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On the Lattice of Quasivarieties of Sugihara Algebras

Abstract. Let $\boldsymbol{S}$ denote the variety of Sugihara algebras. We prove that the lattice $\Lambda(\boldsymbol{K})$ of subquasivarieties of a given quasivariety $\boldsymbol{K} \subseteq \boldsymbol{S}$ is finite if and only if $\boldsymbol{K}$ is generated by a finite set of finite algebras. This settles a conjecture by Tokarz [6]. We also show that the lattice $\Lambda(\boldsymbol{S})$ is not modular.

1. Preliminaries

Let $\mathfrak{S}$ be the algebra $\langle Z, \wedge, \vee, \rightarrow, -\rangle$ of type $\langle 2, 2, 2, 1\rangle$ where Z is the set of integers with the usual ordering, $\bar{x} = -x$, and

$$x \rightarrow y = \begin{cases} \bar{x} \vee y & \text{if } x \leqslant y \\ \bar{x} \wedge y & \text{otherwise.} \end{cases}$$

By a *Sugihara algebra* we will understand any algebra in the variety $\boldsymbol{S}$ generated by $\mathfrak{S}$. The variety $\boldsymbol{S}$ is closely related to the deductive system RM of relevant logic (cf. Anderson and Belnap [1], Dunn [3]). More precisely, RM is strongly algebraizable in the sense of Blok and Pigozzi [2], and its associated variety is $\boldsymbol{S}$.

Let $\mathfrak{S}_{2n}$ denote the subalgebra of $\mathfrak{S}$ whose domain is $S_{2n} = \{-n, \ldots, -1, 1, \ldots, n\}$, and $\mathfrak{S}_{2n+1}$ the subalgebra with domain $S_{2n+1} = S_{2n} \cup \{0\}$, for $n \geqslant 1$. We will use $\mathfrak{S}_1$ to denote the 1-element Sugihara algebra. The variety generated by $\mathfrak{S}_n$ will be denoted by $\boldsymbol{S}_n$. For a class K of similar algebras we denote the quasivariety generated by K by $Q(K)$. For notions of lattice theory and universal algebra we refer to [4] and [5].

Lemma 1.1.
(i) *$\boldsymbol{S}$ is locally finite.*
(ii) *Up to isomorphism, the only finite subdirectly irreducible algebras in $\boldsymbol{S}$ are the $\mathfrak{S}_i$, $2 \leqslant i < \omega$.*

Proof. (i): As any n-generated subalgebra of $\mathfrak{S}$ is isomorphic to a subalgebra of $\mathfrak{S}_{2n+1}$, the free algebra $\mathfrak{F}_{\boldsymbol{S}}(n)$ in $\boldsymbol{S}$ with n free generators belongs to $ISP(\mathfrak{S}_{2n+1})$ therefore, $\mathfrak{F}_{\boldsymbol{S}}(n)$ is finite.

(ii): That every $\mathfrak{S}_i$ is subdirectly irreducible follows from the observation that the least congruence relation on $\mathfrak{S}_i$ that identifies -1 with $+1$ is a unique atom in the congruence lattice of $\mathfrak{S}_i$.

Let $\mathfrak{A} \in S$ be finite and subdirectly irreducible, and let $|\mathfrak{A}| = n$. Then $\mathfrak{A} \in HSP(\mathfrak{S}_{2n+1})$ since $\mathfrak{F}_S(n) \in ISP(\mathfrak{A}_{2n+1})$. Hence, by Jónsson's Lemma, $\mathfrak{S} \in HS(\mathfrak{S}_{2n+1})$. But every nontrivial algebra from $HS(\mathfrak{S}_{2n+1})$ is isomorphic to one of $\mathfrak{S}_i$'s, where $2 \leqslant i \leqslant 2n+1$. Thus $\mathfrak{A} \cong \mathfrak{S}_n$ for some $n, n \geqslant 2$.

