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Vibration isolation using open or filled trenches

Part 3: 2-D non-homogeneous soil

K. L. Leung
Department of Civil and Mineral Engineering, University of Minnesota, Minneapolis, MN 55455, USA

D. E. Beskos
Department of Civil Engineering, University of Patras, Patras 26110, Greece

I. G. Vardoulakis
Department of Civil and Mineral Engineering, University of Minnesota, Minneapolis, MN 55455, USA

Abstract. The problem of isolating structures from surface waves by open or filled trenches under conditions of plane strain is numerically studied. The soil is assumed to be an isotropic, linear elastic or viscoelastic nonhomogeneous (layered) half-space medium. Waves generated by the harmonic motion of a rigid surface machine foundatin are considered. The formulation and solution of the problem are accomplished by the frequency domain boundary element method. The Green's function of Kausel-Peek-Hull for a thin layered half-space is employed and this essentially requires only a discretization of the trench perimeter and the soil-foundation interface. The proposed methodology is used for the solution of a number of vibration isolation problems and the effect of soil inhomogeneity on the wave screening effectiveness of trenches is discussed.

1. Introduction

Use of open or infilled trenches as barriers for ground-transmitted waves generated by surface disturbances, succeeds in vibration isolation of structures through wave diffraction and consequent wave amplitude reduction behind the trench (Richart et al. 1970). A comprehensive account of the literature on the experimental, analytical and numerical treatment of vibration isolation and associated wave diffraction problems under plane and three-dimensional conditions can be found in two previous papers (Beskos et al. 1986; Dasgupta et al. 1990) published in this journal as parts 1 and 2 and dealing with homogeneous soils. The present paper is part 3 of this series and deals with vibration isolation analysis in nonhomogeneous soils under plane strain conditions.

Most of the analytical and numerical work in soil mechanics and soil-structure interaction in the framework of linear theories is based on the assumption that the elastic parameters are uniform throughout the half-space soil medium. However, in reality the stiffness of soils generally increases with depth as a consequence of the increasing effective overburden pressure. Moreover, it is observed that real soils are often stratified. Thus, the most general nonhomogeneous soil model in the framework of isotropic, linear elastic behavior is the one consisting of a number of layers with moduli varying with depth. Density and Poisson's ratio are usually assumed to have constant values in every layer. Two types of inhomogeneous soil medium are usually employed in applications. The layered half-space with constant moduli values in each layer and the half-space with the shear modulus varying with depth. However, when a numerical method of analysis is employed, both of these soil types as well as the general soil model consisting of layers with variable moduli are usually treated as layered or piecewise homogeneous media.

Wave propagation analysis in inhomogeneous soils presents many difficulties, especially when analytical methods of solution are employed. Analytic solutions of wave propagation problems in a half-space with the shear modulus varying with depth are very much involved and can lead to very complicated closed forms, only for special cases of inhomogeneity (e.g., Meissner 1921; Stoneley 1936; Hook 1961, 1962; Rao 1967, 1970, 1978; Vardoulakis 1981). Among the very many analytical-numerical and numerical works on wave propagation in layered media with constant material properties in every layer, one can mention here the classical text of Ewing et al. (1957) and the works of Thomson (1950), Haskell (1953), Harkrider (1964), Gupta (1966), Waas (1972), Luco (1974, 1976), Kausel et al. (1975), Gazetas and Roesset (1979), Apsel (1979), Gazetas (1980), Tassoulas (1981), Apsel and Luco (1987), and Luco and Wong (1987).

Analytic or even analytic-numerical treatment of wave propagation problems in layered media is restricted to very simple geometries and boundary conditions. Realistic wave diffraction problems involving complex geometries and boundary conditions, such as vibration isolation ones, can only be solved numerically. The Boundary Element Method (BEM) is ideally suited for elastic wave diffraction problems involving infinite or semi-infinite domains and, as it has been demonstrated in various places (e.g., Kobayashi 1987; Beskos 1987; Manolis and Beskos 1988), it is more advantageous than either the Finite Element Method (FEM) or the Finite Difference Method (FDM). Waas (1972), Segol et al. (1978) and May and Bolt (1982) developed special FEM's to study the amplitude reduction of surface waves by open or infilled trenches in an isotropic, linear elastic or viscoelastic, layered half-plane. Their methods, however, required the use of complicated non-reflecting boundaries, which are sometimes applicable only when the layered soil is supported on rigid bedrock.

The most important ingredient of the BEM is the fundamental solution or Green's function. Use of the full-space time or frequency domain Green's function in a vibration isolation problem involving a foundation and a trench in a layered soil medium, requires not only a discretization of the trench surface and the soil-foundation interface, but also a discretization of a portion of the free soil surface and the layer interfaces around the region of interest. This increases considerably the size of the problem and effectively restricts the BEM to a small number of layers, especially in the three-dimentional case. Thus, treatment of the continuously nonhomogeneous case by this approach may not be practically possible due to the large number of layers required to model the soil as piecewise homogeneous. However, it presents the advantage of treating layers of any geometry and not just horizontal ones. This approach has been successfully used in the frequency domain for vibration isolation analysis in layered soil media with a small number of layers by Beskos et al. (1986) and Banerjee et al. (1988) under plane strain and three-dimensional conditions, respectively. On the other hand, use of the layered half-space Green's function in vibration isolation analysis, requires only a discretization of the trench surface and the soil-foundation interface, thereby permitting the efficient treatment of a large number of horizontal layers. Among the various existing frequency domain layered half-space Green's functions one can mention those of Apsel (1979), Luco and Apsel (1983), Apsel and Luco (1983), Kausel and Peek (1982), Hull and Kausel (1984), Wolf (1985), Herrmann and Wang (1985), Kundu and Mal (1985), Xu and Mal (1987), Chapel and Tsakalidis (1985), Chapel (1987) and Kawase (1988).

