Andalusian Astronomy: al-Zîj al-Muqtabis *of Ibn al-Kammâd*

JOSÉ CHABÁS & BERNARD R. GOLDSTEIN

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Contents

Introduction 1
I. Ibn al-Kammâd: Life and Works 2
II. The Solar Equation in *al-Muqtabis* 4
III. Eclipse Theory in *al-Muqtabis* 10
IV. Other Tables in *al-Muqtabis* 24
V. Other Tables in MS Madrid 10023 36
References 38

Introduction

In 1956 E. S. KENNEDY published his *A Survey of Islamic Astronomical Tables* in which he described, briefly in most cases, over 100 sets of tables, called zijes (after the Arabic: *zîj*), from the 8th to the 15th centuries and from all parts of the medieval Islamic world. At that time only two of them had been published, and it was clear that our understanding of scientific activity in the Middle Ages would be greatly enhanced by detailed treatment of the others. Indeed, such has proven to be the case, as we learn from the many studies that have followed this pioneering essay.

Astronomers in Islamic Spain, al-Andalus, composed zijes and, beginning in the 12th century, they were adapted and translated into Hebrew, Latin, Castilian, and Catalan, the most famous examples being the Toledan Tables (see TOOMER (1968)) and the Alfonsine Tables. In Spain, as elsewhere in the Islamic world, these zijes were largely based on the work of predecessors going back to PTOLEMY on the one hand, and Hindu astronomers on the other. More often than not, a table comes with instructions for using it, rather than the method used to construct it. For this reason, much scholarly energy has been devoted to describing the methods underlying these tables, as well as their lines

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of descent. By such analysis, guided by textual material, one can now distinguish tables that are based on entirely new models, tables that are merely copies of tables in previous zijes (at the two extremes), from tables based on previous models but with new parameters and tables composed by modified or new mathematical methods.

The work of IBN-AL-KAMMÂD, an Andalusian astronomer of the 12th century, illustrates most of these characteristics. He composed 3 zijes, none of which survives in the original Arabic, but a Latin manuscript contains a translation of what appears to be one complete zij with references to the others. IBN-AL-KAMMÂD depended on sources that ultimately go back to PTOLEMY and to Hindu astronomers as they were known in zijes prior to his time, and his influence was felt by later astronomers writing in Arabic, Latin, and Hebrew. As is often the case, this work preserves parts of otherwise lost texts: in particular, we gain valuable information on the solar theory of the Andalusian astronomer AZARQUIEL (or AL-ZARQÂLLUH), who lived in the 11th century (see TOOMER (1969)). In this article our primary intention is to describe the astronomical work of IBN-AL-KAMMÂD as it is preserved in Latin in MS Madrid 10023.

IBN-AL-KAMMÂD is cited or criticized by a number of his successors: IBN AL-HÂ'IM writing in Arabic *ca.* 1205 (see SAMSÓ (1992), pp. 321 ff); ABÛ L-ḤASAN ᶜALÎ AL-MARRÂKUSHÎ (*ca.* 1262) (see MILLÁS (1950), p. 347); JUAN GIL (*ca.* 1350) whose astronomical tables are preserved in a Hebrew version (see GOLDSTEIN (1985), p. 237); AL-ḤADIB, a 14th century astronomer from Spain who went to Sicily where he wrote astronomical works in Hebrew (see GOLDSTEIN (1985), p. 239); JOSEPH IBN WAQÂR, a Spanish astronomer of the mid-14th century who wrote in Arabic and Hebrew (see GOLDSTEIN (1985), p. 237); and most importantly, as we shall see, IBN-AL-KAMMÂD had a profound influence on the Tables of Barcelona (see MILLÁS (1962)).

In the discussions that follow new information of particular interest for the history of astronomy in Spain is presented: see, *e.g.*, the solar equation table (Section II); the preservation of material that goes back to the zij AL-MUMTAḤAN of YAḤYÂ IBN ABÎ MANṢÛR who lived in the 9th century (Section III, C, F and J); the table for trepidation (Section IV, B); and the tables for planetary latitude (Section IV, E).

In sum, IBN-AL-KAMMÂD was a major player in medieval Spanish astronomy; his achievements and the extent of his legacy have not yet been sufficiently appreciated.

I. IBN AL-KAMMÂD: Life and Works

This is the name by which the astronomer ABÛ JAᶜFAR AḤMAD BEN YÛSUF IBN AL-KAMMÂD is known, although a variety of names has been associated with him (VERNET (1949), pp. 72–73). He was probably from Sevilla, but active in Córdoba in the 12th century. In a remarkable study, MILLÁS (1942), pp. 231–247, called attention to a 14th century Latin manuscript at the Biblioteca Nacional de Madrid, MS 10023, containing one of IBN-AL-KAMMÂD's works; he described it and gathered all the information then available on this astronomer. Not much more is known now.

IBN-AL-KAMMÂD is the author of 3 zijes:

1. *al-Kawr* ᶜ*alâ al-dawr* (the periodic rotations [?]) (in 60 chapters). This treatise has been partially preserved in Arabic in MS Escorial Ar. 939,4 (not seen by us). There is also a short text in Castilian in Segovia, Biblioteca de la Catedral, MS 115, ff. 218vb–220vb, attributed to YUÇAF BENACOMED, and entitled "Libro sobre çircunferencia de moto".
2. *al-Amad* ᶜ*alâ al-abad* (the eternally valid [tables])
3. *al-Muqtabis* (the compilation [of the two previous works]). MS Madrid 10023 contains the Latin translation of *al-Muqtabis*. In the explicit it is clearly stated that the translation was done by JOHN OF DUMPNO in 1260 in Palermo. In Arabic only chapter 28 has survived: see MS Alger 1454, 2, ff. 62–63.

The dates for IBN-AL-KAMMÂD are uncertain. MILLÁS (1950), p. 346, considered the period "towards the end of the 12th century", and suggested the year 1195 as that of his death, probably following AHLWARDT (1893), p. 219, where this date is given with no specification of his source. More recently, IBN-AL-KAMMÂD has been taken to be a "direct disciple of AZARQUIEL". This claim is based on a note in the margin of f. 30r of MS lat. 7281, a 15th century manuscript at the Bibliothèque Nationale de Paris, and it has been argued that this claim is supported by the date A.H. 480 (1087–88 A.D.) that appears in MS Madrid 10023, f. 65v: see Section V, M, below. The marginal note, already transcribed in MILLÁS (1950), p. 14, reads: "Post uenit ALCAMET discipulus . . .", referring to AZARQUIEL. According to MILLÁS, the same hand, or a similar one, has added: "Similiter discipulus MESSALLE". It does not seem at all warranted to deduce from this expression that IBN-AL-KAMMÂD was a "direct pupil" of AZARQUIEL. Instead, we understand this to mean only that IBN-AL-KAMMÂD was a follower of AZARQUIEL's methods. On the other hand, the date in MS Madrid 10023, f. 65v, is not the only one mentioned in the last section of this manuscript (*e.g.*, on f. 66r there is a table for A.H. 550: see Section V, R, below); as we shall see, the last section of this MS contains a variety of tabular material not directly related to *al-Muqtabis*. SAMSÓ (1992), p. 322, noted that IBN-AL-HÂ'IM (fl. 1205) criticized IBN-AL-KAMMÂD. Since the available evidence suggests that IBN-AL-KAMMÂD lived after AZARQUIEL and before IBN-AL-HÂ'IM, we conclude that IBN-AL-KAMMÂD lived in the 12th century, without offering any greater precision.

Texts in MS Madrid 10023

a) *al-Muqtabis*

Text in Latin, presented in two columns.
– Introduction (ff. 1ra-1va). Transcribed in MILLÁS (1942), pp. 231–232.
– Index (ff. 1va-2rb). Transcribed in MILLÁS (1942), pp. 234–235.

- Canons, 1 to 30, each of them called "porta" (ff. 2rb–18vb). Three chapters have been published so far: canon 1 is transcribed in MILLÁS (1942), pp. 235–236, canon 28 in VERNET (1949), pp. 74–78, and canon 30 in MILLÁS (1942), pp. 237–238. Note that canon 30 explicitly mentions the other two works of IBN-AL-KAMMÂD: *al-Kawr ᶜalâ al-dawr* and *al-Amad ᶜalâ al-abad.*
- Explicit (f. 18vb). Transcribed in MILLÁS (1942), p. 238.

b) Other texts

From f. 18vb to f. 24rb there is a set of chapters, in some disorder, that are distinct from those of *al-Muqtabis.* Some of them are associated with *al-Kawr ᶜalâ al-dawr*, and were also translated by JOHN OF DUMPNO in 1262 in Palermo; their incipits and explicits were published in MILLÁS (1942), pp. 238–242. MILLÁS also transcribed some of the texts therein, and TOOMER (1969), pp. 323–324, transcribed and translated a text concerning the variation of solar eccentricity.

Tables in MS Madrid 10023

Two sets of tables can easily be distinguished in the manuscript:

a) *al-Zîj al-Muqtabis* (ff. 27r–54v).

The tables are mentioned, or their use is explained, in the text in 30 canons called *al-Muqtabis.* The tables are calculated for the meridian of Córdoba. Folio 54v contains the last table of *al-Muqtabis* (a geographical table), as we learn from canon 10 (f. 6va): ". . . tabulam longitudinum terrarum positam in ultimo huius canonis". We will discuss all the tables in *al-Muqtabis* as follows: the solar equation in Section II, eclipse theory in Section III, and the remaining tables in Section IV.

b) Other tables (ff. 55r–66r)

These tables do not form a homogeneous set. They are not mentioned in the 30 canons of *al-Muqtabis,* and are presumably distinct from that work. Some of the tables are related to *al-Kawr ᶜalâ al-dawr*, some are attributed to astronomers other than IBN-AL-KAMMÂD, and still others are calculated for places other than Córdoba. These tables will be discussed in Section V.

II. The Solar Equation in *Al-Muqtabis*

(f. 35r) "Tabula directionis centri solis et augis eius ad primordium annorum seductionis"

Above the heading, the longitude of the solar apogee, presumably for the Hijra, is given: "Aux" 2s 16;45,21°. This table displays the solar equation in degrees, minutes and seconds as a function of mean solar anomaly. The maximum solar equation, which occurs at 92°, is 1;52,44°, thus differing from the more common values: 1;59° (ḤABASH AL-ḤÂSIB, YAḤYÂ IBN ABÎ MANṢÛR), 1;59,10° (Toledan Tables, AL-BATTÂNÎ), 2;14° (AL-KHWÂRIZMÎ), 2;23° (PTOLEMY).

The use of this table is explained in canon 13 (f. 7vb).

In his *Tractatus super totam astrologiam*, BERNARDUS DE VIRDUNO (ca. 1300) attributes an eccentricity of 1;58 to AZARQUIEL, which yields a maximum equation of 1;52,42° (see TOOMER (1987), pp. 515–517 and SAMSÓ (1992), p. 216). However, this is not the only parameter used by AZARQUIEL for, in the Alfonsine translation of his treatise on the construction of the equatorium, 1;52,30° is explicitly called a rounded parameter for the solar eccentricity (see SAMSÓ (1987), p. 468). It is therefore likely that IBN AL-KAMMÂD accepted a parameter from AZARQUIEL.

The columns in table 1, The Solar Equation in *al-Muqtabis*, are arranged as follows:

(1) Mean solar anomaly, $\bar{\kappa}$, for each integer degree from 1° to 180°.

(2) Entries in the text (f. 35r): solar equation, $c(\bar{\kappa})$, in degrees, minutes and seconds.

(3) Line-by-line differences in (2): $c(\bar{\kappa}+1) - c(\bar{\kappa})$. This column allows us to recognize quite a number of errors in the entries for the solar equation in the text, for these should increase monotonically from anomaly 0° to reach a maximum at anomaly 92°, and then decrease monotonically to anomaly 180°. Consequently, the set of line-by-line differences should follow a smooth decreasing pattern. In a great number of cases, pairs of successive line-by-line differences in (3), whether erroneous or not, are identical, thus strongly suggesting that the table for the solar equation was originally computed for every other degree, and that interpolation was used to derive the rest of the entries.

