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INSTANTIAL TERMS, ANAPHORA AND ARBITRARY OBJECTS*

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Frege, as is well known, opposed the view that just as genuine proper names have definite objects as their meanings, the variables used in mathematics have variable or indefinite objects as their meanings. In a typical passage, Frege puts the matter this way:

> '*n*' is not the proper name of any number, definite or indefinite . . . We write the letter '*n*' in order to achieve generality . . . Of course we may speak of indefiniteness here; but here the word 'indefinite' is not an adjective of 'number', but ['indefinitely'] is an adverb, e.g. of the verb 'to indicate'. We cannot say that '*n*' designates an indefinite number, but we *can* say that it indicates numbers indefinitely.[1]

In opposing the view that variables name indefinite numbers, we can see Frege as providing an alternative explanation as to how sentences in mathematics containing variables express *general* claims. Indefinite number theorists hold that a statement like '$(a + b) + c = a + (b + c)$' expresses a universal claim, whereas '$(2 + 3) + 4 = 2 + (3 + 4)$' doesn't, simply because of the *kind* of objects that '*a*', '*b*', and '*c*', as opposed to '2', '3', and '4', name. The semantic relation between '2' and 2 is the same as the semantic relation between '*a*' and (the indefinite number) *a*: each expression names the corresponding object. It is the indefiniteness of the objects named by variables that endows sentences containing variables with generality. Frege thought that he had a better explanation of the generality of sentences containing variables and other expressions of generality. Instead of locating the source of generality in the kind of objects named by an expression of generality, Frege sought to explain the generality as resulting from a difference in the semantics of such expressions and genuine proper names (note the contrast drawn between *designating* and *indefinitely indicating* in the above quotation).

A principal virtue of the Fregean account of expressions of generality is that it highlights the notions that are not needed for a semantics

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of proper names but are crucial to the development of a systematic account of the truth conditions of sentences containing multiple expressions of generality: the notions of scope and ranging over a domain of individuals.[2] By contrast, an account of expressions of generality according to which they refer to indefinite objects, even if it gets the truth conditions to come out right, assimilates such expressions to proper names and so obscures the distinctive features of such expressions.

In recent work, Kit Fine has sought to resurrect the view that variables in mathematics refer to indefinite or, as he calls them, arbitrary objects.[3] Indeed Fine holds that instantial terms figuring in universal generalization and existential instantiation in systems of natural deduction, and some anaphoric pronouns in natural languages, refer to arbitrary objects as well. As against this, I intend to argue that the arbitrary objects account, like the theories of indefinite numbers that preceded it, obscures rather than highlights the distinctive features of the various expressions it claims to handle; and that there is another view of the semantics of these expressions which is preferable to the arbitrary objects account.[4]

The plan of the present essay is first, to sketch Fine's theory of arbitrary objects; second, to sketch an alternative to Fine's account; third, to argue that Fine's arguments in support of arbitrary object theory also support the alternative; and finally, to argue that this alternative is preferable to arbitrary object theory.

Because my arguments against Fine will not concern the more technical features of his theory, I shall sketch the theory informally. The easiest way to introduce the theory is to look at some of the data to which the theory is primarily applied, and consider (in rough form) the account the theory gives of the data. Hence, we shall begin by looking at instantial terms in natural deduction and English arguments. Let us suppose that we are using a system whose formulations of UG and EI are as follows:

$$\text{UG}\ \frac{P(u)}{(x)P(x)} \qquad \text{EI}\ \frac{(\mathrm{E}x)P(x)}{P(u)}$$

('$P(u)$' is obtained by replacing all free occurrences of 'x' in '$P(x)$' by occurrences of 'u' — in actual applications of UG and EI when a variable plays the role played by 'u' in the above formulations we shall call it the *instantial variable/term.*) The necessary restrictions are simpler to state if, following Kalish and Fine, we define a relation of *dependence* between the variables occurring in a derivation. A variable x depends (in a given derivation) on a variable y if x is the instantial variable of a formula at a line resulting from the application of EI and y is free in that formula or x depends on some variable that depends on y. The restrictions now are: EI: 'u' must have no prior occurrrences in the derivation; UG: no instantial term in an application of EI is also an instantial term in an application of UG, and 'u' and any variable depending on 'u' must not occur free in '$(x)P(x)$' or any supposition governing '$(x)P(x)$'.[5]

Suppose that in a derivation in our system we infer 'Hu' from '$(\mathrm{E}x)Hx$' by EI. What semantic account are we to give of the instantial term 'u' here? Fine answers that 'u' refers to an arbitrary object (an arbitrary H, in this case). In general, arbitrary objects are associated with ranges of individuals (the *value range* of the object). In this case, for example, the object referred to by 'u' (in a given model) is associated with the set of individuals (if there are any) assigned to 'H' (by the model). If we infer '$(x)Fx$' from 'Fv' by UG, 'v' refers to the "completely arbitrary" object (i.e. the object 'v' refers to is associated with the set of all individuals). It is by associating arbitrary objects with ranges of individuals that statements containing expressions referring to arbitrary objects express general claims. As Fine puts it, ". . . a statement concerning A-objects is true just in case it is true for all their values.", the admissible values for an arbitrary object being the individuals in the set associated with the object.[6]

Above, in formulating restrictions on UG and EI, we defined a relation of dependence between the variables in a given derivation of the system. Corresponding to this, Fine defines a relation of dependence between *arbitrary objects.* This idea can be illustrated intuitively by considering an arbitrary natural number n. The individual natural numbers are the object n's values. n^2 is an arbitrary object that depends on n. This dependence between the objects is reflected in the fact that the assignment of a value to n^2 requires a prior assignment of a value to n. Now consider the following, rather trivial, derivations:

(A)	1. $(x)(\mathrm{E}y)Fxy$	(B)	1. $(\mathrm{E}y)(x)Fxy$
	2. $(\mathrm{E}y)Fzy$		2. $(x)Fxw$
	3. Fzw		3. Fzw
	4. $(\mathrm{E}w)Fzw$		4. $(z)Fzw$
	5. $(z)(\mathrm{E}w)Fzw$		5. $(\mathrm{E}w)(z)Fzw$

By the definition of dependence between *variables* given above, the variable 'w' depends on the variable 'z' in (A), whereas neither variable depends on the other in (B), (the restrictions on UG thus prevent applying UG to line 3 in (A) and hence reattaching the quantifiers in the wrong order). And the *object* referred to by 'w' in (A), on Fine's definition of the relation of dependence between objects, depends on the object referred to by 'z' in (A), whereas neither of the objects referred to by 'z' and 'w' in (B) depends on the other. As before, this is reflected in the fact that assigning a value to 'w' in (A) intuitively requires the prior assignment of a value to 'z'. In general, when a variable 'x' depends on a variable 'y' in a derivation, the arbitrary object referred to by 'x' will depend on the arbitrary object referred to by 'y'. That the object 'w' refers to in (A) depends on that referred to by 'z' in (A) and not so in (B) entails that 'w' refers to a different object in (A) than it does in (B). This explains why (e.g.) the inference from 'Fzw' to '$(z)Fzw$' is valid (with respect to Fine's semantics) in (B) but would not be in (A).

Fine applies his arbitrary object semantics to various systems of natural deduction. Different restrictions on UG and EI in the different systems, according to Fine, reflect the fact that the instantial terms used in UG and EI in the different systems refer to different arbitrary objects (we shall return to this point). Fine then proves that all rules of inference of the systems he treats, including UG and EI, are truth-preserving with respect to the semantics he provides (of course, if one treats the instantial term 'u' in UG ($F(u)/(x)F(x)$) as the name of an individual, the rule is not truth-preserving; Fine's proofs show that construing 'u' as the name of an arbitrary object results in UG being truth-preserving – similar remarks apply to EI).[7]

I wish to stress again that Fine also wants to apply the theory of arbitrary objects to instantial terms in English "UG and EI", certain anaphoric pronouns (and perhaps definite descriptions) in natural language (see (F2) below), and to variables in mathematics. These expressions too are to be construed as referring to arbitrary objects.

