Graham Oddie and Pavel Tichý

The Logic of Ability, Freedom and Responsibility

Abstract. The aim of this paper is to offer a rigorous explication of statements ascribing ability to agents and to develop the logic of such statements. A world is said to be *feasible* iff it is compatible with the actual past-and-present. W is a *P-world* iff W is feasible and P is true in W (where P is a proposition). P is a *sufficient condition* for Q iff every P world is a Q world. P is a *necessary condition* for Q iff Q is a sufficient condition for P. Each individual property S is shown to generate a rule for an agent X. X *heeds* S iff X makes all his future choices in accordance with S. (Note that X may heed S and yet fail to have it). S is a *P-strategy for X* iff X's heeding S together with P is a necessary and sufficient condition for X to have S. (P-strategies are thus rules which X is able to implement on the proviso P). *Provisional opportunity*: X has the *opportunity* to A *provided* P iff there is an S such that S is a P-strategy for X and X's implementing S is a sufficient condition for X's doing A. P is *etiologically complete* iff for every event E which P reports P also reports an etiological ancestry of E, and P is true. *Categorical opportunity*: X has the *opportunity* to A iff there is a P such that P is etiologically complete and X has the opportunity to A provided P. For X to have the ability to A there must not only *be* an appropriate strategy, but X must have a command of that strategy. X *steadfastly intends* A iff X intends A at every future moment at which his doing A is not yet inevitable. X has a *command* of S w.r.t. A and P iff X's steadfastly intending A together with P is a sufficient condition for X to implement S. *Provisional ability*: X *can* A *provided* P iff there is an S such that S is a P-strategy for X, X's implementing S is a sufficient condition for X's doing A, and X has a command of S w.r.t. A and P. *Categorical ability*: X *can* A iff there is a P such that P is etiologically complete and X can A provided P. X is *free* w.r.t. to A iff X can A and X can non-A. X is *free* iff there is an A such that X is free w.r.t. A.

0. Introduction

The aim of this article is to offer a rigorous explication of statements ascribing ability to agents, and to develop the logic of such statements. The explication is framed in the system of transparent intensional logic (t.i.l.) which is briefly sketched in sections 1 and 2. A detailed exposition of t.i.l. can be found in [4]. For other applications of t.i.l. see [1], [2], and [3].

The philosophical background and motivation for the present theory are expounded in detail in [5]. The key concept of the theory is that of a *strategy for* an agent. An individual property S is a strategy for an agent X just in case S provides X with a plan for any contingency that may arise. The instantiation of S by X at the present moment thus depends solely on whether X makes choices in accordance with the instructions given by S. Roughly speaking, X has an *opportunity* to A if there is an S such that S is a strategy for X and the implementation of S by X is a sufficient condition for X to A. X is said to have a *command of S relative to A* if he is disposed to follow S in any situation in which he consistently intends to A. X has the *ability* to A – i.e., X *can* A – if there is an S such that S is a strategy for X, X's implementing S is a sufficient condition for X's Aing, and X has a command of S relative to A. X is *free with respect to A* if he can both A and non-A. X is a *free agent* if there is an A such that X is free w.r.t. A. Finally, X is (partially) *responsible for* a state of affairs Q if there has been a time at which X could have done something to avert Q.

1. Objects and constructions

Any conceptual scheme (such as the one underlying ordinary language) is based on a *universe of discourse*, i.e., the collection of the lowest-level objects (called *individuals*) coming under the purview of the scheme, and an *intensional base*, i.e. , the collection of primitive attributes that can, within the framework, be ascribed to various set-theoretical objects over the universe. Together a universe of discourse and an intensional base form what we shall call an *epistemic framework.*

Once an epistemic framework is given, a range of possibilities arise as to how the attributes in the intensional base are in fact distributed through the objects over the universe. As the possession of an attribute by an object is a time-dependent affair, the possibilities are, more particualrly, possible *histories* of the distribution. It is customary to speak of those possibilities as *possible worlds*, and of the collection of all possible worlds as the *logical space* of the epistemic framework. One of the possible worlds is the *actual world*; it is, however, no part of the definition of the framework to specify which world it is. To locate the actual world in the logical space is the ultimate (and probably unattainable) aim of *factual* inquires conducted within the framework.

Let o be the two-element class of truth-values (*truth*, **T**, and *falsehood*, **F**), ι the universe of discourse, and ω the logical space. Moreover, let τ be the time scale, i.e., the linearly ordered class of moments of time. Note that if an origin and a unit of duration – say, one second – are fixed, moments of time can be represented in a one-to-one fashion by real numbers. Thus τ can be looked upon as the class of real numbers.

o, ι, ω, and τ are *types*. Besides, where η, $\xi_1, \dots, \xi_n$ are arbitrary *types*, the class of all (total and partial) n-ary functions from $\xi_1, \dots, \xi_n$ into η – symbolically, $(\eta\xi_1 \dots \xi_n)$ – is also a *type*. (Nothing is a *type* unless it so follows from the above.) A member of a type ξ is also called an object of type ξ, or briefly a *ξ-object*.

Objects of type $(o\xi)$ are called classes of ξ-objects, or briefly *ξ-classes*. Where C is a ξ-class and X a ξ-object, X is said to be an *element* or *counter-element* of C according as the value of C at X is **T** or **F**. Objects of type

$(o\xi\eta)$ are called (binary) linkages between ξ-objects and η-objects, or briefly *ξ, η-linkages*. Where L is a ξ, η-linkage, X a ξ-object, and Y an η-object, X is said to be *linked* or *counterlinked* by L to Y according as L takes X and Y to $\mathbf{T}$ or $\mathbf{F}$.

Some particular objects deserve special mention. $\mathbf{0}$, of type τ, is the number nought (or the origin of the time scale). $\sim$, of type (oo), and $\supset$, of type (ooo), are the familiar truth-functions, and $+$, of type $(\tau\tau\tau)$ is addition. $=\xi$, of type $(o\xi\xi)$, is identity between ξ-objects, and $<$, of type $(o\tau\tau)$, is the less-then (or before) relation between numbers (or times). Π^ξ and Σ^ξ, both of type $(o(o\xi))$, are the universal and existential quantifiers over ξ-objects: the value of Π^ξ (resp. Σ^ξ) at a ξ-class is $\mathbf{T}$ or $\mathbf{F}$ according as the class does or does not contain all (resp. some) ξ-objects. $\Delta^{\eta\xi}$ is the $(\eta\xi)$, ξ, η-linkage which links or counterlinks F with X and Y according as F does or does not take X to Y.

Objects of type $((\xi\tau)\omega)$ are called *ξ-intensions*. Where I is a ξ-intension and W a world, the value (if any) of I at W is called the *chronology* of I in W; moreover, if I's chronology takes a moment T to ξ-object X, we say that X *occupies* (or *embodies*) *I in W at T*. If nothing occupies I in W at T, we say that I is *vacant* in W at T. A ξ-intension can thus be regarded as an office occupiable by ξ-objects. The American presidency, for example, is an office occupiable by individuals; call it $\mathbf{A}$. If W is the actual world and T any moment between August 9, 1974 and January 20, 1977, $\mathbf{A}$ is held by Gerald R. Ford in W at T.

Some kinds of intension are particularly noteworthy. o-intensions are known as *propositions*. We shall use the letter 'π' to denote the type $((o\tau)\omega)$ of propositions. Where A is a proposition, A is said to be *true* or *false* in W at T according as it is occupied by $\mathbf{T}$ or $\mathbf{F}$ in W at T; if A is neither true nor false in W at T, then A is said to be *vacuous* in W at T. $(o\xi)$-intensions are known as properties of ξ-objects, or briefly as *ξ-properties*. Where S is a ξ-property and X a ξ-object, X is said to *instantiate* or *counterinstantiate* S in W at T according as X is an element or counterelement of the occupant of S in W at T. We shall use the letter 'σ' to denote the type $(((o\iota)\tau)\omega)$ of ι-properties. Redness, call it $\mathbf{R}$, is an example of a ι-property, i.e., of a σ-object; an individual instantiates $\mathbf{R}$ in W at T just in case it is red in W at T. $(o\xi\eta)$-intensions are known as (binary) relations between ξ-objects and η-objects, or briefly as ξ, η-relations. Where R is a ξ, η-relation, X a ξ-object and Y an η-object, X is said to be *related* or *counterrelated* by R to Y in W at T according as X is linked or counterlinked to Y by the occupant of R in W at T.

Given some objects, other objects can often be *constructed* from them. Thus if F is a function of type $(\eta\xi)$ and is defined at a ξ-object X, then an object — namely the value of F at X — can be constructed by *applying* F to X. We shall speak of this construction as $[FX]$. (Note that the notation '$[FX]$' does not stand for whatever η-object is the value of at X, but rather for a particular *way* of arriving at that object, namely by applying F to X). By replacing F or X in $[FX]$ with an appropriate variable we obtain an *open* construction ($[fX]$ or $[Fx]$ or $[fx]$), a construction, that is, which depends for what it constructs on values assigned to its variable(s). Variables in an open construction can be *abstracted upon*. For example, abstracting upon f in $[fX]$ we get a function which takes any $(\eta\xi)$-object to the value taken by that object at X. This particular construction of that function from $[fX]$ will be spoken of as $[\lambda f[fX]]$. A construction may be *improper* in the sense of constructing nothing at all. For example, if F is not defined at X, then the construction $[FX]$ is improper; but it is, nevertheless, a construction.

The proposition that the American president is red, for example, can be constructed thus: $\left[\lambda\omega\left[\lambda t\left[[[\mathbf{R}w]t]\,[[\mathbf{A}w]t]\right]\right]\right]$ (w and t being variables ranging over ω and τ respectively). As it constructs a π-object, the construction will be also called a π-construction.

We shall now define the notion of ξ-construction inductively. In doing so, it will be convenient to write '$\tilde{\mathrm{x}}_n$', '$\tilde{\mathrm{X}}_n$', '$\tilde{\xi}_n$' etc. respectively for '$\mathrm{x}_1\mathrm{x}_2\ldots\ldots\mathrm{x}_n$', '$\mathrm{X}_1\mathrm{X}_2\ldots X_n$', '$\xi_1\xi_2\ldots\xi_n$' etc., and '$\overline{\mathrm{x}}_n$', '$\overline{\mathrm{X}}_n$' etc. for '$\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_n$', '$\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_n$' etc.

\#1. Let x be a ξ-object or a variable of type ξ. Then x is a *ξ-construction*.

\#2. Let F be a *$(\eta\tilde{\xi}_n)$-construction*, X_1 a *ξ_1-construction*, ..., and X_n a *ξ_n-construction*. Then the application $[\mathrm{F}\tilde{\mathrm{X}}_n]$ of F to $\overline{\mathrm{X}}_n$ is an *η-construction*.