(4) Reconstructed line-by-line differences. Not all the line-by-line differences which do not fit smoothly in (3) have been reconstructed; we have only made suggestions in those cases where the tabulated values for the solar anomaly can be easily derived from the reconstructed values. A question mark indicates that, although there is a peculiar value in (3), we do not offer any alternative value.

(5) Reconstructed values for the solar equation. These values result from an easy explanation of the way the copying, or the calculational error was made. There follows a list of various kinds of such errors:

- misreading of a digit: $c(22) = 0;40,19$ instead of 0;40,59 (19 and 59 are easily confused in Arabic); $c(28) = 0;50,27$ instead of 0;51,27;
- inversion of the order of the entries: in the text, the seconds for $c(175)$ and $c(176)$ are apparently inverted;
- incorrect interpolation between computed entries: $c(30)$ and $c(32)$ were computed correctly, but $c(31)$ seems to result from an incorrect interpolation between them;
- displacement of columns; from $c(145)$ to $c(150)$ the entries for the minutes have been shifted one line downwards.

We have left unchanged those "erroneous" entries of the solar equation for which we do not have an easy explanation; some of them may be due to incorrect computation by the author of the table rather than to copyist errors.

(6) Recomputed values: for the recomputation of the entries, we used a simple eccentric model, and the following expression:

[1] $$\tan(c) = e * \sin(\bar{\kappa})/(60 + e * \cos(\bar{\kappa}))$$

where the eccentricity used is $e = 1;58,2 = 60 * \sin(1;52,44)$.

(7) Differences in seconds: *T*(ext) – *C*(omp.), *i.e.*, col. (2) – col. (6); in those cases where we have confidence in our reconstructed values, this column displays the differences between the reconstructed values in col. (5), and computation in col. (6).

Table 1. The Solar Equation in *al-Muqtabis*

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	0; 1,53	0;1,53			0; 1,54	– 1
2	3,47	1,54			3,49	– 2
3	5,40	1,53			5,43	– 3
4	7,34	1,54			7,37	– 3
5	9,25	1,51	1,56	9,30	9,31	– 1
6	11,25	2, 0	1,55		11,24	+ 1
7	13,18	1,53			13,18	0
8	15,11	1,53			15,12	– 1
9	17, 4	1,53			17, 5	– 1
10	0;18,57	0;1,53			0;18,58	– 1
11	20,50	1,53			20,50	0
12	22,42	1,52			22,42	0
13	24,39	1,57	?		24,34	+ 5
14	26, 9	1,30	?		26,26	– 15
15	28,13	2, 4	?		28,16	– 3
16	30,17	2, 4	?	30, 7	30, 7	0
17	32, 1	1,44	?		31,57	– 4
18	33,45	1,44	?		33,46	– 1
19	35,35	1,50			35,35	0
20	0;37,25	0;1,50			0;37,24	+ 1
21	39,12	1,47			39,11	+ 1
22	40,19	1, 7	1,47	40,59	40,58	+ 1
23	42,45	2,26	1,46		42,45	0
24	44,30	1,45			44,30	0
25	46,–	–	1,46	46,16	46,15	+ 1
26	48,16	–	1,44	48, 0	47,59	+ 1
27	49,44	1,28	1,44		49,43	+ 1
28	50,27	0,44	1,44	51,27	51,25	+ 2
29	53, 7	2,40	1,40		53, 7	0
30	0;54,48	0;1,41			0;54,48	0

Table 1. *(Continued)*

(1)	(2)	(3)	(4)	(5)	(6)	(7)
31	56,58	2,10	1,40	56,28	56,27	+ 1
32	58, 7	1, 9	1,39		58, 7	0
33	0;59,45	1,38			0;59,44	+ 1
34	1; 1,23	1,38			1; 1,21	+ 2
35	2,38	1,15	1,35	2,58	2,57	+ 1
36	4,33	1,55	1,35		4,32	+ 1
37	6, 7	1,34			6, 68	+ 1
38	7,37	1,30			7,38	− 1
39	9, 9	1,32			9,10	− 1
40	1;10,41	0;1,32			1;10,40	+ 1
41	12,10	1,29			12, 9	+ 1
42	13,39	1,29			13,37	+ 2
43	15, 7	1,28			15,4	+ 3
44	16,35	1,28			16,29	+ 6
45	17,56	1,21			17,53	+ 3
46	19,17	1,21			19,15	+ 2
47	20,33	1,16	1,21	20,38	20,37	+ 1
48	21,58	1,25	1,20		21,47	+ 1
49	23,45	1,47	1,17	23,15	23,15	0
50	1;24,32	0;0,47	0;1,17		1;24,33	− 1
51	25,47	1,15			25,48	− 1
52	27, 2	1,15			27, 3	− 1
53	28,19	1,17	1,13	28,15	28,15	0
54	29,27	1, 6	1,12		29,26	+ 1
55	30,37	1,10			30,36	+ 1
56	31,46	1, 9			31,44	+ 2
57	32,50	1, 4			32,51	− 1
58	33,54	1, 4			33,56	− 2
59	34, 7	0,13	1, 3	34,57	34,59	− 2
60	1;36, 0	0;1,53	0;1, 3		1;36, 1	− 1
61	36,58	0,58	?		37, 1	− 3
62	37,59	1, 1	?		37,59	0
63	38,53	0,54			38,56	− 3
64	39,49	0,56			39,51	− 2
65	40,48	0,59			40,44	+ 4
66	41,37	0,49			41,35	+ 2
67	42,25	0,48			42,25	0
68	43,13	0,48			43,12	+ 1
69	43,59	0,47			43,58	+ 1
70	1;44,43	0;0,44			1;44,43	0
71	45,25	0,42			45,25	0
72	46, 6	0,41			46, 5	+ 1
73	46,44	0,38			46,44	0
74	47,21	0,37			47,21	0

(Continued)

Table 1. *(Continued)*

(1)	(2)	(3)	(4)	(5)	(6)	(7)
75	47,57	0,36			47,55	+ 2
76	48,33	0,36			48,28	+ 5
77	49, 1	0,28			48,59	+ 2
78	49,29	0,28			49,28	+ 1
79	49,54	0,25			49,55	− 1
80	1;50,19	0;0,25			1;50,20	− 1
81	50,51	0,32	0,22	50,41	50,43	− 2
82	51, 3	0,12	0,22		51, 4	− 1
83	51,22	0,19			51,23	− 1
84	51,41	0,19			51,40	+ 1
85	51,58	0,17			51,56	+ 2
86	52,15	0,17	0,12	52,10	52, 9	+ 1
87	52,14	− 0, 1	0, 9	52,19	52,20	− 1
88	52,19	0, 5	0,10	52,29	52,29	0
89	52,29	0,10	0, 6	52,35	52,35	0
90	1;52,40	0;0,11	0;0, 5		1;52,40	0
91	52,42	0, 2			52,43	− 1
92	52,44	0, 2			52,44	0
93	52,42	− 0, 2			52,43	− 1
94	52,39	− 0, 3			52,39	0
95	52,33	− 0, 6			52,34	− 1
96	52,27	− 0, 6			52,26	+ 1
97	52,16	− 0,11			52,17	− 1
98	52, 6	− 0,10			52, 6	0
99	51,50	− 0,16			51,52	− 2
100	1;51,34	− 0;0,16			0;51,36	− 2
101	51,22	− 0,12	?		51,18	+ 4
102	50,59	− 0,23	?		50,58	+ 1
103	50,41	− 0,18			50,36	+ 5
104	50,22	− 0,19		50,12	50,12	0
105	49,48	− 0,34	− 0,26		49,46	+ 2
106	49,18	− 0,30			49,18	0
107	48,50	− 0,28	?		48,48	+ 2
108	48,16	− 0,34			48,16	0
109	47,31	− 0,45	− 0,35	47,41	47,41	0
110	1;47, 5	− 0;0,26	− 0,36		0;47, 5	0
111	46,55	− 0,10	− 0,40	46,25	46,27	− 2
112	46,45	− 0,10	− 0,40	45,45	45,41	+ 4
113	45,33	− 1,12	− 0,42	45, 3	45, 4	− 1
114	44,20	− 1,13	− 0,43		44,20	0
115	43,12	− 1, 8	− 0,48	43,32	43,33	− 1
116	42,22	− 0,50		42,42	42,45	− 3
117	42,22	0, 0	− 0,50	41,52	41,55	− 3
118	41, 1	− 1,21	− 0,51		41, 3	− 2

Table 1. *(Continued)*

(1)	(2)	(3)	(4)	(5)	(6)	(7)
119	40,56	– 0, 5	– 0,55	40, 6	40, 9	– 3
120	1;39,52	–0;1, 4	–0;0,54	1;39,12	1;39,13	– 1
121	38,52	– 1, 0		38,12	38,15	– 3
122	37,53	– 0,59		37,13	37,15	– 2
123	36,51	– 1, 2		36,11	36,13	– 2
124	35,51	– 1, 0		35,11	35,10	+ 1
125	33,25	– 1,26	– 1, 6	34, 5	34, 5	0
126	32,58	– 0,27	– 1, 7		32,57	+ 1
127	31,50	– 1, 8			31,48	+ 2
128	30,39	– 1,11			30,38	+ 1
129	29,25	– 1,14			29,25	0
130	1;28,11	– 0;1,14			1;28,11	0
131	27,25	– 0,46	– 1,15	26,55	26,55	0
132	26,40	– 0,45	– 1,15	25,40	25,37	+ 3
133	24,50	– 1,50	– 1,20	24,20	24,20	0
134	23, 0	– 1,50	– 1,20		22,57	+ 3
135	21,54	– 1, 6	– 1,26	21,34	21,35	– 1
136	20,11	– 1,43	– 1,23		20,10	+ 1
137	18,44	– 1,27			18,45	– 1
138	17,18	– 1,26			17,17	+ 1
139	15,49	– 1,29			15,49	0
140	1;14,20	– 1,29			1;14,18	+ 2
141	12,47	– 1,33			12,17	0
142	11,13	– 1,34			11,13	0
143	9,39	– 1,34			9,39	0
144	8, 6	– 1,33			8, 3	+ 3
145	7,26	– 0,40	– 1,40	6,26	6,26	0
146	6,49	– 0,37	– 1,37	4,49	4,47	+ 2
147	5, 8	– 1,41		3, 8	3, 7	+ 1
148	3,27	– 1,41		1; 1,27	1; 1,26	+ 1
149	1; 1,10	– 2,17	– 1,37	0;59,50	0;59,43	+ 7
150	0;59,19	– 1,51		0;57,59	0;58, 0	– 1
151	57,21	– 1,58	– 1,38	56,21	56,15	+ 6
152	54,31	– 2,50	– 1,50		54,29	+ 2
153	52,28	– 2, 3	?		52,42	– 14
154	50,37	– 1,51	?		50,54	– 17
155	48,47	– 1,50	?		49, 6	– 19
156	47,17	– 1,30	?		47,15	+ 2
157	44,55	– 2,22	– 1,52	45,25	45,24	+ 1
158	43,32	– 1,23	– 1,53		43,33	+ 1
159	41,39	– 1,53			41,40	– 1
160	0;39,56	– 1,53			0;39,46	0

(Continued)

Table 1. *(Continued)*

(1)	(2)	(3)	(4)	(5)	(6)	(7)
161	37,51	– 1,55			37,52	– 1
162	35,57	– 1,54			35,57	0
163	34, 2	– 1,55			34, 1	+ 1
164	32, 6	– 1,56			32, 5	+ 1
165	30,14	– 1,52	?		30, 8	+ 6
166	28,19	– 1,55			28,10	+ 9
167	26,16	– 2, 3	?		26,11	+ 5
168	24,13	– 2, 3	?		24,13	0
169	22,14	– 1,59			22,13	+ 1
170	0;20,15	– 1,59			0;20,14	+ 1
171	18,14	– 2, 1			18,13	+ 1
172	16,43	– 1,31	– 2, 1	16,13	16,13	0
173	14,12	– 2,31	– 2, 1		14,12	0
174	12,11	– 2, 1			12,11	0
175	10, 8	– 2, 3	– 2, 1	10,10	10,10	0
176	8,10	– 1,58	– 2, 2	8, 8	8, 8	0
177	6, 6	– 2, 4	– 2, 2		6, 6	0
178	4, 4	– 2, 2			4, 4	0
179	2, 2	– 2, 2			2, 2	0
180	0; 0, 0	– 2, 2			0; 0, 0	0

III. Eclipse Theory in *Al-Muqtabis*

All the tables related to eclipse theory are mentioned in canons 25 (ff. 13rb–14rb) and 27 (ff. 15ra–16va). Some specific terminology in Latin is used in canon 25: "longitudo" for elongation, "preuentio" for opposition, "peruenencia solis" for the fraction of the elongation that "belongs to" the Sun, "precessio" for the hourly relative velocity of the luminaries.

