields the following, restrictedly valid formula-schema:

(SUB) $J_2(a=b) \rightarrow J_1(\Phi(a/x) \leftrightarrow \Phi(b/x)$ (where $\Phi(x)$ is a semantically specific sentential function containing one free variable x)

(SUB), in effect, licenses the substitution of b for a in semantically specific contexts simply on the grounds that '$J_2(a=b)$' is true, i.e., on the grounds that '$a=b$' is indeterminate. We take it that this, in itself, is an untoward consequence of Pelletier's system(s). Any reasonable response to Pelletier should block not only the derivation of a contradiction but also the derivation of (SUB).

We argue that the contradiction and the untoward (SUB) should be laid at the feet of the unreasonably strong principle (LL). It seems to us to be eminently reasonable to adopt a version of Leibniz' Law that applies *only* when the identity formula '$a=b$' is determinately true:

(ClassicalLL) $J_1(a=b) \leftrightarrow J_1(\forall F)(Fa \leftrightarrow Fb)$.

This principle certainly does everthing that the Leibniz' Law, as *classi-*

cally understood, does. We shall grant that (ClassicalLL), along with Pelletier's other principles, should be accepted by the vagueness-in-reality theorist. But we claim that it is *enough* for the vagueness-in-reality theorist to accept (ClassicalLL), along with Pelletier's other principles. Doing so results in an extensional system. And it also yields the following eminently plausible theorem:

$$(\mathrm{J}_3\mathrm{LL}\rightarrow)\quad \mathrm{J}_3(a=b) \rightarrow \mathrm{J}_3(\forall F)(Fa \leftrightarrow Fb)$$

But accepting only (ClassicalLL), in conjunction with Pelletier's other principles, blocks his derivation of a contradiction from the assumption that a vague object a exists (i.e., the assumption that, for some names 'a' and 'b', the formula '$\mathrm{J}_2(a=b)$' is true).

We propose to show that acceptance of (ClassicalLL) (which, of course, follows from Pelletier's stronger (LL)) entails that the resulting system will have as a theorem a "no-indeterminacy-with-respect-to-all-predicates" principle.

$$(\mathrm{NoJ}_2\forall)\qquad \sim(\exists a)(\exists b)\mathrm{J}_2(\forall F)(Fa \leftrightarrow Fb).$$

This result, we shall argue, shows that Pelletier's (LL) should be eschewed. After these proofs, we shall return to the general logical/philosophical issue of the representation of Leibniz' Law in many-valued systems such as Pelletier's.

First we have the derivation:

1. $\mathrm{J}_1(a=b)$	Assumption
2. $\mathrm{J}_1(\forall F)(Fa \leftrightarrow Fb)$	1, (ClassicalLL)
3. $\sim\mathrm{J}_2(\forall F)(Fa \leftrightarrow Fb)$	2, (USV)

Since steps 1 and 3 are semantically specific formulas, we can use conditional proof to obtain the theorem

$$(\mathrm{A})\qquad \mathrm{J}_1(a=b) \rightarrow\ \sim\mathrm{J}_2(\forall F)(Fa \leftrightarrow Fb).$$

Next we have the derivation:

1. $\mathrm{J}_2(a=b)$	Assumption
2. $\mathrm{J}_1(a=a)$	(refl′)
3. $\sim(\exists F)\mathrm{J}_3(Fa \leftrightarrow Fb)$	*Reductio* assumption
4. $(\forall F)\sim\mathrm{J}_3(Fa \leftrightarrow Fb)$	3, quantifier duality

5.	$\sim J_3(J_2(a=a) \leftrightarrow J_2(a=b))$	4, instantiate $(\forall F)$ to $(\lambda x)J_2(a=x)$ and λ convert
6.	$\sim J_2(J_2(a=a) \leftrightarrow J_2(a=b))$	(Lemma)
7.	$J_1(J_2(a=a) \leftrightarrow J_2(a=b))$	5, 6 (USV)
8.	$J_2(a=a) \leftrightarrow J_2(a=b)$	7, (J_1)
9.	$J_2(a=a)$	1, 8 $\leftrightarrow$ elim.
10.	$\sim J_2(a=a)$	2, (USV)
11.	$(\exists F)J_3(Fa \leftrightarrow Fb)$	3–10, indirect proof
12.	$\sim J_2(\forall F)(Fa \leftrightarrow Fb)$	11, $(J_2\text{-}\forall)$