The present work employs the frequency domain direct BEM for the solution of vibration isolation problems under conditions of plane strain. The time harmonic motion of a rigid surface foundation generates surface waves which are diffracted by an open or filled rectangular trench. The soil medium is assumed to be an isotropic linearly elastic or viscoelastic layered half-plane. Use is made of the Kausel–Peek–Hull Green's function (Kausel and Peek 1982; Hull and Kausel 1984) which assume the layers to be very thin, thereby requiring the sublayering of every thick layer. After validation of the method, the screening effectiveness of open or filled trenches in some layered soil models is studied in detail and the effect of inhomogeneity is assessed. The case of nonhomogeneous soils characterized by a shear modulus exhibiting a linear or nonlinear variation with depth is considered in another publication (Leung et al. 1990). The work presented in this paper is part of the doctoral dissertation of the first author (Leung 1989) which can be consulted for more details. Some preliminary results of his work have also been reported in Leung et al. (1987).

2 Frequency domain BEM for layered half-plane

For a time harmonic excitation, the response of a plane elastic body B with boundary S is also harmonic and the boundary integral equation (Manolis and Beskos 1988)

$$c u_j(\xi) = \int_S U_{ji}(\mathbf{x}, \xi, \omega) t_i(\mathbf{x}) \mathrm{d}S(\mathbf{x}) - \int_S T_{ji}(\mathbf{x}, \xi, \omega) u_i(\mathbf{x}) \, \mathrm{d}S(\mathbf{x}), \tag{1}$$

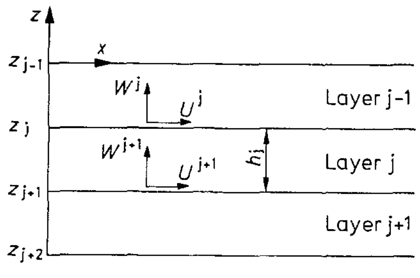


Fig. 1. Geometry of a layered stratum

connects the amplitudes of boundary displacements u_i and tractions t_i with the aid of the displacement U_{ji} and traction T_{ji} Green's tensors, which depend on frequency ω and are usually defined for the infinite elastic plane. In the above, body forces have been assumed to be zero, $\mathbf{x}$ and ξ are points on S and $c = 1/2$ if S is smooth at ξ. For the numerical solution of Eq. (1), the boundary S is first discretized into a number of line boundary elements over which displacements and tractions are assumed to vary in a specific way and Eq. (1) is then written in the discrete matrix form

$$\tfrac{1}{2}\{u\} = [U]\{t\} - [T]\{u\}, \tag{2}$$

where $\{t\}$ and $\{u\}$ are the vectors of the nodal values of boundary traction and displacement amplitudes, respectively and $[U]$ and $[T]$ are influence matrices with entries of the form

$$U_{ji}(m, n) = \int\limits_{S_{Q-1/2}}^{S_{Q+1/2}} U_{ji}(\mathbf{x}, \xi_m) N(\mathbf{x}) \, \mathrm{d}S(\mathbf{x}) \tag{3}$$

$$T_{ji}(m, n) = \int\limits_{S_{Q-1/2}}^{S_{Q+1/2}} T_{ji}(\mathbf{x}, \xi_m) N(\mathbf{x}) \, \mathrm{d}S(\mathbf{x}) \tag{4}$$

with $N(\mathbf{x})$ being the shape function for node n in segment Q, ξ_m being the node m in segment P and $S_{Q+1/2}$ and $S_{Q-1/2}$ being the end points of segment Q over which the integration takes place. For constant boundary elements, the node n is chosen to be at the midpoint of the element and $N(x) = 1$, while for linear elements the nodes are chosen to be at the two end points of the element and $N(x)$ is a linear interpolation function.

In this work body B is a layered stratum resting on a half-space base under conditions of plane strain with N horizontal layer interfaces defined by $z = z_1, z_2, \ldots, z_N$ and with layer j defined by $z_j < z < z_{j+1}$, as shown in Fig. 1. The medium within each layer j of thickness h_j is assumed to be homogeneous, isotropic and linearly elastic. For this body, the frequency domain Green's function is obtained with the aid of the thin-layer theory of Waas (1972) and Kausel and Peek (1982) in conjunction with the half-plane impedance expression of Hull and Kausel (1984). Actually the Green's function for body B is obtained by an inversion of the thin-layer stiffness matrix through a spectral decomposition procedure (Kausel and Peek 1982). The advantage of this thin-layer stiffness matrix technique over the classical Haskell–Thomson transfer matrix technique for finite layers (Haskell 1953; Thomson 1950) and the finite layer stiffness matrix technique of Kausel and Roesset (1981) is that the transcendental functions in the layered stiffness matrix are linearized.