A. Solar and lunar velocities

(f. 51v) "Tabula diuersi motus solis in una hora quod est respectus"
(f. 51v) "Tabula diuersi motus lune in una hora quod est respectus"

The use of these two tables is explained in canon 25 (f. 13va). Both tables coincide, except for copying errors, with those in AL-KHWÂRIZMÎ (SUTER (1914), pp. 175–180, tables 61–66). The extremal values are:

$$v_s(1^\circ) = 0;2,22^\circ/\text{h}, \text{ and } v_s(180^\circ) = 0;2,24^\circ/\text{h} \text{ (read: } 0;2,34^\circ/\text{h).}$$

$$v_m(1^\circ) = 0;30,12^\circ/\text{h}, \text{ and } v_m(180^\circ) = 0;35,40^\circ/\text{h}.$$

The same two tables are also found in the tables of JUAN GIL (London, Jews College, MS Heb. 135, f. 91r), and the Tables of Barcelona (MILLÁS (1962), table 43), as well as in a manuscript containing the Tables of Toulouse (Paris, B. N., MS Lat. 16658, ff. 90v–93r). The Toledan Tables (*cf.* TOOMER (1968), p. 82) and the Almanac of AZARQUIEL (MILLÁS (1950), p. 174), have tables for solar and lunar velocities that agree with those in AL-BATTÂNÎ, but differ from the present ones.

For the solar velocity table, the formula used in recomputing the entries is:

$$v = \bar{v} + \bar{v} * \Delta, \qquad [2]$$

where $\bar{v} = 0;59,8/24$ and $\Delta = c(\bar{\kappa} + 1) - c(\bar{\kappa})$; c is the solar equation as a function of the mean anomaly, $\bar{\kappa}$, taken from AL-KHWÂRIZMÎ's tables 21–26, col. 2. The recomputed values (which are not displayed here) are in good agreement with the entries in the table. Equation [2] has been used by analogy with equation [3], used for computing the lunar velocity table (*cf.* GOLDSTEIN (1992); GOLDSTEIN *et al.*):

$$v(\alpha) = 0;32,56 + 0;32,40 * \Delta, \qquad [3]$$

where $\Delta = c(\alpha + 1) - c(\alpha)$; c is the lunar equation as a function of the anomaly, α, taken from AL-KHWÂRIZMÎ's tables 21–26, col. 3. The results are shown in Table 2, together with a comparison between the entries in the tables of IBN-AL-KAMMÂD and AL-KHWÂRIZMÎ. The original table seems to be better preserved by AL-KHWÂRIZMÎ than by IBN AL-KAMMÂD. It should be noticed that previously this table had not been recomputed successfully (*cf.* AS-SALEH (1970), p. 162, and NEUGEBAUER (1962), p. 106).

The columns in table 2, Lunar Velocity, are arranged as follows:

(1) Lunar anomaly, α, for each integer degree from 1° to 180°.

(2) Entries in the text (f. 51v): lunar velocity in minutes and seconds of arc per hour.

(3) Variant readings in AL-KHWÂRIZMÎ's lunar velocity table (SUTER (1914), tables 61–66, col. 3); only the seconds are displayed here, except for 72°.

(4) Recomputed values at multiples of 5° of anomaly; only the seconds are displayed here, except for 135°.

Table 2. Lunar Velocity

(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
1	30;12		12	31	30;36		
2	30;12			32	30;37		
3	30;12			33	30;38		
4	30;13			34	30;40		
5	30;13		13	35	30;41		39

(Continued)

Table 2. *(Continued)*

(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
6	30;13			36	30;43		
7	30;14	13		37	30;44		
8	30;14			38	30;46		
9	30;14			39	30;47		
10	30;15	14	14	40	30;49		57
11	30;15			41	30;51		
12	30;15			42	30;52		
13	30;16			43	30;54		
14	30;17			44	30;56		
15	30;18	17	17	45	30;58		56
16	30;19	18		46	31; 0		
17	30;20	19		47	31; 2		
18	30;21	20		48	31; 4		
19	30;22	21		49	31; 6		
20	30;23	22	22	50	31; 8		7
21	30;24	23		51	31;10		
22	30;25	24		52	31;12		
23	30;26	25		53	31;14		
24	30;27			54	31;16		
25	30;28		26	55	31;21	18	17
26	30;29			56	31;23	21	
27	30;30			57	31;25	23	
28	30;32			58	31;27	25	
29	30;33			59	31;29	27	
30	30;34		34	60	31;29		29
61	31;32			91	32;58		
62	31;35			92	33; 0		
63	31;37			93	33; 2		
64	31;40			94	33; 5		
65	31;42		42	95	33; 7		15
66	31;45			96	33;10		
67	31;47			97	33;13		
68	31;49			98	33;16		
69	31;51	· 52		99	33;20		
70	31;54		55	100	33;23		26
71	31;57			101	33;26		
72	31; 1	32; 1		102	33;29		
73	32; 5			103	33;32		
74	32; 8			104	33;36		
75	32;12		12	105	33;39		44
76	32;17	16		106	33;42		
77	32;23	20		107	33;45		
78	32;27	24		108	33;48		
79	32;27			109	33;51		
80	32;31		32	110	33;54		54

Table 2. *(Continued)*

(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
81	32;34			111	33;57		
82	32;37			112	34; 0		
83	32;39			113	34; 3		
84	32;42			114	34; 5	6	
85	32;46	45	55	115	34; 9		12
86	32;47			116	34;12		
87	32;49			117	34;15		
88	32;52			118	34;17		
89	32;54			119	34;20		
90	32;56		57	120	34;22		24
121	34;26	25		151	35;21		
122	34;27			152	35;22		
123	34;30			153	35;24		
124	34;32			154	35;25		
125	34;36	34	36	155	35;26		49
126	34;37			156	35;27		
127	34;40			157	35;28		
128	34;45	42		158	35;29		
129	34;45			159	35;30		
130	34;48		49	160	35;31		31
131	34;50			161	35;32		
132	34;53			162	35;33		
133	34;55			163	35;33		
134	34;58			164	35;34		
135	35; 0		34;57	165	35;35		36
136	35; 1			166	35;35		
137	35; 3			167	35;36		
138	35; 4			168	35;37		
139	35; 5			169	35;37		
140	35; 6		7	170	35;38		38
141	35; 8			171	35;38		
142	35; 9			172	35;38		
143	35;10			173	35;39		
144	35;12			174	35;39		
145	30;13		17	175	35;39		39
146	35;14			176	35;39		
147	35;16			177	35;39		
148	35;17			178	35;40		
149	35;18			179	35;40		
150	35;20		22	180	35;40		40

B. Time from mean to true syzygy

(f. 52r) "Tabula horarum longitudinis ad dirigendum tempus coniunctionis et preuentionis"

The use of this table is explained in canon 25 (f. 13va). It is a double argument table: the vertical argument is the elongation (e), given in degrees and minutes, from 0;30° to 12;0° at intervals of 0;30°. The horizontal argument is the velocity of the Moon relative to that of the Sun ($v_m - v_s$), in minutes and seconds of arc per hour, from 0;27,30°/h to 0;33,30°/h, at intervals of 0;0,30°/h.

Each entry (t) can be computed from the following equation:

$$t = e/(v_m - v_s), \qquad [4]$$

where t is the time, given in hours and minutes, that the Moon takes to travel the longitudinal arc between the Sun and the Moon at mean syzygy, *i.e.* the time interval from mean to true syzygy. For a discussion of tables for finding the time from mean to true syzygy, see CHABÁS & GOLDSTEIN.

The entries in the column for 0;32,0°/h are also copied, erroneously, in the column for 0;32,30°/h. A similar, but not quite identical, table is found in the Tables of Barcelona (MILLÁS (1962), table 42).

C. Solar eclipses

(f. 52v) "Tabula rectitudinum ad eclipses solares"

This table in 7 columns (see table 3), is mentioned in canon 27 (f. 15rb), and gives the declination of midheaven, *i.e.*, the intersection of the ecliptic and the local meridian, in degrees and minutes, as a function of the longitude of the ascendant. The manuscript uses "septentrional" for north, and "meridional" for south, which we have transcribed by assigning positive or negative signs, respectively, to the tabulated entries.

The declination reaches its maximum of 23;50° at Libra 1° and its minimum of −23;51° at Aries 0°. Note that 23;51° is the value used by PTOLEMY in the Handy Tables for the obliquity of the ecliptic. This table is closely related to a table found in an Arabic manuscript (MS Escorial Ar. 927, ff. 9v, 12r, 12v: see KENNEDY (1986)), where it is called "the Table of *samt* for determining solar eclipses" (*jadwal al-samt li-ᶜilm kusûf al-shams*). KENNEDY & FARIS (pp. 21–24) have described – and presented a graph – of this table, based on the copy in the Escorial where it appears among tables attributed to YAḤYÂ IBN ABÎ MANṢÛR (9th century). The entries in this table in MS Madrid 10023, f. 52v, are displayed in table 3 and, for comparison, those in the Arabic manuscript in the Escorial are displayed in a separate table (see table 4), where the underlining of entries indicates that they differ from those in table 3. Both tables are clearly variants of the same archetype despite many discrepancies that can be ascribed to copyist errors. The Arabic copy displays 180 additional entries because it fails to recognize the symmetries (see KENNEDY & FARIS, p. 24). In addition to the

Table 3. Table of the *samt* for Determining Solar Eclipses (MS Madrid 10023, f. 52v)

Degree of the ascendant		Lib	Sco	Sag	Cap	Aqu	Pis
1	29	+ 23;50	+ 19;25	+ 6;47	− 8;35	− 18;34	− 22;47
2	28	23;50	19; 5	6;23	8;32	18;50	22;49
3	27	23;48	18;50	6; 0	9;27	19; 5	22;52
4	26	23;47	18;35	5;53	9;50	19;19	22;57
5	25	23;45	18;33	5;50	10;12	19;20	22;59
6	24	23;42	18; 2	4; 1	10;32	19;35	23; 0
7	23	23;39	17;46	3;37	10;53	19;50	23; 5
8	22	23;36	17;29	3;33	11;18	20; 3	23;12
9	21	23;32	17;12	2;50	11;59	20;16	23;17
10	20	23;23	16;55	2; 1	12; 1	20;33	23;18
11	19	23;16	16;36	1;36	12;22	20;35	23;23
12	18	23;12	16;18	0;48	12;48	20;39	23;24
13	17	23; 5	15;41	+ 0;24	13; 4	20;47	23;26
14	16	22;59	15; 3	0; 0	13;23	20;52	23;27
15	15	22;52	14;24	− 0;24	13;43	20;55	23;32
16	14	22;44	14; 5	0;48	14; 5	20;58	23;36
17	13	22;37	13;24	1;36	14;24	21; 0	23;37
18	12	22;23	13; 4	1; 1	14;44	21; 4	23;39
19	11	22;18	12;22	2;20	15; 3	21; 7	23;39
20	10	22;11	12;39	3;53	15;23	21;18	23;42
21	9	21;51	11;39	3;37	15;41	21;29	23;42
22	8	21;40	11;33	4; 1	16; 0	21;40	23;45
23	7	21;38	10;36	4;25	16;18	21;41	23;45
24	6	21;18	10;12	5;13	16;36	21;51	23;47
25	5	21; 7	9;52	5;36	16;44	22; 1	23;48
26	4	20;55	9;44	6; 0	16;56	22;10	23;48
27	3	20;42	8;42	6;24	17;12	22;10	23;50
28	2	20;29	8;19	7;10	17;27	22;19	23;50
29	1	20;16	7;23	7;18	17;46	22;28	23;51
30	0	+ 20; 3	+ 7;10	− 7;16	− 18; 2	− 22;35	− 23;51
		Vir	Leo	Can	Gem	Tau	Ari

value adopted for the obliquity of the ecliptic, the table depends on the latitude of the place for which it is intended. In this case, the latitude used seems to be 35;55,48° (*cf.* MS Escorial Ar. 927, f. 8v), even though the use of seconds for geographical latitude was not meaningful at the time; this latitude corresponds to YAḤYÂ's native Ṭabaristân (*cf.* KENNEDY & FARIS, p. 24). However, it is quite likely that IBN AL-KAMMÂD, who is associated with Córdoba (latitude = 38;30° as it appears in the geographical table on f. 54v), did not know the geographical latitude for which the table was computed. It is worth noting that the table for solar declination (f. 35v) is based on a different value for the obliquity of the ecliptic: 23;33°.