Again, we use conditional proof to obtain

(B) $\quad J_2(a=b) \rightarrow \sim J_2(\forall F)(Fa \leftrightarrow Fb)$

Finally, we sketch a proof of the following theorem:

(C) $\quad J_3(a=b) \rightarrow \sim J_2(\forall F)(Fa \leftrightarrow Fb).$

1.	$J_3(a=b)$	Assumption
2.	$J_2(\forall F)(Fa \leftrightarrow Fb)$	*Reductio* assumption
3.	$\sim(\exists F)J_3(Fa \leftrightarrow Fb)$	2, $(J_2\text{-}\forall)$
4.	$(\forall F)[J_1(Fa \leftrightarrow Fb) \vee J_2(Fa \leftrightarrow Fb)]$	3, (USV)
5.	$J_1(J_1(a=a) \leftrightarrow J_1(a=b)) \vee J_2(J_1(a=a) \leftrightarrow J_1(a=b))$	4, instantiate $(\forall r)$ to $(\lambda x)(J_1(a=x)$ and λ convert
6.	$J_1(J_1(a=a) \leftrightarrow J_1(a=b))$	5, (lemma) and disj. syllogism
7.	$J_1(a=a) \leftrightarrow J_1(a=b)$	6, (J_1)
8.	$J_1(a=a)$	(refl′)
9.	$J_1(a=b)$	7, 8, $\leftrightarrow$ elim
10.	$\sim J_3(a=b)$	9, (USV) —contradiction with 1
11.	$\sim J_2(\forall F)(Fa \leftrightarrow Fb)$	2–10, indirect proof

Consequently, theorem $(\text{NoJ}_2\forall)$ follows from (USV), and theorems (A), (B), and (C) by propositional logic, universal generalization, and quantifier duality.

The problem, it seems to us, is not the existence of vague objects, nor Leibniz' Law as classically understood, nor the J-operators, nor any other of the "reasonable" logical principles assumed by Pelletier. Rather, the problem is the distribution of the J-operators over Pelletier's (LL). The result is not merely the harmless (ClassicalLL) version of Leibniz' Law but also such aberrant principles as

$$(\mathrm{J_2LL})\quad \mathrm{J}_2(a=b) \leftrightarrow \mathrm{J}_2(\forall F)\,(Fa \leftrightarrow Fb).$$

We have shown that, with the classical version of Leibniz' Law — our (ClassicalLL) — and Pelletier's other principles, the *denial* of the right side of this biconditional is provable. So it is not surprising that the admission of ($\mathrm{J_2LL}$) into Pelletier's system allows him to derive a contradiction on the assumption of the truth of its left side, for some *a* and *b*.

As a result of accepting (LL), Pelletier must — on pain of contradiction — give up the existence of any vague objects. By substituting the slightly weaker (ClassicalLL), we can keep the existence of vague objects. We have demonstrated that Pelletier's own argument for denying the existence of vague objects will not work when (ClassicalLL) is substituted for (LL). It can also be proved that there is no derivation within a logic satisfying his principles, when this substitution has been effected, of a formula that may be taken to express the nonexistence of vague objects:

$$(\mathrm{NVO})\quad \sim(\exists a)\,(\exists b)\mathrm{J}_2(a=b).$$

A model-theoretic argument establishes this result. That is, we specify a (bivalent) model-theoretic structure that (a) validates all Pelletier's principles except (LL), (b) validates (ClassicalLL), (c) makes all the rules of inference Pelletier wishes to use truth-preserving, but (d) falsifies (NVO).