The view that I shall champion over the theory of arbitrary objects has grown out of consideration of the semantics of certain anaphoric pronouns/definite descriptions, and instantial terms in natural deduction and informal arguments in English.[8] We shall begin to explain this view by contrasting its account of instantial terms in UG and EI in systems of natural deduction with that of Fine. The view is perhaps best explained by considering a system of natural deduction in which scope lines are used to indicate which suppositions a given formula depends on, and scope lines flagged by variables are used in applications of UG and EI. In particular, suppose that UG and EI are formulated roughly as follows ('$P(x)$' differs from '$P(y)$' only in that '$P(x)$' contains free 'x' where and only where '$P(y)$' contains free 'y' and A contains no occurrences of free 'y' — for a more detailed statement of these rules see the Appendix):

$$
\begin{array}{llcll}
\text{EI} & (\mathrm{E}x)P(x) & \qquad & \text{UG} & y \mid . \\
 & y \mid P(y) & & & \mid . \\
 & \mid . & & & \mid . \\
 & \mid . & & & \mid P(y) \\
 & \mid . & & & (x)P(x) \\
 & \mid A & & & \\
 & A & & &
\end{array}
$$

Now consider the following derivation:

$$
\begin{array}{ll}
\mid (\mathrm{E}x)\,(y)Fxy & \text{hypothesis} \\
\vdash\!\!\text{———} & \\
\mid z \mid (\mathrm{E}x)\,(y)Fxy & \text{reiteration} \\
\mid \mid \quad w \mid (y)Fwy & \text{EI} \\
\mid \mid \quad \mid Fwz & \text{UI} \\
\mid \mid \quad \mid (\mathrm{E}x)Fxz & \text{EG} \\
\mid \mid (\mathrm{E}x)\,Fxz & \text{E discharge} \\
\mid (y)\,(\mathrm{E}x)\,Fxy & \text{UG} \\
(\mathrm{E}x)\,(y)Fxy \rightarrow (y)\,(\mathrm{E}x)Fxy & \rightarrow \text{intro}
\end{array}
$$

As already indicated, Fine would construe free occurrences of 'z' and

'w' in such a derivation as referring to arbitrary objects; and it is in virtue of referring to such objects that formulas containing free occurrences of these variables express general claims such that (e.g.) '(Ex)Fxz' at line 6 entails '(y) (Ex)Fxy' at line 7. The present view holds that free occurrences of 'z' and 'w' in this derivation are quantifier-like expressions of generality. Standard quantifiers in first order formal languages have their forces (universal or existential) and their scopes marked syntactically in the formulas in which they occur. By contrast, nothing in the *formula* 'Fwz' at line 4 above tells us whether 'w' and 'z' should be understood as having universal or existential force, and what the relative scopes of these two expressions of generality are.[9] Intuitively, it is *the structure of the derivation* that determines these features of the semantic significance of free occurrences of 'z' and 'w' in the above derivation. In particular, the fact that 'w' flags an application of EI and 'z' flags an application of UG determines that 'w' and 'z' have existential and universal force respectively. And that the scope line 'w' flags is subordinate to the scope line 'z' flags determines the relative scopes of free occurrences of 'z' and 'w' at line 4.

On the present view, then, instantial terms in UG and EI such as free occurrences of 'z' and 'w' in the above derivation are quantifier-like expressions of generality such that various features of their semantic significance are determined by the structure of the derivations they occur in. Instead of having their scopes and forces marked by syntactical features of the *sentences* in which they occur, features of the *derivational setting* (e.g. order of subordination of scope lines, etc.) of these expressions keep track of these things. We shall put this point by referring to these instantial terms as *context dependent quantifiers*, thus emphasizing the role linguistic context (derivational setting) plays in determining their semantic significance and that they are expressions of generality; and we shall call the view of these terms presently being sketched *CDQ*.

The formal semantics provided by CDQ for instantial terms in UG and EI comprises two components. First there is a definition of the *context of an occurrence of a formula at a line in a derivation*. This context encodes all information about the structure of the derivation that the occurrence of the formula occurs in relevant to the semantic significance of any instantial terms occurring in the formula. Next, there is a definition of *truth in a context under an interpretation.* Together, these definitions assign truth conditions to lines of derivations of the system in such a way that for any inference in a derivation (including UG and EI) if the premises of the inference are true in their contexts under an interpretation, the conclusion to the inference is true in its context under the interpretation. In the appendix just such a semantics is formulated for a system with UG and EI formulated as above.

Given what has been said, it should be clear that in a given derivation, the contexts of some lines will need to encode more information than those of other lines. For if more context dependent quantifiers occur in a formula at one line than in a formula at another, more information about discourse structure will have to be encoded in the context of the first line than in that of the second. We shall express this state of affairs by saying that the context of the first line is *more complex* than that of the second line. Just as clearly, the application of some rules of inference will yield conclusions whose contexts are more complex than those of the premises, whereas some will yield conclusions whose contexts are less complex. For example, in applying EI, a new context dependent quantifier is introduced. Hence the context of the line in which it is introduced must encode new information that was not encoded in the context of the previous line, viz. information about derivational structure relevant to properly interpreting the new context dependent quantifier. We might, then, think of EI as a *context changing rule* which increases context complexity. By contrast, in applying UG a context dependent quantifier is eliminated. Hence, the context of the conclusion of an application of UG need not encode information about derivational structure relevant to properly interpreting the eliminated term, though the context of the premise to the inference had to encode this information. Hence, we might think of UG as a context changing rule which decreases context complexity.

Thus far, we have discussed a system of natural deduction whose formulations of UG and EI involve flagged scope lines because in such a system features of the derivational setting necessary to properly interpret the context dependent quantifiers are perspicuously displayed (e.g. one can *see* the order of subordination of the scope lines which determines the relative scopes of the context dependent quantifiers). But CDQ applies to any system of natural deduction; it is just that the features of derivational structure that must be encoded in the context of a line in the derivation are more subtle.

Some readers will have been asking about the point of applying the semantics of CDQ (or arbitrary object semantics) to instantial terms in derivations. Given soundness and completeness results for such systems, what is the semantics for? The benefits of such an application will be discussed in the sequel. For a moment I want to dwell on the way CDQ increases our understanding of *natural* language. For whether instantial terms in natural deduction are understood along the lines suggested by CDQ or not, a variety of expressions of natural language appear to behave in the way that CDQ holds that instantial terms do. First, consider "instantial terms" in English "UG" and "EI" such as those underlined in the following argument:

(F1) premise 1) Every professor has a bad student.
premise 2) Every bad student hates each of his/her professors.
3) Consider an arbitrary professor.
4) By 1), <u>the professor</u> has a bad student.
5) Consider <u>the professor's bad student</u>.
6) By 2), <u>the student</u> hates <u>the professor</u>.
7) So every professor is hated by some student.

Whatever our views on lines in derivations, surely we want to say that as we read through (or hear) such an argument we understand each line and the claim it makes. Hence, to the extent that we believe that understanding each line consists in grasping its truth conditions, we are forced to assign truth conditions to the lines in this argument, and assign them in such a way that the inferences made are truth-preserving (e.g. if the truth conditions assigned to line 6 obtain, those assigned to line 7 obtain). According to CDQ, the underlined descriptions are, once again, context dependent quantifiers. As before, their forces, and their scopes relative to one another are fixed by features of their linguistic environments (in this case, the domains over which they range are determined by such features as well — e.g. the 'Consider . . .' clause at line 5 determines the domain over which subsequent occurrences of 'the student' ranges). And once again we define the context of a sentence in a discourse/argument such that it encodes all the information about the linguistic environment relevant to the significance of the context dependent quantifiers in the sentence. Again, the semantics is completed by a definition of truth in a context.

