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Triangular and rectangular plate elements based on generalized compatibility conditions

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Abstract. Three generalized conforming elements for thin plate bending based on generalized compatibility conditions are derived: a triangular element with 9 d.o.f. (LT element) and two rectangular elements with 12 d.o.f. (LR-1 and LR-2). Their formulation is as simple as the conventional non-conforming element and they can be implemented in routine manner. By satisfying the generalized compatibility condition under constant moments, these new elements can always pass the patch test. Numerical examples show that high accurate result is obtained with fewer degrees of freedom by using these new elements of thin plate.

1 Introduction

To formulate a non-conforming element or a hybrid-displacement element of thin plates, the modified potential energy principle (Pian and Tong 1987) should be used:

$$\pi_{mp} = \pi_p - \sum_e H = \text{stationary}, \tag{1}$$

where

$$H = \oint_{\partial A_e} \left[Q_n(w - \bar{w}) - M_n\left(\frac{\partial w}{\partial n} - \bar{\psi}_n\right) - M_{ns}\left(\frac{\partial w}{\partial s} - \bar{\psi}_s\right)\right] ds. \tag{2}$$

π_p and π_{mp} are functionals of minimum and modified potential energy theorems, w is interior deflection, $\bar{w}, \bar{\psi}_n, \bar{\psi}_s$ are interelement displacements, Q_n, M_n, M_{ns} are boundary tractions on ∂A_e.

Based on the functional π_{mp} of Eq. (1), Tong (1970) formulated the hybrid-displacement element which has been used successfully in plate bending and other problems. Since the functional π_{mp} is a multifield variational functional which contains three kinds of independent variables (i.e. element displacements, boundary displacements and boundary tractions), the formulation of the hybrid-displacement element is rather complicated.

Long (1988, 1989) developed the generalized conforming element which can be formulated by using the degenerated form of π_{mp}. The procedure consists of the following steps. Firstly, the interelement displacement compatibility conditions are relaxed and thus the variational principle initially used is the modified potential energy principle. Secondly, the following generalized compatibility condition

$$H = \oint_{\partial A_e} \left[Q_n(w - \bar{w}) - M_n\left(\frac{\partial w}{\partial n} - \bar{\psi}_n\right) - M_{ns}\left(\frac{\partial w}{\partial s} - \bar{\psi}_s\right)\right] ds = 0 \tag{3}$$

is assumed to be satisfied, then the multifield functional π_{mp} degenerates to the single-field functional π_p. Finally, based on the simple form of π_p, the element stiffness matrix can be easily obtained by the conventional procedure.

In this paper, three generalized conforming elements of thin plates are derived: a triangular element with 9 d.o.f. (LT) and two rectangular elements with 12 d.o.f. (LR-1 and LR-2). The stiffness matrices of these elements are rank sufficient and pass Irons' patch test. The performance is demonstrated to assess these elements.

2 Generalized conforming triangular element LT

The nodal displacement vector of a triangular element of thin plate with 9 d.o.f. (Fig. 1) is

$$\{q\}^e = [w_1 \quad \psi_{x1} \quad \psi_{y1} \quad w_2 \quad \psi_{x2} \quad \psi_{y2} \quad w_3 \quad \psi_{x3} \quad \psi_{y3}]^T \tag{4}$$

where d_1, d_2, d_3 denote the side lengths of the triangle, w denotes deflection, and $\psi_x = \partial w/\partial x$ and $\psi_y = \partial w/\partial y$ denote rotations. Along each side of the element the deflection $\bar{w}$ is assumed to be cubic and the normal slope $\bar{\psi}_n$ linearly distributed. The deflection field w of the element is expressed in terms of area coordinates L_1, L_2, L_3:

$$w = [F_\lambda]\{\lambda\}, \tag{5}$$

where

$$\{\lambda\} = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6 \quad \lambda_7 \quad \lambda_8 \quad \lambda_9 \quad \lambda_{10} \quad \lambda_{11} \quad \lambda_{12}]^T$$

$$[F_\lambda] = [L_1 \quad L_2 \quad L_3 \quad L_1L_2 \quad L_2L_3 \quad L_3L_1 \quad L_1^2L_2 \quad L_2^2L_3 \quad L_3^2L_1 \quad L_1^2L_2L_3 \quad L_2^2L_3L_1 \quad L_3^2L_1L_2]. \tag{6}$$