According to the thin-layer theory (Waas 1972), the thickness of each layer in chosen to be small, i.e., less than 1/10 of the Rayleigh wavelength in that layer, so that the displacements within the layer can be assumed to vary linearly with depth, but be continuous in the x direction. Thus, the displacements in the frequency-Fourier transformed with respect to x domain can be represented by a linear interpolation of the discrete "nodal" displacements at the layer interfaces and read, e.g., in layer j as

$$\begin{aligned} \bar{U}^{(j)}(z) &= [(z_{j+1} - z)\bar{U}^j + (z - z_j)\bar{U}^{j+1}]/h_j \\ \bar{W}^{(j)}(z) &= [(z_{j+1} - z)\bar{W}^j + (z - z_j)\bar{W}^{j+1}]/h_j \end{aligned} \tag{5}$$

where $\bar{U}^{(j)}$ and $\bar{W}^{(j)}$ are the transformed displacements along x and z directions as functions of z in layer j and $\bar{U}^j$ and $\bar{W}^j$ are their nodal values at layer interface $z = z_j$. Application of the principle of virtual work to different virtual displacement states in each layer results in a set of equilibrium equations which can be assembled in a finite element fashion to form the global matrix equation for the layered stratum

$$[\bar{K}]\{\phi\} = 0 \tag{6}$$

where

$$[\bar{K}] = [A]\zeta^2 + [B]\zeta + [C] \tag{7}$$

is the $2N \times 2N$ global stiffness matrix, $\{\varphi\}$ is the $2N \times 1$ vector with $\varphi^{2j-1} = U^j$ and $\varphi^{2j} = W^j$ for $1 \leqslant j \leqslant N$, where N is the number of layers and $[A]$, $[B]$ and $[C] = [G] - \omega^2[M]$ are symmetric complex matrices involving algebraic expressions in terms of material properties. Equations (6) and (7) constitute a quadratic eigenvalue problem with ζ being the eigenvalue corresponding to the eigenvector $\{\varphi\}$. A generalized Rayleigh quotient iteration scheme is used to solve for the $2N$ eigenvalues and eigenvectors. For the case of a layered half-plane, a second-order paraxial approximation $[\bar{K}(\zeta)]_h$ for the exact impedance of the half-plane derived in Hull and Kausel (1984) is added directly to the global stiffness matrix $[\bar{K}]$ of the layered system to obtain $[\bar{K}]_t$.

The discrete displacement Green's tensor for body B is defined in the frequency-Fourier transform domain as the response vector $\{\overline{GU}\}$ to unit line loads $\{\overline{GP}\}$ acting in both horizontal and vertical directions at each discrete layer interface of the layered half-plane B, for which equilibrium equations read

$$[\bar{K}]_t\{\bar{U}\} = \{\bar{P}\}. \tag{8}$$

Computation of $\{\bar{U}\}$ involves an inversion of $[\bar{K}]_t$ which is achieved by a spectral decomposition procedure based on knowledge of the eigenvalues ζ and eigenvectors $\{\varphi\}$ of the system. Thus, the frequency domain displacement Green's tensor, after inversion of the Fourier transform, can finally take the form (Kausel and Peek 1982)

$$U_{jk}^{mn} = \sum_{l=1}^{2N} \alpha_{jk} \phi_j^{ml} (1/\zeta_l) e^{-(i\zeta_l|x|)} \phi_k^{nl} \tag{9}$$

where j, k take the values $x, z, \alpha_{xx} = \alpha_{zz} = -i/2$, $i = \sqrt{-1}, \alpha_{xz} = \pm 1/2, \alpha_{zx} = \mp 1/2$, the $\pm$ sign indicates a + when $x \geqslant 0$ and a − when $x < 0$, U_{jk}^{mn} denotes the jth displacement component at the mth layer interface corresponding to a unit load acting in the kth direction at the nth layer interface and φ_k^{nl} denotes the eigenvector component in the kth direction at the nth layer interface of the lth wave mode. Once the displacement Green's tensor u_{jk} is known, the traction one T_{jk} can be easily obtained. For more details about the computation of the above Kausel–Peek–Hull Green's tensors one can consult the aforementioned original references or Leung (1989). Viscoelastic soil behavior can be easily introduced in the present formulation by simply replacing the elastic constants λ and μ by their complex values

$$\lambda^* = \lambda(1 + 2i\beta), \quad \mu^* = \mu(1 + 2i\beta), \tag{10}$$

where β is the constant hysteretic damping coefficient.

3 Vibration isolation by trenches in layered soil

Consider the vibration isolation system of Fig. 2 consisting of a layered half-space under conditions of plane strain, a rigid, surface footing bonded on the soil and subjected to a vertical harmonic force

$$P = P_o e^{i\omega t}, \tag{11}$$

where ω is the circular operational frequency, $i = \sqrt{-1}$ and t is time, and an open rectangular

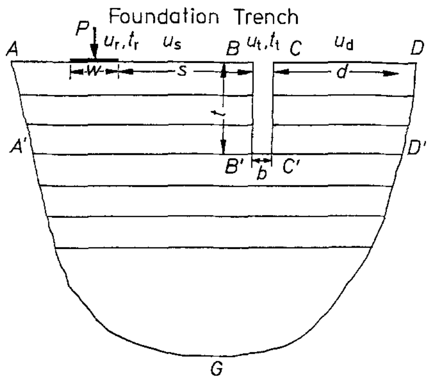


Fig. 2. Vibration isolation system on a thin layered halfspace

trench, which reduces the amplitude of the surface waves generated by the motion of the machine foundation through wave diffraction.