Finally, consider the underlined anaphoric pronouns/descriptions in the following discourse:

(F2) 1) Every professor at the University of San Clemente$_1$ teaches a large lecture class$_2$.
2) The professor$_1$ does all the grading for the class$_2$.
3) The class$_2$ has a final exam$_3$.
4) The final$_3$ is comprehensive.
5) It$_3$ need not be long, however.[10]

Sentences containing underlined pronouns/descriptions express general claims as they occur in this discourse: 2), for example, expresses the claim that every professor at the University of San Clemente teaches a large lecture class for which he/she does all the grading. And it seems clear that these sentences express general claims in virtue of containing the underlined pronouns/descriptions. Hence, these terms are functioning as expressions of generality. Further, as in the other cases we have considered, these expressions of generality are sensitive to linguistic context. For if we immerse one of the sentences of (F2) (e.g. sentence 2)) in an appropriately different linguistic context

(F3) 1) Some professor in Western Australia$_1$ teaches a class on surf-skiing$_2$.
2) The professor$_1$ does all the grading for the class$_2$.

it expresses a quite different claim. CDQ again holds that these pronouns and descriptions are context dependent quantifiers. Again, these context dependent quantifiers have their scopes relative to other context dependent quantifiers, the domains over which they range, and their forces determined by characteristics of the discourses they occur in.[11] The semantic theory for these anaphoric pronouns/descriptions again is constituted by definitions of the context of an occurrence of a sentence in a discourse, and of truth in a context. This semantic theory, I claim, assigns intuitively plausible truth conditions to sentences such as those in (F2)–(F3) as they occur in those discourses.[12] The sentences can contain any number of pronouns/descriptions anaphoric to quantifiers which may occur in a number of different sentences which themselves may contain an arbitrary number of pronouns/ descriptions anaphoric to quantifiers in other sentences.

Now that we are familiar with the theory of arbitrary objects and its competitor (at least in broad outline), let us consider Fine's arguments in favor of his theory and see how the competition fares with respect to these arguments. Here we must distinguish Fine's arguments in favor of adopting arbitrary object semantics for instantial terms in UG and EI in systems of natural deduction (and for the instantial terms in "ordinary language" UG and EI) from his arguments in favor of what we might call the general theory of arbitrary objects. Let us first consider Fine's arguments for applying arbitrary object theory to instantial terms in UG and EI.

Fine lists three benefits to be gained from adopting arbitrary object semantics for natural deduction.[13] The first concerns formulating proper restrictions (i.e. those that result in a sound system) on UG and EI. Finding proper restrictions is by no means a trivial matter, as witnessed by the fact that a number of incorrect formulations have appeared in print.[14] Arbitrary object semantics, however, provides a method for discovering a variety of correct restrictions. If we decide to interpret 'u' in $(\mathrm{E}x)A(x)/A(u)$ as referring to the arbitrary A, if anything A's (i.e. associate the range of individuals that A, if any does, with the arbitrary object 'u' refers to) and interpret 'w' in $A(w)/(x)A(x)$ as referring to the arbitrary non-A, if anything is non-A, the restrictions on UG and EI formulated by Quine result.[15] If we interpret the instantial term in EI as above, and interpret 'u' in $A(u)/(x)A(x)$ as the "completely" arbitrary object (i.e. associate the range of all individuals with the arbitrary object referred to by 'u'), the Copi–Kalish restrictions result.[16] In general, then, different correct restrictions on UG and EI issue from different decisions as to what arbitrary objects the instantial terms refer to.

CDQ yields the same benefit, though it views the restrictions in a quite different light. From the standpoint of CDQ, in attempting to formulate proper restrictions on, for example, UG, one is attempting to characterize conditions such that if the derivational setting of an occurrence of a formula A and the derivational setting of an occurrence of A's universal generalization (with respect to a particular variable) satisfy the conditions, then if A is true in that context under an interpretation, the universal generalization of A (with respect to a particular variable) will be true in its context under the interpretation. And in general, different sets of proper restrictions on UG will result from noting that in the semantics provided by CDQ there are a variety of conditions that A, its context and the universal generalization of A and its context may satisfy, any of which insure that if A is true under an interpretation in the context, its universal generalization will be true under that interpretation in its context.[17] Distinct sets of proper restrictions on UG and EI thus turn out to be distinct sets of constraints on formulas and their contexts which result in the truth of a formula in a context insuring the truth of another formula in a related context, when UG or EI licenses the inference from the former to the latter. Thus, on the question of providing a method of discovering different restrictions on UG and EI, CDQ seems at least on par with arbitrary object semantics.

Fine's second argument for doing arbitrary object semantics for instantial terms in natural deduction is that it renders syntactic features of derivations, and the derivations themselves, meaningful in a way that "classical" semantics for first order logic doesn't. In the first place, as we have seen, arbitrary object semantics provides an explanation and justification of particular restrictions on UG and EI. In the absence of arbitrary object semantics, the only explanation and justification that can be given for such restrictions, Fine says, is that they work. Perhaps most importantly, derivations themselves can be viewed as meaningful. Through arbitrary object semantics, lines in derivations are endowed with truth conditions such that all rules of inference are truth-preserving. Thus we need not take an instrumentalist view of natural deduction rules (particularly, EI and UG) according to which the steps in a derivation are simply meaningless transformations which are only a means to confirm some result.

Once again, CDQ fares as well as arbitrary object semantics. We have already seen how the theory explains and justifies different restrictions on UG and EI. And the theory allows derivations to be viewed as meaningful, just as does the arbitrary objects approach, by assigning truth conditions to lines in derivations such that the rules of inference of the system are truth-preserving with respect to the semantics (see appendix). Indeed, CDQ emphasizes the meaningfulness of derivations by holding that the structure of the derivation stores information concerning the proper interpretation of the context dependent quantifiers in it.

The final argument Fine gives in favor of arbitrary object semantics for natural deduction is that it allows for simple and direct proofs of the classical soundness of the systems in question (i.e. proofs that if neither the last formula of a derivation, nor any formula it depends on, contains instantial terms, the latter formulas together entail the former formula). As before, CDQ yields the same benefit. I refer the reader to the appendix, where it is shown how classical soundness for a system of natural deduction (theorem 2) follows straightforwardly from the proof that all rules of inference in the system are truth-preserving with respect to the semantics (theorem 1).

It seems, then, that the arguments that Fine gives for applying the theory of arbitrary objects to natural deduction equally support applying CDQ to this range of data.

This brings us to the arguments Fine offers in favor of the *general* theory of arbitrary objects. Interestingly, Fine admits that approaches other than arbitrary object semantics (though he doesn't discuss CDQ) reap all the benefits in application to natural deduction that arbitrary object semantics does.[18] Given that other theories perform as well as arbitrary object semantics in application to natural deduction, one cannot argue for the general theory simply on the basis of the fruitfulness of the application to natural deduction. As Fine puts the point:

> The decisive reason [in favor of the theory of arbitrary objects] lies not in instantial reasoning itself but in its relation to the general use of variables in mathematics or of pronouns in ordinary language . . . When one looks at these other uses, it becomes clear that variables and pronouns alike are used to signify arbitrary objects. Considerations of uniformity then force one to adopt the same view of instantial reasoning.[19]

So Fine claims that it is the fruitfulness of these other applications of the theory of arbitrary objects, as compared with its competitors, that provides an argument for the theory in general and, indirectly, for the application of the theory to natural deduction. I wish to argue that, on the contrary, CDQ works as well in application to pronouns in ordinary language and variables in mathematics as does arbitrary object semantics.