\#3. Let Y be an *η-construction* and $\overline{x}_n$ distinct variables of the respective types $\tilde{\xi}_n$. Then the abstraction $[\lambda\tilde{x}_n\mathrm{Y}]$ of Y on $\overline{x}_n$ is a *$(\eta\tilde{\xi}_n)$-construction*.

\#4. Nothing is a *ξ-construction* unless it so follows from #1–3.

The lettering used so far will be used throughout the paper. In particular, small Roman letters will stand for unspecified objects or variables, capital Roman letters for unspecified constructions, small italics for unspecified variables, capital italics for unspecified objects, and small Greek letters other than o, ι, τ, ω, π, and σ for unspecified types. Besides, brackets will be omitted where no confusion can result, and a dot will represent a left-hand bracket whose right-hand mate is to be imagined as far to the right as is consistent with other pairs of brackets. By $\mathrm{X}(\overline{Z}_n/\overline{z}_n)$ we shall understand the result of supplanting the free occurrences of $\overline{z}_n$ by $\overline{Z}_n$ respectively.

Parentheses and the superscripts which go with the symbols 'Π', 'Σ', '=', and 'Δ' will also be omitted where possible. Moreover, by $\mathrm{X} = \mathrm{Y}$, $\mathrm{A} \supset \mathrm{B}$, etc. we shall understand $= \mathrm{XY}$, $\supset \mathrm{AB}$ etc, and by $(\forall x)\mathrm{A}$ and $(\exists x)\mathrm{A}$ shall understand $\Pi^{\xi}\lambda x\mathrm{A}$ and $\Sigma^{\xi}\lambda x\mathrm{A}$ (where x/ξ). Further notational economy will be achieved by writing 'X_Y', 'θ_ξ' etc. for '[XY]', '$(\theta\xi)$' etc, and 'X_{YZ}', '$\theta_{\xi\eta}$' etc. for '[[XY]Z]', '$((\theta\xi)\eta)$' etc. For example, $\iota_{\tau\omega}$ is the same as $((\iota\tau)\omega)$ and $\lambda\omega\lambda t.\mathbf{R}_{wt}\mathbf{A}_{wt}$ is the above construction of the proposition that the American president is red.

2. Derivations

An ordered couple whose first component is a ξ-object or a ξ-variable a and whose second component is a ξ-construction A, symbolically a:A, will be called a *match*. An assignment v of values to the variables is said to *satisfy* a:A if on v a and A construct one and the same object. We shall also allow for matches whose first components are missing, symbolically: A. Assignment v *satisfies* :A if A is improper on v. Two matches are said to be *patently incompatible* if they are of the form A_1:A, A_2:A, where A_1 and A_2 are distinct objects, or of the form a:A, :A. Patently incompatible matches are clearly never satisfied by the same valuation.

A couple whose first component is a finite set Φ of matches and whose second component is a match $\mathfrak{M}$ is called a *sequent* and symbolized thus: $\Phi \to \mathfrak{M}$. The members of Φ are called the *antecedents* and $\mathfrak{M}$ is called the *succedent* of $\Phi \to \mathfrak{M}$. We shall write $\mathfrak{M}_1, \ldots, \mathfrak{M}_n \to \mathfrak{M}$ for $\{\mathfrak{M}_1, \ldots, \mathfrak{M}_n\} \to \mathfrak{M}$. Assignment v *satisfies* Φ if it satisfies every member of Φ. $\Phi \to \mathfrak{M}$ is *valid* if every assignment which satisfies Φ also satisfies $\mathfrak{M}$.

In what follows we shall state a number of validity-preserving operations on sequents, called *rules of derivation*. Rules of derivation will be stated in the following form:

(*) $\quad \Phi_1 \to \mathfrak{M}_1;\ \Phi_2 \to \mathfrak{M}_2;\ \ldots;\ \Phi_k \to \mathfrak{M}_k \vDash \Phi \to \mathfrak{M}.$

In (*), $\Phi_1 \to \mathfrak{M}_1$, $\Phi_2 \to \mathfrak{M}_2, \ldots, \Phi_k \to \mathfrak{M}_k$, and $\Phi \to \mathfrak{M}$ represent sequents of certain specific forms, peculiar to the rule in question. A clause may be attached to (*) whereby some additional constraints are imposed on those sequents. Such clauses will be called *conditions*. (*) affirms that whenever the sequents on the left of $\vDash$ are valid, the sequent on the right is also valid. The latter sequent is then said to be *derivable from* the former sequents *according to* the rule (*).

A finite string of sequents is said to be a *derivation with respect to* (*w.r.t.*) a given set $\mathfrak{R}$ of rules of derivation if each term of the string — also called a *step* of the derivation — is derivable from earlier steps according to a member of $\mathfrak{R}$. If $\Phi \to \mathfrak{M}$ is a step of a derivation w.r.t. $\mathfrak{R}$ it is said to be *derivable w.r.t.* $\mathfrak{R}$, symbolically $\vdash_{\mathfrak{R}} \Phi \to \mathfrak{M}$.

The following are some basic rules of derivation (stated without proof).

2.1 $\quad \vDash \Phi \to \mathfrak{M}$. *Condition*: $\mathfrak{M}$ belongs to Φ.

2.2 $\quad \Psi \to \mathfrak{M} \vDash \Phi \to \mathfrak{M}$. *Condition*: Ψ is a subset of Φ.

2.3 $\quad \Phi, \mathfrak{N} \to \mathfrak{M};\ \Phi \to \mathfrak{N} \vDash \Phi \to \mathfrak{M}$.

2.4 $\quad \vDash$ a:a.

2.5 $\Phi \to \mathfrak{Q}_1$; $\Phi \to \mathfrak{Q}_2 \models \Phi \to \mathfrak{M}$. *Condition*: $\mathfrak{Q}_1$ and $\mathfrak{Q}_2$ are patently incompatible.

2.6 $\Phi, :\mathrm{A} \to \mathfrak{M}$; $\Phi, a:\mathrm{A} \to \mathfrak{M}$. $\models \Phi \to \mathfrak{M}$ *Condition*: a is not free in Φ, A, and $\mathfrak{M}$.

2.7 $\Phi \to y:\mathrm{F}\tilde{\mathrm{X}}_m$; $\Phi, f:\mathrm{F}, x_1:\mathrm{X}_1, \ldots, x_m:\mathrm{X}_m \to \mathfrak{M} \models \Phi \to \mathfrak{M}$. *Condition*: f, $\tilde{x}_m$ are distinct and not free in Φ, $\mathrm{F}\tilde{\mathrm{X}}_m$, and $\mathfrak{M}$.

2.8 $\Phi \to y:\mathrm{F}\tilde{\mathrm{X}}_m$; $\Phi \to \mathrm{x}_1:\mathrm{X}_1; \ldots; \Phi \to \mathrm{x}_m:\mathrm{X}_m \models \Phi \to y:\mathrm{F}\tilde{\mathrm{x}}_m$.

2.9 $\Phi \to y:\mathrm{F}\tilde{\mathrm{x}}_m$; $\Phi \to \mathrm{x}_1:\mathrm{X}_1; \ldots; \Phi \to \mathrm{x}_m:\mathrm{X}_m \models \Phi \to y:\mathrm{F}\tilde{\mathrm{X}}_m$.

2.10 $\Phi,\ y:\mathrm{f}\tilde{x}_m \to y:\mathrm{g}\tilde{x}_m$; $\Phi,\ y:\mathrm{g}\tilde{x}_m \to y:\mathrm{f}\tilde{x}_m \models \Phi \to \mathrm{f}:\mathrm{g}$. *Condition*: $\tilde{x}_m$ and y are distinct and not free in Φ, f, g.

2.11 $\Phi, f:\lambda\tilde{x}_m\mathrm{Y} \to \mathfrak{M} \models \Phi \to \mathfrak{M}$. *Condition*: f is not free in Φ, $\lambda\tilde{x}_m\mathrm{Y}$, and $\mathfrak{M}$.

2.12 $\Phi \to \mathrm{a}:[\lambda\tilde{z}_m\mathrm{A}]\tilde{\mathrm{Z}}_m \models \Phi \to \mathrm{a}:\mathrm{A}(\bar{\mathrm{Z}}_m/\bar{z}_m)$. *Condition*: for $1 \leqslant i \leqslant m$, every variable free in Z_i is free for z_i in A.

2.13 $\Phi \to \mathrm{z}_1:\mathrm{Z}_1; \ldots; \Phi \to \mathrm{z}_m:\mathrm{Z}_m$; $\Phi \to \mathrm{a}:\mathrm{A}(\bar{\mathrm{Z}}_m/\bar{z}_m) \models \Phi \to \mathrm{a}:[\lambda\tilde{z}_m\mathrm{A}]\tilde{\mathrm{Z}}_m$. *Condition*: $\bar{z}_m$ are distinct and for $1 \leqslant i \leqslant m$ all variables free in Z_i are free for z in A.

The following seventeen rules are based on sundry properties of the objects **T**, **F**, $\sim$, $\supset$, Π^ξ, Σ^ξ, $\Delta^{\eta\xi}$ and $=^\xi$ (i, j, I, i/o; x, z, X, Y, x/ξ; y/η; $\mathrm{F}/\eta\xi$; c, $\mathrm{C}/o\xi$):

2.14 $\Phi, \mathbf{T}:\mathrm{i} \to \mathfrak{M}$; $\Phi, \mathbf{F}:\mathrm{i} \to \mathfrak{M} \models \Phi \to \mathfrak{M}$.

2.15 $\Phi, i: \sim\mathrm{i} \to \mathfrak{M} \models \Phi \to \mathfrak{M}$. *Condition*: i is not free in Φ, i, and $\mathfrak{M}$.

2.16 $\Phi, \mathrm{i}:\mathrm{j} \to \mathfrak{Q}_1$; $\Phi, \mathrm{i}:\mathrm{j} \to \mathfrak{Q}_2 \models \Phi \to \mathrm{i}: \sim\mathrm{j}$. *Condition*: $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are patently incompatible.

2.17 $\Phi, i:\ \mathrm{i} \supset \mathrm{j} \to \mathfrak{M} \models \Phi \to \mathfrak{M}$. *Condition*: i is not free in Φ, u, v, $\mathfrak{M}$.

2.18 $\Phi, \mathbf{T}:\ \mathrm{i} \to \mathbf{T}:\mathrm{j} \models \Phi \to \mathbf{T}:\mathrm{i} \supset \mathrm{j}$.

2.19 $\Phi \to \mathbf{T}:\mathrm{I} \supset \mathrm{J}$; $\Phi \to \mathbf{T}:\mathrm{I} \models \Phi \to \mathbf{T}:\mathrm{J}$.

2.20 $\Phi, i:Q\mathrm{c} \to \mathfrak{M} \models \Phi \to \mathfrak{M}$. *Condition*: Q is Π^ξ or Σ^ξ and i is not free in Φ, $\mathfrak{M}$.