2. Directly indecomposable Sugihara algebras

If $\mathfrak{A} \in S$ is finite, then $\mathfrak{A}$ has a smallest and a largest element, denoted by $0_{\mathfrak{A}}$ and $1_{\mathfrak{A}}$ respectively. For $a \in A$ let $a^* = a \to 0_{\mathfrak{A}}$, and let the *center of* $\mathfrak{A}$, denoted $C(\mathfrak{A})$, be the set $\{a^* : a \in A\}$. If $\mathfrak{A} \cong \mathfrak{S}_n, n \geqslant 2$, then $C(\mathfrak{A}) = \{0_{\mathfrak{A}}, 1_{\mathfrak{A}}\}$. More generally we have:

LEMMA 2.1. *Let $\mathfrak{A}$ be a subdirect product of a finite family $\langle \mathfrak{A}_i : i \in I\rangle$ of finite subdirectly irreducibles. Then $a \in C(\mathfrak{A})$ if and only if $a(i) \in \{0_{\mathfrak{A}_i}, 1_{\mathfrak{A}_i}\}$ for all $i \in I$.*

PROOF. If $a \in C(\mathfrak{A})$, then $a = b^*$ for some $b \in A$. If $b(i) = 0_{\mathfrak{A}_i}$, then $a(i) = b(i) \to 0_{\mathfrak{A}_i} = 1_{\mathfrak{A}_i}$. If $b(i) > 0_{\mathfrak{A}_i}$, then $a(i) = b(i) \to 0_{\mathfrak{A}_i} = 0_{\mathfrak{A}_i}$. Conversely, if $a(i) \in \{0_{\mathfrak{A}_i}, 1_{\mathfrak{A}_i}\}, i \in I$, then $a = a^{**}$, and hence $a \in C(\mathfrak{A})$.

Observe that $C(\mathfrak{A})$ is closed under $\vee, \wedge, \to$, and $^-$, and hence is a subuniverse of $\mathfrak{A}$ which is a Boolean algebra.

THEOREM 2.2. *Let $\mathfrak{A} \in S$ be finite. Then $\mathfrak{A}$ is directly indecomposable if and only if $C(\mathfrak{A}) = \{0_{\mathfrak{A}}, 1_{\mathfrak{A}}\}$.*

PROOF. If $\mathfrak{A} = \mathfrak{B} \times \mathfrak{C}$, with $|B| > 1, |C| > 1$, then $\langle 0_{\mathfrak{B}}, 1_{\mathfrak{C}}\rangle \in C(\mathfrak{A})$, while $\langle 0_{\mathfrak{B}}, 1_{\mathfrak{C}}\rangle \neq 0_{\mathfrak{A}}, \langle 0_{\mathfrak{B}}, 1_{\mathfrak{C}}\rangle \neq 1_{\mathfrak{A}}$. For the converse, suppose $C(\mathfrak{A}) \neq \{0_{\mathfrak{A}}, 1_{\mathfrak{A}}\}$ say, $a \in C(\mathfrak{A}), a \neq 0_{\mathfrak{A}}, 1_{\mathfrak{A}}$. We may assume $\mathfrak{A}$ is a subdirect product of a finite family of finite subdirectly irreducibles $\langle \mathfrak{A}_i : i \in I\rangle$. By the previous lemma there is a $J \subsetneq I, \varnothing \neq J$, such that

$$a(i) = \begin{cases} 1_{\mathfrak{A}_i} & \text{if } i \in J \\ 0_{\mathfrak{A}_i} & \text{if } i \notin J \end{cases}$$

Let $\mathfrak{B} = \pi_J[\mathfrak{A}]$, $\mathfrak{C} = \pi_{I \setminus J}[\mathfrak{A}]$. Then $|B| > 1$, $|C| > 1$, and clearly $A \subseteq B \times C$. Conversely, if $x \in B, y \in C$, then $x = \pi_J(x'), y = \pi_{I\setminus J}(y')$ for some $x', y' \in A$, and $\langle x, y\rangle = (x' \wedge a) \vee (y' \wedge a^*)$. Hence $\langle x, y\rangle \in A$, and thus $\mathfrak{A} = \mathfrak{B} \times \mathfrak{C}$, i.e., $\mathfrak{A}$ is directly decomposable. This completes the proof of the theorem.

In the remainder of this section we will characterize the finite Sugihara algebras $\mathfrak{A}$ whose center consists of $0_{\mathfrak{A}}, 1_{\mathfrak{A}}$ only. Clearly the $\mathfrak{S}_n, n \geqslant 2$ are among them, but these are not the only ones.