Taking pronouns in ordinary language first, the only example I have found in Fine's writings on arbitrary objects of natural language pronouns to which he thinks arbitrary object semantics will apply is the following:

Every man owns a donkey. He beats it. He feeds it rarely . . .[20]

But this is just the sort of anaphora that CDQ was designed to handle (as suggested above, it can handle much more complex examples as well)! Hence, if anything, it must be shown that arbitrary object semantics can match the performance of CDQ: it has already been successfully applied to this type of data, whereas arbitrary object semantics has not.

Turning now to variables in mathematics, it would appear that these behave just like the pronouns/descriptions in (F1), and thus pose no difficulty for CDQ. Consider, for example, the following bit of reasoning:

(F4) Let n be a composite positive integer. Then there is an integer less than n and greater than 0 that divides n. Let m be such an integer. Then m must be prime or composite. If m is the smallest number between 1 and n that divides n, m is prime . . .

Occurrences of 'n' and 'm' function in (F4) in just the way that occurrences of 'the professor' and 'the student' function in (F1) (this is more clear if we reformulate (F4) to more closely resemble (F1): "1) Every composite integer is divisible by an integer less than it and greater than 0. 2) Let n be a composite integer. Then by 1), some integer less than n and greater than 0 . . ." – the only difference is the use of the (syntactically) name-like terms 'n' and 'm' (and the "let" clauses that introduce them) in (F4) instead of the definite desriptions employed in (F1)). Since CDQ is capable of handling the instantial terms in (F1), it can do the same for (F4) and it would appear that variables pose no difficulties.

Yet Fine has argued that certain uses of variables in mathematics cannot be handled without arbitrary object semantics. It behooves us to consider such uses and determine whether they pose difficulties for

CDQ. Fine considers the following example, taken from a mathematics text:

> Let $y = f(x)$ be a continuous function. Take any real h. Then for some k, $f(x + h) = y + k$. Now since f is continuous, $k \to 0$ as $h \to 0$. So . . .[21]

Fine claims that the statement '$k \to 0$ as $h \to 0$', though understandable on the arbitrary objects account, is impossible to make sense of on an account without arbitrary objects. Before explaining how CDQ does make sense of this example, I wish to note that arbitrary object semantics as Fine has formulated it is not capable of handling the problematic statement '$k \to 0$ as $h \to 0$'! For if we construe 'k' and 'h' in Fine's example as names of (the appropriate) arbitrary objects, Fine's definition of truth tells us (roughly) that '$k \to 0$ as $h \to 0$' is true if the statement is true of all pairs of values (i.e. pairs of individual numbers) that the arbitrary objects k and h can jointly assume. So in order to determine the truth of '$k \to 0$ as $h \to 0$', Fine's theory requires us to evaluate the truth of nonsensical things like '$4 \to 0$ as $2 \to 0$'. In fairness, Fine says that the statement '$k \to 0$ as $h \to 0$' "attributes a certain property to the *set* of all pairs of values assumed by the A-objects k and h" (NDAO p. 106, my emphasis). This suggests that the definition of truth he provides should not be applied to the statement, since that definition is intuitively for statements attributing properties to arbitrary objects (or their values) themselves. Still, this is just to say that the only explicitly formulated semantics provided by Fine won't handle the example. I find it surprising that Fine invokes an example that his theory as formulated doesn't handle in defense of the theory.

Turning now to CDQ's ability to handle Fine's example, neither CDQ nor any semantic theory needs to handle the example as it stands. For the passage is highly elliptical in two respects and needs to be unpacked before being treated by a semantic theory.[22] First, there is the '$\to$' notation. This notation is an abbreviation of talk about limits, with the notion of a limit being defined by means of a so-called "epsilon/delta" formulation of which the following is an example:[23] Let a_1, a_2, . . . be a sequence of reals. A is the limit of the sequence if for each $e > 0$ there is a $d > 0$ such that $|a_d - A|$, $|a_{d+1} - A|$, $|a_{d+2} - A|$,

... are all less than e. Using '$\rightarrow$' notation, we express the claim that A is the limit of our sequence $a_1, a_2, \ldots$ as follows: $a_n \rightarrow A$ as $n \rightarrow \infty$. It is worth stressing that this is the way virtually all contemporary mathematicians understand '$\rightarrow$' notation, and not simply the view of mathematicians and philosophers concerned with the foundations of mathematics. When the notation is used at all in mathematics texts, it is explicitly introduced as an abbreviation for talk about limits, with limits being defined by means of epsilon/delta formulations of the sort just mentioned.[24] Hence before providing a semantic treatment for the passage Fine cites, we need to replace the '$\rightarrow$' notation with what it abbreviates.

If we now try to replace the '$\rightarrow$' notation in Fine's example with the talk about limits it abbreviates, we see that there is a second respect in which the passage is elliptical. For it isn't clear what limit the author is considering in saying '$k \rightarrow 0$ as $h \rightarrow 0$'. Intuitively, the author seems to be imagining choosing a real (i.e. h) and then choosing other reals thus generating an infinite sequence of reals $h_0, h_1, \ldots$. For each h_n in the sequence there will be a k_n such that $f(x + h_n) = y + k_n$. So each sequence $h_1, h_2, \ldots$ determines a sequence $k_1, k_2, \ldots$. Hence, it seems reasonable to suppose that when the author says '$k \rightarrow 0$ as $h \rightarrow 0$' he is saying that for any sequence $h_1, h_2, \ldots$ whose limit is 0, the limit of the corresponding sequence $k_1, k_2, \ldots$ is 0. If something like this is correct, then with all abbreviations and ellipses eliminated, the passage would read roughly as follows: Let $y = f(x)$ be continuous. Let $h_0, h_1, \ldots$ be any sequence of reals. Then there is a sequence of reals $k_1, k_2, \ldots$ such that $f(x + h_n) = y + k_n$. Since f is continuous, if for each $e > 0$ there is a $d > 0$ such that $|h_d - 0|, |h_{d+1} - 0|, |h_{d+2} - 0|, \ldots$ are all less than e, then for each $j > 0$ there is a $g > 0$ such that $|k_g - 0|, |k_{g+1} - 0|, |k_{g+2} - 0|, \ldots$ are all less than j.

The reader may or may not agree with the details of my particular "reconstruction" of the passage. In either case, the point is that we must eliminate the '$\rightarrow$' notation in favour of the limit talk (epsilon/delta formulation) it abbreviates prior to treating the passage semantically. In so doing, we must make explicit whatever it is we are considering the limit of. I claim that we will be left with a passage containing only standard quantifiers ('for each $e > 0$ ...') and context dependent quantifiers (introduced by 'Take any ...' clauses and 'Let' clauses) of the sort that occur in (F4), which we have already argued CDQ can handle. Hence Fine's passage poses no difficulty for CDQ.