2.21 $\Phi \to \mathbf{T}:\mathrm{C}x \models \Phi \to \mathbf{T}:\Pi\mathrm{C}$. *Condition*: x is not free in Φ, C.

2.22 $\Phi \to \mathbf{T}:\Pi\mathrm{C} \models \Phi \to \mathbf{T}:\mathrm{Cx}$

2.23 $\Phi \to \mathbf{T}:\mathrm{CX} \models \Phi \to \mathbf{T}:\Sigma\mathrm{C}$.

2.24 $\Phi \to \mathbf{T}:\Sigma\mathrm{C}$; $\Phi, \mathbf{T}:\mathrm{C}x \to \mathfrak{M} \models \Phi \to \mathfrak{M}$. *Condition*: x is not free in Φ, C, and $\mathfrak{M}$.

2.25 $\Phi, i:\Delta\mathrm{fxy} \to \mathfrak{M} \models \Phi \to \mathfrak{M}$. *Condition*: i is not free in Φ, x, y, and $\mathfrak{M}$

2.26 $\Phi \to \mathbf{T}:\Delta\mathrm{FXy} \models \Phi \to \mathrm{y}:\mathrm{FX}$.

2.27 $\Phi\rightarrow \mathrm{y}:\mathrm{FX} \models \Phi\rightarrow \mathbf{T}:\Delta\mathrm{FXy}$.

2.28 $\Phi, i:\mathrm{x} = \mathrm{z}\rightarrow\mathfrak{M} \models \Phi\rightarrow\mathfrak{M}$.

2.29 $\Phi\rightarrow\mathbf{T}:\mathrm{x} = \mathrm{X} \models \Phi\rightarrow\mathrm{x}:\mathrm{X}$.

2.30 $\Phi\rightarrow\mathrm{x}:\mathrm{X} \models \Phi\rightarrow\mathbf{T}:\mathrm{x} = \mathrm{X}$.

The following is an example of a derivation w.r.t. rules 2.1–2.30. For brevity, the numeral denoting the ordinal number of a step is also used as an abbreviation of the succedent of that step, and steps justifiable by 2.2 are omitted.

$1.1\rightarrow\mathbf{T}$: $.\Sigma\lambda x .\sim \mathrm{I}(2.1)/2.2\rightarrow\mathbf{T}$: $[\lambda x .\sim \mathrm{I}]x(2.1)/3.2\rightarrow\mathbf{T}$: $\sim\mathrm{I}(2, 2.12)/4.4 \rightarrow i$: $\mathrm{I}(2.1)/5.2, 4\rightarrow\mathbf{T}$: $\sim i(3, 4, 2.8)/6.6\rightarrow c$: $\lambda x\mathrm{I}(2.1)/7.7\rightarrow j$: $\Pi c(1.1)/ /8.6, 7\rightarrow j$: $\Pi\lambda x\mathrm{I}(7, 6, 2.9)/9.9\rightarrow\mathrm{T}$: $j(2.1)/10.\rightarrow j$: $j(2.4)/11.9\rightarrow\mathbf{T}$: $[\lambda jj]j(9, 10, 2.13)/12.7, 9\rightarrow\mathbf{T}$: $[\lambda jj]$. $\Pi c(9, 7, 2.9)/13.6, 7, 9\rightarrow\mathbf{T}$: $\Pi\lambda x\mathrm{I}(12, 6, 2.9)/ /14.6, 7, 9\rightarrow\mathbf{T}$: $[\lambda x\mathrm{I}]x(13, 2.2)/15.6, 7, 9\rightarrow\mathbf{T}$: $\mathrm{I}(14, 2.12)/16.6, 7, 9\rightarrow\mathbf{T}$: $[\lambda ii]\mathrm{I}(15, 2.13)/17.4, 6, 7, 9\rightarrow\mathbf{T}$: $[\lambda ii]i(16, 4, 2.8)/18.4, 6, 7, 9\rightarrow\mathbf{T}$: $i(17, 2.12)/19.2, 4, 6, 7, 9\rightarrow\mathbf{T}$: $\sim\mathbf{T}(5, 18, 2.8)/20.20\rightarrow\mathbf{F}$: $\mathbf{T}(2.1)/21.\rightarrow\mathbf{F}$: $\mathbf{F}(2.4)/ /22.\rightarrow\mathbf{F}$: $\sim\mathbf{T}(20, 21, 2.16)/23.2, 4, 6, 7\rightarrow\mathbf{T}$: $\sim j(19, 22, 2.16)/24.2, 4, 6, 7 \rightarrow\mathbf{T}$: $\sim. \Pi\lambda x\mathrm{I}(23, 8, 2.9)/25.2, 4, 6\rightarrow 24(24, 2.20)/26.2, 4\rightarrow 24(25, 2.11)/ /27.2\rightarrow 24(3, 26, 2.7)/28.1\rightarrow 24(1, 27, 2.24)$.

Thus the sequent $\mathbf{T}$: $.\Sigma\lambda x .\sim \mathbf{I}\rightarrow\mathbf{T}$: $\sim. \Pi\lambda x\mathrm{I}$ is derivable w.r.t. the rules 2.1–2.30. We shall often deal with sequents which, like the above example, are of the form x: $\mathrm{A}_1, \ldots, \mathrm{x}:\mathrm{A}_n\rightarrow\mathrm{x}:\mathrm{A}$. In order to save space, we shall symbolize such sequents thus: $\mathrm{A}_1, \ldots, \mathrm{A}_n\rightarrow_{\mathrm{x}}\mathrm{A}$. Extra notational economy will be achieved by writing $\mathrm{A}\leftrightarrow_{\mathrm{x}}\mathrm{B}$ to denote the pair of sequents $\mathrm{A}\rightarrow_{\mathrm{x}}\mathrm{B}$ and $\mathrm{B}\rightarrow_{\mathrm{x}}\mathrm{A}$. Moreover, where x is $\mathbf{T}$ we shall drop the subscript altogether.

A new object is often conveniently introduced into discourse by means of a rule of derivation of the form $\models \mathrm{A}\leftrightarrow_x\mathrm{B}$, where the introduced object appears in A but not in B and x is not free in A or B. Subtraction, for example, can be introduced by laying down the following rule, where $(k, l, t/\tau;\ i/o)$:

2.31 $\models [k-l] = t\leftrightarrow_i[t+l] = k$.

For the subtraction function $-$ is the only object X such that the sequents $[kXl] = t\leftrightarrow_i[t+l] = k$ are valid. Similarly, we can introduce conjunction, disjuction, truth and implication (symbolically: &, $\vee$, **Tr**, and $\prec$ respectively) by means of the following rules of derivation (where $h, j/o$):

2.32 $\models i \mathbin{\&} j\leftrightarrow_h \sim. i \supset. \sim j$ *Condition*: h is not free in $[i \mathbin{\&} j]$.

In subsequent derivation rules of this form we shall leave the appropriate condition unstated.

2.33 $\models i\vee j\leftrightarrow_h \sim. [\sim i] \mathbin{\&} .\sim j$.

2.34 $\models \mathbf{Tr}_{wt}p\leftrightarrow_h(\exists c).[\Delta pwc] \mathbin{\&} . \Delta ct\mathbf{T}$ $(p/\pi, c/o\tau)$.

2.35 $\models p \prec q\leftrightarrow_h(\forall w)(\forall t).[\mathbf{Tr}_{wt}p] \supset. \mathbf{Tr}_{wt}q$ (q/π).

In the following sections we shall state the derivability of various sequents from hitherto stated rules. Where $\mathfrak{R}$ is the class of rules which have been defined before such a statement is made we shall write simply $\vdash \mathfrak{S}$ for $\vdash_{\mathfrak{R}} \mathfrak{S}$. Limitations of space preclude exhibition of full derivations in the style exemplified above. Instead, we shall confine ourselves to citing whatever previously stated derivability results are crucial in deriving the sequent in question and assume that the reader can construct a full derivation in terms of previously stated rules of derivation and the well known rules of elementary number theory governing 0, $<$, $+$, and $-$.

In order to avoid repeated type indications, a small italic letter will always be used to refer to unspecified variables of the same type, in conformity with the following schedule: a/σ, b/σ, $c/o\pi$, $d/o\pi$, e/π, g/π, h/o, i/o, j/o, k/τ, l/τ, $n/(o\pi(o\pi))_{\tau\omega}$, p/π, q/π, r/σ, s/σ, t/τ, u/ω, v/ω, w/ω, x/ι, y/ι.

3. The Past, Present and Future

Consider a member of the intensional base, say a monadic attribute instantiable by ξ-objects. There is clearly a unique function which takes every world to the function which takes every moment of time to the extension of the attribute in that world at that time. This function is a ξ-property, and we shall say that it *corresponds* to the attribute. ξ-properties which correspond to the members of the intensional basis are *primary*. In general, we have the class $\mathbf{Pr}^{\bar{\xi}_n}$ $\left(\text{of type } o\left((o\bar{\xi}_n)_{\tau\omega}\right)\right)$ of primary $\bar{\xi}_n$-relations. By what has been said in Section 1 a possible world is completely specified by the chronologies of the primary attributes.

A *basic* proposition is one which says of some specifiec objects that they do or do not display a specific attribute in the intensional base. Each member of the set of basic propositions ($\mathbf{Ba}/o\pi$) is thus constructible from some primary $\bar{\xi}_n$-relation R and some specific objects $\bar{X}_n$ of the respective types $\bar{\xi}_n$, in one of the following ways: $\lambda w\lambda t.R_{wt}\tilde{X}_n$, $\lambda w\lambda t\,.\!\sim\!.\,R_{wt}\tilde{X}_n$. (Consequently, the negations of basic propositions are themselves basic.) We assume that basic propositions are defined at all world-times, and that worlds are individuated by which basic propositions are true in them at each moment. We thus have:

3.1 (a) $\models (\forall p)(\forall t).\mathbf{Ba}_p \supset .\, [\mathbf{Tr}_{wt}p] =.\, \mathbf{Tr}_{vt}p \leftrightarrow w = v$,

(b) $\models \mathbf{Ba}_p \rightarrow \mathbf{Ba}\,\lambda w\lambda t\,.\!\sim\!.\,p_{wt}$,

(c) $\models \mathbf{Ba}_p \rightarrow p_{wt} \vee \,.\!\sim\!.\,p_{wt}$.

The *K-shift* ($\mathbf{Sh}/\pi\tau\pi$) of a proposition P is the proposition that P will take place in K seconds' time:

3.2 $\models \mathbf{Sh}\,kp \leftrightarrow_q \lambda w\lambda t.p_{w[t+k]}$.