Given $\mathfrak{A} \in S$, we define an extension $\mathfrak{A}^+$ of $\mathfrak{A}$ as follows. The domain A^+ of $\mathfrak{A}^+$ consists of A together with two new elements, $\bot$ and $\top$. The lattice order of $\mathfrak{A}$ is extended by stipulating

$$\bot \leqslant x, \quad x \in A^+$$
$$\top \geqslant x, \quad x \in A^+.$$

The operations $\to$ and $^-$ are extended by the clauses:

$$x \to y = \begin{cases} \top & \text{if } x = \bot \text{ or } y = \top \\ x \to y \ \ (\text{in } \mathfrak{A}) & \text{if } x, y \in A \\ \bot & \text{otherwise} \end{cases}$$

and

$$\bar{x} = \begin{cases} \top & \text{if } x = \bot \\ \bar{x} \ \ (\text{in } \mathfrak{A}) & \text{if } x \in A \\ \bot & \text{otherwise.} \end{cases}$$

Note that for $n \geqslant 1$, $\mathfrak{S}_n^+ \cong \mathfrak{S}_{n+2}$.

LEMMA 2.3. *If $\mathfrak{A} \in \boldsymbol{S}$, then $\mathfrak{A}^+ \in \boldsymbol{S}$ as well.*

PROOF. In view of Lemma 1.1(i), it suffices to show that every finite subalgebra $\mathfrak{B}$ of $\mathfrak{A}^+$ belongs to $\boldsymbol{S}$. If B contains only $\bot$ and $\top$, then $\mathfrak{B} \cong \mathfrak{S}_2$ and so, in this case, $\mathfrak{B} \in \boldsymbol{S}$. Otherwise, let $C = B \setminus \{\bot, \top\}$. Then C is the domain of some subalgebra $\mathfrak{C}$ of $\mathfrak{A}$. $\mathfrak{C}$ is a subdirect product of a finite family of finite algebras $\langle \mathfrak{A}_i : i \in I \rangle$, where $\mathfrak{A}_i$ is trivial or subdirectly irreducible, and hence $\mathfrak{B}$ is a subdirect product of the family $\langle \mathfrak{A}_i^+ : i \in I \rangle$.

If $\mathfrak{A} \cong \mathfrak{B}^+$ for some $\mathfrak{B} \in \boldsymbol{S}$, let $\mathfrak{A}^-$ be the subalgebra of $\mathfrak{A}$ with domain $A \setminus \{0_{\mathfrak{A}}, 1_{\mathfrak{A}}\}$. In particular, for $n \geqslant 3$, $\mathfrak{S}_n^- = \mathfrak{S}_{n-2}$.

It follows easily from the definition of $\mathfrak{A}^+$ that $C(\mathfrak{A}^+) = \{\bot, \top\} = \{0_{\mathfrak{A}^+}, 1_{\mathfrak{A}^+}\}$. Conversely we have:

THEOREM 2.4. *Let $\mathfrak{A} \in \boldsymbol{S}$ be finite and non-trivial. Then $C(\mathfrak{A}) = \{0_{\mathfrak{A}}, 1_{\mathfrak{A}}\}$ if and only if $\mathfrak{A} \cong \mathfrak{S}_2$ or $\mathfrak{A} \cong \mathfrak{B}^+$ for some $\mathfrak{B} \in \boldsymbol{S}$. Furthermore, if $\mathfrak{A} \cong \mathfrak{B}^+$ for some $\mathfrak{B} \in \boldsymbol{S}$ and $\mathfrak{A} \in \boldsymbol{S}_n$, then $\mathfrak{B} \in \boldsymbol{S}_{n-2}$.*

PROOF. Only one direction needs verification.