Table 4. Table of the *samt* of Determining Solar Eclipses (MS Escorial Ar. 927, ff. 9v, 12r, 12v)*

Degree of the ascendant		Lib	Sco	Sag	Cap	Aqu	Pis
1	29	+ 23;50	+ 19;25	+ 6;47	− 8;39	− 18;34	− 22;39
2	28	–	19; 5	6;23	8;42	18;50	22;45
3	27	23;48	18;50	6; 0	9;27	19; 5	22;51
4	26	23;46	18;35	5;53	9;50	19;15	22;56
5	25	23;45	18;33	5; 7	10;12	19;25	23; 0
6	24	23;42	18; 2	5; 1	10;32	19;35	23; 4
7	23	23;39	17;56	4;37	10;56	19;50	23; 8
8	22	23;36	17;29	4;13	11;18	20; 3	23;11
9	21	23;32	17;12	3;50	11;39	20;18	23;14
10	20	23;23	16;55	3; 1	12; 1	20;33	23;17
11	19	23;16	16;36	2;36	12;22	20;35	23;20
12	18	23;12	16;18	1;48	12;48	20;39	23;23
13	17	23; 5	15;41	+ 0;24	13; 5	20;42	23;27
14	16	22;59	15; 3	0; 0	13;23	20;47	23;28
15	15	22;52	14;24	− 0;24	13;43	20;55	23;30
16	14	22;44	14; 5	0;48	14; 5	20;58	23;32
17	13	22;37	13;24	1;36	14;24	21; 0	23;34
18	12	22;27	13; 4	2; 1	14;44	21; 5	23;36
19	11	22;18	12;22	2;25	15; 3	21; 7	23;38
20	10	22;11	12;30	3;53	15;23	21;18	23;40
21	9	21;51	11;39	3;36	15;41	21;29	23;41
22	8	21;40	11;33	4; 1	16; 0	21;40	23;42
23	7	21;30	10;36	4;24	16;18	21;41	23;43
24	6	21;18	10;12	5;13	16;36	21;51	23;44
25	5	21; 7	9;50	5;36	16;44	22; 1	23;45
26	4	20;55	9; 5	6; 0	16;55	22;10	23;46
27	3	20;42	8;42	6;24	17;12	22;14	23;47
28	2	20;29	8;19	7;10	17;26	22;19	23;48
29	1	20;16	7;13	7;13	17;56	22;28	23;49
30	0	+ 20; 3	+ 7;10	− 7;16	− 18;10	− 22;32	− 23;50
		Vir	Leo	Can	Gem	Tau	Ari

* The columns for Libra, Scorpio, and Sagittarius are transcribed from f. 9v; the columns for Capricorn and Aquarius from f. 12v; and the column for Pisces from f. 12r, for only one entry appears on f. 12v while the rest of the column on that page is blank. In the following list we display all entries on f. 12r, that differ from those of f. 12v. Variant readings from f. 12r:

Cap	2 = 9; 2	Cap	6 = 10;34	Cap	8 = 11;17	Cap	12 = 12;43
Cap	14 = 13;25	Cap	15 = 13;45	Cap	18 = 14;43	Aqu	2 = 18;47
Aqu	3 = 19; 1	Aqu	4 = 19;15	Aqu	5 = 19;26	Aqu	6 = 19;38
Aqu	8 = 20; 4	Aqu	9 = 20;18	Aqu	11 = 20;36	Aqu	13 = 20;42
Aqu	14 = 20;47	Aqu	15 = 20;52	Aqu	17 = 21; 5	Aqu	18 = 21;12
Aqu	19 = 21;19	Aqu	20 = 21;26	Aqu	21 = 21;33	Aqu	23 = 21;47
Aqu	24 = 21;54	Aqu	26 = 22; 7	Aqu	27 = 22;18	Aqu	29 = 22;25

Clearly, in the Arabic copy f. 12v has the better readings; the copyist apparently realized that f. 12r had many errors, and so he tried again on f. 12v. The table of the *samt* discussed here is also found in the Tables of Barcelona (MILLÁS (1962), table 47), and suffers from the same errors as the table of *al-Muqtabis* in MS Madrid 10023.

A method for recomputing the entries of the table has been suggested by NEUGEBAUER (*cf.* KENNEDY & FARIS, p. 24), and it consists of 3 steps:

(i) for each integer value of the argument (the longitude of the ascendant) find its oblique ascension by means of a table for the appropriate latitude (NEUGEBAUER suggested using a table for 36° since no such table is known for 35;55,48°);
(ii) in a table for normed right ascensions (see ff. 48v–49r; *cf.* AL-KHWÂRIZMÎ's table in SUTER (1914), pp. 171–173), find the longitude for which its normed right ascension equals the value obtained previously;
(iii) the declination corresponding to that longitude in a table of declinations is the entry sought.

D. *Lunar eclipses*

(f. 52v) "Tabula eclipsium lunarium"

The use of this table (see table 5) is described in canon 27 (f. 15vb). Columns 2–4 display the lunar eclipse magnitude (digits), the duration of the eclipse (hours) and the duration of totality (hours), as functions of the argument of latitude of the Moon (degrees). The headings are "prima porta", "secunda porta", "tercia porta", respectively, while that for the argument is "longitudo a capite et cauda".

Among the material appearing at the end of this manuscript after the tables associated with *al-Muqtabis*, folio 57v has the same table under the title: "Hec tabula est quam extraxit et composuit ALKEMED, in eclipsibus lunaris in canone suo que est extracta a canone EBI IUSUFI cognoscitur BYN TARACH, que est ualde uerax". This author is probably to be identified with the late 8th century astronomer YA'QÛB IBN ṬÂRIQ, a collaborator of AL-FAZÂRÎ at Baghdad, and whose zij was called the Sindhind (*cf.* PINGREE (1968b) and (1970)). Note that EBI IUSUFI, or ABÛ YÛSUF, means the father of JOSEPH, and, in Arabic nomenclature, this "nickname" can be substituted for JACOB, who was the father of the Biblical JOSEPH. Previously, MILLÁS (1942), p. 245, had suggested that the name here was a corrupt form of the name of the 11th century astronomer MUḤAMMAD BEN YÛSUF BEN AḤMAD IBN MU'ÂDH, from Jaén.

The two versions of this table are displayed in table 5. The entries in both versions of this table seem to be quite corrupt, and do not allow us to derive the parameters underlying them. Another version of this table is found in the Tables of Barcelona (MILLÁS (1962), table 50), and the entries in it are also corrupt.

Note that there is a single table for lunar eclipses, which is quite uncommon, for almost all zijes have two such tables (one for minimum, and one

Table 5. Table for Lunar Eclipses

	Col. 2 (d)				Col. 3 (h, min, s)						Col. 4 (h, min, s)					
	f. 52v		f. 57v		f. 52v			f. 57v			f.52v			f. 57v		
12	0	55	0	54	0	6	0	0	6	0	0	0	0	0	0	0
11	1	35	1	35	0	14	0	0	54	5	0	0	0	0	0	0
10	3	50	3	50	0	50	40	1	50	40	0	0	0	0	0	0
9	4	16	4	56	1	6	50	2	6	50	0	0	0	0	0	0
8	6	47	6	47	1	43	9	2	2	9	0	0	0	0	0	0
7	8	19	8	39	1	57	50	2	36	50	0	0	0	0	0	0
6	10	29	10	29	2	0	9	3	0	20	0	0	0	0	0	0
5	12	0	11	0	2	0	50	3	0	50	0	0	48	0	0	40
4	12	0	12	0	2	0	0	3	0	0	1	2	40	0	2	40
3	12	0	12	0	2	20	0	4	20	0	1	6	48	1	6	48
2	12	0	12	0	2	25	0	4	40	0	1	8	48	1	8	43
1	12	0	12	0	2	30	0	4	50	0	1	10	12	1	10	42

for maximum, lunar distance). The astronomical work of JACOB BEN DAVID BONJORN, as astronomer of the 14th century from Perpignan, has only one table for lunar eclipses, but the entries in it are unrelated to those here (*cf.* CHABÁS (1991), p. 309).

E. Color of eclipses

(f. 52v) "Colores"

This table (see table 6) is arranged in 6 columns: column 1 displays the argument of lunar latitude; col. 2 gives the color of solar eclipses as a function of the argument of lunar latitude, in degrees; col. 3 displays lunar latitude in minutes; col. 4 gives the color of lunar eclipses in terms of lunar latitude; cols. 5 and 6 (not shown here) display the magnitudes of solar eclipses in area digits as a function of the magnitude of the eclipse in linear digits. Canon 27 (f. 16va) refers to the first four columns, whereas the magnitudes of eclipses are treated on f. 15vb. All six columns are also found in the Tables of Barcelona (MILLÁS (1962): cols. 1–4 appear in table 51, and cols. 5–6 in table 48).

KENNEDY (1956a), p. 159, has noted that Treatise VIII of AL-BÎRÛNÎ's *Qânûn al-Mas^cûdî* has a chapter on the colors of solar and lunar eclipses. On the colors of lunar eclipses, see GOLDSTEIN (1967), pp. 234–235. For IBN AL-MUTHANNÂ (10th century), the color changes during the eclipse, whereas for IBN EZRA (MILLÁS (1947), p. 167) color is a function of latitude, as is the case here. Chapter 151 of *Kitâb al-ᶜAmal bi'l-Asturlâb*, by the Persian AL-ṢÛFÎ (903–986) contains a similar list for the colors of lunar eclipses, but with different entries from those presented here (see KENNEDY & DESTOMBES, p. 413).

Table 6. Table for the Color of Eclipses

(1) Arg. lat.	(2) Solar eclipse	(3) Lat.	(4) Lunar eclipse
1	valde niger	10	niger valde
2	niger clarus		in nigredine
3	turbatus rubeus	20	niger
4	turbatus croceus		cum rubedi
5	turbatus clarus	30	niger
6	turbatus cinereus		cum rubediem
7	cinereus	40	niger
8	cinereus		cum croceo
9	cinereus	50	turbatus
10	cinereus		
11	croceus	60	cinereus
12	rubeus albus		

Chapter 35 of the *Libro de las Taulas Alfonsíes*, "De qué color sera ell eclipsy", concerns lunar eclipses, and gives two different rules for their colors. These rules show similarities with those in the above-mentioned tables, but do not fully agree with them.

The columns concerning the areas of eclipses (cols. 5–6) appear in a number of earlier tables: the Toledan Tables (Toomer (1968), p. 113, table 76), the zij of al-Battânî (Nallino, II, p. 89), the Almanac of Azarquiel (Millás (1950), p. 233), and in al-Khwârizmî's zij (Suter (1914), p. 190, table 76, columns 6–7). In fact, this table is already found in Ptolemy's Almagest (VI, 8) and Handy Tables (Stahlman (1959), p. 258). The entries in all these tables agree, except in the Toledan Tables, where the entries for 3 and 5 (linear) digits are, respectively, 1;50 d and 3;20 d instead of 1;45 d and 3;40 d.