One such structure, due to an anonymous reader of this paper, is the following. Adopt a possible-worlds structure for the S5 modal predicate calculus without identity but with the Barcan formula (so the accessibility relation among worlds will be an equivalence relation and there will be the same domain of objects at each possible world). Define J_1 to be modal necessity, J_3 to be modal impossibility, and J_2 to be modal contingency (i.e., $\mathrm{J}_2\Phi$ is $(\mathrm{M}\Phi \wedge \sim\mathrm{L}\Phi)$). Now define the identity predicate as follows: $a = b$ is true at a world w iff, for each sentential function $\Phi(x)$ containing no modal operators and exactly one free variable x, $[\Phi(a/x) \leftrightarrow \Phi(b/x)]$ is true at w. Because of limitation of space, we leave it as one of those slightly irritating 'exercises for the reader' to verify that it is possible to construct such a structure, in which a and b are indistinguishable by nonmodal formulas in some worlds but not in others, that validates (ClassicalLL) and all of Pelletier's principles except (LL), but which falsifies (NVO).

Since there are no free lunches in logic, what must *we* give up in substituting (ClassicalLL) for (LL)? The answer is (J_2LL) and the 'right-to-left' direction of (J_3LL):

$$(J_3LL\leftarrow)\quad J_3(\forall F)(Fa \leftrightarrow Fb) \rightarrow J_3(a=b).$$

The admission of the semantic-value or J operators into a many-valued logical system creates semantically specific sentential functions, i.e., sentential functions which, for each object a in the domain (whether a "vague object" or not), must take either the value 'definite truth' (1) or the value 'definite falsity' (3). As our derivation of the theorem (No$J_2\forall$) shows, a consequence in such a system is that, for any objects a and b in the domain, it will either be definitely true or definitely false that they agree with respect to their possession of *all* properties (satisfaction of *all* sentential functions).

What of ojbects a and b such that it is indeterminate that $a = b$? Well, because of the existence of semantically specific predicates/sentential functions, it will be definitely false that a and b agree with respect to possession of all their properties. For example, there will be one property '$(\lambda x)\ J_1(a = x)$' (the property of its being definitely true of something that it is identical to a) which it is definitely true that a possesses but definitely false that b possesses. Hence, there is at least one property with respect to the possession of which it is definitely false that a and b agree; and, according to the semantical interpretation of the universal quantifier implicitly accepted by Pelletier, it follows that it is definitely false that they agree with respect to the possession of *all* properties. Does this result show that, despite our initial assumption, a and b are really definitely distinct (non-identical) objects? We believe that such a conclusion (subtly) begs the question against the vagueness-in-reality theorist.

By adopting (ClassicalLL), as opposed to Pelletier's (LL), the vagueness-in-reality theorist can maintain the following propositions:

(i) If it is definitely true that *a* and *b* are identical, then it is definitely true that they share all the same properties; and if it is definitely true that *a* and *b* share all the same properties, then it is definitely true that they are identical (Classical LL);
(ii) If it is definitely false than *a* and *b* are identical, then it is definitely false that *a* and *b* share all the same properties ($J_3LL\rightarrow$);
(iii) But if it is definitely false that *a* and *b* share *all* the same properties (including ones expressed by semantically specific sentential functions), it is *either* definitely false that *a* and *b* are identical *or* indeterminate whether *a* and *b* are identical.

Since the introduction of J operators and semantically specific sentential functions entails that, when it is indeterminate that $a = b$, there will be *some* properties with respect to the satisfaction of which *a* and *b* definitely disagree, (iii) should not be a disturbing consequence for the vagueness-in-reality theorist.[6] And, of course, acceptance of (III) precludes the truth of both (J_2LL) and ($J_3LL\leftarrow$), which are precisely the principles sacrificed by the substitution of our (ClassicalLL) for Pelletier's (LL).