Suppose $\mathfrak{A}$ is a subdirect product of the finite family $\langle \mathfrak{A}_i : i \in I \rangle$ of finite subdirectly irreducibles. Observe that $0_{\mathfrak{A}} = \langle 0_{\mathfrak{A}_i} : i \in I \rangle$, and $1_{\mathfrak{A}} = \langle 1_{\mathfrak{A}_i} : i \in I \rangle$. If $b \in A$, $b \succ 0_{\mathfrak{A}}$ then for some $i \in I$ $b(i) > 0_{\mathfrak{A}_i}$, and hence $b^*(i) = 0_{\mathfrak{A}_i}$. It follows that $b^* \neq 1_{\mathfrak{A}}$, and since $C(\mathfrak{A}) = \{0_{\mathfrak{A}}, 1_{\mathfrak{A}}\}$ we conclude that $b^* = 0_{\mathfrak{A}}$, and thus that $b(i) > 0_{\mathfrak{A}_i}$, $i \in I$. Since $\mathfrak{A}$ is subdirectly embedded in $\prod_{i \in I} \mathfrak{A}_i$, we see that $b(i) \succ 0_{\mathfrak{A}_i}$, $i \in I$, and hence that b is the only cover of $0_{\mathfrak{A}}$. If $b(j) = 1_{\mathfrak{A}_j}$ for some $j \in I$, then $\bar{b}(j) = 0_{\mathfrak{A}_j} \not\succeq b(j)$, and hence $\bar{b} = 0_{\mathfrak{A}}$ and $b = \bar{\bar{b}} = 1_{\mathfrak{A}}$. In this case $\mathfrak{A} \cong \mathfrak{S}_2$. If $b(i) < 1_{\mathfrak{A}_i}$ for all $i \in I$, then we have $0_{\mathfrak{A}_i} \prec b(i) \leqslant \bar{b}(i) \prec 1_{\mathfrak{A}_i}$, $i \in I$, and it follows that $[b, \bar{b}] \subseteq \prod_{i \in I} \langle \mathfrak{A}_i^- : i \in I \rangle$, and hence is a subuniverse of $\mathfrak{A}$. Let $\mathfrak{B}$ be the Sugihara algebra with universe $B = [b, \bar{b}]$, then it is easy to verify that $\mathfrak{A} \cong \mathfrak{B}^+$. Finally, if, in this case, $\mathfrak{A} \in \boldsymbol{S}_n$, $n \geqslant 3$, then $\mathfrak{A}_i \in \boldsymbol{S}_n \setminus \boldsymbol{S}_2$ for all $i \in I$, and hence $\mathfrak{A}_i^- \in \boldsymbol{S}_{n-2}$. Thus $\mathfrak{B} \in \boldsymbol{S}_{n-2}$.

COROLLARY 2.5. *Let $\mathfrak{A} \in S$ be finite. Then $\mathfrak{A}$ is directly indecomposable if and only if $\mathfrak{A} \cong \mathfrak{S}_2$ or $\mathfrak{A} \cong \mathfrak{B}^+$ for some $\mathfrak{B} \in S$.*

We noticed before that if $\mathfrak{A} \in S$ is finite, then $C(\mathfrak{A})$ is a Boolean algebra, say, $C(\mathfrak{A}) \cong (\mathfrak{S}_2)^n$. In view of Corollary 2.5, $\mathfrak{A}$ is then isomorphic to $\prod_{i=1}^{n} \mathfrak{B}_i$, where each $\mathfrak{B}_i$ is either isomorphic to $\mathfrak{S}_2$ or to $(\mathfrak{B}_i^-)^+$.

3. The lattice of quasi-varieties of Sugihara algebras

In this section we will characterize the quasivarieties of Sugihara algebras which have only a finite number of subquasivarieties. If K is a quasivariety, let $\Lambda(K)$ denote the lattice of its subquasivarieties, ordered by inclusion. A finite algebra $\mathfrak{A}$ is *critical* if it does not belong to the quasivariety generated by all its proper subalgebras.

LEMMA 3.1. *Assume K is a locally finite quasivariety. Then $\Lambda(K)$ is finite if and only if K contains, up to isomorphism, a finite number of critical algebras.*

PROOF. $\Rightarrow$: Notice that if $\mathfrak{A}$ is critical, then it is subdirectly irreducible in $Q(\mathfrak{A})$. Hence, for critical algebras $\mathfrak{A}$ and $\mathfrak{B}$ we have: $Q(\mathfrak{A}) = Q(\mathfrak{B})$ implies $\mathfrak{A} \cong \mathfrak{B}$. From this the statement follows.

$\Leftarrow$: Since K is locally finite, each subquasivariety of K is generated by its critical members.

The proof of the following lemma is straightforward.