The *K-shift of a class* $(\mathbf{Shc}/(o\pi)\tau(o\pi))$ of propositions C is the class of all K-shifts of members of C:

3.3 $\models \mathbf{Shc}\,kc \leftrightarrow_q \lambda p\,.(\exists q)\,.c_q\ \&.\ p = \mathbf{Sh}\,kq.$

By the *conjunction* $(\mathbf{Cj}/\pi(o\pi))$ of a class of propositions C we shall understand the proposition which is true just in case every member of C is true, and false otherwise:

3.4 $\models \mathbf{Cj}c \leftrightarrow_q \lambda w \lambda t\,.(\forall p)\,.[\Delta cp\mathbf{T}] \supset .\ \mathbf{Tr}_{wt}p$

The rules of derivation stated so far yield:

3.5 $\vdash c_p \rightarrow \mathbf{Cj}_c \prec p.$
3.6 $\vdash \mathbf{Sh}k\mathbf{Cj}_c \leftrightarrow_q \mathbf{Cj}\,.\mathbf{Shc}kc.$
3.7 $\vdash (\forall p)\,.[\Delta cp\mathbf{T}] \supset .\ \Delta dp\mathbf{T} \rightarrow \mathbf{Cj}_d \prec \mathbf{Cj}_c.$
3.8 $\vdash \mathbf{T} \leftrightarrow_i [\mathbf{Cj}\lambda p\ .\sim.\ p = p]_{wt}.$

The *tautology* $(\mathbf{Taut}/\pi)$ is that proposition which is true in all worlds at all times:

3.9 $\models \mathbf{Taut} \leftrightarrow_p \lambda w \lambda t \mathbf{T}.$

We have the following derivability results:

3.10 $\vdash \mathbf{T} \leftrightarrow_i \mathbf{Taut}_{wt}.$
3.11 $\vdash \mathbf{Taut} \leftrightarrow_q \lambda w \lambda t [\mathbf{Cj}\lambda p\ .\sim.\ p = p]_{wt}$ (by 3.8 and 3.10).

A proposition P is a *basic past-shift* $(\mathbf{Bap}/o\pi)$ (a *basic future-shift* $(\mathbf{Baf}/o\pi)$) just in case it is in the set of K-shifts of basic propositions for some K less (or greater) than 0:

3.12 $\models \mathbf{Bap}p \leftrightarrow_i (\exists k)\,.[k < 0]\ \&\ [\mathbf{Shc}k\mathbf{Ba}]p.$
3.13 $\models \mathbf{Baf}p \leftrightarrow_i (\exists k)\,.[0 < k]\ \&\ [\mathbf{Shc}k\mathbf{Ba}]p.$

P is a *basic shift* $(\mathbf{Bas}/o\pi)$ just in case it is basic, or a basic past-shift or a basic future shift:

3.14 $\models \mathbf{Bas}p \leftrightarrow_i .\mathbf{Ba}_p \vee .\mathbf{Bap}_p \vee \mathbf{Baf}_p.$

C is a *class of basic shifts* $(\mathbf{Bac}/o(o\pi))$ if all members of C are basic shifts:

3.15 $\models \mathbf{Bac}c \leftrightarrow_i (\forall p)\,.[\Delta cp\mathbf{T}] \supset \mathbf{Bas}_p.$

The above rules yield:

3.16 $\vdash \rightarrow \mathbf{Bac}\lambda p\ .\sim.\ p = p.$
3.17 $\vdash 0 < k,\ (\exists q).\mathbf{Bap}_q\ \&.\ q_{wt}\ \&.\ p = \mathbf{Sh}-_k q \rightarrow \mathbf{Bap}_p\ \&\ p_{w[t+k]}.$

The *present, past, future, past-and-present, and past-present-and-future* $(\mathbf{Pre}, \mathbf{Pst}, \mathbf{Fut}, \mathbf{Pstp}, \mathbf{Ppf}/\pi_{\tau\omega})$ are the conjunction of all the true members, respectively, of **Ba**, **Bap**, **Baf**, the union of **Ba** and **Bap**, and the union of **Ba Bap**, and **Baf**:

3.18 $\models \mathbf{Pre}_{wt} \leftrightarrow_q \mathbf{Cj}\lambda p\,.\,\mathbf{Ba}_p \;\&\; p_{wt}.$
3.19 $\models \mathbf{Pst}_{wt} \leftrightarrow_q \mathbf{Cj}\lambda p\,.\,\mathbf{Bap}_p \;\&\; p_{wt}.$
3.20 $\models \mathbf{Fut}_{wt} \leftrightarrow_q \mathbf{Cj}\lambda p\,.\,\mathbf{Baf}_p \;\&\; p_{wt}.$
3.21 $\models \mathbf{Pstp}_{wt} \leftrightarrow_q \mathbf{Cj}\lambda p\,.\,[p = \mathbf{Pst}_{wt}] \vee [p = \mathbf{Pre}_{wt}].$
3.22 $\models \mathbf{Ppf}_{wt} \leftrightarrow_q \mathbf{Cj}\lambda p\,.\,[p = \mathbf{Pstp}_{wt}] \vee [p = \mathbf{Fut}_{wt}].$

We have:

3.23 Where P is **Pre**, **Pst**, **Fut**, **Pstp**, or **Ppf**, (a) $\vdash \rightarrow [P_{wt}]_{wt}$, (b) $\vdash [P_{wt}]_{vk} \leftrightarrow P_{wt} = P_{vk}$ (by 3.5).
3.24 $\vdash \mathbf{Ppf}_{wt} \leftrightarrow_q \mathbf{Cj}\lambda p\,.\,\mathbf{Bas}_p \;\&\; p_{wt}.$
3.25 $\vdash \mathbf{Ppf}_{wt} = \mathbf{Ppf}_{vt} \rightarrow w = v$ (by 3.5, 3.1, and 3.24).
3.26 $\vdash 0 \leqslant k \rightarrow \mathbf{Pst}_{w[t+k]} \prec .\; \mathbf{Sh}-_k\mathbf{Pst}_{wt}$ (by 3.6, 3.7, and 3.17).
3.27 $\vdash 0 \leqslant k,\; \mathbf{Pstp}_{w[t+k]} = \mathbf{Pstp}_{v[t+k]} \rightarrow \mathbf{Pre}_{wt} = \mathbf{Pre}_{vt}$.
3.28 $\vdash$ Where P is **Pst** or **Pstp**, $\vdash 0 \leqslant k,\; P_{w[t+k]} = P_{v[t+k]} \rightarrow P_{wt} = P_{vt}$ (by 3.26, 3.27)

A world is *weakly* (or *strongly*) *K-feasible* if it will be a candidate for actuality unitl and excluding (or including) K seconds hence $\big(\mathbf{Fea}, \mathbf{Feas}/ /(o\tau\omega)_{\tau\omega}\big)$:

3.29 $\models \mathbf{Fea}_{wt}kv \leftrightarrow_i \mathbf{Pst}_{w[t+k]} = \mathbf{Pst}_{v[t+k]}.$
3.30 $\models \mathbf{Feas}_{wt}kv \leftrightarrow_i \mathbf{Pstp}_{w[t+k]} = \mathbf{Pstp}_{v[t+k]}.$

We have:

3.31 $\vdash \mathbf{Feas}_{wt}kv \rightarrow \mathbf{Fea}_{wt}kv.$
3.32 Where F is **Feas** or **Fea**, (a) $\vdash \rightarrow F_{wt}kw$, (b) $\vdash F_{wt}kv \rightarrow F_{vt}kw$, (c) $F_{wt}kv, F_{vt}ku \rightarrow F_{wt}ku.$
3.33 $\vdash l < k,\; \mathbf{Fea}_{wt}kv \rightarrow \mathbf{Feas}_{wt}lv$ (by 3.28).

A proposition is *weakly* (or *strongly*) *K-feasible* if it is true in some weakly (or strongly) K-feasible world $\big(\mathbf{Fs}, \mathbf{Fss}/(o\tau\pi)_{\tau\omega}\big)$:

3.34 $\models \mathbf{Fs}_{wt}kp \leftrightarrow_i (\exists v)\,.\,[\mathbf{Fea}_{wt}kv] \;\&\; p_{vt}.$
3.35 $\models \mathbf{Fss}_{wt}kp \leftrightarrow_i (\exists v)\,.\,[\mathbf{Feas}_{wt}kv] \;\&\; p_{vt}.$

We have:

3.36 $\vdash \mathbf{Fss}_{wt}kp \rightarrow \mathbf{Fs}_{wt}kp$ (by 3.31).
3.37 $\vdash p_{wt} \rightarrow \mathbf{Fss}_{wt}kp$ (by 3.32).
3.38 $\vdash \mathbf{Fea}_{wt}kv, \mathbf{Fs}_{wt}kp \rightarrow \mathbf{Fs}_{vt}kp$ (by 3.32).
3.39 $\vdash \mathbf{Feas}_{wt}kv, \mathbf{Fss}_{wt}kp \rightarrow \mathbf{Fss}_{vt}kp$ (by 3.32).
3.40 $\vdash k < l,\; \sim .\, \mathbf{Fss}_{wt}kp \rightarrow\; \sim .\, \mathbf{Fs}_{wt}lp$ (by 3.33).
3.41 $\vdash \rightarrow \mathbf{Fss}_{wt}k\mathbf{Ppf}_{wt}$ (by 3.23 and 3.3).
3.42 $\vdash p_{vk} \rightarrow [\lambda w\lambda t\,.\,[\mathbf{Ppf}_{vk}]_{wt} \;\&.\; k = t] \prec p.$
3.43 $\vdash p_{vk} \rightarrow \mathbf{Fss}_{vk}l\lambda w\lambda t\,.\,[\mathbf{Ppf}_{vk}]_{wt} \;\&\; p_{wt}$ (by 3.23 and 3.3).

A proposition is *K-inevitable* $\left(\mathbf{Inev}/(o\tau\pi)_{\tau\omega}\right)$ if it is in true every strongly K-feasible world:

3.44 $\models \mathbf{Inev}_{wt}kp \leftrightarrow_i (\forall v).[\mathbf{Feas}_{wt}kv] \supset . \mathbf{Tr}_{wt}p.$

We have:

3.45 (a) $\mathbf{Pstp}_{wt} \prec p \rightarrow \mathbf{Inev}_{wt}0p$ (by 3.23).
(b) $\mathbf{Inev}_{vk}0p \rightarrow [\lambda w\lambda t.[\mathbf{Pstp}_{vk}]_{wt} \ \&. \ k = t] \prec p$ (by 3.23).
3.46 $\vdash 0 \leqslant k, \ \mathbf{Inev}_{\omega t}0p \rightarrow \mathbf{Inev}_{w[t+k]}0.\mathbf{Sh}-_{k}p$ (by 3.28).
3.47 $\vdash \mathbf{Inev}_{wt}0\,\mathbf{Ppf}_{wt}, \mathbf{Feas}_{wt}0v \rightarrow w = v$ (by 3.23 and 3.25).
3.48 $\vdash (\forall p).c_p \supset \mathbf{Inev}_{wt}kp \rightarrow \mathbf{Inev}_{wt}k\mathbf{Cj}_c.$
3.49 $\vdash \rightarrow \mathbf{Inev}_{wt}0\mathbf{Pstp}_{wt}$ (by 3.45).
3.50 $\vdash \mathbf{Inev}_{wt}0\mathbf{Fut}_{wt}, \mathbf{Feas}_{wt}0v \rightarrow w = v$ (by 3.47, 3.48, and 3.49).