In order to solve $\{\lambda\}$ in terms of $\{q\}^e$, it is necessary to establish 12 compatibility conditions. First, three compatibility conditions for w at each node are used:

$$w(x_i, y_i) - w_i = 0 \quad (i = 1, 2, 3) \tag{7}$$

from which λ_1, λ_2 and λ_3 are solved:

$$\lambda_1 = w_1, \quad \lambda_2 = w_2, \quad \lambda_3 = w_3. \tag{8}$$

Further, in order to solve $\lambda_4, \lambda_5, \ldots, \lambda_{12}$, nine generalized compatibility conditions are established according to Eq. (3). The assumed element moments are linear:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} L_1 & L_2 & L_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_1 & L_2 & L_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_1 & L_2 & L_3 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_9 \end{bmatrix}, \tag{9}$$

where $\beta_1, \ldots, \beta_9$ are nine arbitrary parameters. Substituting Eqs. (5) and (9) into (3), the following nine generalized compatibility conditions are obtained:

$$\oint_{\partial A_e} \left[\frac{b_i l}{2A}(w - \bar{w}) - L_i l^2 \left(\frac{\partial w}{\partial n} - \bar{\psi}_n \right) + L_i lm \left(\frac{\partial w}{\partial s} - \bar{\psi}_s \right) \right] ds = 0$$

$$\oint_{\partial A_e} \left[\frac{c_i m}{2A}(w - \bar{w}) - L_i m^2 \left(\frac{\partial w}{\partial n} - \bar{\psi}_n \right) - L_i lm \left(\frac{\partial w}{\partial s} - \bar{\psi}_s \right) \right] ds = 0$$

$$\oint_{\partial A_e} \left[\frac{c_i l + b_i m}{2A}(w - \bar{w}) - 2L_i lm \left(\frac{\partial w}{\partial n} - \bar{\psi}_n \right) - L_i(l^2 - m^2) \left(\frac{\partial w}{\partial s} - \bar{\psi}_s \right) \right] ds = 0 \quad (i = 1, 2, 3), \tag{10}$$

where l and m are directional cosines of the outer normal on ∂A_e, A is the area of the element and

$$b_1 = y_2 - y_3, \quad b_2 = y_3 - y_1, \quad b_3 = y_1 - y_2$$

$$c_1 = x_3 - x_2, \quad c_2 = x_1 - x_3, \quad c_3 = x_2 - x_1. \tag{11}$$

From Eq. (10), $\lambda_4, \ldots, \lambda_{12}$ can be solved in terms of $\{q\}^e$. Combining this result and Eq. (8), we have

$$\{\lambda\} = [A]\{q\}^e, \tag{12}$$

in which

$$[A]=[[A_1]\quad [A_2]\quad [A_3]] \tag{13}$$

$$[A_1]=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ (-3+r_2)/2 & (-3r_2c_2+3c_2+2c_1)/12 & -(-3r_2b_2+3b_2+2b_1)/12 \\ -(r_2+r_3)/2 & (3r_2c_2-3r_3c_3-c_1)/12 & -(3r_2b_2-3r_3b_3-b_1)/12 \\ (3+r_3)/2 & (3r_3c_3+15c_3+14c_1)/12 & -(3r_3b_3+15b_3+14b_1)/12 \\ 3-r_2 & (3r_2c_2-9c_2-8c_1)/6 & -(3r_2b_2-9b_2-8b_1)/6 \\ r_2+r_3 & (-3r_2c_2+3r_3c_3+c_1)/6 & -(-3r_2b_2+3r_3b_3+b_1)/6 \\ -(3+r_3) & (-3r_3c_3-9c_3-8c_1)/6 & -(-3r_3b_3-9b_3-8b_1)/6 \\ -r_2+r_3 & (r_2c_2+r_3c_3-c_1)/2 & -(r_2b_2+r_3b_3-b_1)/2 \\ -r_2+r_3 & (r_2c_2+r_3c_3-c_1)/2 & -(r_2b_2+r_3b_3-b_1)/2 \\ -r_2+r_3 & (r_2c_2+r_3c_3-c_1)/2 & -(r_2b_2+r_3b_3-b_1)/2 \end{bmatrix}$$