F. Parallax in latitude

(f. 53r) "Tabula latitudinis solis indicate que est diuersitas respectus lune in latitudine specialiter"

The use of this table (see table 7, below) is described in canon 27 (f. 15rb). The table displays the adjusted parallax in latitude, P_β, in minutes and seconds of arc, where "adjusted parallax" means the difference between the lunar and the solar parallax (see KENNEDY (1956b), p. 35). Its maximum value is 0;48,32° at 90°. The same table appears in AL-KHWÂRIZMÎ's zij (SUTER (1914), pp. 191–192, tables 77 and 77a, col. "Diversitas respectus in latitudine"), but the values tabulated there differ in all cases, *e.g.*, the entry for 90° is 0;48,45°. This specific table was discussed by NEUGEBAUER (1962), pp. 121–123.

Among the eclipse tables attributed to YAḤYÂ IBN ABÎ MANṢÛR, and analysed by KENNEDY & FARIS (pp. 20–38), there is a table entitled "table for the solar latitude" (*jadwal ᶜard al-shams*) which coincides with this one (MS Escorial Ar. 927, ff. 10v and 71v). Note the absurdity of the title for this table which, in fact, deals with the latitudinal component of the adjusted parallax. KENNEDY & FARIS (p. 25) give the function that underlies the entries of the table:

[5] $$P_\beta = 0;48,32 * \sin(\theta),$$

where θ is the solar zenith distance. They also state that the author of the calculations necessary for this table did "an extraordinarily bad job", as the results almost never agree with the recomputed ones. These irregularities, which also occur in this table by IBN AL-KAMMÂD, allow us to relate it with confidence to that of YAḤYÂ.

The same table is also found in the Tables of Barcelona (MILLÁS (1962), table 46), and it exhibits the same inconsistencies as our text. Comparison between the entries in *al-Muqtabis* and the recomputed values seem to indicate that some shifts of entries occurred in the copying process (entries for 23°–30° have been shifted one place downwards, entries for 33°–35° have also been shifted one place, and entries around 67° may have been shifted two places downwards). In any case, we are far from the smoothness of the analogous table in AL-KHWÂRIZMÎ where the maximum value is 0;48,45°.

The columns in table 7 for the adjusted parallax in latitude are the following:

(1) Solar zenith distance for each integer degree from 1° to 90°.

(2) Entries in the text: adjusted parallax in latitude (P_β), in minutes and seconds of arc.

(3) Variant readings in the Tables of Barcelona (MS Ripoll 21).

(4) Recomputed values by means of equation [5].

G. Lunar latitude

(f. 53r) "Tabula latitudinis lune indicate"

This table is mentioned in canon 27 (f. 15rb), and it gives the lunar latitude (β) as a function of the argument of lunar latitude (ω). It is also found, with minor variant readings, in AL-KHWÂRIZMÎ's zij (SUTER (1914), pp. 132–134, tables 21–26; cf. NEUGEBAUER (1962), pp. 95–98), where the maximum latitude is 4;30°.

Table 7. Table for the Adjusted Parallax in Latitude

(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
1	0;50			46	34;30		
2	1;41			47	35;56	34;56	
3	2;32	2;33		48	35; 0		
4	3;23			49	35;52		
5	4;13		4;14	50	36; 8		37;11
6	5; 3	5; 4		51	36;44		
7	5;55			52	37;21		
8	6;45			53	38; 7		
9	7;37			54	38;59	38;39	
10	8;23	8;27	8;26	55	39;28	38;48	39;45
11	9;16			56	39;22	39;21	
12	10; 5			57	39;36		
13	10;54			58	39;50		
14	11;44			59	40;56	40;16	
15	12;33		12;34	60	40;44		42; 2
16	13;23			61	40;55		
17	14;12			62	41;43		
18	15; 1			63	41;54	41;53	
19	15;51			64	42;54	42;34	43;37
20	16;41		16;36	65	42;57		43;59
21	17;32	17;38		66	43;57	43;17	44;20
22	18;22	18;26	18;11	67	43;59	43;39	
23	18;12	19;14	18;58	68	44; 0		
24	18;58	19;58	19;44	69	44;22		
25	19;44	20;44	20;31	70	44;43		45;36
26	20;31		21;17	71	45; 5		
27	21;17		22; 2	72	45;27		
28	22; 3		22;47	73	45;48		
29	22;49		23;32	74	46; 2		46;39
30	23;35		24;16	75	46;16		46;53
31	23;35	24;22	25; 0	76	46;29	49;39	47; 6
32	24;22	25; 4	25;43	77	46;42		
33	25;46		26;26	78	46;55		
34	26;28		27; 8	79	47; 8		
35	27;10		27;50	80	47;21		47;48
36	28;32		28;32	81	47;34		
37	29;34	28;44		82	47;40		
38	29;56			83	47; 0	48; 0	
39	30;37			84	47; 5	48; 5	
40	31;38	31;22	31;12	85	47; 9	48; 9	48;25
41	31;22	31;42		86	47;14	48;14	
42	32;48			87	47;18	48;18	
43	33;52	33;12		88	47;22	48;22	
44	33;58			89	48;27		
45	34; 4		34;19	90	48;32	48;34	48;32

KENNEDY & UKASHAH, pp. 95–96, have shown that the entries in this table were computed according to the "method of sines" given by the formula:

$$\beta = 4;30 * \sin(\omega). \qquad [6]$$

KENNEDY (1956a), p. 146, mentions that a similar table, with the same maximum value, appears in the zij of YAḤYÂ IBN ABÎ MANṢÛR.

The maximum value in the table on f. 53*r* is 4;29° instead of 4;30°. This does not suggest a different parameter; rather, it should be interpreted as a variant reading of 4;30°. The same table is also found in the Tables of Barcelona (MILLÁS (1962), table 44), and exhibits the same characteristics.

The table for the lunar latitude on f. 35v of this MS displays a different maximum: 5;0°, which is the value used by PTOLEMY, AL-BATTÂNÎ, AZARQUIEL, among many others.

H. Elongation

(f. 53v) "Tabula longitudinis et dimidii sexti eius"

Canon 27 (f. 13rb) explains the use of this table. Two sets of entries are tabulated, the lunar longitude (L_m) and the solar longitude (L_s); both sets of entries are functions of the elongation (e) between the Moon and the Sun, given in degrees and minutes, from 0;30° to 12;0°, at intervals of 0;30°. The entries L_m and L_s are such that $e = L_m - L_s$, where $L_m = 13e/12$ and $L_s = e/12$.

A similar, but more extensive table, is found in the Tables of Barcelona (MILLÁS (1962), table 41): the elongation is given at intervals of 0;6° instead of 0;30°, and it ranges from 0° to 13;12° instead of from 0° to 12;0°.

I. Parallax in longitude

(f. 53v) "Tabula diuersitatis respectus lune in longitudine"

This table, mentioned in canon 25 (f. 14ra), gives the longitudinal component of adjusted parallax (P_λ), in hours and minutes, as a function of the argument, given in time from 0;15 h to 9 h, at intervals of 0;15 h.

The entries reach a maximum of 1;36 h, and can be easily derived from the column with the heading "Horae diversorum/diversitatis respectuum lunae [in longitudine]" in AL-KHWÂRIZMÎ's zij (SUTER (1914), pp. 191–192, tables 77 and 77a; *cf.* NEUGEBAUER (1962), pp. 121–126). However, in AL-KHWÂRIZMÎ's zij, (i) the argument is not given in time, but in degrees, and (ii) the parallax in longitude is given to seconds. KENNEDY (1956b), pp. 49–50, has shown that AL-KHWÂRIZMÎ's table can be computed by means of the following formula:

$$P_\lambda = 1;36 * \sin(\theta(t)) \qquad [7]$$

where

[8] $$t = \theta - (\varepsilon * \sin(\theta)),$$

such that θ, the argument (in degrees), meets the condition that $0° \leqslant \theta \leqslant 150°$. The coefficient 1;36 = 0;4 * 24 = 24/15 contains the standard Hindu value for the obliquity of the ecliptic, 24°. In the case of IBN AL-KAMMÂD's table, the entries can be recomputed by means of equation [7], where

[9] $$t = (\theta - (24 * \sin(\theta))/15,$$

for all $0° \leqslant \theta \leqslant 135°$; now, 135° = 9 h * 15°/h, and 9 h is the maximum value of the argument, expressed in time.

Only two such adjusted longitudinal parallax tables of this kind are known: that of AL-KHWÂRIZMÎ and the one discovered by KENNEDY (KENNEDY (1956b), p. 48) in the zij of IBN AL-SHÂṬIR (*ca.* 1350), explicitly paraphrasing an early Islamic source that has not been identified (KENNEDY & FARIS, pp. 33–38). We can now add to that short list the parallax table of IBN AL-KAMMÂD and that in the Tables of Barcelona (MILLÁS (1962), table 45).

J. "Tabula eclipsium solarium" (f. 54r)

Column 2 of this table gives the magnitude of the eclipse, in (linear) digits and minutes, as a function of the adjusted latitude (at conjunction) of the Moon displayed in column 1, in minutes and seconds of arc, from 0;34,13° to 0°. The explanation of this table in canon 27 (f. 15va) confirms the above value for the eclipse limit: "si fuerit minus 34 minutis et 13 secundis erit eclipsis".

Columns 3–9 form a double argument table. The vertical argument is the eclipse magnitude in digits (col. 2); the horizontal argument is the relative velocity of the Moon with respect to the Sun ($v_m - v_s$), in minutes and seconds of arc per hour, from 0;27,30°/h to 0;33,30°/h, at intervals of 0;1°/h.

This table, as some previous ones, seems to derive from YAḤYÂ IBN ABÎ MANṢÛR (MS Escorial Ar. 927, f. 13r). KENNEDY & FARIS (pp. 27–30), once again, have transcribed and explained this table.

The adjusted latitude of the Moon is a linear function of the magnitude of the eclipse, so that the graph of the function relating columns 1 and 2 should be a straight line. However, this is not the case: there are "jumps" at 4;20 and 9;20 digits; and one should consider the first three entries in col. 2 and 0 *d*, 20 *d*, and 40 *d*, instead of 0;15 *d*, 0;30 *d*, and 0;45 *d* (these errors appear in this table in the Arabic MS as well as in our Latin text).

The second part of the table displays the half-duration of the eclipse, in hours and minutes, as a function of its magnitude and the relative velocity of the Moon with respect to the Sun. The recomputations made by KENNEDY & FARIS (p. 29) give good results, but fail to reproduce the entries of the table precisely.

IV. Other Tables in *Al-Muqtabis*

A. *Calendaric tables* (ff. 27r–v)

The purpose of these tables is to convert dates from the Arabic calendar to "Roman" (*i.e.*, Julian) and Egyptian calendars. The epoch of the radix given is the Hijra: noon of July 14 A.D. 622. Canon 9 (f. 6ra) states that the epoch of the radix is in "the day of Mercury" (Wednesday). The tables display two correspondences: A.H. 0 = 932 julian years 9 months 17;0 days from the beginning of the Seleucid era = 9 Egyptian years 11 months 9 days from the beginning of the Yazdijird era.

These tables entirely or partially reproduce calendaric tables in the zij of AL-KHWÂRIZMÎ/MASLAMA (SUTER (1914), p. 110, table 2; p. 111, table 2a; and p. 113, table 3), and/or in pseudo-BATTÂNÎ (MASLAMA) (NALLINO, II, pp. 301, 304–305).

B. *Trepidation* (ff. 28v, 35v)

(f. 28v) "Tabula aduentionis puncti capitis arietis"

The same radix and equivalent entries for the mean motion of the vernal point are found in AZARQUIEL's *Treatise on the motion of the fixed stars* (Paris, B.N. MS Heb. 1036; see MILLÁS (1950), pp. 266, 324); the *Liber de motu octave sphere*, attributed to THÂBIT IBN QURRA displays a similar table (MILLÁS (1950), p. 507); *cf.* MORELON (1987), p. xix). On the theory of trepidation see GOLDSTEIN (1964), DOBRZYCKI (1965), NORTH (1967), NORTH (1976), vol. 3, pp. 155–158, MERCIER (1976–77) and SAMSÓ (1992).