By 3.50, if the future is inevitable then the actual world is now the only feasible world. As a consequence, if the future is always inevitable then the actual world is the only world which is ever feasible:

3.51 $\vdash (\forall t).\mathbf{Inev}_{wt}0\mathbf{Fut}_{wt}, (\exists t).\mathbf{Feas}_{wt}0v \rightarrow w = v$ (by 3.50).

A proposition P is *K-settled* $\left(\mathbf{Set}/(o\tau\pi)_{\tau\omega}\right)$ if either P itself or the negation of P is K-inevitable:

3.52 $\models \mathbf{Set}_{wt}kp \leftrightarrow_i [\mathbf{Inev}_{wt}kp] \vee .\mathbf{Inev}_{wt}k\lambda w\lambda t \,.\!\sim p_{wt}.$

A proposition P is *weakly* (or *strongly*) *K-sufficient for* proposition Q [and Q *weakly* (or *strongly*) *K-necessary for P*] if Q is true in every weakly (or strongly) K-feasible world in which P is true $\left(\mathbf{Suf}, \mathbf{Sufs}/(o\tau\pi\pi)_{\tau\omega}\right)$:

3.53 $\models \mathbf{Suf}_{wt}kpq \leftrightarrow_i (\forall v).[\mathbf{Fea}_{wt}kv \ \&. \ \mathbf{Tr}_{vt}p] \supset \mathbf{Tr}_{vt}q.$
3.54 $\models \mathbf{Sufs}_{wt}kpq \leftrightarrow_i (\forall v).[\mathbf{Feas}_{wt}kv \ \&. \ \mathbf{Tr}_{vt}p] \supset \mathbf{Tr}_{vt}q.$

A proposition P is *realisably weakly* (or *strongly*) *K-sufficient for* proposition Q [and Q *realisably weakly* (or *strongly K-necessary for P*] if P is weakly (or strongly) K-feasible and P is weakly (or strongly) K-sufficient for Q, $\left(\mathbf{Sufr}, \mathbf{Sufsr}/(o\tau\pi\pi)_{\tau\omega}\right)$:

3.55 $\models \mathbf{Sufr}_{wt}kpq \leftrightarrow_i \mathbf{Fs}_{wt}kp \ \&. \ \mathbf{Suf}_{wt}kpq.$
3.56 $\models \mathbf{Sufsr}_{wt}kpq \leftrightarrow_i \mathbf{Fss}_{wt}kp \ \&. \ \mathbf{Sufs}_{wt}kpq.$

We have:

3.57 Where S is **Suf**, **Sufs**, **Sufr**, **Sufsr**,
$\vdash S_{wt}kpq \rightarrow \mathbf{Inev}_{wt}k\lambda w\lambda t.S_{wt}kpq.$
3.58 Where S is **Sufs** or **Suf**, (a) $\vdash \rightarrow S_{wt}kp\mathbf{Taut}.$
(b) $\vdash S_{wt}kpq, S_{wt}kqr \rightarrow S_{wt}kpr.$
3.59 $\vdash p_{wt} \rightarrow \mathbf{Sufsr}_{wt}0\,[\mathbf{Ppf}_{wt}]p$ (by 3.23, 3.32 and 3.41).
3.60 $\vdash \rightarrow \mathbf{Sufs}_{wt}k\left[\lambda w\lambda t.[\lambda w\lambda t\lambda x.p_{wt}]_{wt}x\right]p.$

4. Connections

A proposition is inevitable (by 3.45) if it is implied by the past-and-present. Hence all true past-shifts of basic propositions are now inevitable. However, *future*-shifts may also be inevitable. This is because worlds consist not only of observable events, but also of *connections* between such events. One event's having the power to bring about or *cause* another event is an example of such a connection. The extension of cause-effect relation is part of what makes a possible world the world it is. In other words, the cause-effect relation is *primary*.

Causation is just one example of what will be called nexuses. A *nexus* $\left(\mathbf{Nex}/o\left((o\pi(o\pi))\tau\omega\right)\right)$ is any primary relation N between events and classes of events satisfying the following condition: the proposition that event E is related by N to a class of events D implies that one of the members of D occurs:

4.1 $\models \mathbf{Nex}n \leftrightarrow_i \mathbf{Pr}_n^{\pi,(o\pi)}$ & $(\forall e)(\forall d).\left[\lambda w\lambda t.[n_{wt}ed]\ \&\ e_{wt}\right]$ $\prec \lambda w\lambda t(\exists q).d_q\ \&\ q_{wt}$.

Causation is an example of what might be called *strict* nexuses: nexuses which always relate events with one-element classes of events. An example of a nexus which is not strict is the *disposition* relation.

Event E is *connected* $\left(\mathbf{Con}/(o\pi(o\pi))_{\tau\omega}\right)$ to class D if E brings about (one member of) D.

4.2 $\models \mathbf{Con}_{wt}ed \leftrightarrow_i \mathbf{Bac}_d\ \&\ (\exists n).\mathbf{Nex}_n\ \&\ (\exists k).n_{w[t+k]}[\mathbf{Sh}-_k e].\mathbf{Shc}-_k d.$

Proposition Q *forces* $\left(\mathbf{For}/(o\pi(o\pi))\right)D$ if there is an event E such that Q implies that E brings about D:

4.3 $\models \mathbf{For}qd \leftrightarrow_i (\exists e).q \prec \lambda w\lambda t.[\mathbf{Con}_{wt}ed]\ \&\ e_{wt}$.

The *latitude* $\left(\mathbf{Lat}/(o\pi)\pi\pi\right)$ of an event P *relative* to a proposition Q is the intersection of all classes which contain P and are forced by Q.

4.4 $\models \mathbf{Lat}pq \leftrightarrow_c \lambda e.(\forall d).[\mathbf{For}qd\ \&.\ \Delta dp\mathbf{T}] \supset .\ \Delta de\mathbf{T}.$

Thus the latitude of P relative to Q is the set of all alternatives to P given all the connections and events reported by Q. The latitude of P relative to the actual past-present-and-future is the *actual latitude* of P. The actual latitude is thus the set of real alternatives to P given the obtaining connections and history of the actual world. If the latitude of P relative to a true proposition Q is the same as its actual latitude, it is clear that Q reports an etiological ancestry of P. Q is said to be *etiologically complete* $\left(\mathbf{Ecom}/(o\pi)_{\tau\omega}\right)$ if Q is true and is the conjunction of a set of basic shifts whose latitude relative to Q is their actual latitude:

4.5 $\models \mathbf{Ecom}_{wt}q \leftrightarrow_i q_{wt}\ \&.\ (\exists d).\mathbf{Bac}_d\ \&.\ [q = \mathbf{Cj}_d]\ \&.\ (\forall p).d_p \supset .\ [\mathbf{Lat}pq]$ $=.\ \mathbf{Lat}p\mathbf{Ppf}_{wt}$.

An etiologically complete proposition thus entails a full etiological ancestry of each event which it reports.
We have:

4.6 $\vdash \rightarrow \mathbf{Ecom}_{wt}\mathbf{Taut}$ (by 3.11 and 3.16).
4.7 $\vdash \rightarrow \mathbf{Ecom}_{wt}\mathbf{Ppf}_{wt}$ (by 3.23 and 3.24).

5. Choices, rules, and strategies

Willing $(\mathbf{Vol}/(o\iota\sigma)_{\tau\omega})$ is a relation between an agent and a ι-property. Agent X bears the willing relation to a property just in case he wills to have that property. X's *K-choice* $(\mathbf{Choi}/(\pi\tau\iota)_{\tau\omega})$ is the conjunction of all volitions X will perform in K seconds' time:

5.1 $\models \mathbf{Choi}_{wt}kx \leftrightarrow_p \mathbf{Cj}\lambda q(\exists r).[q = \lambda w\lambda t.\, \mathbf{Vol}_{w[t+k]}xr] \;\&\; q_{wt}.$

5.1 yields:

5.2 $\vdash \rightarrow [\mathbf{Choi}_{wt}kx]_{wt}.$

G is a *feasible K-choice* $(\mathbf{Fchoi}/(o\tau\iota\pi)_{\tau\omega})$ for an agent X if it will still be feasible in K seconds' time for G to be X's choice.

5.3 $\models \mathbf{Fchoi}_{wt}kxg \leftrightarrow_i \mathbf{Fs}_{wt}k\lambda w\lambda t.g =. \mathbf{Choi}_{wt}kx.$
5.4 $\vdash \rightarrow \mathbf{Fchoi}_{wt}kx.\mathbf{Choi}_{wt}kx$ (by 3.36, and 3.37).

Each property S represents a *rule* for an agent — that is to say, S generates a set of instructions which the agent may or may not follow. Basically the instructions will be of a conditional form, specifying a range of admissible choices for the agent for each of the ways in which the world may develop. Let us say that the agent *implements* the rule if he now has the property. (The present possession of the property may, of course, depend on what happens in the future.) Then the admissible choices generated by S are just those which, whenever it is still weakly feasible for the agent to implement S, do not of themselves rule out his implementing S. If the choices which the agent makes are all admissible then he is said to *heed* S. Note that an agent may well heed a rule (that is, do his best to implement it) and yet fail to implement it.

A rule (or property) S is *K-applicable* $(\mathbf{Appl}/(o\tau\iota\sigma)_{\tau\omega})$ for X if it is weakly K-feasible for X to have S:

5.5 $\models \mathbf{Appl}_{wt}kxs \leftrightarrow_i \mathbf{Fs}_{wt}k\lambda w\lambda t.s_{wt}x.$

G is an *admissible K-choice* $(\mathbf{Adm}/o\tau\pi\iota\sigma)_{\tau\omega})$ for X w.r.t. S if it is weakly K-feasible that G is X's K-choice and X has S:

5.6 $\models \mathbf{Adm}_{wt}kgxs \leftrightarrow_i \mathbf{Fs}_{wt}k\lambda w\lambda t.[g = \mathbf{Choi}_{wt}kx] \;\&.\; s_{wt}x.$

X *K-heeds* $(\mathbf{Heed}/(o\tau\iota\sigma)_{\tau\omega})S$ if either S is not K-applicable for X or X's K-choice is admissible for X w.r.t. S:

5.7 $\models \mathbf{Heed}_{wt}kxs \leftrightarrow_i [\mathbf{Appl}_{wt}kxs] \supset. \mathbf{Adm}_{wt}k[\mathbf{Choi}_{wt}kx]xs.$

We have:

5.8 $\vdash \mathbf{Adm}_{wt}kgxs \to \mathbf{Sufr}_{wt}kg\lambda w\lambda t.\mathbf{Heed}_{wt}kxs$ (by 3.32 and 5.2).
5.9 $\vdash \sim.\ \mathbf{Fs}_{wt}k\lambda w\lambda t.s_{wt}x \to (\forall v).[\mathbf{Fea}_{wt}kv] \supset .\ \mathbf{Heed}_{wt}kxs$ (by 3.38).
5.10 $\vdash s_{wt}x \to \mathbf{Adm}_{wt}k[\mathbf{Choi}_{wt}kx]xs$ (by 3.32).
5.11 $\vdash \to (\exists g).[\mathbf{Fchoi}_{wt}kxg]\ \&.\ \mathbf{Sufr}_{wt}kg\lambda w\lambda t.\mathbf{Heed}_{wt}kxs$ (by 5.2, 5.4, 5.8, 5.9 and 5.10).