$$[A_2]=\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ (3+r_1)/2 & (3r_1c_1+15c_1+14c_2)/12 & -(3r_1b_1+15b_1+14b_2)/12 \\ (-3+r_3)/2 & (-3r_3c_3+3c_3+2c_2)/12 & -(-3r_3b_3+3b_3+2b_2)/12 \\ -(r_3+r_1)/2 & (3r_3c_3-3r_1c_1-c_2)/12 & -(3r_3b_3-3r_1b_1-b_2)/12 \\ -(3+r_1) & -(3r_1c_1+9c_1+8c_2)/6 & +(3r_1b_1+9b_1+8b_2)/6 \\ 3-r_3 & (3r_3c_3-9c_3-8c_2)/6 & -(3r_3b_3-9b_3-8b_2)/6 \\ r_1+r_3 & (-3r_3c_3+3r_1c_1+c_2)/6 & -(-3r_3b_3+3r_1b_1+b_2)/6 \\ r_1-r_3 & (r_3c_3+r_1c_1-c_2)/2 & -(r_3b_3+r_1b_1-b_2)/2 \\ r_1-r_3 & (r_3c_3+r_1c_1-c_2)/2 & -(r_3b_3+r_1b_1-b_2)/2 \\ r_1-r_3 & (r_3c_3+r_1c_1-c_2)/2 & -(r_3b_3+r_1b_1-b_2)/2 \end{bmatrix}$$

$$[A_3]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ -(r_1+r_2)/2 & (3r_1c_1-3r_2c_2-c_3)/12 & -(3r_1b_1-3r_2b_2-b_3)/12 \\ (3+r_2)/2 & (3r_2c_2+15c_2+14c_3)/12 & -(3r_2b_2+15b_2+14b_3)/12 \\ (-3+r_1)/2 & (-3r_1c_1+3c_1+2c_3)/12 & -(-3r_1b_1+3b_1+2b_3)/12 \\ (r_1+r_2) & (-3r_1c_1+3r_2c_2+c_3)/6 & -(-3r_1b_1+3r_2b_2+b_3)/6 \\ -(3+r_2) & -(3r_2c_2+9c_2+8c_3)/6 & (3r_2b_2+9b_2+8b_3)/6 \\ 3-r_1 & (3r_1c_1-9c_1-8c_3)/6 & -(3r_1b_1-9b_1-8b_3)/6 \\ -r_1+r_2 & (r_1c_1+r_2c_2-c_3)/2 & -(r_1b_1+r_2b_2-b_3)/2 \\ -r_1+r_2 & (r_1c_1+r_2c_2-c_3)/2 & -(r_1b_1+r_2b_2-b_3)/2 \\ -r_1+r_2 & (r_1c_1+r_2c_2-c_3)/2 & -(r_1b_1+r_2b_2-b_3)/2 \end{bmatrix}$$

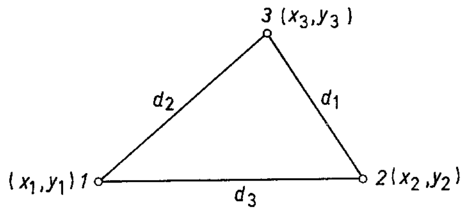


Fig. 1. A triangular element

and

$$r_1 = \frac{d_2^2 - d_3^2}{d_1^2}, \quad r_2 = \frac{d_3^2 - d_1^2}{d_2^2}, \quad r_3 = \frac{d_1^2 - d_2^2}{d_3^2}.$$

Substituting Eq. (12) into Eq. (5), we obtain

$$w = [F_\lambda][A]\{q\}^e. \tag{14}$$

The element stiffness matrix $[k]^e$ may be derived by the conventional procedure. This element is called LT element.

3 Generalized conforming rectangular elements LR-1 and LR-2

The nodal displacement vector $\{q\}^e$ of a 12-d.o.f. rectangular element $2a \times 2b$ (Fig. 2) is

$$\{q\}^e = [w_1 \quad \psi_{x1} \quad \psi_{y1} \quad w_2 \quad \psi_{x2} \quad \psi_{y2} \quad w_3 \quad \psi_{x3} \quad \psi_{y3} \quad w_4 \quad \psi_{x4} \quad \psi_{y4}]^T. \tag{15}$$