For a rigid footing in vertical motion with a displacement amplitude Δ, the compatibility and equilibrium equations take the form

$$\{u_{rx}\} = 0, \quad \{u_{rz}\} = \{I\}\Delta \tag{12}$$

$$P_o = -m\omega^2\Delta + \sum_{k=1}^{n} l_k \sigma_{zz}^k, \tag{13}$$

where $\{u_{rx}\}$ and $\{u_{rz}\}$ are the horizontal and vertical components, respectively, of the rigid foundation displacement amplitude $\{u_r\}$, m is the foundation mass and l_k and $\sigma_{zz}^k = t_z^k$ denote the length and the interface traction, respectively, of the kth foundation element. Equations (12) and (13) can be written in a compact matrix form as

$$\begin{aligned} \{u_r\} &= \{0, \{I\}\}^T \Delta \\ \{F\} &= -\omega^2 [M]\Delta + [L]\{t_r\} \end{aligned} \tag{14}$$

where $\{I\}$ stands for the unit vector and $\{t_r\}$ for the interface traction vector.

Using Eq. (2) one can write for the soil medium

$$\frac{1}{2}\begin{Bmatrix} \{u_r\} \\ \{u_t\} \end{Bmatrix} = \begin{bmatrix} [U_{11}][U_{12}] \\ [U_{21}][U_{22}] \end{bmatrix} \begin{Bmatrix} \{t_r\} \\ \{t_t\} \end{Bmatrix} - \begin{bmatrix} [T_{11}][T_{12}] \\ [T_{21}][T_{22}] \end{bmatrix} \begin{Bmatrix} \{u_r\} \\ \{u_t\} \end{Bmatrix} \tag{15}$$

where the subscripts r and t correspond to the soil-foundation interface and the trench perimeter, respectively. Equation (15) does not involve any discretization of the free soil surface because use is made of the half-plane Green's function. One can rewrite Eq. (15) in the form

$$\begin{bmatrix} [K_{11}][K_{12}] \\ [K_{21}][K_{22}] \end{bmatrix} \begin{Bmatrix} \{u_r\} \\ \{u_t\} \end{Bmatrix} = \begin{Bmatrix} \{t_r\} \\ \{0\} \end{Bmatrix}, \tag{16}$$

where the boundary condition $\{t_t\} = \{0\}$ has been taken into account and where

$$\begin{bmatrix} [K_{11}][K_{12}] \\ [K_{21}][K_{22}] \end{bmatrix} = \begin{bmatrix} [U_{11}][U_{12}] \\ [U_{21}][U_{22}] \end{bmatrix}^{-1} \left[\begin{bmatrix} [T_{11}][T_{12}] \\ [T_{21}][T_{22}] \end{bmatrix} + \frac{1}{2}[I] \right]. \tag{17}$$

Combining (16) with (14) one receives

$$\begin{bmatrix} -\omega^2[M] + [L][K_{11}], & [L][K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \begin{Bmatrix} \{I\}\Delta \\ \{u_t\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{0\} \end{Bmatrix}. \tag{18}$$

Solution of (18) provides Δ and $\{u_t\}$. Then $\{t_r\}$ can be obtained from (16). Finally the soil surface displacements $\{u_s\}$ and $\{u_d\}$ before and after the trench, respectively, needed for assessing the screening effectiveness of the trench, can be evaluated by using Eq. (1) with $c = 1$ in a pointwise fashion without any additional discretization. Thus one has

$$\begin{Bmatrix} \{u_s\} \\ \{u_d\} \end{Bmatrix} = \begin{bmatrix} [U_{33}][U_{34}] \\ [U_{43}][U_{44}] \end{bmatrix} \begin{Bmatrix} \{t_r\} \\ \{t_t\} \end{Bmatrix} - \begin{bmatrix} [T_{33}][T_{34}] \\ [T_{43}][T_{44}] \end{bmatrix} \begin{Bmatrix} \{u_r\} \\ \{u_t\} \end{Bmatrix}. \tag{19}$$

In view of the fact that $\{t_t\} = \{0\}$ and $[T_{33}] = [T_{43}] = [0]$, Eq. (19) becomes

$$\begin{Bmatrix} \{u_s\} \\ \{u_d\} \end{Bmatrix} = \begin{bmatrix} [U_{33}] \\ [U_{43}] \end{bmatrix} \{t_r\} - \begin{bmatrix} [T_{34}] \\ [T_{44}] \end{bmatrix} \begin{Bmatrix} \{u_r\} \\ \{u_t\} \end{Bmatrix}. \tag{20}$$

Constant boundary elements are used for the horizontal surfaces and linear boundary elements for the vertical trench walls in order to conform with the assumed displacement variation inside every thin layer.