It appears, then, that the arguments Fine offers in support of arbitrary object semantics fail to provide grounds for favoring that approach over CDQ. In application to instantial terms in natural deduction, CDQ yields the benefits Fine claims for the arbitrary object approach; and CDQ is capable of handling variables in mathematics and the anaphoric pronouns of natural language mentioned by Fine. In spite of this apparent parity, I believe that CDQ is preferable to arbitrary object semantics and should be viewed as the standard account of these various phenomena.[25] To see why, suppose that someone were to formulate a semantics for (ordinary) quantifiers according to which quantifier phrases are referring terms which refer to objects like Fine's arbitrary objects.[26] Further suppose that this account was on par with the usual account of the semantics of quantifiers (e.g. the truth conditions for quantifier-containing sentences that this account delivers agree with those of the usual account, etc.). Which of the two accounts should we accept as the standard account of expressions of generality? It seems to me that we should prefer the usual account. After all, there are significant differences between expressions of generality and standard names. The notions of a *domain* of individuals over which an expression of generality ranges and the *scope* of such an expression need not be invoked for a semantics of standard referring terms and are required for a satisfactory semantics of quantifier expressions.[27] Conceptual clarity and perspicuity dictate that our semantics be formulated in such a way as to highlight these important differences between names and expressions of generality.

In introducing new objects for quantifier expressions to refer to one assimilates the semantics of expressions of generality to that of standard names. Of course, if the account is adequate, as we have hypothesized it is, it must somehow capture the notions of scope and domain (with Fine's approach, these notions would be captured by the relation of dependence between arbitrary objects, and the value range of an arbitrary object respectively). But it can hardly be said that such an account highlights or clarifies these notions and hence the respects in which expressiohns of generality differ from names. Surely, if anything, the introduction of referents for quantifiers obscures the differences between genuine names and expressions of generality by construing both sorts of expressions as referring terms.

Much the same point can be made by considering two distinct ways to explain differences in the behavior of linguistic expressions. On the one hand, the fact that native speakers (whose idiolects include the names 'Woody Allen' and 'Los Angeles') would hold that the first, but not the second, of the following pair of sentences is somehow deviant is to be explained by differences in the *objects named* by 'Woody Allen' and 'Los Angeles' and not by differences in the *semantics* of the two expressions:

> Woody Allen has a population of three million.
> Los Angeles has a population of three million.

By contrast, a number of differences in the behavior of 'I' and 'Woody Allen' (e.g. the contextual sensitivity of 'I') seem naturally viewed as resulting from a *semantic* difference between the two expressions/a difference in the way the two expressions are "hooked up" to the world. Perhaps some object could be constructed for 'I' to refer to such that the truth conditions for occurrences of sentences containing it come out right. But to do so is to represent the dissimilarities in the behavior of 'I' and 'Woody Allen' as like the dissimilarities in the behavior of 'Los Angeles' and 'Woody Allen': both sets of dissimilarities resulting from differences in the objects the expressions refer to. It seems to me we should ask of such an account: why should we view dissimilarities in behavior that are naturally viewed as arising from different sources as though they all arise from the same source (differences in objects named)? In so doing we risk obscuring important distinctions between classes of expressions and hence failing to ask important theoretical questions. Similarly, dissimilarities in the behavior of expressions of generality and referring terms seem naturally viewed as resulting from differences in the semantics of the two terms. In inventing objects for expressions of generality to refer to we assimilate the differences in behavior between expressions of generality and standard names to those between names of different kinds of things. Yet the differences in the behavior of 'every man' and 'Woody Allen' seem so unlike differences in behavior resulting from expressions naming different kinds of objects (such as that exemplified in our pair of sentences above) that it is hard to see why we should go along with such an assimilation when it is unnecessary to do so.[28]

Returning to the question of whether arbitrary object semantics or CDQ ought to be the preferred theory for instantial terms in English and natural deduction, variables in mathematics, and (at least some) anaphoric pronouns, arguments in favor of CDQ analogous to those just given seem even more decisive. As before the notions of scope and domain are crucial to providing a semantics for the terms in question; and the introduction of complicated objects as references for these expressions assimilates such expressions to names and in so doing obscures rather than highlights these notions. Again, in viewing the differences in behavior between these expressions and standard names as arising from the same source as differences in the behavior of names referring to objects of different kinds, we seem to group together cases that are quite dissimilar. But here we can go even further. *Ordinary* quantifiers possess scopes, and range over domains. What is distinctive about the terms that we have been discussing? Surely it is the fact that the semantic significance of an occurrence of one of these expressions is largely determined by the structure of the discourse it occurs in (whether the discourse be an English argument, a derivation of natural deduction, or a non-argumentative English discourse). Information necessary for the proper interpretation of occurrences of these expressions is, in effect, stored in the structure of the discourse. Of course this point is central to CDQ's conception of these expressions, and so is strongly emphasized by CDQ. The arbitrary objects approach, by contrast, obscures this point. Indeed, the only remarks one finds in RAO (Fine's most detailed work on the subject) that bear on the issue of sensitivity to context are that "... the interpretation of A-letters [instantial terms] gets determined in the course of the derivation." and that "The derivation provides us, in effect, with a definition of the A-letters it uses."[29] As in the previous case, arbitrary object semantics obscures the distinctive features of the expressions it claims to treat, while CDQ highlights these features. As a result, CDQ encourages us to ask important questions (such as: are there other expressions which exploit the information storage capacity of discourse structure in the way that these terms do?), that aren't raised by arbitrary object semantics. For these reasons, CDQ is the preferable theory of this data.

APPENDIX

The formulations of UG and EI are as follows:

$$
\begin{array}{ll}
\text{EI} & (\mathrm{E}x)P(x) \\
y & \begin{array}{|l} P(y) \\ . \\ . \\ . \\ A \end{array} \\
 & A
\end{array}
\qquad
\begin{array}{ll}
\text{UG} \; y & \begin{array}{|l} . \\ . \\ . \\ P(y) \end{array} \\
 & (x)P(x)
\end{array}
$$

Restrictions:[30] (i) A and $P(y)$ (in EI) and $P(y)$ (in UG) occur *immediately* subordinate to the y-flagged scope lines (i.e. since suppositions are entered as follows $\begin{array}{|l} P \\ \hline \\ \end{array}$ this means that A and $P(y)$ are subordinate to no temporary supposition or flagged scope line which is itself subordinate to the y-flagged scope line); (ii) $P(y)$ and $P(x)$ are formulas which differ only in that the latter contains occurrences of free x where and only where the former contains occurrences of free y; (iii) formulas containing free y cannot be reiterated across y-flagged scope lines; (iv) no formula may be entered by EI immediately subordinate to a flagged scope line which terminates in an application of UG (e.g. $P(y)$ in the UG schema above cannot have been entered by EI); (v) A (in the EI schema) must not contain occurrences of free y. We shall, in effect, suppose an EI strategy as schematized above to consist of two separate inferences: the inference of $P(y)$ from $(\mathrm{E}x)P(x)$ we shall call *EI* and we shall say that y was *introduced* in $P(y)$; the inference from A (containing no free occurrences of y) subordinate to the y-flagged scope line to A outside the y-flagged scope line we shall call *E discharge*. To save space, we shall not bother to formulate the classically valid rules of inference for the system (universal instantiation, existential generalization, and the propositional rules), since our concern is to construct a semantics for instantial terms that results in UG and EI being truth preserving with respect to the semantics.