An agent *heeds* $(\mathbf{Hed}/(o\iota\sigma)_{\tau\omega})$ S if he K-heeds S for every positive K:

5.12 $\models \mathbf{Hed}_{wt}xs \leftrightarrow_i (\forall k).[0 < k] \supset .\ \mathbf{Heed}_{wt}kxs.$
5.13 $\vdash (\forall k).\ \mathbf{Adm}_{wt}k[\mathbf{Choi}_{wt}kx]xs \to \mathbf{Hed}_{wt}xs.$
5.14 $\vdash s_{wt}x \to \mathbf{Hed}_{wt}xs$ (by 5.10 and 5.13).
5.15 $\vdash \sim.\ \mathbf{Fss}_{wt}0\lambda w\lambda t.s_{wt}x,\ 0 < k \to \mathbf{Heed}_{wt}kxs$ (by 3.40).
5.16 $\vdash \to \mathbf{Fss}_{wt}0\lambda w\lambda t.\mathbf{Hed}_{wt}xs$ (by 3.37, 5.14 and 5.15).
5.17 $\vdash \to \mathbf{Sufs}_{wt}0[\lambda w\lambda t.[\mathbf{Hed}_{wt}x\lambda w\lambda t\lambda xp_{wt}]\ \&\ p_{wt}]\lambda w\lambda t.[\lambda w\lambda t\lambda xp_{wt}]_{wt}x.$

Let **Hd** (of type $\sigma\sigma$) be the function which takes each property S to the property of heeding S:

5.18 $\models \mathbf{Hd}s \leftrightarrow_r \lambda w\lambda t\lambda x.\mathbf{Hed}_{wt}xs.$

We have:

5.19 $\vdash \mathbf{Appl}_{wt}kxs \to \mathbf{Appl}_{wt}kx\mathbf{Hd}_s$ (by 5.14, 3.36, and 3.37).
5.20 $\vdash \mathbf{Hed}_{wt}x\mathbf{Hd}_s \to \mathbf{Hed}_{wt}xs$ (by 3.32, 3.38, 5.10 and 5.19).
5.21 $\vdash \mathbf{Hd}.\mathbf{Hd}_s \leftrightarrow_r \mathbf{Hd}_s$ (by 5.14 and 5.20).

From 5.20 it follows that the heeding of S is a property that the agent can acquire simply by making the right choices at the right times. It does not matter what the world does in response to the agent's choices; there is always a feasible K-choice for the agent to make which will ensure that he K-heeds S. By continuing to make these choices the agent heeds S. Thus the *heeding of S* could be called a *strict strategy*. It generates a rule whose implementation by the agent is just a matter of the agent making appropriate choices.

Besides strategies of this sort which require for their implementation no cooperation on the part of the world, we will consider *provisional strategies* — that is to say, rules which the agent is able to implement on some proviso.

Where S is a property and P a proposition, S is a *P-strategy* $(\mathbf{Str}/(o\iota\sigma\pi)_{\tau\omega})$ for X just in case X's having S is strongly sufficient for P and X's heeding S together with P is realisably strongly sufficient for X to implement S:

5.22 $\models \mathbf{Str}_{wt}xsp \leftrightarrow_i [\mathbf{Sufs}_{wt}0[\lambda w\lambda t.s_{wt}x]p]\ \&.\ \mathbf{Sufsr}_{wt}0[\lambda w\lambda t.[\mathbf{Hed}_{wt}xs]\ \&\ p_{wt}]\lambda w\lambda t.s_{wt}x.$

It follows that the property of heeding S is a P-strategy for X just in case

X's heeding S is strongly sufficient for P:

5.23 $\vdash \mathbf{Str}_{wt}x\mathbf{Hd}_s p \leftrightarrow_i \mathbf{Sufs}_{wt}0[\lambda w\lambda t.\mathbf{Hed}_{wt}xs]p$ (by 5.16, 5.20).
5.24 $\vdash \rightarrow \mathbf{Str}_{wt}x\mathbf{Hd}_s\mathbf{Taut}$ (by 5.23 and 3.58).
5.25 $\vdash \mathbf{Str}_{wt}xsp \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Str}_{wt}xsp$ (by 3.57).
5.26 $\vdash \mathbf{Fss}_{wt}0p \rightarrow \mathbf{Str}_{wt}x[\lambda w\lambda t\lambda x.p_{wt}]p$ (by 3.60, 5.14, and 5.17).
5.27 $\vdash \mathbf{Str}_{wt}xsp \rightarrow \mathbf{Fss}_{wt}0\lambda w\lambda t.[s_{wt}x]\ \&\ p_{wt}$.
5.28 $\vdash \mathbf{Str}_{wt}xsp, \mathbf{Inev}_{wt}0p \rightarrow \mathbf{Str}_{wt}xs\mathbf{Taut}$ (by 3.58, 5.16).
5.29 $\vdash \rightarrow \mathbf{Str}_{vk}x[\lambda w\lambda t\lambda x[\mathbf{Ppf}_{vk}]_{wt}]\mathbf{Ppf}_{vk}$ (by 3.41 and 5.26).

A property S *ensures* $(\mathbf{Ens}/(o\iota\sigma\sigma)_{\tau\omega})$ a property A for X just in case X's having S is strongly sufficient for X to have A:

5.30 $\models \mathbf{Ens}_{wt}xsa \leftrightarrow_i \mathbf{Sufs}_{wt}0[\lambda w\lambda t.s_{wt}x]\lambda w\lambda t.a_{wt}x$.
5.31 $\vdash \rightarrow \mathbf{Ens}_{wt}xss$.
5.32 $\vdash \mathbf{Ens}_{wt}xsa, \mathbf{Ens}_{wt}xab \rightarrow \mathbf{Ens}_{wt}xsb$ (by 3.58).
5.33 $\vdash \mathbf{Ens}_{wt}xsa \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Ens}_{wt}xsa$ (by 3.57).
5.34 $\vdash \mathbf{Sufs}_{wt}0p\lambda w\lambda t.a_{wt}x \rightarrow \mathbf{Ens}_{wt}x[\lambda w\lambda t\lambda x p_{wt}]a$.
5.35 $\vdash a_{vk}x \rightarrow \mathbf{Ens}_{vk}x[\lambda w\lambda t\lambda x.[\mathbf{Ppf}_{vk}]_{wt}]a$ (by 3.59 and 5.34).

6. Opportunity

We can now define what it takes for an agent to have an opportunity to perform a given task on a goven proviso. Agent X has the *opportunity to A provided P* $(\mathbf{Opp}/(o\iota\sigma\pi)_{\tau\omega})$ just in case there is a P-strategy S for X such that S ensures A for X:

6.1 $\models \mathbf{Opp}_{wt}xap \leftrightarrow_i (\exists s).[\mathbf{Str}_{wt}xsp]\ \&.\ \mathbf{Ens}_{wt}xsa$.

We have:

6.2 $\vdash \rightarrow \mathbf{Opp}_{wt}x\mathbf{Hd}_s\mathbf{Taut}$ (by 5.31 and 5.24).

Hence any agent has the opportunity to heed any rule come what may.

6.3 (a) $\vdash \mathbf{Opp}_{wt}xap \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Opp}_{wt}xap$ (by 5.33 and 5.25).
(b) $\vdash \sim.\mathbf{Opp}_{wt}xap \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\sim.\mathbf{Opp}_{wt}xap$ (by 6.3a).

Accordingly, opportunity is independent of the future:

6.4 $\vdash \rightarrow \mathbf{Set}_{wt}0\lambda w\lambda t.\mathbf{Opp}_{wt}xap$ (by 6.3).
6.5 $\vdash \mathbf{Ens}_{wt}xab, \mathbf{Opp}_{wt}xap \rightarrow \mathbf{Opp}_{wt}xbp$ (by 5.32).

Hence weakening of the task preserves opportunity to perform it.

6.6 $\vdash \mathbf{Sufsr}_{wt}0p\lambda w\lambda t.a_{wt}x \rightarrow \mathbf{Opp}_{wt}xap$ (by 5.26 and 5.34).
6.7 $\vdash \mathbf{Opp}_{wt}xap \rightarrow \mathbf{Fss}_{wt}0\lambda w\lambda t.[a_{wt}x]\ \&\ p_{wt}$ (by 3.57).
6.8 $\vdash \mathbf{Inev}_{wt}0\lambda w\lambda t.a_{wt}x, \mathbf{Fss}_{wt}0p \rightarrow \mathbf{Opp}_{wt}xap$ (by 6.6).
6.9 $\vdash \mathbf{Opp}_{wt}xap \leftrightarrow \mathbf{Opp}_{wt}x[\lambda w\lambda t\lambda x.\mathbf{Opp}_{wt}xap]p$ (by 6.3, 6.7, and 6.8).
6.10 $\vdash a_{wt}x \rightarrow \mathbf{Opp}_{wt}xa\mathbf{Ppf}_{wt}$ (by 3.59, and 6.6).
6.11 $\vdash \mathbf{Opp}_{wt}xap, \mathbf{Inev}_{wt}0p \rightarrow \mathbf{Opp}_{wt}xa\mathbf{Taut}$ (by 5.28).
6.12 $\vdash \mathbf{Inev}_{wt}0\mathbf{Fut}_{wt}, \mathbf{Opp}_{wt}xap \rightarrow a_{wt}x$ (by 3.50 and 6.7).

It might be tempting to define categorical opportunity as the possession of a provisional opportunity on an obtaining proviso. In [5] this is shown to be inadequate. To have the *categorical opportunity* $(\mathbf{Op}/(o\iota\sigma)_{\tau\omega})$ to do A an agent must have the opportunity to do A on some etiologically complete proviso:

6.13 $\quad \models \mathbf{Op}_{wt}xa \leftrightarrow_{t} (\exists p).[\mathbf{Ecom}_{wt}p] \;\&.\; \mathbf{Opp}_{wt}xap.$

We have:

6.14 $\quad \vdash \mathbf{Opp}_{wt}xa\mathbf{Taut} \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Op}_{wt}xa$ (by 6.3 and 4.6).
6.15 $\quad \vdash \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Op}_{wt}x\mathbf{Hd}_s$ (by 6.14 and 6.2).