When the trench is filled with some other material, e.g., bentonite or concrete, an additional BEM equation describing the frequency domain dynamic behavior of the filling has to be written down and conditions of compatibility and equilibrium at the soil-filling interface have to be enforced. For details one should consult Dasgupta et al. (1990), who describe this coupling procedure under three-dimensional conditions. In this work, however, the frequency domain BEM in conjunction with the Kausel–Peek Green's function for a layered stratum is employed for the filling and this requires no discretization of the top surface of the filling and succeeds in a perfect node-matching between soil-infill at their interface along the vertical walls and the bottom of the trench.

The special nature of the semi-analytic-numeric Green's function of Kausel-Peek-Hull employed here [Eq. (9)], makes possible the explicit closed form evaluation of the integrals (3) and (4) needed in the present BEM. Details can be found in Leung (1989).

In the numerical examples of the following section, the methodology just described was applied in conjunction with substructuring in order to increase the accuracy of the results. This substructuring involves the subdomains ABB′A′, CDD′C′ and A′B′C′D′G shown in Fig. 2, which are connected to each other through equilibrium and compatability at their interfaces. It was found in Leung (1989) that displacements obtained without substructuring are not very accurate, especially along the horizontal direction where constant elements are used. This indicates that the order of the elements employed in the discretization affects the accuracy of the solution.

4 Numerical examples and discussion

The methodology presented in the two previous sections is utilized here for solving some typical vibration isolation problems in layered soil after validation of this methodology on the basis of the homogeneous soil case.

Example 1: Consider the passive vibration isolation problem of Fig. 2 assuming that the soil is homogeneous with shear modulus $\mu = 132\,\mathrm{MN/m^2}$, weight density $\gamma = 17.5\,\mathrm{KN/m^3}$, Poisson's ratio $\nu = 0.25$ and hysteretic damping coefficient $\beta = 5\%$. It is further assumed that $P_o = 1000\,\mathrm{KN/m^2}$, $f = \omega/2\pi = 100\,\mathrm{Hz}$, Rayleigh wavelength $L_R = 2.5\,\mathrm{m}$, $W = w/L_R = 0.4$, $T = t/L_R = 1$ and $B = b/L_R = 0.2$, where w is the width of the massless footing [it was found in Beskos et al. (1986) that the footing mass does not affect the screening effectiveness of the trench], and t and b are the depth and the width of the open trench. The trench is located at a dimensionless distance $S = s/L_R = 4$ from the footing, while the dimensionless distance of interest after the trench is taken to be $D = d/L_R = 6$. Discretization along the horizontal direction involves 20, 40, 4 and 60 constant boundary elements for w, s, b and d, respectively, while discretization along the vertical direction involves 94 thin layers in a depth of $7\,L_R$ arranged from the top to bottom as follows: 36 elements of $(L_R/30)$ size from 0–3.0 m, 12 of $(L_R/20)$ from 3.0–4.5 m 8 of $(L_r/13.33)$ from 4.5–6.0 m, 6 of $(L_R/10)$ from 6.0–7.5 m and 32 $(L_R/8)$ from 7.5–17.5 m. Figure 3 shows the real and imaginary parts of the vertical surface displacement amplitudes versus the dimensionless distance $\xi = x/L_R$ as computed by the present thin-layer BEM and the BEM of Beskos et al. (1986) employing the full space Green's function. The agreement between the two methods is very good. The same is also true for the

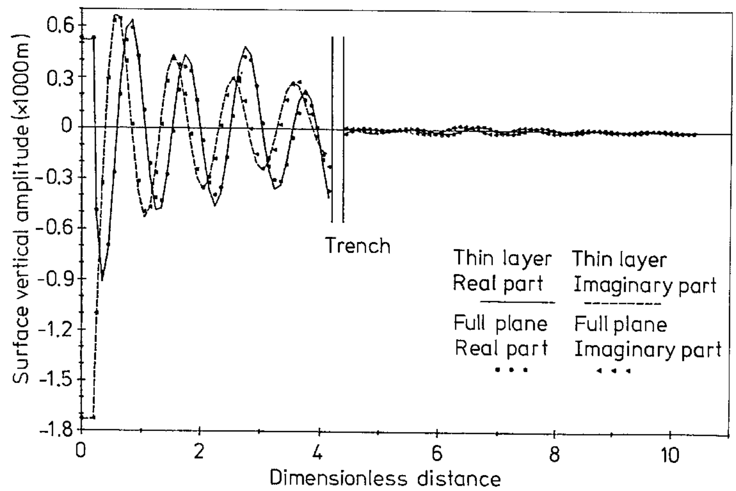
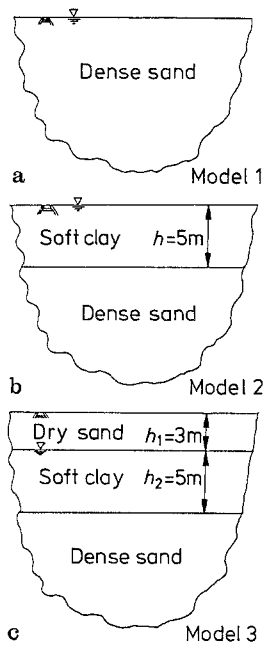


Fig. 3. Surface vertical displacement versus nondimensional distance for the case with trench, $S = 4.0$ and $T = 1.0$