(f. 35v) "Tabula directionis aduentionis capitis arietis"

The entries (see table 8) display very nearly a sine function whose maximum is 9;59° at 90°; we have not succeeded in explaining the deviations from the sine function, *e.g.* the entry for 30° is not half the entry for 90°. TOOMER (1968), p. 118, gives two tables, which are also sine functions, associated with the Toledan Tables, and in fact they already appear in the *Liber de motu octave sphere* (MILLÁS (1950), pp. 507–508). The radix and the mean motion of the first point in Aries (see f. 28v and the comments on it, above) are taken from AZARQUIEL's *Treatise on the motion of the fixed stars*. The use of this table is briefly outlined in canon 12 (f. 7v), where another work by IBN AL-KAMMÂD is explicitly mentioned: *al-Amad ᶜalâ al-abad*. This table is also found in the Tables of Barcelona (MILLÁS (1962), table 20), a treatise by AL-MARRÂKUSHÎ (SÉDILLOT (1834), p. 131), and in the tables of JUAN GIL (London, Jews College, MS Heb. 135, f. 78b).

We have tried to recompute this table in many ways, and by far the best fit comes from formula [10] which is intended to represent AZARQUIEL's somewhat vaguely defined second model (MILLÁS (1950), pp. 287–289, 317–318; *cf.* SAMSÓ (1992), p. 230):

[10] $$\sin(e) = r * \sin(i)/60$$

Table 8. Table for the Motion of the First Point in Aries

Degrees		0/6	1/7	2/8
1	29	0;10	5;16	8;52
2	28	0;20	5;25	8;56
3	27	0;31	5;34	9; 1
4	26	0;41	5;43	9; 5
5	25	0;53	5;52	9;10
6	24	1; 3	6; 4	9;14
7	23	1;54	6;16	9;17
8	22	1;25	6;29	9;21
9	21	1;35	6;41	9;24
10	20	1;45	7;53	9;28
11	19	1;56	7;57	9;31
12	18	2; 8	7; 2	9;35
13	17	2;19	7; 6	9;38
14	16	2;30	7;10	9;41
15	15	2;41	7;14	9;45
16	14	2;50	7;21	9;47
17	13	2;18	7;28	9;49
18	12	3; 6	7;35	9;51
19	11	3;15	7;42	9;52
20	10	3;25	7;48	9;55
21	9	3;36	7;54	9;56
22	8	3;47	8; 0	9;56
23	7	3;47	8; 7	9;57
24	6	4; 8	8;13	9;58
25	5	4;19	8;20	9;59
26	4	4;29	8;25	9;59
27	3	4;39	8;31	9;59
28	2	4;49	8;37	9;59
29	1	4;38	8;41	9;59
30	0	5; 7	8;47	9;59
		5/11	4/10	3/9

Variant readings in the Tables of Barcelona (MS Ripoll 21, f. 137r):

e (7) = 1;14 e(16) = 2;51 e(17) = 2;58 e(29) = 4;58
e(40) = 6;43 e(41) = 6;57 e(50) = 7;54 e(51) = 7;58
e(53) = 8; 6 e(55) = 8;19

where e is the entry in the table, r = 10;24, and i is the argument. The maximum entry in the table, 9;59°, is indeed a rounded value for arcsin (10;24/60) = 9;58,54°.

Geometrically, we can understand the underlying model by referring to Figure 1: a small circle or epicycle, *BCE*, whose center is *A*, lies in the plane of

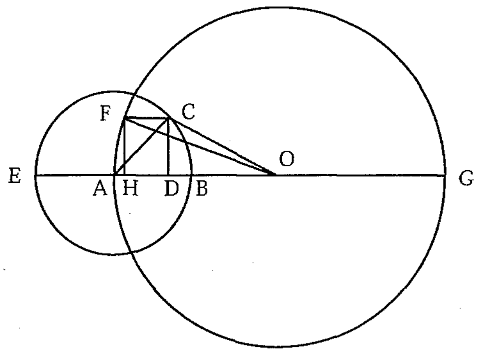

Fig. 1. The geometrical model underlying IBN AL-KAMMÂD's table for trepidation, as reconstructed.

Table 9. Trepidation According to IBN AL-KAMMÂD

(1)	(2)	(3)	(4)
10	1;45	1;43	2
20	3;25	3;24	1
30	5; 7	4;58	9
35	5;52	5;42	10
40	6;43*	6;24	19
45	7;14	7; 2	12
50	7;48	7;38	10
60	8;47	8;38	9
70	9;28	9;22	6
80	9;55	9;50	5
90	9;59	9;59	0

* There is textual problem with this entry (see Table 8); hence we display surrounding values.

the ecliptic, circle AG, and the center of the sphere is O. Note that the small circle BCE is partly inside, and partly outside, the sphere. The angle i is equal to arc BC; radius $AB = 10;24$, and $OA = 60$. Through C, draw a line parallel to BA, reaching the large circle AG at F. Draw OF, and let AOF be the angle e, which we seek. To compute angle e, we drop a perpendicular FH from F to AO; then $FH = CD$, and $CD = r * \sin(\mathrm{i})$. In right triangle FHO, $FO = 60$, and $\sin(e) = FH/FO$; equation [10] follows. We cannot account for the remaining differences between text and computation. The columns in table 9 for trepidation according to IBN AL-KAMMÂD are as follows:

(1) i: argument
(2) e: text of IBN AL-KAMMÂD

(3) e: computed, using equation [10]
(4) the difference: T(ext) – C(omp.), in minutes

The value 10;24 for r in equation [10] is made plausible by the two other models for trepidation associated with AZARQUIEL and his followers. SAMSÓ (1992), pp. 235–236, describes the third model of AZARQUIEL and shows how it yields a maximum value, P_{max} very close to 10;24° (10;23,29°), based on the equations:

[11] $$\delta = r * \sin(i),$$

[12] $$\sin(P) = \sin(\delta)/\sin(23;33°)$$

where P is the amount of precession corresponding to an argument i, r is 4;7,58°, and 23;33° is the value for the obliquity. Moreover, NEUGEBAUER (1962), p. 184, presents a formula based on AZARQUIEL's first model:

[13] $$\sin(P) = \sin(P_{max}) * \sin(i),$$

If we substitute 10;24° for P_{max} in equation [13], we find a set of values that agree very well with those preserved by the late 13th century astronomers IBNAL-BANNÂ' (MS Escorial Ar. 909, f. 22v) and IBN AL-RAQQÂM (MS Kandilli 249, f. 66v) who only tabulated these entries to degrees and minutes. However, since the agreement is equally good using equations [11] and [12] on the one hand, and equation [13] on the other, we cannot decide which procedure was used for computing this table. Table 10 displays these computations, where the columns are the following:

(1) i: argument
(2) e: text of IBN AL-BANNÂ' and IBN AL-RAQQÂM
(3) e: computed, using equation [13]
(4) e: computed, using equations [11] and [12]

Table 10. Trepidation According to IBN AL-BANNÂ' and IBN AL-RAQQÂM

(1)	(2)	(3)	(4)
10	1;48	1;47,47	1;47,48
20	3;32	3;32,23	3;32,24
30	5;11	5;10,43	5;10,40
40	6;40	6;39,48	6;39,40
50	7;57	7;56,55	7;56,42
60	8;59	8;59,39	8;59,20
70	9;46	9;45,59	9;45,35
80	10;14	10;14,25	10;13,56
90	10;24	10;24	10;23,29

C. *Mean motion tables* (ff. 28r–34v).

The headings of the tables indicate that they were intended for the meridian of Córdoba and calculated for Arabic years, months, *etc.* We have computed all mean motions from the corresponding tabulated values for A.H. 720, except for that of the lunar anomaly. The method is simply to subtract the corresponding radix from each value for A.H. 720, taking into account the full rotations, and then to divide the result by the number of days elapsed from the epoch of the Hijra calendar.

Sun (f. 28r)

The tabulated values for the mean motions are given to seconds. They do not agree with either AL-KHWÂRIZMÎ (SUTER (1914), p. 115, table 4), or AL-BATTÂNÎ (NALLINO, II, p. 20). The entry for the radix is 1s 6;35,9°, and the tabulated value for A.H. 720 is 7s 16;13,19°. The difference between these two values is 6s 9;38,10° = 189;38,10°. Now, in 720 Arabic years, the Sun has completed 698 full rotations; therefore, the arc length traveled by the Sun in 720 Ar. y. (= 255,144 days) is 189;38,10 + (698 * 360) degrees, and the daily mean motion of the Sun resulting is 0;59, 8, 9, 21, 15, . . . °/d, which yields a year-length of 365;15, 36, 34, . . . d. The value deduced from the table is sidereal. It differs from the daily mean motion embedded in the Toledan Tables, and attributed to AZARQUIEL (0;59, 8, 11, 28, 27, . . . °/d) by an amount which is exactly equal to AZARQUIEL's value for the daily motion of the solar apogee (0;0, 0, 2, 7, 10, 39, . . . °/d, *cf.* TOOMER (1969), p. 319).

The radix given here (1s 6;35,9°) corresponds to the solar centrum (the distance from a sidereally fixed apogee). To obtain the longitude of the Sun at epoch, add the given radix to the longitude of the apogee (2s 16;45,21°, *f.* 35r); the result is 3s 23;20,30°, a value which is close to, but not identical with, those of AZARQUIEL or AL-KHWÂRIZMÎ/MASLAMA (see TOOMER (1968), p. 44).

Solar apogee (f. 28v)

The daily mean motion of the solar apogee resulting from the tabulated value for A.H. 720 (0s 2;24,24°) is 0;0, 0, 2, 2, 14, 46, . . . °/d. This implies a progress of 1° in about 299 Arabic years or in about 290 Julian years, a value which differs from the daily motion of the apogee used by AZARQUIEL (0;0, 0, 2, 7, 10, 39, . . . °/d), which corresponds to a progress of 1° in about 279 Julian years. A parameter very similar to that of AZARQUIEL is also found in the works of IBN ISḤÂQ, IBN AL-BANNÂ' and ABÛ I-ḤASAN ᶜALÎ B. ABÎ ᶜALÎ AL-QUSANṬAYNÎ (MILLÁS (1950), pp. 352–353; SAMSÓ (1992), p. 212).

The recomputations show that IBN AL-KAMMÂD used AZARQUIEL's value for the mean motion of the Sun, but that he incorporated a different parameter for the mean motion of the apogee which does not appear in any known text prior to this one.

Moon (ff. 29r–v)

The daily mean motion of the Moon in longitude resulting from the tabulated value for 720 A.H. (10s 8;37,1°) is 13;10,34,52,46,... °/d. This is exactly AL-KHWÂRIZMÎ's value (*cf.* NEUGEBAUER (1962), pp. 42, 92), and very nearly that in the Toledan Tables (*cf.* TOOMER (1968), p. 44).

The daily mean motion of the Moon in anomaly resulting from the tabulated value for 900 A.H. (9*s* 13;12°) is 13;3,53,56,19,... °/d. This is very nearly the value in the Toledan Tables (*cf.* TOOMER (1968), p. 44), which differs from that of AL-KHWÂRIZMÎ (*cf.* NEUGEBAUER (1962), p. 92).

The tabulated values for the mean motion in longitude agree almost exactly with those in AL-KHWÂRIZMÎ (SUTER (1914), pp. 117–119, table 6–8), and exhibit very small differences with those in the Toledan Tables (TOOMER (1968), p. 48). However, the tabulated values for the mean motion in anomaly are in agreement with those of AL-BATTÂNÎ (NALLINO, II, p. 20), and in the Toledan Tables (TOOMER (1968), p. 49), except for the fact that the motion in anomaly is given to seconds there.

Planets (ff. 30r–34v)

For Saturn, Jupiter, Mars and Mercury, the tables give entries for the mean motion of the centrum (*i.e.*, the distance from a fixed apogee) and anomaly. For Venus, the entries display the mean motion in anomaly only, and for the lunar node, the entries are the complement in 360° of the mean motion of the ascending node in longitude. All entries are given to minutes. Those for the yearly progress of the centrum of Mercury agree with those for the Sun (f. 28*r*), except for the fact that the latter are given to seconds. The values for the radices are for the Hijra (see table 11, below), and differ from those in the zij of AL-KHWÂRIZMÎ/MASLAMA and those in the Toledan Tables.