Contexts

The context of a formula at a line in a derivation is to encode information about derivational structure affecting the semantic significance of the formula as it occurs in the derivation. Intuitively, the things that affect the semantic significance of an occurrence of a formula are the suppositions and flagged scope lines to which it is subordinate, and the order of subordination of the suppositions and flagged scope lines with respect to each other. Hence, we take the *context of a formula A at a line in a derivation* to be a sequence whose elements are the temporary suppositions and flags heading scope lines to which the formula is subordinate, in the order in which they occur in the derivation. To illustrate, the context of A at line i in the following derivation schema is $\langle P, y, Q, x \rangle$ ('P' and 'Q' stand for temporary suppositions; lower case letters are variables flagging scope lines):

```
    | P
    |—
    | y | | Q
    |   | |—
    |   | | x | .
    |   | |   | .
i)  |   | |   | A
```

Interpretations

$\langle U, f \rangle$ is an *interpretation* of our language, where U is a non-empty set and f is a function which maps individual constants, 0-place and n-place predicates ($n \geqslant 1$) to elements of U, T or F, and subsets of U^n respectively. $\langle\langle U, f \rangle, g\rangle$ is an *extended interpretation* and an *extension of* $\langle U, f \rangle$, where $\langle U, f \rangle$ is an interpretation and g is a function mapping variables to elements of U. The definition of truth under $\langle\langle U, f \rangle, g\rangle$ is standard (e.g. $(x)A(x)$ is true under $\langle\langle U, f \rangle, g\rangle$ iff $A(x)$ is true under every $\langle\langle U, f \rangle, g'\rangle$, where g' differs from g at most on x, etc.), except that free variables get treated like individual constants. A formula is true under $\langle U, f \rangle$ iff it is true under every extension of $\langle U, f \rangle$.

Levels of Information

Let $\langle c_1, \ldots, c_n \rangle$ be the context of A at line i in derivation D (henceforth '$A_{\mathrm{i},D}$' indicates the occurrence of formula A at line i in derivation D). Then:

1) The 0^{th} level of information of A in $\langle c_1, \ldots, c_n \rangle =$
$\{I \mid I$ is an extended interpretation and A is true under $I\}$
2) The m^{th} level of information of A in $\langle c_1, \ldots, c_n \rangle$ $(0 < m \leqslant n) =$
case a): $c_{(n+1)-m}$ is a formula B:
$\{I \mid I$ belongs to the $m - 1^{\text{st}}$ level of A in $\langle c_1, \ldots, c_n \rangle$ or I is an extended interpretation which makes B false$\}$
case b): $c_{(n+1)-m}$ is the variable x and flags an application of EI (let $P(x)$ be the formula in which x was introduced and be inferred from $(\mathrm{E}y)P(y)$ by EI):
$\{I \mid$ *either* I belongs to the $m-1^{\text{st}}$ level of A in $\langle c_1, \ldots, c_n \rangle$, makes $(\mathrm{E}y)P(y) \rightarrow P(x)$ true, and all I' differing from I at most on x which make $(\mathrm{E}y)P(y) \rightarrow P(x)$ true belong to the $m - 1^{\text{st}}$ level of A in $\langle c_1, \ldots, c_n \rangle$ *or* some I^* differing from I at most on x belongs to the $m - 1^{\text{st}}$ level of A in $\langle c_1, \ldots, c_n \rangle$, makes $(\mathrm{E}y)P(y) \rightarrow P(x)$ true, and all $I^{*\prime}$ differing from I^* at most on x which make $(\mathrm{E}y)P(y) \rightarrow P(x)$ true belong to the $m - 1^{\text{st}}$ level of A in $\langle c_1, \ldots, c_n \rangle\}$[31]
case c): $c_{(n+1)-m}$ is the variable x and flags an application of UG:
$\{I \mid I$ and every I' differing from I at most on x belongs to the $m - 1^{\text{st}}$ level of A in $\langle c_1, \ldots, c_n \rangle\}$

Truth in a Context Under an Interpretation

Let the context of $A_{\mathrm{i},D}$ be $\langle c_1, \ldots, c_n \rangle$. Then A *is true in* $\langle c_1, \ldots, c_n \rangle$ *under* $\langle U, f \rangle$ iff every extension of $\langle U, f \rangle$ belongs to the highest (i.e. the n^{th}) level of information of A in $\langle c_1, \ldots, c_n \rangle$.

CDQ Validity

Let B at line i be inferred from $B_1, \ldots, B_m$ at lines $j_1, \ldots, j_m$ in derivation D. The inference $B_1, \ldots, B_m/B$ is *CDQ valid in D* iff for any $\langle U, f \rangle$ such that $B_1, \ldots, B_m$ at lines $j_1, \ldots, j_m$ are true in their contexts under $\langle U, f \rangle$, B at i is true in its context under $\langle U, f \rangle$.

THEOREM 1: All inferences in derivations of our system are CDQ valid in those derivations.

To save space, we simply claim that classically valid rules of inference are CDQ valid in all derivations (again our concern is to show this for UG and EI).[32] This leaves UG and EI.

Case 1): $(x)A(x)_{k,D}$ was inferred from $A(y)_{\mathrm{i},D}$ by UG:

Note that the context of $A(y)_{\mathrm{i},D}$ differs from the context of $(x)A(x)_{k,D}$ only in containing y, and y is the last element of the context of $A(y)_{\mathrm{i},D}$ (since by restriction (i), $A(y)_{\mathrm{i},D}$ is immediately subordinate to the y-flagged scope line). Let the context of $A(y)_{\mathrm{i},D}$ be $\langle c_1, \ldots, c_n, y \rangle$. Then that of $(x)A(x)_{k,D}$ is $\langle c_1, \ldots, c_n \rangle$. To show that if $A(y)_{\mathrm{i},D}$ is true in $\langle c_1, \ldots, c_n, y \rangle$ under $\langle U, f \rangle$, $(x)A(x)_{k,D}$ is true in $\langle c_1, \ldots, c_n \rangle$ under $\langle U, f \rangle$, it suffices to show that any $\langle\langle U, f \rangle, g \rangle$ that belongs to the highest level of $A(y)_{\mathrm{i},D}$ in its context, belongs to the highest level of $(x)A(x)_{k,D}$ in its context. We shall prove the stronger claim that the highest level of $A(y)_{\mathrm{i},D}$ in its context is identical to the highest level of $(x)A(x)_{k,D}$ in its context. To show this, it suffices to show that the first level of $A(y)_{\mathrm{i},D}$ (i.e. that formed by appeal to y) is identical to the 0^{th} level of $(x)A(x)_{k,D}$ (for the contexts of the two formulas are term by term identical from this point on — hence the first, second, third, $\ldots$, n^{th} levels of $(x)A(x)_{k,D}$ will be identical to the second, third, fourth, $\ldots$, $n+1^{\text{st}}$ levels of $A(y)_{\mathrm{i},D}$ respectively). Suppose $\langle\langle U, f \rangle, g \rangle$ belongs to the 0^{th} level of $(x)A(x)_{k,D}$. Then $\langle\langle U, f \rangle, g \rangle$ makes $(x)A(x)$ true and hence every $\langle\langle U, f \rangle, g' \rangle$, where g' differs from g at most on x, makes $A(x)$ true. Suppose (for reductio) that $\langle\langle U, f \rangle, g \rangle$ doesn't belong to the first level of $A(y)_{\mathrm{i},D}$. Then some $\langle\langle U, f \rangle, g^* \rangle$, where g^* differs from g at most on y, isn't in the 0^{th} level of $A(y)_{\mathrm{i},D}$ and hence doesn't make $A(y)$ true. But then the $\langle\langle U, f \rangle, g' \rangle$ where g' differs from g at most on x and $g'(x) = g^*(y)$ makes $A(x)$ false (here we rely on the fact that $A(x)$ and $A(y)$ differ only in that the former contains free x where and only where the latter contains free y; restriction (ii) in the statement of UG and EI insures this). Contradiction. Hence $\langle\langle U, f \rangle, g \rangle$ belongs to the first level of $A(y)_{\mathrm{i},D}$. Suppose $\langle\langle U, f \rangle, g \rangle$ belongs to the first level of $A(y)_{\mathrm{i},D}$. Then every $\langle\langle U, f \rangle, g' \rangle$ where g' differs from g at most on y

belongs to the 0^{th} level of $A(y)_{\mathrm{i},D}$ and hence makes $A(y)$ true. Suppose (for reductio) that $\langle\langle U, f\rangle, g\rangle$ doesn't belong to the 0^{th} level of $(x)A(x)$ and hence doesn't make $(x)A(x)$ true. Then some $\langle\langle U, f\rangle, g^*\rangle$ where g^* differs from g at most on x doesn't make $A(x)$ true. But then the $\langle\langle U, f\rangle, g'\rangle$ where g' differs from g at most on y and $g'(y) = g^*(x)$ doesn't make $A(y)$ true. Contradiction. Hence, $\langle\langle U, f\rangle, g\rangle$ belongs to the 0^{th} level of $(x)A(x)_{k,D}$.
Case 2: $A(y)_{k,D}$ was inferred from $(\mathrm{E}x)A(x)_{\mathrm{i},D}$ by EI:
Note that the context of $A(y)_{k,D}$ differs from the context of $(\mathrm{E}x)A(x)_{\mathrm{i},D}$ only in containing y, and y is the last element of $A(y)_{k,D}$'s context. The proof is similar to the UG case.