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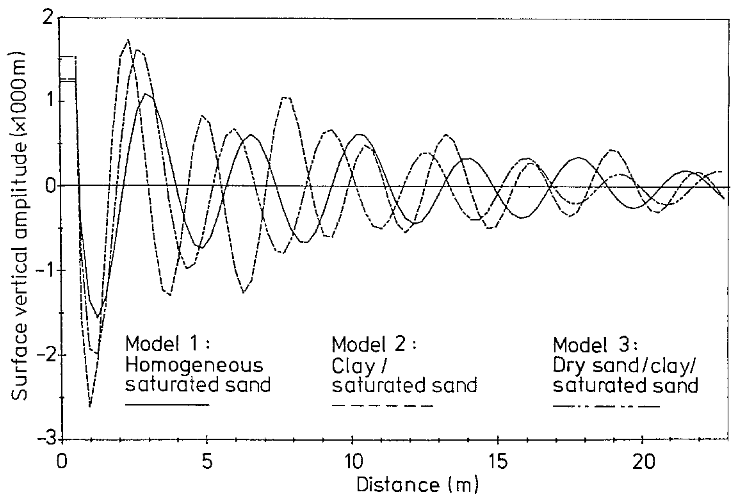

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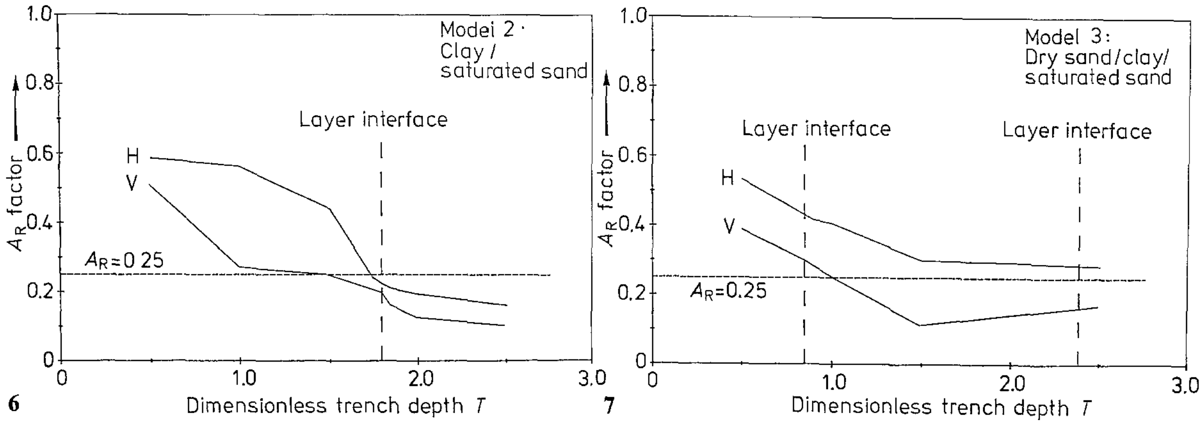
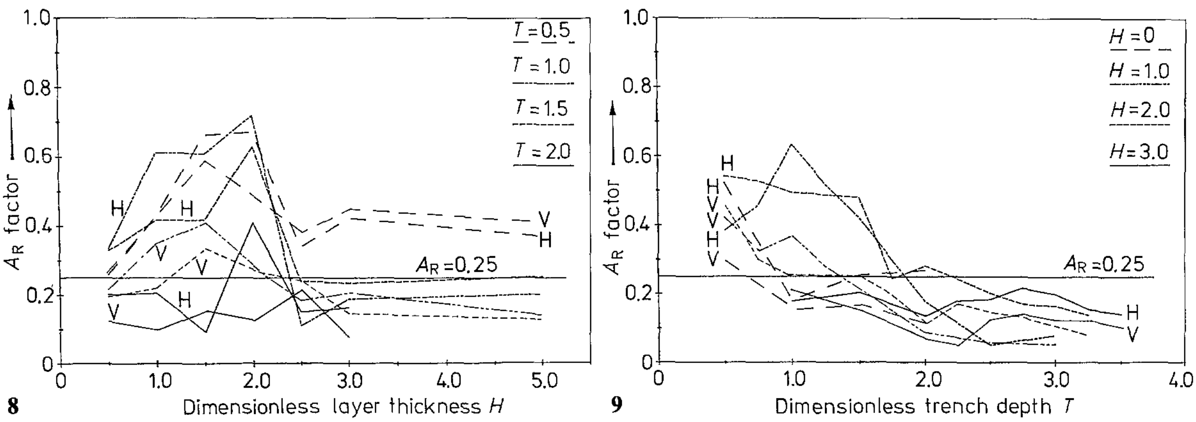
Figs. 4 and 5a–c. Layered soil models. **5** Real parts of the free field surface vertical displacement versus distance for all three soil models

Table 1. Values of actual L_R and computed L'_R Rayleigh wavelengths for the three soil models

Model 1, computed L'_R	= 3.71 m
Saturated sand, actual L_R	= 3.728 m
Difference	−0.48%
Model 2, computed L'_R	= 2.77 m
Soft clay, actual L_R	= 2.796 m
Difference	−0.93%
Model 3, computed L'_R	= 3.33 m
Dry sand, actual L_R	= 3.355 m
Difference	−0.75%