The mean motions in anomaly of the superior planets are not generally tabulated in other sets of astronomical tables (and this is also the case for almost all copies of the Toledan Tables). The recomputed values obtained here can be compared, however, with those in one copy of the Toledan Tables (Oxford, Bodleian Library, MS Laud. Misc. 644, cf. TOOMER (1968), p. 45): 0;57,8°/d (Saturn), 0;54,9°/d (Jupiter) and 0;27,41°/d (Mars). The same values are explicitly found in the work of ḤABASH (DEBARNOT (1987), p. 44), where the value for Mars is 0;27,42°/d.

All parameters computed from the tabulated entries for the mean motions show close, although not perfect, agreement with those derived from the Toledan Tables, the differences never being greater than 0;0,0,1°.

D. Spherical astronomy

Declination (f. 35v)

This is a table giving the declination of the Sun for each integer degree; the maximum entry is 23;33°. This value for the obliquity of the ecliptic is

Table 11. Summary of the Mean Motions and Radices

	Mean motion (°/d)	Radix (°)
Solar longitude	0;59,8,9,21,15,...	1s 6;35,9 *
Solar apogee	0;0,0,2,2,14,46,...	2s 16;45,21 **
Vernal point	0;0,0,54,56,57,...	0s 3;51,11
Lunar longitude	13;10,34,52,46,...	4s 0;34,42 ***
Lunar anomaly	13;3,53,56,19,...	3s 18;11
Double elongation		0s 14;33
Lunar node	0;3,10,46,41,...	4s 6;30
Saturn (longitude)	0;2,0,25,36,...	7s 26;52 *
Saturn (anomaly)	0;57,7,44,57,...	11s 27;48
Jupiter (longitude)	0;4,59,6,43,...	5s 21;58 *
Jupiter (anomaly)	0;54,9,3,37,...	4s 23;2
Mars (longitude)	0;31,26,31,40,...	3s 1;46 *
Mars (anomaly)	0;27,41,40,34,...	8s 22;14
Venus (anomaly)	0;36,59,29,21,...	1s 15;21
Mercury (longitude)	0;59,8,11,23,...	9s 4;58 *
Mercury (anomaly)	3;6,24,7,19,...	2s 14;1

* The values of the radices for the Sun and the planets correspond to their centrum, *i.e.*, their distance from apogee. Note that the MS does not provide this information for Venus.

** This value for the longitude of the solar apogee is given on f. 35r. However, in the table for the mean motion of the solar apogee (f. 28v) one finds 0s 0;0,0 opposite "Radix".

*** The radix for the Moon is called its "centrum", here meaning longitude.

associated with the zij AL-MUMTAḤAN (VERNET (1956), p. 515). The entries in this table agree, except for minor differences, with those in the Almanac of AZARQUIEL (MILLÁS (1950), p. 174), where the argument is given only at intervals of 3°. The value found in the Toledan Tables, and attributed to AZARQUIEL, is 23;33,30° (TOOMER (1968), p. 30).

Daylight (f. 47v)

The heading of the table mentions Córdoba, but it does not specify a value for its latitude. The geographical table on f. 54v gives its latitude, φ, as 38;30°, the most prevalent value for it at the time. This table gives the half length of daylight as a function of the solar longitude. The maximum entry represents half of the longest daylight (M/2), and it is 7;21 h for Cancer 0°. This value follows from the formula:

[14] $$\tan(\varphi) = -\cos(M/2) * \cot(\varepsilon),$$

where φ = 38;30° and ε = 23;33°.

Normed right ascension (ff. 48v–49r)

The same table of right ascension, beginning with Capricorn 0°, is found in AL-BATTÂNÎ (NALLINO, II, pp. 61–64), for an obliquity (ε) of 23;35°. It is also found in the Toledan Tables (TOOMER (1968), p. 34, table 17) and, with copying errors in an abridged version, in the Almanac of AZARQUIEL (MILLÁS (1950), pp. 220–221, tables 69–70). The table differs from that in the Handy Tables (STAHLMAN (1959), pp. 206–209, table 1) and in AL-KHWÂRIZMÎ's zij (SUTER (1914), pp. 171–173, tables 59–59b), both calculated for higher values of the obliquity.

Oblique ascension for Córdoba (ff. 49v–51r)

The mean values between the rising times of Aries and Virgo, of Taurus and Leo, and of Gemini and Cancer, are respectively 27;50°, 29;54,30° and 32;15,30°. They are almost identical with those derived from the Toledan Tables for the seven climates: 27;50°, 29;54° and 32;16°, which are the Ptolemaic values for the right ascensions (*cf.* TOOMER (1968), p. 42). They are also embedded in the zij of AL-KHWÂRIZMÎ, but differ from those in the zij of AL-BATTÂNÎ.

When recomputing the entries, close, although not exact, agreement is obtained with $\varphi = 38;30°$ and $\varepsilon = 23;51°$. With the two other values of the obliquity found in our text (23;33° and 23;35°), the agreement is worse.

Another column in this table represents the length of the seasonal hours. The entry for Cancer 0° is 18;24°. Now $18;24 * 12/15 = 14;43$ h, which gives a half daylight of 7;21, 30 h, and this is quite close to the value 7;21 h for Cancer 0° (see the table for the half length of daylight as a function of the solar longitude on f. 47v).

A similar table for Salé, a place in North Africa near Rabat, whose latitude is given here as 33°, is found on ff. 59v–61r.

E. Latitude

Moon (f. 35v)

The maximum value in the table for the latitude of the Moon is 5;0°. The same value for the inclination of the lunar orbit is found in the Almagest V, 8, and in many other texts, including the Toledan Tables and the Almanac of AZARQUIEL (*cf.* MILLÁS (1950), p. 173, where the argument is given only at intervals of 6°). For another table for the lunar latitude in *al-Muqtabis*, see Section III. G.

Planets (ff. 45r–v)

This table for the latitude of the superior planets is the same as the one in the Almagest XIII, 5, and in AL-BATTÂNÎ (NALLINO, II, p. 140 (columns 1–3) and p. 141 (column 4)). The pattern of this table differs greatly from the corresponding one in the Handy Tables. TOOMER listed some MSS associated with the Toledan Tables that contain such a table, but concluded that it is not part of the original Toledan Tables (TOOMER (1968), p. 72).

In contrast to the superior planets, the latitude table for the inferior planets does not conform to the pattern of the Almagest, the zij of AL-BATTÂNÎ, or the tables associated with the Toledan Tables. Rather, this table reproduces, with variant readings, the entries which are multiples of 6° in the Handy Tables (STAHLMAN (1959), pp. 331–334, tables 49–50, where the entries are given at 3-degree intervals). In particular, the maximum values for the mean latitude of Mercury (3;52°) agree in both sets of tables, but those for the mean latitude of Venus differ (8;35° in our text and 8;51° in the Handy Tables). However, canon 16 (f. 10va) gives 8;36° and 4;18° as the values for the maximum latitude of Venus and Mercury. KENNEDY (1956a), p. 173, reports maximum values for Venus (8;56°) and Mercury (4;18°), and associates the zij AL-MUMTAḤAN and IBN HIBINTÂ with them.

Nevertheless, the outstanding feature here is the juxtaposition of different Ptolemaic tabular material: the Almagest for the superior planets, and the Handy Tables for the inferior planets. The source for such a mixed approach has not been determined.

Note that in the tables associated with *al-Muqtabis* no values are given for the longitudes of the planetary nodes, although MS 10023 (f. 66r) has a list of them.

F. Equations

Moon (ff. 36r–37r)

The same table is found in the Almagest V, 8, in the Handy Tables, as well as in many medieval tables, such as the zij of YAḤYÂ BEN ABÎ MANṢÛR (SALAM & KENNEDY, pp. 495–496), the zij of AL-BATTÂNÎ (NALLINO, II, pp. 78–83) and the Toledan Tables (TOOMER (1968), pp. 58–59). The table lists columns for the equation for mean to true apogee, an interpolation function, the increment in the equation of center, and the equation of center. There is no column here for the lunar latitude, which is tabulated separately (see f. 35v). The entries for the equation of center agree with those in AL-BATTÂNÎ's zij, but differ slightly from those in the Toledan Tables (*e.g.*, in our table the maximum of 5;1,0° is reached at 95°, while in the Toledan Tables the value is 5;0,59°, and it occurs at 94–95°). On the other hand, in our table the order of the columns is the same as in the Toledan Tables, and differs from that in AL-BATTÂNÎ's zij.

Planets (ff. 37v–44v)

The tables for the equations of the five planets are essentially those found in the Almagest XI, 11, in the Handy Tables, as well as in the zij of AL-BATTÂNÎ (NALLINO, II, pp. 108–137) and the Toledan Tables (TOOMER (1968), pp. 60–68), among many others. Although in most respects the zij of AL-BATTÂNÎ and the Toledan Tables agree for the planetary equations, there are some differences between them: for instance, there is a column for the planetary stations in the Toledan Tables, which is neither in AL-BATTÂNÎ's zij nor in our table. All we can deduce from these tables is that IBN AL-KAMMÂD accepted the PTOLEMAIC tradition, as displayed in the Handy Tables, followed by most Muslim astronomers.

It is worthwhile to notice that in the tables of IBN AL-KAMMÂD the maximum solar equation (1;52,44°: f. 35r) differs from the maximum equation of center for Venus (1;59°). This value for Venus is not that of the Almagest, but follows AL-BATTÂNÎ, *etc.* (*cf.* GOLDSTEIN & SAWYER). This indicates that IBN AL-KAMMÂD's contribution was restricted to solar theory and that he did not introduce any changes in planetary theory.

Values for the sidereally fixed apogees appear above the tables for the equations on ff. 37v–38v (Saturn), 39r–40r (Jupiter), 40v–41v (Mars), 42r–43r (Venus), and 43v–44v (Mercury). The apogee for Venus is that ascribed to the Sun on f. 35r.

Saturn	238;38,30°
Jupiter	158;21,0°
Mars	119;41,0°
Venus	76;45,21°
Mercury	198;21,0°

G. Stations (f. 46r)

Only four values are given for each planet: the positions of the first and the second stationary points for arguments of 0° and 180° (see table 12). The tabulated values for the same argument add up correctly to 360°. In all cases, they coincide with those found in the Toledan Tables (TOOMER (1968), pp. 60–68), and the zij of AL-KHWÂRIZMÎ (SUTER (1914), pp. 138–167, tables 27–56), both zijes displaying tables for each integer degree of the argument. Nearly the same values are also found in the zij of AL-BATTÂNÎ (NALLINO, II, pp. 138–139), which have a PTOLEMAIC origin. However, ALMAGEST XII, 8, and the Handy Tables (STAHLMAN (1959), pp. 335–339, tables 51–55), have slightly different tables for the stations: in the latter the argument was modified, as well as the interval for the calculation of the entries (6° in the Almagest, 3° in the Handy Tables).

Table 12. Planetary Stations

	Saturn	Jupiter	Mars	Venus	Mercury
1st st. at apogee	3s22;44	4s 4; 5	5s 7;28	5s15;51	4s27;14
2nd st. at apogee	8s 7;16	7s25;55	6s22;32	6s14; 9	7s 2;46
1st st. at perigee	3s25;30	4s 7;11	5s19;15	5s18;21	4s24;42
2ndt st. at perigee	8s 4;30	7s22;49	6s10;45	6s11;39	7s 5;18

H. *Equation of time* (f. 46r)

Beneath the table we read: "Mediatus solis in radice posita ad directionem dierum cum noctibus: 10.23.24.50. a puncto capitis arietis". This value agrees with that appearing in canon 11 (f. 7rb), and seems to correspond to the argument (in signs and degrees) for the minimum entry in the table. Hence it is to be understood as 10s 23;24,50°. The entries in this table, in time-degrees, rounded to the nearest integer, may have been taken from the more precise values given in the zij of AL-BATTÂNÎ (NALLINO, II, pp. 61–64) or in the Toledan Tables (TOOMER (1968), pp. 34–35).