We have come to realize, however, that some philosophers find intuitive the principles we sacrifice — perhaps particularly the left-to-right direction of (J_2LL). While we doubt that there is any knock-down argument that could entirely overturn such intuitions, we believe that we can produce 'possible counterexamples' to diminish their vivacity. In conclusion, we construct one such case. Suppose that we think of an object, Mt. Ranier, as a neighborhood or set of 'physical' points — but as a fuzzy set: some points are determinately members, some are determinately not members, and it is indeterminate whether some points are members. Consider an object (= fuzzy set), Mt. Ranier*, which is 'exactly like' Mt. Ranier with respect to point-constitution, except that, for one of the points *p* such that it is indeterminate whether Mt. Ranier contains *p, p is* determinately contained by Mt. Ranier*. *Our* intuitions tell us that it is indeterminate whether Mt. Ranier is identical to Mt. Ranier*. After all, there is no point such that it is determinately the case that one mountain contains it and determinately the case that the 'other' mountain fails to contain it. However, there is a 'property', e.g., its being indeterminate whether *x* contains point *p* $[(\lambda x)J_2(x \text{ contains } p)]$ such that it is determinately true that Mt. Ranier possesses the 'property' and determinately false that Mt. Ranier* possesses it. And similarly, *mutatis mutandis*, for other 'properties' constructed using the other J operators. Since we have admitted into our logic(s) such properties constructed from J operators, it is not the case that it is indeterminate whether Mt. Ranier and Mt. Ranier* agree with respect to the possession of all their properties: there are some properties with respect to the possession of which they determinately disagree. We take our example to be evidence for the falsity of the left-to-right direction of (J_2LL). While it seems reasonable to us to say that the left side, J_2(Mt Rainier = Mt. Ranier*), is true, the right side clearly is false.

Of course, we realize that one philosopher's *modus ponens* in another's *modus tollens*. If one is deeply enough committed to maintaining the truth of left-to-right (J_2LL), one obviously must affirm the falsity of its left side, for *any* objects *a* and *b*. While we recognize the possibility of such a commitment to (J_2LL$\rightarrow$), we find ourselves unable to empathize with strong intuitions concerning a principle that, in the context of a logic containing semantic value operators, is surely very much a matter of art. In short, our skepticism concerning attempts to resolve 'deep' metaphysical questions solely by means of logical maneuvers remains unshaken.

NOTES

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1 *The Journal of Philosophy*, vol. LXXXVI, no. 9 (September, 1989), pp. 481–92. All parenthetical page references in the text refer to this publication.

2 Of course, it is possible to define the matrix for the biconditional in such a way that a biconditional is true just in case both of its constituents have the same semantic value; doing so will conflate (E) and (E′). But it is difficult to so define the biconditional while prserving the 'classical relationships' among the connectives. In many standard 3-valued (and many-valued) logics, a number or classical 'meta-principles' do not hold. For example, a semantic analogue of the deduction theorem does not always hold: that is, it does not always follow from the fact that Φ entails Ψ that '$\Phi \rightarrow \Psi$' is valid. And from the fact that Φ entails Ψ, it does not always follow that '$\sim\Psi$' entails '$\sim\Phi$'.

3 Actually, it is a second-order version of ($J_2 - \forall$) that both Pelletier and we employ.

4 For a useful survey of many-valued logic, see Alasdair Urquhart, "Many-Valued Logic," in *Handbook of Philosophical Logic*, Vol. III, *Alternatives to Classical Logic*, ed. D. Gabbay and F. Guenthner (Dordrecht and Boston, 1986), pp. 71–116.

5 Pelletier's text reads "equivalently $J_3\Phi$." We take this to be a typographical error: what is wanted is either "equivalently $J_3J_2\Phi$" or, perhaps, "equivalently, $J_1\Phi \vee J_3\Phi$."

[6] So far as we can see, it is possible for the vagueness-in-identity theorist consistently to accept a restricted version of ($J_3LL\leftarrow$), which arguably is "in the spirit of" Leibniz' Law: for any a and b, if there is some sentential function F *not containing any semantic-value or J operators* such that it is definitely true that a satisfies F and definitely false that b satisfies F, then it is definitely false that a is identical to b.

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