LEMMA 3.2. (i) *Let $\mathfrak{B}_1, \ldots, \mathfrak{B}_n \in S$. If $\prod_{i=1}^{n} \mathfrak{B}_i$ is critical, then $\mathfrak{B}_1, \ldots, \mathfrak{B}_n$ are critical and pairwise non-isomorphic.*

(ii) *Let $\mathfrak{B}$ be a finite and non-trivial Sugihara algebra. If $\mathfrak{B}^+$ is critical, then so is $\mathfrak{B}$.*

We are now ready to prove the main result of the paper:

THEOREM 3.3. *Let K be a subquasivariety of S. Then $\Lambda(K)$ is finite if and only if K is generated by a finite set of finite algebras.*

PROOF. $\Rightarrow$: This is immediate from Lemma 3.1.

$\Leftarrow$: Since K is generated by a finite set of finite algebras, it follows from Lemma 1.1 (ii) that $K \subseteq S_n$ for some $n < \omega$. We claim that the varieties S_n possess, up to isomorphism, only finitely many critical algebras. In virtue of Lemma 3.2(i), it suffices to show that each S_n has only a finite number of algebras which are both critical and directly indecomposable. Since $\mathfrak{S}_2$ is the only directly indecomposable algebra in S_2, our claim holds true for S_2. Now suppose it has been verified for $k < n$, where $2 < n < \omega$. If $\mathfrak{A} \in S_n$ is critical and directly indecomposable, then by Corollary 2.5

$\mathfrak{A} \cong \mathfrak{S}_2$ or $\mathfrak{A} \cong \mathfrak{B}^+$ for some $\mathfrak{B} \in \boldsymbol{S}$. In view of Lemma 3.2 (ii), in the latter case $\mathfrak{A} \cong \mathfrak{S}_3$ or $\mathfrak{A} \cong \mathfrak{B}^+$ for some non-trivial critical algebra $\mathfrak{B}$. Furthermore, by the second part of Theorem 2.4, $\mathfrak{B} \in \boldsymbol{S}_{n-2}$. Since by our induction hypothesis $\boldsymbol{S}_{n-2}$ has only a finite number of critical algebras, it follows that $\boldsymbol{S}_n$ has only a finite number of critical, directly indecomposable algebras. This completes the proof of the claim.

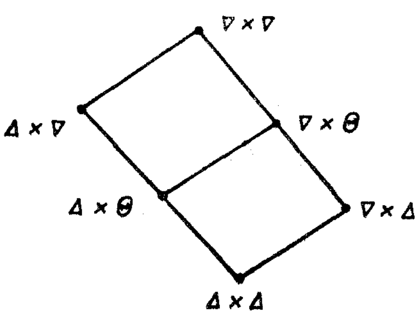
By the claim and Lemma 3.1, the lattices $\Lambda(\boldsymbol{S}_n)$ are finite, $n < \omega$, and thus so is $\Lambda(\boldsymbol{K})$.