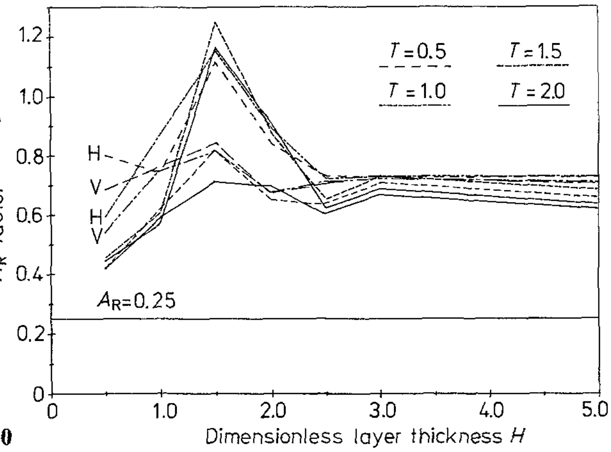
Figs. 6 and 7. Horizontal (H) and vertical (V) amplitude reduction factors versus nondimensional trench depth **6** for model 2, **7** for model 3



Figs. 8 and 9. Horizontal (H) and vertical (V) amplitude reduction factor **8** versus nondimensional layer thickness for open trench: $H = h/L_R$, $T = 0.5, 1.0, 1.5$ and 2.0; **9** versus nondimensional open trench depth for the cases of $H = 0.0, 1.0, 2.0$ and 3.0

horizontal surface displacement amplitudes (Leung 1989). The full-space BEM required a discretization involving the same number of constant elements along the horizontal direction as in the thin-layer BEM and 10 elements along every sidewall of the trench. It was found that more CPU time is required for the thin-layer BEM than for the full-space BEM, indicating that for vibration isolation problems in plane strain, the thin-layer BEM should be mainly used for layered soil.

Example 2: In this example, the effect of horizontal layering of soils on the screening effectiveness of the rectangular trench of Fig. 2 is studied by examining the three soil models of Fig. 4, on the assumption that $P_0 = 1000\,\mathrm{KN/m^2}$, $f = 50\,\mathrm{Hz}$ and $B = 0.1$. These models involve saturated dense sand ($\mu = 85616.5\,\mathrm{KN/m^2}$, $\gamma = 21.0\,\mathrm{KN/m^3}$, $\nu = 0.33$, $\beta = 5\%$, $L_R = 3.728\,\mathrm{m}$, $v_R = 186.40\,\mathrm{m/s}$), saturated soft clay ($\mu = 44518.0\,\mathrm{KN/m^2}$, $\gamma = 19.41\,\mathrm{KN/m^3}$, $\nu = 0.33$, $\beta = 5\%$, $L_R = 2.796\,\mathrm{m}$, $v_R = 139.80\,\mathrm{m/s}$) and dry sand ($\mu = 59449.5\,\mathrm{KN/m^2}$, $\gamma = 18.00\,\mathrm{KN/m^3}$, $\nu = 0.33$, $\beta = 5\%$, $L_R = 3.355\,\mathrm{m}$, $v_R = 167.76\,\mathrm{m/s}$). The thin-layer BEM is applied here with the discretization pattern of the previous example. A dynamic analysis of the three soil models without the trench is done first for further validation of the method and better understanding of their behavior to the dynamic disturbance of the foundation. Figure 5 depicts the real parts of the vertical component of the free field surface diaplacement amplitude versus distance for all three soil models. By measuring the distance between two successive peaks from the graphs (at a location of several wavelengths from the source of disturbance), the effective Rayleigh wavelength L_R' for each model



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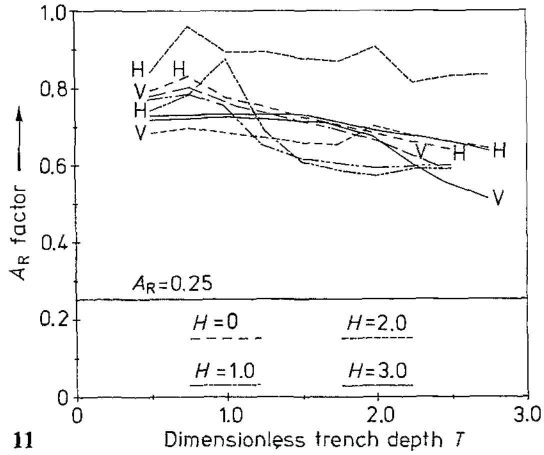

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Figs. 10 and 11. Horizontal (H) and vertical (V) amplitude reduction factor **10** versus nondimensional layer thickness for concrete filled trench: $H = h/L_R$, $T = 0.5$, 1.0, 1.5 and 2.0; **11** versus nondimensional concrete filled trench depth for the cases of $H = 0.0$, 1.0, 2.0 and 3.0