THEOREM 2: (Classical Soundness) Let $B_1, \ldots, B_n$ be the permanent premises of derivation D, let B be the final formula in D, and let B be subordinate to no flagged scope lines in D. Then if $B_1, \ldots, B_n$ are true under $\langle U, f\rangle$, B is true under $\langle U, f\rangle$.[33]
PROOF: Note that the contexts of $B, B_1, \ldots, B_n$ in D will all be $\langle\,\rangle$, since these depend on no temporary assumptions and are subordinate to no flagged scope lines in D. Suppose that $B_1, \ldots, B_n$ are true under $\langle U, f\rangle$. Then $B_1, \ldots, B_n$ are true in their (null) contexts in D under $\langle U, f\rangle$ (i.e. every extension of $\langle U, f\rangle$ makes $B_1, \ldots, B_n$ true and hence belongs to the 0^{th} level of each of these formulas in its null context). So, by theorem 1, any formula C inferred from one or more of $B_1, \ldots, B_n$ is true in its context in D. By theorem 1 again, any formula inferred from C (and any other formulas inferred from one or more of $B_1, \ldots, B_n$) is true in its context in D under $\langle U, f\rangle$. So, by repeated applications of theorem 1, it will follow that B is true in its (null) context in D under $\langle U, f\rangle$. Thus B is true under $\langle U, f\rangle$. Q.E.D.

Roles of Restrictions on EI and UG in the Above Proofs

Restrictions (i) and (ii) were appealed to explicitly in the proof of theorem 1. Restriction (iii) is required to show that the rule of reiteration (which allows one to move formulas "inward" across scope lines) is CDQ valid (which we have not done). Restriction (iv) enables us to hold that each flag flags an application of UG or EI but not both: if the flag flags a scope line to which a formula entered by EI is immediately subordinate, it flags an application of EI; otherwise, it flags an application of UG (note that a "vacuous" flag — that is, a flag flagging a line to which no formula entered by EI is immediately subordinate and which does not end in an application of UG — gets construed as flagging an application of UG). Restriction (iv) was implicitly appealed to twice above. First, in defining *levels of information* we assumed that each variable flags an application of EI or UG but not both (see cases b) and c)). Second, in the proof of theorem 1, we assumed that if $(x)A(x)$ was inferred from $A(y)$ by UG, the variable y in the context of $A(y)$ flags an application of UG (only) and hence is to be treated in accordance with case c) in the definition of levels of information. Restriction (iv) justifies this assumption (and the analogous assumption in the EI case). Restriction (v) is required to prove the CDQ validity of what I above called E discharge (see note 32).

NOTES

* I wish to thank Michael Liston, Mark Wilson, and John Vickers for helpful discussions of the issues addressed herein.

[1] *Translations from the Philosophical Writings of Gottlob Frege* (eds. Geach and Black), Basil Blackwell, Oxford 1977, p. 110.

[2] Indeed, in *Begriffschrift*, Frege gives as the reason for introducing his quantifier notation the necessity of marking the scope of an expression of generality.

[3] See 'A Defence of Arbitrary Objects' (henceforth, DAO), *Proceedings of the Aristotelian Society* Supplementary Volume 57, 1983, pp. 55—77; 'Natural Deduction and Arbitrary Objects' (henceforth, NDAO), *Journal of Philosophical Logic* 14 (1985) 57—107; and *Reasoning with Arbitrary Objects* Aristotelian Society Series Volume 3 (henceforth, RAO), Basil Blackwell, Oxford, 1985.

[4] It may seem provincial of me to limit my attention to Fine's view and the alternative I intend to discuss, given the existence of sophisticated theories of anaphoric pronouns such as that outlined in Hans Kamp's 'A Theory of Truth and Semantic Representation' (in *Formal Methods in the Study of Language*, J. Groenendijk, T. Janssen, M. Stockhof (eds.), Mathematisch Centrum, Amsterdam, 1981, pp. 277—322). But Kamp's theory is incapable of handling much of the data that is at issue here. For example, that theory is incapable of handling the anaphoric pronoun in the following discourse

> Every female professor has a computer. She is financially responsible for it.

(I intend 'a computer' to have narrow scope with respect to 'Every female professor'). Kamp's rules of DRS construction prevent the construction of what Kamp calls a *complete DRS* for this discourse; and one must construct a complete DRS for a discourse for Kamp's semantics to handle the discourse. (In particular, Kamp's rule CR3 (p. 311) prevents substituting the "discourse referent" introduced by the processing of 'a computer' for 'it' in the second sentence, in effect preventing anaphoric connection between 'it' and 'a computer'; similarly for 'Every female professor' and 'she'.) More importantly, even if the rules were changed in order to allow the construction of a complete DRS for this discourse, the semantics would not come out right. For the value of the pronoun 'it' in the second sentence depends on the value of the pronoun 'She' in the sense that the truth of the sentence requires that for any female professor we choose (value of 'She') there must be a computer (value of 'it') such that the professor is financially responsible for the computer. Kamp's semantics does not include a device for keeping track of such dependence between the values of pronouns. This lack prevents Kamp's theory from handling instantial terms in natural deduction and English arguments, and variables in mathematics as well. But this is just the sort of data that is the subject of this essay.

[5] The resulting formulations of UG and EI are essentially those of Copi (*Symbolic Logic*, second edition; The Macmillan Company, New York, 1965) as reformulated by Kalish (review of Copi in *Journal of Symbolic Logic* 1967, vol. 32, p. 254).

[6] NDAO p. 64.

[7] What Fine actually does is to extend classical models for first order languages to what he calls *A-models* by adding (i) a set A of arbitrary objects; (ii) a relation on the set A (intuitively, the relation of dependence between arbitrary objects); (iii) and a designation function which maps A-letters (instantial terms in EI and UG) to elements of the set A (arbitrary objects). He then provides a definition of what it is for an A-model to be suitable for a given derivation D — roughly, suitable A-models properly interpret the instantial terms in D — and shows that any classical model can be extended to a suitable A-model for a given derivation D. Finally, Fine proves, for each system of natural deduction he treats, that if the premises of some rule of inference (including UG and EI) in a derivation D are true under a suitable A-model for D, the conclusion is true under that model.

[8] This account was first suggested in George Wilson's 'Pronouns and Pronomial Descriptions: A New Semantical Category' in *Philosophical Studies* 45 (1984), 1—30; and subsequently developed in more detail in my 'Pronouns, Descriptions, and the Semantics of Discourse' in *Philosophical Studies* 51 (1987), 341—363.