The deflection field w of an element is described by

$$w = [F_\lambda]\{\lambda\} \tag{16}$$

in which

$$\{\lambda\}^T = [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6 \quad \lambda_7 \quad \lambda_8 \quad \lambda_9 \quad \lambda_{10} \quad \lambda_{11} \quad \lambda_{12}]$$

$$[F_\lambda] = [1 \quad \xi \quad \eta \quad \xi^2 \quad \xi\eta \quad \eta^2 \quad \xi^3 \quad \xi^2\eta \quad \xi\eta^2 \quad \eta^3 \quad \xi^3\eta \quad \xi\eta^3] \tag{17}$$

and $\xi = x/a$, $\eta = y/b$ denote dimensionless coordinates. First, three compatibility conditions for nodal deflections w_i $(i = 1, 2, 3, 4)$ can be established:

$$\sum_{i=1}^{4} w_i = 4(\lambda_1 + \lambda_4 + \lambda_6), \quad \sum_{i=1}^{4} w_i \xi_i = 4(\lambda_2 + \lambda_7 + \lambda_9), \quad \sum_{i=1}^{4} w_i \eta_i = 4(\lambda_3 + \lambda_8 + \lambda_{10}). \tag{18}$$

Further, nine generalized compatibility conditions are established according to Eq. (3) in which two sets of element moments are assumed:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} 1 & \xi & \eta & \xi\eta & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \xi & \eta & \xi\eta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_9 \end{bmatrix} \tag{19a}$$

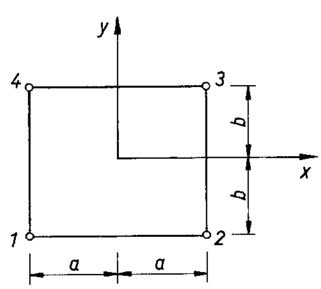


Fig. 2. A rectangular element

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} 1 & \xi & \eta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \xi & \eta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \xi^2 & \eta^2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_9 \end{bmatrix}. \tag{19b}$$

Thus for the first set of element moment (19a), the relation between $\{\lambda\}$ and $\{q\}^e$ can be written as follows:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \\ \lambda_9 \\ \lambda_{10} \\ \lambda_{11} \\ \lambda_{12} \end{bmatrix} = \frac{1}{120} \begin{bmatrix} 30 & 15a & 15b & 30 & -15a & 15b & 30 & -15a & -15b & 30 & 15a & -15b \\ -45 & -15a & -20b & 45 & -15a & 20b & 45 & -15a & -20b & -45 & -15a & 20b \\ -45 & -20a & -15b & -45 & 20a & -15b & 45 & -20a & -15b & 45 & 20a & -15b \\ 0 & -15a & 0 & 0 & 15a & 0 & 0 & 15a & 0 & 0 & -15a & 0 \\ 66 & 18a & 18b & -66 & 18a & -18b & 66 & -18a & -18b & -66 & -18a & 18b \\ 0 & 0 & -15b & 0 & 0 & -15b & 0 & 0 & 15b & 0 & 0 & 15b \\ 15 & 15a & 5b & -15 & 15a & -5b & -15 & 15a & 5b & 15 & 15a & -5b \\ 0 & 15a & 0 & 0 & -15a & 0 & 0 & 15a & 0 & 0 & -15a & 0 \\ 0 & 0 & 15b & 0 & 0 & -15b & 0 & 0 & 15b & 0 & 0 & -15b \\ 15 & 5a & 15b & 15 & -5a & 15b & -15 & 5a & 15b & -15 & -5a & 15b \\ -18 & -15a & -3b & 18 & -15a & 3b & -18 & 15a & 3b & 18 & 15a & 3b \\ -18 & -3a & -15b & 18 & -3a & 15b & -18 & 3a & 15b & 18 & 3a & -15b \end{bmatrix} \begin{bmatrix} w_1 \\ \psi_{x1} \\ \psi_{y1} \\ w_2 \\ \psi_{x2} \\ \psi_{y2} \\ w_3 \\ \psi_{x3} \\ \psi_{y3} \\ w_4 \\ \psi_{x4} \\ \psi_{y4} \end{bmatrix}. \tag{20}$$