can be determined. The L'_R is very close to the actual L_R of the soil material at a depth of $L'_R/2$ (Richart et al. 1970). For model 2, a depth of $L'_R/2 = 1.385$ m is in the soft clay region, while for model 3, a depth of $L'_R/2 = 1.665$ m is in the dry sand region. The comparison of the computed L'_R and the actual L_R at $L'_R/2$ described in Table 1, clearly shows a close agreement. The wave screening effectiveness of trenches is studied next for the soil models 2 and 3 as a function of trench depth. Nondimensionalization is affected by the L_R of the superficial layer of the layered soil medium and surface displacements are plotted with origin the middle of the rigid foundation. Figure 6 portrays the horizontal and vertical amplitude reduction factors A_R versus nondimensional trench depth for soil model 2, while Fig. 7 the same things for soil model 3. The factor A_R is defined as the average normalized surface displacement amplitude behind the trench over a distance $D = d/L_R = 4$. Adopting the suggestion of Richarts et al. (1970) that $A_R \leqslant 0.25$ for a successful vibration isolation design, one can conclude from the above figures, that trench depths of $T \geqslant 1.75$ and $T \geqslant 1.5$ for soil models 2 and 3, respectively, are required for this to be achieved. In view of the fact that only $T \geqslant 0.6$ is required for homogeneous soil cases (Beskos et al. 1986), one can conclude that deeper trenches are required in layered soils (soft layer(s) on strong half-plane) to achieve the same level of screening as in homogeneous soils (stong half-plane).

Example 3: Since the possible formations of multilayered soils and the parameters involved are numerous, it is very difficult to perform general parametric studies for assessing the screening effectiveness of trenches in multilayered soils. This example concentrates on the case of soil model 2 with variable soft clay depth. All the other material and geometric parameters are as in Example 1. Figure 8 depicts the horizontal and vertical A_R factors versus the nondimensional layer thickness $H = h/L_R$, where L_R corresponds to soft clay and for various values of T. It can be observed that the soil layer thickness has a significant effect on A_R, especially in the range $1.0 \leqslant H \leqslant 2.5$ where A_R shows a considerable increase for decreasing values of T. When $H > 2.5$, a trench with $T \geqslant 1.0$ can provide satisfactory screening. Figure 9 shows the horizontal and vertical A_R factors versus T for various values of H. It can be observed that a trench of $T \geqslant 1.0$ is effective for $H = 0.0$ and 3.0, while a trench of $T \geqslant 1.75$ is needed for $H = 1.0$ and 2.0. For the case of $H = 0.0$, which corresponds to a homogeneous half-space, the finding that a $T \geqslant 1.0$ is required for acceptable design is in agreement with Beskos et al. (1986). For the case of $H = 3.0$, since the bulk of Rayleigh waves travels through a zone near the surface, the presence of a different soil material at a depth of three wavelengths is not as significant as it is at shallower depths. However, if the half-space is composed of a stronger material or hard rock, different results may be expected. Finally, it should be noticed that the finding that for $H = 1.0$ and 2.0 the trench is effective for $T \geqslant 1.75$. is in agreement with the results of the previous example.

Example 4: Here the effect of filling the trench of the previous example with concrete having material properties as described in Beskos et al. (1986) is investigated. Figure 10 shows the horizontal and vertical A_R factors versus H for various values of T. As before, the soil layer thickness has a significant effect on A_R in the range $1.0 \leqslant H \leqslant 2.5$. In this case, however, this effect, especially for the horizontal A_R, consists of much higher values of A_R than before and is independent of T. It is also observed that for $H > 2.5$, A_R factors are close to 0.70 for almost all T indicating that it is not possible to really improve the screening effectiveness by increasing the trench depth, as in the case of the open trench. Figure 11 provides the horizontal and vertical A_R factors versus T for various values of H. It clearly shows that the decrease of A_R, for increasing T is, in general, small and that the effect of H is more pronounced for the horizontal A_R than the vertical A_R, as also observed in Fig. 10. It is also shown in Fig. 11 that for $1.0 \leqslant T \leqslant 2.5$ it is the case with $H = 2.0$ that leads to the best ($A_R \approx 0.6$) results. For $T \geqslant 2.5$, best results can be achieved for $H = 0.0$ or $H \geqslant 3.0$.

5 Conclusions

On the basis of the results presented in this paper, the following conclusions can be drawn:

(1) The present thin-layer BEM presents a very good tool for studying wave propagation problems in multi-layered soils. The soil layer interfaces do not need any discretization since the soil body, which is assumed to be composed of thin layers, is treated as a whole.

(2) In layered soils (soft layer(s) on strong half-plane) deeper trenches are required to achieve the same level of screening as in homogeneous soils (strong half-plane) with this trend being more pronounced for concrete filled trenches.

(3) From the case study of a saturated soft clay layer resting on the top of a half-space of saturated dense sand, it s found that (a) if the clay layer is shallower than 2.5 Rayleigh wavelengths, the screening trench effectiveness is significantly reduced and a trench depth of two Rayleigh wavelengths or more is needed for an acceptable design, (b) if the clay layer is deeper than three Rayleigh wavelengths, the trench screening effectiveness is similar to that for the homogeneous half-space case.

(4) The screening effectiveness of concrete filled trenches in a soil medium consisting of a soft clay layer on the top of a half-space is much lower than in the case of open trenches. Furthermore, increase of the trench depth does not help much to improve the situation.

(5) More extensive parametric studies covering a wide range of geometrical and material properties of layered soil media are required for a thorough investigation of the screening effectiveness of open or filled trenches. The case of transient disturbances has also to be investigated. This can be easily done by the present methodology in conjunction with Laplace transform as explained in Beskos et al. (1986).

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