[9] I use the term 'relative scopes' here on analogy with the use of the term as applied to ordinary quantifiers. If, as I am suggesting, 'z' and 'w' are expressions of generality with universal and existential force respectively, 'Fwz' could express the claim that for any z, there is a w such that Fwz or there is a w, such that for any z Fwz. The different readings, as I shall say, result from giving 'z' and 'w' different *relative scopes*. In the sequel, whenever I talk about the relative scopes of what I shall call context dependent quantifiers, I intend the term 'relative scopes' in the same way. The point being made here is that which of these two possible readings is intended is not marked by syntactical features of the *formula* 'Fwz', but by syntactical features of the derivation it occurs in (i.e. the order of subordination of the flagged scope lines).

[10] I intend sentence 1) to be understood in such a way that 'Every professor' takes wide scope over 'a large lecture class'. Each anaphoric pronoun/description is intended to be anaphoric to the noun phrase with the same numerical subscript.

[11] The force of one of these context dependent quantifiers/anaphoric descriptions is determined by the force of its quantifier antecedent. Similarly for the domain over which the expression ranges. The relative scopes of these context dependent quantifiers are determined by the relative scopes of their quantifier antecedents (they take wide scope with respect to all other operators in sentence). Hence in sentence 2) of (F2) 'The professor' has wide scope with respect to 'the class' (i.e. the sentence claims that for any professor at the University of San Clemente, there is a large lecture class he/she teaches for which he/she does all the grading).

[12] I haven't explained the details of this semantic for the same reason I avoided the technical details of Fine's theory: these details aren't relevant to the issues I wish to raise. The interested reader may look at the application of this type of semantic theory

to natural deduction in the appendix of the present paper, or consult my paper mentioned in note 8.
[13] RAO pp. 122—26 and NDAO pp. 99—102.
[14] Fine mentions a number of such failures in RAO p. 123.
[15] *Methods of Logic*, revised edition; Holt, Rinehart and Winston, New York 1959, pp. 153—167.
[16] As Fine points out, the system appears incorrectly formulated in Copi's *Symbolic Logic*, second edition, The Macmillan Company, New York 1965 and was correctly formulated in Kalish's review of the former work in *Journal of Symbolic Logic* vol. 32, 1967, p. 254.
[17] The interested reader should consult the appendix to this paper to see how the restrictions on UG insure that when an occurrence of a formula B is inferred from an occurrence of a formula A by UG, A, B, and their contexts have certain properties which allow the proof of the claim that if A is true in its context under an interpretation, B is true in its context under the interpretation. See particularly the last section.
[18] RAO p. 141—142; NDAO p. 105.
[19] RAO p. 142.
[20] DAO p. 75.
[21] RAO p. 142.
[22] These remarks may appear inconsistent with the "criticism" of Fine made above. There I claimed that attempting to apply Fine's theory directly to the example results in non-sensicality. But the fact that Fine talks about the "A-objects k and h" suggests that *he* envisions applying his theory to the example as is (especially since he says nothing to indicate otherwise). My point is that since Fine apparently thinks his theory will apply to the example as it stands, and yet the theory as formulated doesn't handle the example as it is, it seemed odd to cite the example in support of the theory.
[23] Here we define the limit of an infinite sequence of reals. Other sorts of limits (e.g. limit of a function) are also defined by means of epsilon/delta formulations, as are the various notions of a function being continuous.
[24] The following passage, in which the author introduces '$\rightarrow$' notation after giving an epsilon/delta definition of the limit of a sequence and introducing the notation '$\lim_{n \to \infty} a_n = A$' to mean that the limit of the sequence $a_1, a_2, \ldots$ is A, is typical: "As an alternative notation to [$\lim_{n \to \infty} a_n = A$] we sometimes write $a_n \to A$ as $n \to \infty$" (*Calculus with Analytic Geometry* by Burton Rodin, Prentice-Hall, New Jersey, 1970, p. 30).
[25] Recall that Fine has not actually shown that arbitrary objects semantics can handle the pronouns that CDQ is capable of treating, nor has he shown how to reformulate the theory to treat the variables in the passage we have just discussed. In talking about parity between CDQ and arbitrary object semantics here, I am assuming that these promisory notes can be cashed in.
[26] In a number of places Fine suggests that he favors just such a semantics for quantifiers. See RAO pp. vii—viii and p. 130. It is interesting to note that on such a view, quantifier phrases are apparently virtually infinitely ambiguous! For consider the following sentences: 1) John loves a woman. 2) Every man over twenty-one loves a woman. 3) Every male friend of every man over twenty-one loves a woman. (I assume that what we would normally call the scopes of the quantifiers are determined by the left-to-right ordering of the quantifiers — the left-most quantifier taking widest scope, etc.) In Fine's technical terms, the arbitrary object that 'a woman' refers to in 1) doesn't *depend on* any other arbitrary object. In sentence 2), however, the arbitrary object 'a woman' refers to depends on the arbitrary object 'Every man' refers to (we would normally express this by saying that 'every man' has wide scope over 'a woman'). But according to Fine's identity conditions for arbitrary objects, A-objects that depend on

different A-objects are different. Hence 'a woman' refers to a different A-object in 1) than it does in 2). For the same reason, 'a woman' in 3) refers to a different A-object than it does in either 1) or 2). Since 'a woman' can (as we usually say) take narrow scope with respect to an indefinite number of universal quantifiers, there is no upper limit on the number of different A-objects 'a woman' can refer to.

27 Of course these notions can be given different names and captured differently within different formalisms. For example, rather than appealing to the usual notion of quantifier scope somehow marked syntactically, Hintikka (*Anaphora and Definite Descriptions*; D. Reidel, Dordrecht; 1985) in constructing his game theoretical semantics for natural languages, appeals to *principles of rule ordering* which determine the order in which evaluation clauses for different quantifiers are to be applied.

28 It is just this sort of consideration that militates against explaining the fact that 'I' can refer to different individuals by claiming it is ambiguous. The behavior of 'I' in this respect is so unlike genuinely ambiguous expressions that we think it best to provide a different explanation of its behavior.

29 RAO p. 92.

30 Because it reduces clutter and doesn't lead to confusion, I avoid quotation marks (and other devices) to distinguish use and mention throughout the appendix.

31 Some might have expected a simpler clause here according to which the set of extended interpretations would be characterized as: $\{I \mid I$ belongs to the $m - 1^{st}$ level of A in $\langle c_1, \ldots, c_n \rangle$ and makes $(\mathrm{E}y)P(y) \rightarrow P(x)$ true, or some I' differing from I at most on x belongs to the $m - 1^{st}$ level of A in $\langle c_1, \ldots, c_n \rangle$ and makes $(\mathrm{E}y)P(y) \rightarrow P(x)$ true$\}$. Though such a clause would indeed suffice to prove that EI is truth-preserving with respect to our semantics (i.e. CDQ valid), the more complicated clause is required in order that certain truth functional inferences occurring subordinate to scope lines marking an EI strategy be truth preserving. For example, without the more complicated clause, the following sort of inference ('Fx & Gx' from 'Fx' and 'Gx') in general would not be truth-preserving: $(\mathrm{E}x)Hx$

$$
\begin{array}{l|l}
x & Hx \\
 & \\
 & Fx \\
 & \\
 & Gx \\
 & \\
 & Fx \;\&\; Gx
\end{array}
$$

32 The CDQ validity in a given derivation of what I earlier called *E discharge* follows from the following easily proved lemma: If a formula A is true under an interpretation in a context C containing a (flagged) variable y not occurring in A, then A is true in the context resulting from deleting y from C.

33 A more rigorous proof of theorem 2 could be given by first using theorem 1 to prove the following lemma: If $B_1, \ldots, B_n$ are the permanent premises of D, then if $B_1, \ldots, B_n$ are true under $\langle U, f \rangle$, every formula in D is true in its context under $\langle U, f \rangle$. Theorem 2 follows directly.

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