A similar result is obtained for the second set of element moment (19b) as follows:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \\ \lambda_9 \\ \lambda_{10} \\ \lambda_{11} \\ \lambda_{12} \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 6 & 3a & 3b & 6 & -3a & 3b & 6 & -3a & -3b & 6 & 3a & -3b \\ -9 & -3a & -4b & 9 & -3a & 4b & 9 & -3a & -4b & -9 & -3a & 4b \\ -9 & -4a & -3b & -9 & 4a & -3b & 9 & -4a & -3b & 9 & 4a & -3b \\ 0 & -3a & 0 & 0 & 3a & 0 & 0 & 3a & 0 & 0 & -3a & 0 \\ 12 & 3a & 3b & -12 & 3a & -3b & 12 & -3a & -3b & -12 & -3a & 3b \\ 0 & 0 & -3b & 0 & 0 & -3b & 0 & 0 & 3b & 0 & 0 & 3b \\ 3 & 3a & b & -3 & 3a & -b & -3 & 3a & b & 3 & 3a & -b \\ 0 & 3a & 0 & 0 & -3a & 0 & 0 & 3a & 0 & 0 & -3a & 0 \\ 0 & 0 & 3b & 0 & 0 & -3b & 0 & 0 & 3b & 0 & 0 & -3b \\ 3 & a & 3b & 3 & -a & 3b & -3 & a & 3b & -3 & -a & 3b \\ -3 & -3a & 0 & 3 & -3a & 0 & -3 & 3a & 0 & 3 & 3a & 0 \\ -3 & 0 & -3b & 3 & 0 & 3b & -3 & 0 & 3b & 3 & 0 & -3b \end{bmatrix} \begin{bmatrix} w_1 \\ \psi_{x1} \\ \psi_{y1} \\ w_2 \\ \psi_{x2} \\ \psi_{y2} \\ w_3 \\ \psi_{x3} \\ \psi_{y3} \\ w_4 \\ \psi_{x4} \\ \psi_{y4} \end{bmatrix}. \tag{21}$$

Based on the two sets of solutions of $\{\lambda\}$, Eqs. (20) and (21), the corresponding stiffness matrices of two new rectangular elements, LR-1 and LR-2, can be established.

4 Numerical examples

Example 1. Simply supported and clamped square plate under uniform load

Numerical results for central deflection obtained with the triangular element LT are given in Table 1 along with those obtained with the CT element (Fricker 1985)—one of the most accurate

Table 1. Central deflection—triangular element

Mesh ($\frac{1}{4}$ plate)	Simply-supported				Clamped			
	Element LT		Element CT		LT		CT	
	A	B	A	B	A	B	A	B
2 × 2	0.4014 (−1.2%)	0.4024 (−0.9%)	0.39930 (−1.7%)	0.35118 (−13.6%)	0.12288 (−2.9%)	0.10768 (−14.9%)	0.14750 (16.6%)	0.10732 (−15.2%)
4 × 4	0.4051 (−0.3%)	0.4058 (−0.1%)	0.40439 (−0.5%)	0.39280 (−3.3%)	0.12544 (−0.8%)	0.12203 (−3.6%)	0.13221 (4.5%)	0.12232 (−3.3%)
6 × 6	0.40574 (−0.1%)	0.40609 (−0.03%)	0.40540 (−0.2%)	0.40028 (−1.5%)	0.12611 (−0.3%)	0.12452 (−1.6%)	0.12912 (2.0%)	0.12468 (−1.5%)
Analytic solution	0.406235 $ql^4/(100D)$				0.12653 $ql^4/(100D)$			

Table 2. Central deflection—rectangular element

Mesh ($\frac{1}{4}$ plate)	Simply-supported			Clamped		
	LR-1	LR-2	ACM	LR-1	LR-2	ACM
2 × 2	0.4051 (−0.3%)	0.4052 (−0.3%)	0.3939 (−3.0%)	0.1238 (−2.0%)	0.1243 (−1.7%)	0.1403 (11.0%)
4 × 4	0.40616 (−0.02%)	0.40617 (−0.02%)	0.4033 (−0.7%)	0.1260 (−0.4%)	0.1261 (−0.4%)	0.1304 (4.0%)
8 × 8	0.40623 (−0.001%)	0.40623 (−0.001%)	0.4056 (−0.2%)	0.12645 (−0.06%)	0.12646 (−0.05%)	0.1275 (0.8%)
Analytic solution	0.406235($ql^4/100D$)			0.12653 ($ql^4/100D$)		