THEOREM 3.4. *$\Lambda(\boldsymbol{S})$ is not modular.*

PROOF. We prove that $\Lambda(\boldsymbol{S}_4)$ is not modular. As $\mathfrak{S}_3$ is a homomorphic image of $\mathfrak{S}_4$, we have $\mathfrak{S}_3, \mathfrak{S}_4 \in \boldsymbol{S}_4$. Hence $Q(\mathfrak{S}_3), Q(\mathfrak{S}_4)$ and $Q(\mathfrak{S}_3 \times \mathfrak{S}_4)$ are contained in $\boldsymbol{S}_4$. Since $\mathfrak{S}_4$ is embeddable in $\mathfrak{S}_3 \times \mathfrak{S}_4$, we have $Q(\mathfrak{S}_4) \subseteq Q(\mathfrak{S}_3 \times \mathfrak{S}_4)$. Clearly $\mathfrak{S}_3 \times \mathfrak{S}_4 \in Q(\mathfrak{S}_3 \times \mathfrak{S}_4) \wedge \big(Q(\mathfrak{S}_3) \vee Q(\mathfrak{S}_4)\big)$. Assume $\Lambda(\boldsymbol{S}_4)$ is modular. Then $\mathfrak{S}_3 \times \mathfrak{S}_4 \in \big(Q(\mathfrak{S}_3 \times \mathfrak{S}_4) \wedge Q(\mathfrak{S}_3)\big) \vee Q(\mathfrak{S}_4)$. It follows from the proof of the previous theorem that the critical directly indecomposable algebras in $\boldsymbol{S}_3$ are $\mathfrak{S}_2$ and $\mathfrak{S}_3$, and hence $\mathfrak{S}_2$, $\mathfrak{S}_3$ and $\mathfrak{S}_2 \times \mathfrak{S}_3$ are the critical algebras in $Q(\mathfrak{S}_3)$. Now $\mathfrak{S}_2$ and $\mathfrak{S}_2 \times \mathfrak{S}_3$ belong to $Q(\mathfrak{S}_3 \times \mathfrak{S}_4)$, but $\mathfrak{S}_3$ does not, because $\mathfrak{S}_3$ is not embeddable in $\mathfrak{S}_3 \times \mathfrak{S}_4$. We conclude that $Q(\mathfrak{S}_3 \times \times \mathfrak{S}_4) \wedge Q(\mathfrak{S}_3) = Q(\mathfrak{S}_2 \times \mathfrak{S}_3)$. Thus $\mathfrak{S}_3 \times \mathfrak{S}_4 \in Q(\mathfrak{S}_2 \times \mathfrak{S}_3) \vee Q(\mathfrak{S}_4)$. Let

$$\varphi : \mathfrak{S}_3 \times \mathfrak{S}_4 \to (\mathfrak{S}_2 \times \mathfrak{S}_3)^I \times \mathfrak{S}_4^J$$

be an embedding, with I, J finite index sets. Since $\boldsymbol{S}$ is congruence distributive, every congruence relation on $\mathfrak{S}_3 \times \mathfrak{S}_4$ is a product of its factors. But $\mathfrak{S}_3$ is simple and the congruence lattice of $\mathfrak{S}_4$ is a three-element chain $\Delta < \Theta < \nabla$, hence the congruence lattice of $\mathfrak{S}_3 \times \mathfrak{S}_4$ is



The map φ is $1-1$, and therefore $\bigwedge_{i \in I \cup J} \ker \pi_i \circ \varphi = \Delta \times \Delta$. Clearly $\ker \pi_i \circ \varphi > \Delta \times \Delta$, for all i, and hence for some $i \in I \cup J$, $\ker \pi_i \circ \varphi = \Delta \times \Theta$, or $\ker \pi_i \circ \varphi = \Delta \times \nabla$. In the first case, $\pi_i \circ \varphi[\mathfrak{S}_3 \times \mathfrak{S}_4] \cong \mathfrak{S}_3 \times \mathfrak{S}_3$, which is not embeddable in either of $\mathfrak{S}_2 \times \mathfrak{S}_3$ or $\mathfrak{S}_4$. In the second case, $\pi_i \circ \varphi[\mathfrak{S}_3 \times \times \mathfrak{S}_4] \cong \mathfrak{S}_3$, which likewise is not embeddable in $\mathfrak{S}_2 \times \mathfrak{S}_3$ or in $\mathfrak{S}_4$. We have thus arrived at a contradiction, and must conclude that $\Lambda(\boldsymbol{S}_4)$ is not modular.

Theorem 3.3 confirms a conjecture by Tokarz [6]. It follows from the results of Dunn [3] that the system RM of relevant implication (cf. Anderson and Belnap [1]) is strongly algebraizable in the sense of [2], and that the class of algebras associated with it is precisely $\boldsymbol{S}$. Tokarz investigated certain deductive systems, stronger than RM: the standard strengthenings of RM. Their associated classes of algebras are precisely the quasivarieties of Sugihara algebras, and the correspondence is 1–1 and order reversing (see [2]). Let us denote by C_n the deductive system whose associated quasivariety is $Q(\mathfrak{S}_n)$. The cardinality of the set of standard strengthenings of C_n equals that of $\Lambda(Q(\mathfrak{S}_n))$, and hence is finite by Theorem 3.3. It follows that the degree of maximality of C_n, dmC_n, equals $|\Lambda(Q(\mathfrak{S}_n))|$ as well, and hence, in particular, is finite. This was proven in Tokarz [6] for $n \leqslant 4$, and conjectured for $n \geqslant 5$.

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