Thus, anybody inevitably has the categorical opportunity to heed any rule.

6.16 $\quad \vdash \mathbf{Ens}_{wt}xab, \mathbf{Op}_{wt}xa \rightarrow \mathbf{Op}_{wt}xb$ (by 6.5).

Thus weakening of the taks preserves categorical opportunity.

6.17 $\quad \vdash a_{wt}x \rightarrow \mathbf{Op}_{wt}xa$ (by 6.10 and 4.7).

Thus, whatever the agent in fact does he has the opportunity to do.

6.18 $\quad \vdash \mathbf{Opp}_{wt}xap, \mathbf{Inev}_{wt}0p \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Op}_{wt}xa$ (by 6.11 and 4.6).
6.19 $\quad \vdash \mathbf{Op}_{wt}xa \rightarrow \mathbf{Fss}_{wt}0\lambda w\lambda t.a_{wt}x$ (by 6.7).
6.20 $\quad \vdash \mathbf{Op}_{wt}x\lambda w\lambda t\lambda x.\mathbf{Opp}_{wt}xap \rightarrow \mathbf{Opp}_{wt}xap$ (by 6.19 and 6.3).
6.21 $\quad \vdash \mathbf{Inev}_{wt}0\mathbf{Fut}_{wt}, \mathbf{Op}_{wt}xa \rightarrow a_{wt}x$ (by 6.12).

Thus if the future is determined then all opportunities are realised. A number of plausible inference schemata are not derivable w.r.t. the rules stated above. Among them are the following:

$$\mathbf{Op}_{wt}xa \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Op}_{wt}xa,$$
$$\mathbf{Op}_{wt}x\lambda w\lambda t\lambda x.\mathbf{Op}_{wt}xa \rightarrow \mathbf{Op}_{wt}xa,$$
$$a_{wt}x, p_{wt} \rightarrow \mathbf{Opp}_{wt}xap,$$
$$\mathbf{Opp}_{wt}xap, \mathbf{Op}_{wt}x\lambda w\lambda t\lambda x p_{wt} \rightarrow \mathbf{Op}_{wt}xa,$$
$$\mathbf{Opp}_{wt}xap, \mathbf{Opp}_{wt}xaq \rightarrow \mathbf{Opp}_{wt}xa\lambda w\lambda t.p_{wt} \vee q_{wt},$$
$$\mathbf{Opp}_{wt}xap, \mathbf{Opp}_{wt}xaq \rightarrow \mathbf{Opp}_{wt}xa\lambda w\lambda t.p_{wt} \;\&\; q_{wt},$$
$$\mathbf{Sufs}_{wt}0[\lambda w\lambda t.a_{wt}x]p, \sim p_{wt} \rightarrow \sim.\mathbf{Op}_{wt}xa,$$
$$\mathbf{Sufsr}_{wt}0p\lambda w\lambda t.\mathbf{Op}_{wt}xa \rightarrow \mathbf{Opp}_{wt}xap.$$

Closer examination shows that each of these is a fallacy from an intuitive point of view. For counterexamples see [5].

7. Ability

Intending, like willing, is a relation between an individual and a property. A slightly more general relation is that of K-intention. Agent X K-*intends* $(\mathbf{Int}/(o\tau\iota\sigma)_{\tau\omega})$ property A if in K seconds' time he will intend to instantiate A now. X is said to *steadfastly intend* A $\left(\mathbf{Ints}/(o\iota\sigma)_{\tau\omega}\right)$ if X will intend to have A (as of now) at every future moment at which it will not be inevitable that he has A (as of now):

7.1 $\models \mathbf{Ints}_{wt}xa \leftrightarrow_i (\forall k).[[0 < k] \;\&\; .\sim. \mathbf{Inev}_{wt}k\lambda w\lambda t.a_{wt}x] \supset. \mathbf{Tr}_{wt}\lambda w\lambda t.\mathbf{Int}_{wt}kxa.$

X has a *command* $\left(\mathbf{Cmd}/(o\iota\sigma\sigma\pi)_{\tau\omega}\right)$ of a strategy S *with respect to* property A *on proviso* P if X's steadfast intention to A together with P is realisably strongly sufficient for X to heed S:

7.2 $\models \mathbf{Cmd}_{wt}xsap \leftrightarrow_i \mathbf{Sufsr}_{wt}0[\lambda w\lambda t.\mathbf{Ints}_{wt}xa] \;\&\; p_{wt}]\lambda w\lambda t.\mathbf{Hed}_{wt}xs.$

We have:

7.3 $\vdash \mathbf{Cmd}_{wt}xsap \to \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Cmd}_{wt}xsap$ (by 3.57).

7.4 $\vdash \mathbf{Ints}_{vk}xa \to \mathbf{Cmd}_{vk}x\left[\lambda w\lambda t\lambda x[\mathbf{Ppf}_{vk}]_{wt}\right]a\mathbf{Ppf}_{vk}$ (by 3.59 and 5.14)

7.5(a) $\vdash \mathbf{Sufsr}_{wt}0[\lambda w\lambda t.\mathbf{Ints}_{wt}x\mathbf{Hd}_s]\lambda w\lambda t.[\mathbf{Hd}_s]_{wt}x \to \mathbf{Cmd}_{wt}x\mathbf{Hd}_s\mathbf{Hd}_s\mathbf{Taut}.$

(b) $\vdash \mathbf{Cmd}_{wt}xsap, \mathbf{Inev}_{wt}0p \to \mathbf{Cmd}\quad xsa\mathbf{Taut}.$

An agent X *can* A *provided* P $\left(\mathbf{Can}/(o\iota\sigma\pi)_{\tau\omega}\right)$ if there is a P-strategy S for X, S ensures A for X, and X has a command of S with respect to A on P:

7.6 $\models \mathbf{Can}_{wt}xap \leftrightarrow_i (\exists s).[\mathbf{Str}_{wt}xsp] \;\&\; [\mathbf{Ens}_{wt}xsa] \;\&.\; \mathbf{Cmd}_{wt}xsap.$

We have:

7.7 $\vdash \mathbf{Can}_{wt}xap \to \mathbf{Opp}_{wt}xap.$

Thus, one can only do on a certain proviso what one has the opportunity to do on that proviso.

7.8 $\vdash \mathbf{Can}_{wt}xap \to \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Can}_{wt}xap$ (by 5.25, 5.33 and 7.3).

7.9 $\vdash \mathbf{Ints}_{wt}xa, a_{wt}x \to \mathbf{Can}_{wt}xa\mathbf{Ppf}_{wt}$ (by 7.4 5.35 and 5.29).

7.10 $\vdash \mathbf{Can}_{wt}xap \to \mathbf{Fss}_{wt}0\lambda w\lambda t.[\mathbf{Ints}_{wt}xa] \;\&.\; p_{wt} \;\&.\; a_{wt}x.$

7.11 $\vdash \mathbf{Can}_{wt}xap \to \mathbf{Sufsr}_{wt}0[\lambda w\lambda t.[\mathbf{Ints}_{wt}xa] \;\&.\; p_{wt}]\lambda w\lambda t.a_{wt}x$ (by 3.57 and 7.10).

7.12 $\vdash \mathbf{Sufsr}_{wt}0[\lambda w\lambda t.\mathbf{Ints}_{wt}x\mathbf{Hd}_s]\lambda w\lambda t.[\mathbf{Hd}_s]_{wt}x \to \mathbf{Can}_{wt}x\mathbf{Hd}_s\mathbf{Taut}$ (by 5.24, 5.31 and 7.5(a); cf. 6.2).

7.13 $\vdash \mathbf{Can}_{wt}xap, \mathbf{Inev}_{wt}0p \to \mathbf{Can}_{wt}xa\mathbf{Taut}$ (by 5.28 and 7.5(b)).

7.14 $\vdash \mathbf{Inev}_{wt}\mathbf{Fut}_{wt}, \mathbf{Can}_{wt}xap \to a_{wt}x$ (by 7.7 and 6.12).

Hence if the future is determined, no one can do anything (on any proviso) that he does not do in fact.

7.15 $\vdash \mathbf{Can}_{wt}x[\lambda w\lambda t\lambda x.\mathbf{Can}_{wt}xap]p \to \mathbf{Can}_{wt}xap$ (by 7.10 and 7.8).

Hence provisional ability to be provisionally able to do something entails provisional ability to do it.

7.16 $\vdash \mathbf{Inev}_{wt}0\lambda w\lambda t.[a_{wt}x]\ \&\ p_{wt}, \mathbf{Fss}_{wt}0\lambda w\lambda t.\ \mathbf{Ints}_{wt}xa \rightarrow \mathbf{Can}_{wt}xap$ (by 5.14, 5.31).

7.17 $\vdash \mathbf{Inev}_{wt}0\lambda w\lambda t.a_{wt}x, \mathbf{Fss}_{wt}0\lambda w\lambda t.\mathbf{Ints}_{wt}xa \rightarrow \mathbf{Can}_{wt}xa\mathbf{Taut}$ (by 7.16).

Thus, if it is feasible for an agent to intend something he cannot help doing, then he can do it come what may.

Categorical ability can now be defined in terms of provisional ability the way categorical opportunity was defined in terms of provisional opportunity. *X can A* $(\mathbf{Cn}/(o\iota\sigma)_{\tau\omega})$ if there is an etiologically complete *P* such that *X* can *A* provided *P*:

7.18 $\models \mathbf{Cn}_{wt}xa \leftrightarrow_i (\exists p).[\mathbf{Ecom}_{wt}p]\ \&.\ \mathbf{Can}_{wt}xap.$

We have:

7.19 $\vdash \mathbf{Cn}_{wt}xa, \mathbf{Ints}_{wt}xa \rightarrow a_{wt}x$ (by 7.11).

Thus if an agent steadfastly intends something he can do then he will do it.

7.20 $\vdash \mathbf{Ints}_{wt}xa, a_{wt}x \rightarrow \mathbf{Cn}_{wt}xa$ (by 7.9 and 4.7).

By 7.20, an agent can do whatever he intentionally does in fact.

7.21 $\vdash \mathbf{Cn}_{wt}xa \rightarrow \mathbf{Fss}_{wt}0\lambda w\lambda t.a_{wt}x\ \&.\ \mathbf{Ints}_{wt}xa$ (by 7.10).

By 7.21 it is strongly feasible for an agent to do what he can do.

7.22 $\vdash \mathbf{Cn}_{wt}xa \rightarrow \mathbf{Op}_{wt}xa$ (by 7.7).