I. *Trigonometry*

Functions related to the Sine (f. 46v)

We adopt the convention that $\mathrm{Sin}\,\theta = 60 \sin\theta$, and similarly other capitalized trigonometric functions are normed for $R = 60$ (*cf.* KENNEDY (1956a), p. 139). Three functions are given for each integer degree: $\mathrm{Sin}\,(\theta)$, $\mathrm{Cos}\,(\theta)$, and $\mathrm{Vers}\,(\theta) = R - \mathrm{Cos}\,(\theta)$. The Almanac of AZARQUIEL (MILLÂS (1950), p. 229) has a table with these three functions, but given at intervals of 3°. Except for copying errors, the entries agree in both tables.

Cotangent function (f. 48r)

The entries in this table represent the length of a shadow (s) projected by a gnomon of 12 units as a function of the altitude of the Sun (h):

[15] $$s = 12 * \mathrm{cotan}\,(h),$$

in the tradition of AL-KHWÂRIZMÎ (SUTER (1914), p. 174, table 60) and AL-BATTÂNÎ (NALLINO, II, p. 60).

J. *Star table* (f. 47r)

This is a list of 30 stars and it displays the following information for each star: magnitude, name, ecliptic coordinates (longitude and latitude), and the planets associated with it (for astrological purposes).

KUNITZSCH (1966), pp. 99–102, described this list under his type XV: the ecliptic longitude of each star is derived from that in PTOLEMY's star catalogue by adding 6;38°, thus indicating that the epoch of this star list is the Hijra. The type defined by KUNITZSCH only includes two versions of this list: the other version is uniquely represented by MS Vienna 5311, f. 129v. Although the stars in both versions are the same, the coordinates for them do not always agree, thus suggesting a common Arabic ancestor. KUNITZSCH also reports close similarity with the star list of ABÛ L-ḤASAN ᶜALÎ AL-MARRÂKUSHÎ (fl. 1262). We have found additional copies of this list in some copies of the Tables of Barcelona *e.g.*, MS Vatican Heb. 356, f. 65b, where the names of stars are given in Hebrew.

K. Excess of revolution

In the first table on f. 54v, the "excess of revolution" is given in degrees and minutes, for 1, 2, 3, ..., 10, 20, ..., 1(00 years. The entry for 1 year is 92;36°, which seems to be an isolated error. To be coherent with all other entries in the table, one should read 93;36°, leading to a year-length of 365;15,36 days = 365 d 6;14,24 h. The entry for 100 years is 0;5°, and the resulting length of the solar year is 365;15,36,0,30 days.

In the second table on f. 54v, the "excess of revolution" is given in hours and minutes of time, for 1, 2, 3, . . . , 10, 20, . . . , 100 years. The entry for 1 year is 6;14 h, which corroborates the emendation above. The entry for 100 years is 23;59 h, and the resulting length of the solar year is 365;15,35,58,30 days, a value close, but not equal, to the parameter derived from the previous table.

These values for the solar year are to be compared with that appearing in the canons (f. 2va): 365;15,36,19,34,12 days (*cf.* MILLAS (1942), p. 236), as well as with the information given on f. 65v of this manuscript: "Length of the solar year according to IBN AL-KAMMÂD:

365;15,36,19,35,32 days".

The daily mean motion of the Sun resulting from the first value is 0;59,8,9,23,44,53 °/d (the second value yields a very similar parameter: 0;59,8,9,23,44,40 °/d), which agrees quite well with the value derived above from the table for the mean solar anomaly on f. 28r.

On f. 57v of this manuscript there is an analogous table attributed to AZARQUIEL, and for a sidereal year of 365;15,24 d.

There follows a list of the various lengths of the solar year associated with IBN AL-KAMMÂD:

365;15,36, 0,30 d	(computed from the first table: f. 54v)
365;15,35,58,30 d	(computed from the second table: f. 54v)
365;15,36,19,34,12 d	(mentioned in canon 1: f. 2va)
365;15,36,19,35,32 d	(attributed to IBN AL-KAMMÂD: f. 65v)

L. Geographical table (f. 54v)

This is a list of 30 places: for each of them we are given its longitude and latitude, in degrees and minutes. The prime meridian used here is located west of the shore of the Western Ocean. It thus differs from that in the Toledan Tables, where the shore of the Western Ocean seems to have been used for most longitudes (*cf.* TOOMER (1968), p. 136). For a general discussion of the prime meridian in Islamic sources, see KENNEDY & KENNEDY, p. xi.

The entry for the latitude of Córdoba is 38;30°, and that for its longitude 27;0°, which is the same value given in canon 9 (f. 6ra): "longitudo a circulo occidentis ex centro Erin est gradus 27". The same values for the longitude and latitude of Córdoba are found in some other Islamic sources, notably in a work by ABÛ L-ḤASAN ᶜALÎ AL-MARRÂKUSHÎ (see KENNEDY & KENNEDY, p. 95). We note that in the Toledan Tables, the longitude of Córdoba is given as 9;20° and its latitude as 38;30° (TOOMER (1968), p. 134).

M. Miscellaneous

(f. 48r) "Tabula directionis arcus luminis et transitus"

This table has 3 columns: (1) "gradus longitudinis", (2) "directio arcus luminis", and (3) minuta diuersitatis transitus". Col. 1 lists degrees at 3° intervals from 3° to 90° (with some copying errors); col. 2 has entries in degrees and minutes from 0;14° to 1507;0°; and col. 3 has entries in minutes and seconds from 0;1,23° to 0;23,33°. The structure of this table is similar to that of some tables in the Almanac of AZARQUIEL (MILLÁS (1950), p. 226), and most of the entries are the same (the entries in col. 2 agree with those in the corresponding column in the Almanac of AZARQUIEL for arguments from 3° to 84°, but for roundings and copying errors: note that the column in the Almanac displays degrees, minutes and seconds, rather than degrees and minutes). But the headings for the columns in the Almanac of AZARQUIEL are different from those here, and we have not succeeded in determining the purpose for which our table (or the corresponding table in the Almanac of AZARQUIEL) was computed.

V. Other Tables in MS Madrid 10023

A. Just after the last table associated with *al-Muqtabis*, there are two tables related to AZARQUIEL's solar theory: (f. 55r) "Tabula motus centri circuli exeuntis centrum in longitudine longiori et propinquiori a centro terre"; (ff. 55v–56r) "Tabula directionis composite centri circuli solis exeuntis centrum de diuersitate centri eiusdem in longitudine propinquiori et longiori a circulo diuersitatis motus centri morantis tempus". TOOMER (1968), p. 325, has reproduced an excerpt and has explained the two tables.

B. (f. 56v) "Tabula uisuum lunarium post occasum solis in climatibus septem"

C. (f. 57r) "Tabula eclipsis lune et quot digiti eclipsantur ex ea et hore dimidii temporis eclipsis"

D. One of the tables on f. 57v has already been analysed above (see Section IV, K), in connection with a table in *al-Muqtabis* (f. 54v) giving the length of the

Table 13. "Mighty Years" of the Planets

Planet	MS Madrid 10023 (f. 58v)	ABÛ MAᶜSHAR
Sun	1461	1461
Venus	1151	1151
Mercury	480	480
Moon	420*	520
Saturn	625*	265
Jupiter	567*	427
Mars	684*	284

* Note that several of the entries in MS Madrid 10023 are corrupt.

solar year. For another table on f. 57v, an eclipse table, see the comments in Section III, D, concerning lunar eclipses in *al-Muqtabis* (f. 52v). Folio 57v has still another table for the maximum values for latitudes in the seven climates.

E. There follow two tables that are clearly related to table 1 in the zij of AL-KHWÂRIZMÎ: (f. 58r) "Tabula cuius est inter annos gentium et alios annos preter illos ad inuicem"; (f. 58v) "Numeri dimissi per 28, 28 secundum annos romanorum et egyptiacum". According to MILLÂS (1942), p. 245, they derive from MASLAMA rather than from AL-KHWÂRIZMÎ.

F. (f. 58v) "Circuitus planeta (. . .) magni in sectis et divinationibus". These are the "Mighty Years" of the planets: see table 13, where the entries of this table are compared with those of ABÛ MAᶜSHAR as preserved by AL-SIJZÎ (Pingree (1968a), p. 64).

G. (f. 58v) "Tabula dierum prouenientium in retrogradationibus planetarum et directionibus eorum"

H. (f. 59r) "Tabula partis cordatum supereminentium et partis almudarat solis et planetarum"

I. Folios 59v to 61v present tables for the city of Salé: see the comments in Section IV, D, on the table in *al-Muqtabis* for the oblique ascension for Córdoba (ff. 49v–51r).

J. The following table is certainly a part of *al-Kawr ᶜalâ al-Dawr*: (ff. 62v–64r) "Tabula extractionis annorum quantitatis durationis creature in uentre matris per longitudinem lune a gradu occidentis". It deals with astrological obstetrics and has been discussed by VERNET (1949), pp. 273–300.

K. (f. 64v) "Tabula circuituum annorum planetarum in natiuitatibus"

L. There follows information on houses, exaltations, triplicities, and signs, presented in tabular form (f. 65r), but every other entry has been left blank. The information given coincides with that in AL-KHWÂRIZMÎ/MASLAMA (SUTER (1914), p. 231, table 116). The table with the heading, "Tabula terminorum egyptiorum" (f. 65r) is the same as the fourth sub-table in AL-KHWÂRIZMÎ/MASLAMA (SUTER (1914), p. 231, table 116), but some of the entries are blank here.

M. (f. 65v) "Mediatus cursus solis in descensu et ad quartas circuli secundum probationem huius canonis". The data are for A.H. 480 (1087–88 A.D.).

N. (f. 65v) "Tabula terminorum ciuium Babillonie ueteris qui sunt magistrum ymaginum". Two parameters are given in this table: the length of the solar year "according to IBN AL-KAMMÂD" (365;15,36,19,35,32 days, *cf.* our comments in Section IV, K, on the table in *al-Muqtabis* on f. 54v) and his length of the lunar month (29;31,50,5,1 days).

O. (f. 66r) "Tabula extracta per misilme de eo quod confirmatum extitit per cines huius artís yspanenses super diuisionem Yspanie per signa duodecim 12 12 12 12". MILLÂS (1942), p. 256, suggested that "misilme" stands for "Maslama".

P. (f. 66r) "Residuum ascensionum ad reuoluciones annorum solarium secundum Muhad Arcadius". MILLÂS (1942), p. 256, identified MUHAD ARCADIUS with ABÛ ᶜABD ALLÂH MUḤAMMAD BEN YÛSUF BEN AḤMAD IBN MUᶜÂDH AL-JAYYÂNÎ, from JAÊN. The value given for one year, 93;2,15°, corresponds to 365;15,30,22 days. The same value is found in AL-KHWÂRIZMÎ/MASLAMA (*cf.* NEUGEBAUER (1962), p. 132, and GOLDSTEIN (1967), pp. 143, 242). In fact, this table attributed to IBN MUᶜÂDH reproduces, with some scribal errors, the table of conversion for the "years of Nativity" given in units of time-degrees in AL-KHWÂRIZMÎ/MASLAMA (SUTER (1914), p. 230, table 115; *cf.* NEUGEBAUER (1962), p. 131).

Q. (f. 66r) "Tabula uisuum planetarum et absconsionum eorum sub radiis solis"

R. (f. 66r) "Capita draconum planetarum in anno quingentesimo et quinquagesimo ab anni seductionis". Note that A.H. 550 corresponds to 1155–56 A.D. See our comments in Section IV, E, on the table of *al-Muqtabis* for the latitude of the planets (ff. 45r–v).

Note added in proof: We are grateful to ANGEL MESTRES (University of Barcelona) for calling our attention to the following passage in MS Hyderabad, Andra Pradesh State Library 298 (no foliation): chap. 35 "ABÛ L-ᶜABBÂS AL-KAMMÂD said in a horoscope he drew in Córdoba in the year 510 Hijra . . . " (1116/7).

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10 Av. Paul Verheyleweghen
B-1160 Bruxelles
Belgium

and

History and Philosophy of Science
2604 CL, University of Pittsburgh
Pittsburgh, PA 15260, USA

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