Table 3. Central moment—triangular element

Mesh ($\frac{1}{4}$ plate)	SS pl.—uniform load				Clamped pl.—uniform load			
	LT		CT		LT		CT	
	A	B	A	B	A	B	A	B
2 × 2	0.5022 (4.9%)	0.5161 (7.8%)	0.49988 (4.4%)	0.43958 (−8.2%)	0.2909 (27%)	0.2380 (3.9%)	0.29510 (28.8%)	0.20527 (−10.4%)
4 × 4	0.4798 (0.2%)	0.4917 (2.7%)	0.48347 (1.0%)	0.47005 (−1.8%)	0.2386 (4.2%)	0.2343 (2.3%)	0.24671 (7.7%)	0.22389 (−2.3%)
6 × 6	0.47821 (−0.1%)	0.48551 (1.4%)	0.48090 (0.4%)	0.47493 (−0.8%)	0.23277 (1.6%)	0.23155 (1.1%)	0.23751 (3.7%)	0.22605 (−1.3%)
Analytic solution	0.47886 ($ql^2/10$)				0.22905 ($ql^2/10$)			

Table 4. Central moment—rectangular element

Mesh ($\frac{1}{4}$ plate)	SS pl.—uniform load			Clamped pl.—uniform load		
	LR-1	LR-2	ACM	LR-1	LR-2	ACM
2 × 2	0.51245 (7.0%)	0.51233 (7.0%)	0.52169 (8.9%)	0.25523 (11.4%)	0.25323 (10.5%)	0.27783 (11.3%)
4 × 4	0.48730 (1.8%)	0.48732 (1.8%)	0.48920 (2.2%)	0.23689 (3.4%)	0.23696 (3.4%)	0.24050 (5.0%)
8 × 8	0.48098 (0.4%)	0.48098 (0.4%)	0.48166 (0.6%)	0.23109 (0.8%)	0.23110 (0.8%)	0.23191 (1.2%)
Analytic solution	0.47886 ($ql^2/10$)			0.22905 ($ql^2/10$)		

nine—d.o.f. triangular elements currently available. Two mesh orientations, A and B, are used (Fig. 3). Central deflection coefficients obtained with the rectangular elements LR-1 and LR-2 are given in Table 2 along with those obtained with the well-known ACM element. Moment coefficients obtained with the triangular element and rectangular elements LT, LR-1 and LR-2 are given in Table 3 and 4 respectively. These new elements exhibit excellent performance.

In order to test whether the LT element can pass the patch test, two examples, pure twisting and pure bending of plates, are analyzed with the LT element.

Example 2. Pure twisting patch test

A square plate is supported on three nodes A, B, C, and a load $p = 5\,\text{KN}$ is applied at node D (Fig. 4a). An arbitrary grid subdivision is shown in Fig. 4b. Correct results are obtained everywhere with the LT element.

Example 3. Pure bending patch test

A 2-element mesh (with node 1 fixed) is under pure bending, $M_y = 1.0\,\text{KN}\cdot\text{m/m}$ (Fig. 5). The LT element reproduces the exact solution and passes the patch test.

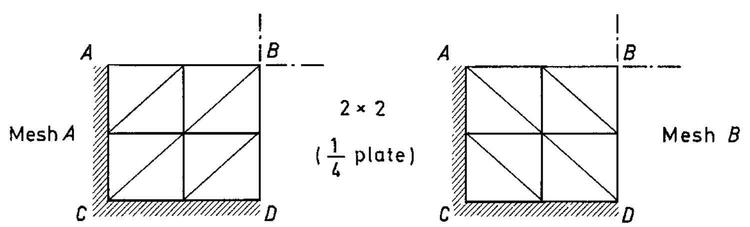
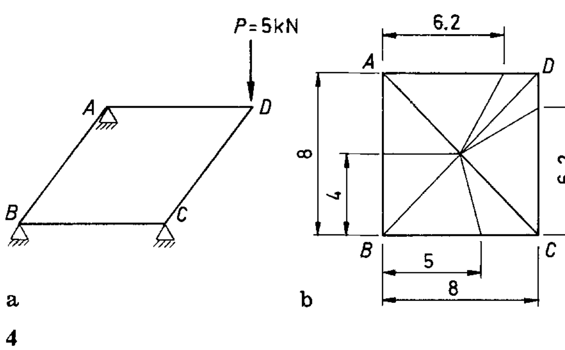
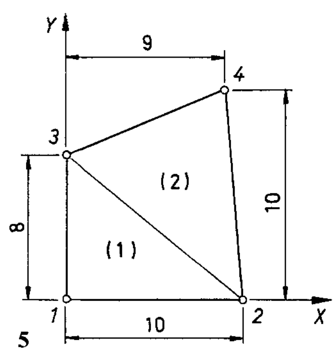


Fig. 3. Two mesh orientations





Figs. 4 and 5. 4 Twisting patch test. 5 Bending patch test

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