By 7.22 one can only do what one has an opportunity to do.

7.23 $\vdash \mathbf{Sufs}_{wt}0[\lambda w\lambda t.\mathbf{Ints}_{wt}xa][\lambda w\lambda t.\sim.a_{wt}x] \rightarrow \sim.\ \mathbf{Cn}_{wt}xa$ (by 7.21).

7.23 could be called the 'bungler theorem'. It says that if intending something prevents an agent from doing that thing, then he cannot do it (even if he accidentally does it).

7.24 $\vdash \mathbf{Cn}_{wt}x\lambda w\lambda t.\mathbf{Can}_{wt}xap \rightarrow \mathbf{Can}_{wt}xap$ (by 7.21 and 7.8).

If an agent can give himself a provisional ability, then he has that provisional ability. (Note that the same does not hold for categorical ability).

7.25 $\vdash \mathbf{Can}_{wt}xa\mathbf{Taut} \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Cn}_{wt}xa$ (by 7.8 and 4.6).

If an agent has the ability to do something come what may, then it is inevitable that he has the categorical ability to do it.

7.26 $\vdash \mathbf{Can}_{wt}xap, \mathbf{Inev}_{wt}0p \rightarrow \mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Cn}_{wt}xa$ (by 7.13 and 7.25).

If an agent can do something on an inevitable proviso then inevitably he has the categorical ability to do it.

7.27 $\vdash \mathbf{Sufs}_{wt}0[\lambda w\lambda t.\mathbf{Ints}_{wt}x\mathbf{Hd}_s][\lambda w\lambda t.[\mathbf{Hd}_s]_{wt}x]$
$\rightarrow\mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Cn}_{wt}x\mathbf{Hd}_s$ (by 7.12 and 7.25).

If intending to heed a strategy is realisably sufficient for an agent to heed it then it is inevitable that he can heed it.

7.28 $\vdash \mathbf{Inev}_{wt}0\lambda w\lambda t.a_{wt}x, \mathbf{Fss}_{wt}0\lambda w\lambda t.\mathbf{Ints}_{wt}xa\rightarrow\mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Cn}_{wt}xa$
(by 7.17 and 7.25).

Thus, if it is feasible for an agent to intend something he cannot help doing then it is inevitable that he can do it.

Some plausible inference schemata involving ability, which are not derivable from the rules stated above, are now listed. Again, it is possible to construct counterexamples which demonstrate that each one is a fallacy from an intuitive point of view. For details see [5].

$a_{wt}x\rightarrow\mathbf{Cn}_{wt}xa$ (cf. 6.17),
$\mathbf{Can}_{wt}xap, \mathbf{Ens}_{wt}xab\rightarrow\mathbf{Can}_{wt}xbp$ (cf. 6.5),
$\mathbf{Cn}_{wt}xa, \mathbf{Ens}_{wt}xab\rightarrow\mathbf{Cn}_{wt}xb$ (cf. 6.16),
$\mathbf{Cn}_{wt}xa\rightarrow\mathbf{Inev}_{wt}0\lambda w\lambda t.\mathbf{Cn}_{wt}xa$,
$a_{wt}x, p_{wt}\rightarrow\mathbf{Can}_{wt}xap$,
$\mathbf{Can}_{wt}xap, \mathbf{Cn}_{wt}x\lambda w\lambda t\lambda xp_{wt}\rightarrow\mathbf{Cn}_{wt}xa$,
$\mathbf{Can}_{wt}xap, \mathbf{Can}_{wt}xaq\rightarrow\mathbf{Can}_{wt}xa\lambda w\lambda t.p_{wt}\vee q_{wt}$,
$\mathbf{Can}_{wt}xap, \mathbf{Can}_{wt}xaq\rightarrow\mathbf{Can}_{wt}xa\lambda w\lambda t.p_{wt}\ \&\ q_{wt}$,
$\mathbf{Sufs}_{wt}0[\lambda w\lambda t.a_{wt}x]p, \sim.p_{wt}\rightarrow\ \sim.\mathbf{Cn}_{wt}xa$,
$\mathbf{Sufsr}_{wt}0p\lambda w\lambda t.a_{wt}x\rightarrow\mathbf{Can}_{wt}xap$ (cf. 6.6),
$\mathbf{Cn}_{wt}x\lambda w\lambda t\lambda x.\mathbf{Cn}_{wt}xa\rightarrow\mathbf{Cn}_{wt}xa$.

8. Freedom and responsibility

An agent is *free* $(\mathbf{Free}/(o\iota\pi)_{\tau\omega})$ *with respect to* A if he can A and he can non-A.

8.1 $\vDash \mathbf{Free}_{wt}xa\leftrightarrow_i[\mathbf{Cn}_{wt}xa]\ \&.\ \mathbf{Cn}_{wt}x\lambda w\lambda t\lambda x.\sim.a_{wt}x.$

We have that if the future is determined then an individual is not free with respect to any property:

8.2 $\vdash \mathbf{Inev}_{wt}\mathbf{Fut}_{wt}\rightarrow\ \sim.\mathbf{Free}_{wt}xa$ (by 7.22 and 6.21).
8.3 $\vdash\ \sim.[\mathbf{Fss}_{wt}0\lambda w\lambda t.\mathbf{Ints}_{wt}xa]\ \&.\ \mathbf{Fss}_{wt}0\lambda w\lambda t.\mathbf{Ints}_{wt}x\lambda w\lambda t\lambda x.\sim.a_{wt}x$
$\rightarrow\ \sim.\mathbf{Free}_{wt}xa$ (by 7.21).

By 8.3, if it is not both strongly feasible for an agent to intend A and strongly feasible for him to intend non-A then the agent is not free with respect to A.

An agent is (minimally) *free* ($\mathbf{Fre}/\sigma$) if he is free w.r.t. some A.

8.4 $\models \mathbf{Fre}_{wt}x \leftrightarrow_i (\exists a).\mathbf{Free}_{wt}xa.$

We have the incompatibility of freedom and determinism:

8.5 $\vdash \mathbf{Inev}_{wt}\mathbf{Fut}_{wt} \rightarrow \sim (\exists x).\mathbf{Fre}_{wt}x$ (by 8.2).

8.6 $\vdash \sim (\exists a).\mathbf{Fss}_{wt}0\lambda w\lambda t.\mathbf{Ints}_{wt}xa \rightarrow \sim .\ \mathbf{Fre}_{wt}x$ (by 8.3).

By 8.6, if it is not feasible for an individual to form intentions then that individual is not free.

An agent is (partially) *responsible* $\left(\mathbf{Resp}/(o\iota\pi)_{wt}\right)$ *for* a state of affairs Q if Q is now inevitable and there was a time at which something the agent could have done would have averted it:

8.7 $\models \mathbf{Resp}_{wt}xq \leftrightarrow_i [\mathbf{Inev}_{wt}0q]\ \&\ (\exists a)(\exists k).[0 < k]\ \&.\ [\mathbf{Ens}_{w[t-k]}xa[\lambda w\lambda t\lambda x\ .\sim .\ q_{w[t+k]}]\ \&.\ \mathbf{Cn}_{w[t-k]}xa.$

We have:

8.8 $\vdash \mathbf{Resp}_{wt}xq \rightarrow (\forall k).[0 \leqslant k].\mathbf{Resp}_{w[t+k]}x.\mathbf{Sh}-_kq$ (by 3.46).

Thus, if an agent is responsible for a state of affairs he will always be responsible for it.

8.9 $\vdash \mathbf{Inev}_{wt}0\lambda w\lambda t.a_{wt}x,\ \mathbf{Free}_{w[t-k]}x\lambda w\lambda t\lambda x.a_{w[t+k]}x \rightarrow \mathbf{Resp}_{wt}x\lambda w\lambda t.a_{wt}x.$

If an action is irrevocably completed and X was free to do it, then X is now responsible for it.

8.10 $\vdash \mathbf{Inev}_{wt}0p,\ [\lambda w\lambda t\ .\sim q_{wt}] \prec \lambda w\lambda t\ .\sim p_{wt},\ \mathbf{Resp}_{wt}xq \rightarrow \mathbf{Resp}_{wt}xp.$

It may seem counterintuitive that if one is responsible for Q and non-Q implies non-P then one is also responsible for P (provided P is also inevitable). But it is clear that if X could have done something to prevent Q then the same thing would have prevented P, where non-Q implies non-P. On the other hand, the following sequent is not derivable:

$$\mathbf{Inev}_{wt}0p,\ q \prec p,\ \mathbf{Resp}_{wt}xq \rightarrow \mathbf{Resp}_{wt}xp.$$

The fact that X is responsible for Q and Q implies P does not entail that X is responsible for P. For it does not follow that X could have done something to prevent P.

8.11 $\vdash \mathbf{Resp}_{wt}x\lambda v\lambda k.q_{vk}\ \&.\ k = t \rightarrow \mathbf{Resp}_{wt}xq.$

The following is a special case of 8.10:

8.12 $\vdash (\exists q).\mathbf{Resp}_{wt}xq \rightarrow \mathbf{Resp}_{wt}x\mathbf{Pstp}_{wt}$ (by 3.49, 3.45(b), 8.10 and 8.11)

Reluctance to accept 8.12 stems from committing the above mentioned fallacy. From the fact that one is responsible for the past-and-present (that is to say, one could have made it different) it does *not* follow that one is responsible for *every* past event.

8.13 $\vdash (\forall t).\sim(\exists a).\mathbf{Fss}_{wt}0\lambda w\lambda t.\mathbf{Ints}_{wt}xa \rightarrow (\forall t).\sim(\exists q).\mathbf{Resp}_{wt}xq$ (by 7.21).

Thus, if it is never feasible for an individual to form intentions then that individual is never responsible for anything.

9. Collective opportunities and abilities

What is not within the power of a single individual may be within the power of a *group* of individuals. Group opportunities and abilities are not reducible to individual opportunities and abilities. We will now briefly outline a way in which the theory can be generalised to deal with groups.

Where Z is an ι-class, Z*'s* K*-choice* is the conjunction of all the volitions members of Z will perform in K seconds' time. A *rule* for Z is any $(o\iota)$-property. The definitions of *applicability*, *admissibility*, *heeding*, *strategy* and *opportunity* can now all be taken over simply by raising the types wherever appropriate. For *ability* a new intention relation is required. An individual X K*-intends* A *for* Z if X will intend in K seconds' time for Z to have A (as of now). Z *steadfastly intends* A if every member of Z intends Z to have A at every future moment at which it is not yet inevitable that Z has A. *Command*, *ability*, *freedom* and *responsibility* for groups can now be defined the way the corresponding notions for individuals were defined above, except for appropriate type-raising. Once this is done the theory of individual ability and freedom expounded above can be reformulated as a special case of the theory of collective ability, where the collective is a one-element class.[1]

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Graham Oddie

Pavel Tichý

University of Otago
Dunedin

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